

Solution to Exercise 2.3 a

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, 100	0, 100	0, 100	0, 100	0, 100
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

a) Determine all strictly dominant strategies.

There are no strictly dominant strategies.

Solution to Exercise 2.3 b

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, 100	0, 100	0, 100	0, 100	0, 100
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

b) Determine all weakly dominant strategies.

There are no weakly dominant strategies.

Solution to Exercise 2.3 c

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, 100	0, 100	0, 100	0, 100	0, 100
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

c) Determine all strictly dominated strategies.

Strategy s_1 is strictly dominated by all other strategies of player 1. By 2.2, every mixed strategy that plays s_1 with positive probability is also strictly dominated.

Solution to Exercise 2.3 d

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, 100	0, 100	0, 100	0, 100	0, 100
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

d) Determine all weakly dominated strategies.

The strategies s_1 , s_2 and s_6 are weakly dominated.

The strategies t_1 and t_2 are weakly dominated.

All mixed strategies that play a weakly dominated strategy with positive probability are also weakly dominated.

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, 100	0, 100	0, 100	0, 100	0, 100
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

- e) Determine which strategies survive the iterative elimination of weakly dominated strategies.

s_1 is weakly dominated \Rightarrow Eliminate s_1 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

t_1 is weakly dominated \Rightarrow Eliminate t_1 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

		t_2	t_3	t_4	t_5
s_2		20, 80	20, 80	20, 80	20, 80
s_3		49, 51	40, 60	40, 60	40, 60
s_4		49, 51	25, 75	60, 40	60, 40
s_5		49, 51	25, 75	9, 91	80, 20
s_6		49, 51	25, 75	9, 91	1, 99

s_2 is weakly dominated \Rightarrow Eliminate s_2 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

		t_2	t_3	t_4	t_5
s_3		49, 51	40, 60	40, 60	40, 60
s_4		49, 51	25, 75	60, 40	60, 40
s_5		49, 51	25, 75	9, 91	80, 20
s_6		49, 51	25, 75	9, 91	1, 99

t_2 is weakly dominated \Rightarrow Eliminate t_2 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

			t_3	t_4	t_5
s_3			40, 60	40, 60	40, 60
s_4			25, 75	60, 40	60, 40
s_5			25, 75	9, 91	80, 20
s_6			25, 75	9, 91	1, 99

s_6 is weakly dominated \Rightarrow Eliminate s_6 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

			t_3	t_4	t_5
s_3			40, 60	40, 60	40, 60
s_4			25, 75	60, 40	60, 40
s_5			25, 75	9, 91	80, 20

t_5 is weakly dominated \Rightarrow Eliminate t_5 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

			t_3	t_4	
s_3			40, 60	40, 60	
s_4			25, 75	60, 40	
s_5			25, 75	9, 91	

s_5 is weakly dominated \Rightarrow Eliminate s_5 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

			t_3	t_4	
s_3			40, 60	40, 60	
s_4			25, 75	60, 40	

t_4 is weakly dominated \Rightarrow Eliminate t_4 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

			t_3		
s_3			40, 60		
s_4			25, 75		

s_4 is weakly dominated \Rightarrow Eliminate s_4 .

Solution to Exercise 2.3 e

2.3 Consider the following normal form game.

			t_3		
s_3			40, 60		

The strategies s_3 and t_3 are the only strategies that survive the iterative elimination of weakly dominated strategies.

Rationalizable Strategies: Definition

Definition (Rationalizable Strategies)

In game $\Gamma_N = [I, \{\Delta(\mathcal{S}_i)\}, \{u_i(\cdot)\}]$, the strategies in $\Delta(\mathcal{S}_i)$ that survive the iterated elimination of strategies that are never a best response are known as player i 's *rationalizable strategies*.

Rationalizable Strategies

		Player 2	
		<i>L</i>	<i>R</i>
Player 1	<i>U</i>	-1 , 0	1 , 0
	<i>M</i>	$\frac{1}{2}, 0$	$\frac{1}{2}, 0$
	<i>D</i>	1, 0	-1, 0

What are the rationalizable strategies?

U is best response to *R* and *D* is best response to *L*.

M is not a best response to a pure strategy, but to the mixed strategy $(1/2L + 1/2R)$.

Rationalizable Strategies

		Player 2	
		<i>L</i>	<i>R</i>
Player 1	<i>U</i>	-1 , 0	1 , 1
	<i>M</i>	$\frac{1}{2}, 1$	$\frac{1}{2}, 0$
	<i>D</i>	1, 0	-1, 1

A mixed strategy can only be a best-response to s_{-i} if all pure strategies that are played with positive probability are a best-response to s_{-i} .

Rationalizable Strategies

		Player 2	
		<i>L</i>	<i>R</i>
Player 1	<i>U</i>	-1 , 1	1 , 1
	<i>M</i>	$\frac{1}{2}, 1$	$\frac{1}{2}, 0$
	<i>D</i>	1, 0	-1, 1

A mixed strategy can only be a best-response if all pure strategies that are played with positive probability are a best-response given the same strategy profile of the others.

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, 100	0, 100	0, 100	0, 100	0, 100
s_2	81, 19	20, 80	20, 80	20, 80	20, 80
s_3	81, 19	49, 51	40, 60	40, 60	40, 60
s_4	81, 19	49, 51	25, 75	60, 40	60, 40
s_5	81, 19	49, 51	25, 75	9, 91	80, 20
s_6	81, 19	49, 51	25, 75	9, 91	1, 99

f) Determine all rationalizable strategies.

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	0, <u>100</u>	0, <u>100</u>	0, <u>100</u>	0, <u>100</u>	0, <u>100</u>
s_2	<u>81</u> , 19	20, <u>80</u>	20, <u>80</u>	20, <u>80</u>	20, <u>80</u>
s_3	<u>81</u> , 19	<u>49</u> , 51	<u>40</u> , <u>60</u>	40, <u>60</u>	40, <u>60</u>
s_4	<u>81</u> , 19	<u>49</u> , 51	25, <u>75</u>	<u>60</u> , 40	60, 40
s_5	<u>81</u> , 19	<u>49</u> , 51	25, 75	9, <u>91</u>	<u>80</u> , 20
s_6	<u>81</u> , 19	<u>49</u> , 51	25, 75	9, 91	1, <u>99</u>

s_1 is never a best response \Rightarrow Eliminate s_1 .

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1					
s_2	<u>81</u> , 19	20, <u>80</u>	20, <u>80</u>	20, <u>80</u>	20, <u>80</u>
s_3	<u>81</u> , 19	<u>49</u> , 51	<u>40</u> , <u>60</u>	<u>40</u> , <u>60</u>	<u>40</u> , <u>60</u>
s_4	<u>81</u> , 19	<u>49</u> , 51	25, <u>75</u>	<u>60</u> , 40	60, 40
s_5	<u>81</u> , 19	<u>49</u> , 51	25, 75	9, <u>91</u>	<u>80</u> , 20
s_6	<u>81</u> , 19	<u>49</u> , 51	25, 75	9, 91	1, <u>99</u>

t_1 is never a best response \Rightarrow Eliminate t_1 .

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1					
s_2		20, <u>80</u>	20, <u>80</u>	20, <u>80</u>	20, <u>80</u>
s_3		<u>49</u> , 51	<u>40</u> , <u>60</u>	40, <u>60</u>	40, <u>60</u>
s_4		<u>49</u> , 51	25, <u>75</u>	<u>60</u> , 40	60, 40
s_5		<u>49</u> , 51	25, 75	9, <u>91</u>	<u>80</u> , 20
s_6		<u>49</u> , 51	25, 75	9, 91	1, <u>99</u>

s_2 is never a best response \Rightarrow Eliminate s_2 .

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1					
s_2					
s_3		<u>49</u> , 51	<u>40</u> , <u>60</u>	40, <u>60</u>	40, <u>60</u>
s_4		<u>49</u> , 51	25, <u>75</u>	<u>60</u> , 40	60, 40
s_5		<u>49</u> , 51	25, 75	9, <u>91</u>	<u>80</u> , 20
s_6		<u>49</u> , 51	25, 75	9, 91	1, <u>99</u>

t_2 is never a best response \Rightarrow Eliminate t_2 .

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1					
s_2					
s_3			<u>40, 60</u>	40, <u>60</u>	40, <u>60</u>
s_4			25, <u>75</u>	<u>60</u> , 40	60, 40
s_5			25, 75	9, <u>91</u>	<u>80</u> , 20
s_6			25, 75	9, 91	1, <u>99</u>

s_6 is never a best response \Rightarrow Eliminate s_6 .

Solution to Exercise 2.3 f

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1					
s_2					
s_3			<u>40, <u>60</u></u>	40, <u>60</u>	40, <u>60</u>
s_4			25, <u>75</u>	<u>60</u> , 40	60, 40
s_5			25, 75	9, <u>91</u>	<u>80</u> , 20
s_6					

The strategies s_3, s_4, s_5, t_3, t_4 and t_6 are the rationalizable strategies.