c) Show that the converse is also true. For any mixed strategy that player 1 might play, there is a realization equivalent behavior strategy.

Suppose that player 1 plays the following mixed strategy σ_1:

$$\begin{aligned}
s_1^1 &= (L, x, x) \text{ with probability } \sigma_1(s_1^1) \\
s_1^2 &= (L, x, y) \text{ with probability } \sigma_1(s_1^2) \\
s_1^3 &= (L, y, x) \text{ with probability } \sigma_1(s_1^3) \\
s_1^4 &= (L, y, y) \text{ with probability } \sigma_1(s_1^4) \\
s_1^5 &= (M, x, x) \text{ with probability } \sigma_1(s_1^5) \\
s_1^6 &= (M, x, y) \text{ with probability } \sigma_1(s_1^6) \\
s_1^7 &= (M, y, x) \text{ with probability } \sigma_1(s_1^7) \\
s_1^8 &= (M, y, y) \text{ with probability } \sigma_1(s_1^8) \\
s_1^9 &= (R, x, x) \text{ with probability } \sigma_1(s_1^9) \\
s_1^{10} &= (R, x, y) \text{ with probability } \sigma_1(s_1^{10}) \\
s_1^{11} &= (R, y, x) \text{ with probability } \sigma_1(s_1^{11}) \\
s_1^{12} &= (R, y, y) \text{ with probability } \sigma_1(s_1^{12})
\end{aligned}$$

with $\sum_{k=1}^{12} \sigma_1(s_1^k) = 1$ and $\sigma_1(s_1^k) \geq 0$ for all $k \in \{1, \ldots, 12\}$.

If player 2 uses the mixed strategy σ_2, i.e. she plays the pure strategy (ℓ) with probability $\sigma_2(\ell)$ and the pure strategy (r) with probability $\sigma_2(r)$, the probability that we reach each terminal node will be

$$\begin{aligned}
P(T_0) &= \sigma_1(s_1^1) + \sigma_1(s_1^2) + \sigma_1(s_1^3) + \sigma_1(s_1^4) \\
P(T_1) &= \Big(\sigma_1(s_1^5) + \sigma_1(s_1^6)\Big)\ \sigma_2(\ell) \\
P(T_2) &= \Big(\sigma_1(s_1^7) + \sigma_1(s_1^8)\Big)\ \sigma_2(\ell) \\
P(T_3) &= \Big(\sigma_1(s_1^5) + \sigma_1(s_1^6)\Big)\ \sigma_2(r) \\
P(T_4) &= \Big(\sigma_1(s_1^7) + \sigma_1(s_1^8)\Big)\ \sigma_2(r) \\
P(T_5) &= \Big(\sigma_1(s_1^9) + \sigma_1(s_1^{11})\Big)\ \sigma_2(\ell) \\
P(T_6) &= \Big(\sigma_1(s_1^{10}) + \sigma_1(s_1^{12})\Big)\sigma_2(\ell) \\
P(T_7) &= \Big(\sigma_1(s_1^9) + \sigma_1(s_1^{11})\Big)\ \sigma_2(r) \\
P(T_8) &= \Big(\sigma_1(s_1^{10}) + \sigma_1(s_1^{12})\Big)\sigma_2(r).
\end{aligned}$$

Then the following behavior strategy for player 1 is realization equivalent:

$$\Big(p_1\,\mathbf{L} + p_2\,\mathbf{M} + p_3\,\mathbf{R}\ ,\ q_1\,\mathbf{x} + q_2\,\mathbf{y}\ ,\ r_1\,\mathbf{x} + r_2\,\mathbf{y}\Big)$$

with

$$
\begin{aligned}
p_1 &= \sigma_1(s_1^1) + \sigma_1(s_1^2) + \sigma_1(s_1^3) + \sigma_1(s_1^4) \\
p_2 &= \sigma_1(s_1^5) + \sigma_1(s_1^6) + \sigma_1(s_1^7) + \sigma_1(s_1^8) \\
p_3 &= \sigma_1(s_1^9) + \sigma_1(s_1^{10}) + \sigma_1(s_1^{11}) + \sigma_1(s_1^{12}) \\
q_1 &= \frac{\sigma_1(s_1^5) + \sigma_1(s_1^6)}{\sigma_1(s_1^5) + \sigma_1(s_1^6) + \sigma_1(s_1^7) + \sigma_1(s_1^8)} \\
q_2 &= \frac{\sigma_1(s_1^7) + \sigma_1(s_1^8)}{\sigma_1(s_1^5) + \sigma_1(s_1^{6)} + \sigma_1(s_1^7) + \sigma_1(s_1^8)} \\
r_1 &= \frac{\sigma_1(s_1^9) + \sigma_1(s_1^{11})}{\sigma_1(s_1^9) + \sigma_1(s_1^{10}) + \sigma_1(s_1^{11}) + \sigma_1(s_1^{12})} \\
r_2 &= \frac{\sigma_1(s_1^{10}) + \sigma_1(s_1^{12})}{\sigma_1(s_1^9) + \sigma_1(s_1^{10}) + \sigma_1(s_1^{11}) + \sigma_1(s_1^{12})}.
\end{aligned}
$$

To verify that this behavior strategy is realization equivalent, one needs to calculate the probabilitiy distribution over the terminal nodes given this behavior strategy.

Terminal node T_0 will be reached whenever player 1 chooses action L. The probability that player 1 plays L is $p_1 = \sigma_1(s_1^1)+\sigma_1(s_1^2)+\sigma_1(s_1^3)+\sigma_1(s_1^4)$ according to the behavior strategy. This corresponds to the probability $P(T_0)$ that we calculated for the mixed strategy σ_1 of player 1.

Terminal node T_1 will be reached whenever player 1 chooses action M, player 2 plays ℓ and player 1 chooses x at information set 2. Player 1 chooses M with probability of $p_2 = \sigma_1(s_1^5) + \sigma_1(s_1^6) + \sigma_1(s_1^7) + \sigma_1(s_1^8)$, player 2 plays ℓ with probability of $\sigma_2(\ell)$ and player 1 chooses x at information set 2 with probability q_1. Hence the probability that these three actions are taken can be calculated by multiplying the three probabilities:

$$
\begin{aligned}
p_2\sigma_2(\ell)q_1 =& \Big(\sigma_1(s_1^5) + \sigma_1(s_1^6) + \sigma_1(s_1^7) + \sigma_1(s_1^8)\Big)\sigma_2(\ell)\left(\frac{\sigma_1(s_1^5) + \sigma_1(s_1^6)}{\sigma_1(s_1^5) + \sigma_1(s_1^6) + \sigma_1(s_1^7) + \sigma_1(s_1^8)}\right) \\
=& \Big(\sigma_1(s_1^5) + \sigma_1(s_1^6)\Big)\sigma_2(\ell)
\end{aligned}
$$

which corresponds to the probability $P(T_1)$ that we calculated for the mixed strategy σ_1 of player 1.

Repeating this procedure for all terminal nodes yields that this behavior strategy is realization equivalent to the mixed strategy σ_1.