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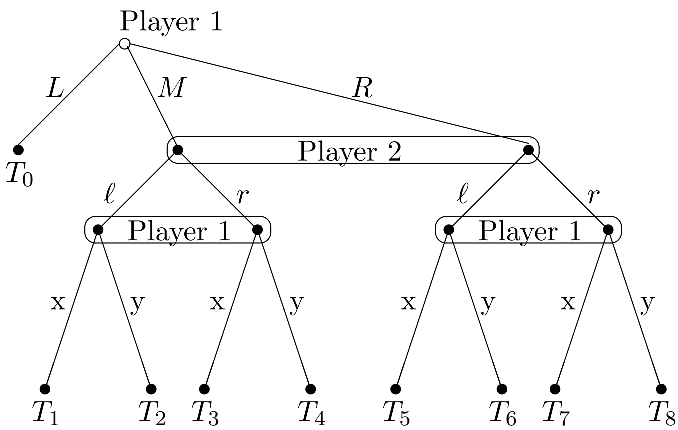
Problem Set I

1.1 (cf. MAS-COLELL, p.233, 7.D.1)
In a game where player i has N information sets indexed $n = 1, \ldots, N$ and M_n possible actions at information set n, how many strategies does player i have?

1.2 (cf. MAS-COLELL, p.233, 7.E.1)
Consider the two-player game whose extensive form representation (excluding payoffs) is depicted below.



a) What are the possible strategies of player 1 and player 2?

b) Show that for any behavior strategy of player i, there is a mixed strategy for that player that yields exactly the same distribution over outcomes for any strategies, mixed or behavior, that might be played by i's rivals [this result is due to Kuhn (1953)].

c) Show that the converse is also true. For any mixed strategy that player 1 might play, there is a realization equivalent behavior strategy.

Enjoy!

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Problem Set II

2.1 (cf. MAS-COLELL, p.262, 8.C.2)
Prove that the order of removal does not matter for the set of strategies that survives a process of iterated deletion of strategies that are never a best response.

2.2 (cf. MAS-COLELL, p.262, 8.B.6)
Prove that if pure strategy s_i is a strictly dominated strategy in game $\Gamma_N = [I, \{\Delta(\mathcal{S}_i)\}, \{u_i(\cdot)\}]$, then so is any strategy that plays s_i with positive probability.

2.3 Consider the following normal form game.

	t_1	t_2	t_3	t_4	t_5
s_1	$0, 100$	$0, 100$	$0, 100$	$0, 100$	$0, 100$
s_2	$81, 19$	$20, 80$	$20, 80$	$20, 80$	$20, 80$
s_3	$81, 19$	$49, 51$	$40, 60$	$40, 60$	$40, 60$
s_4	$81, 19$	$49, 51$	$25, 75$	$60, 40$	$60, 40$
s_5	$81, 19$	$49, 51$	$25, 75$	$9, 91$	$80, 20$
s_6	$81, 19$	$49, 51$	$25, 75$	$9, 91$	$1, 99$

a) Determine all strictly dominant strategies.

b) Determine all weakly dominant strategies.

c) Determine all strictly dominated strategies.

d) Determine all weakly dominated strategies.

e) Determine which strategies survive the iterative elimination of weakly dominated strategies.

f) Determine all rationalizable strategies.

Enjoy!

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Problem Set III

3.1 (cf. MAS-COLELL, p.262, 8.D.2)
Show that if there is a unique profile of strategies that survives iterated removal of strictly dominated strategies, this profile is a Nash equilibrium.

3.2 (cf. MAS-COLELL, p.262, 8.D.4)
Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them 100 dollars in profit, but they must agree on how to split the 100 dollars. Bargaining works as follows: The two individuals each make a demand simultaneously. If their demands sum to more than 100 dollars, they fail to agree, and each gets nothing. If their demand sums to less than 100 dollars, they do the project, each gets his demand, and the rest goes to charity.

a) What are each player's strictly dominated strategies?

b) What are each player's weakly dominated strategies?

c) What are the pure strategy Nash equilibria of this game?

3.3 (cf. MAS-COLELL, p.262, 8.D.9)
Consider the following game:

		Player 2			
		LL	L	M	R
Player 1	U	100,2	-100, 1	0,0	-100, -100
	D	-100,-100	100,-49	1,0	100,2

a) If you were player 2 in this game and you were playing it once without the ability to engage in preplay communication with player 1, what strategy would you choose?

b) What are all the Nash equilibria (pure and mixed) of this game?

c) Is your strategy choice in a) a component of any Nash equilibrium strategy profile? Is it a rationalizable strategy?

d) Suppose now that preplay communication were possible. Would you expect to play something different from your choice in a)?

Enjoy!

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Problem Set IV

4.1 (cf. MAS-COLELL, p.302, 9.B.7)

Consider the finite horizon bilateral bargaining game, but instead of assuming that players discount future payoffs, assume that it costs $c < v$ to make an offer. (Only the player making an offer incurs this cost, and players who have made offers incur this cost even if no agreement is ultimately reached.) What is the (unique) SPNE of this alternative model?

4.2 (cf. MAS-COLELL, p.302, 9.B.9)

Consider a game in which the following simultaneous-move game is played twice:

		Player 2		
		b_1	b_2	b_3
	a_1	10,10	2,12	0,13
Player 1	a_2	12,2	5,5	0,0
	a_3	13,0	0,0	1,1

The players observe the actions chosen in the first play of the game prior to the second play. What are the pure strategy subgame perfect Nash equilibria of this game?

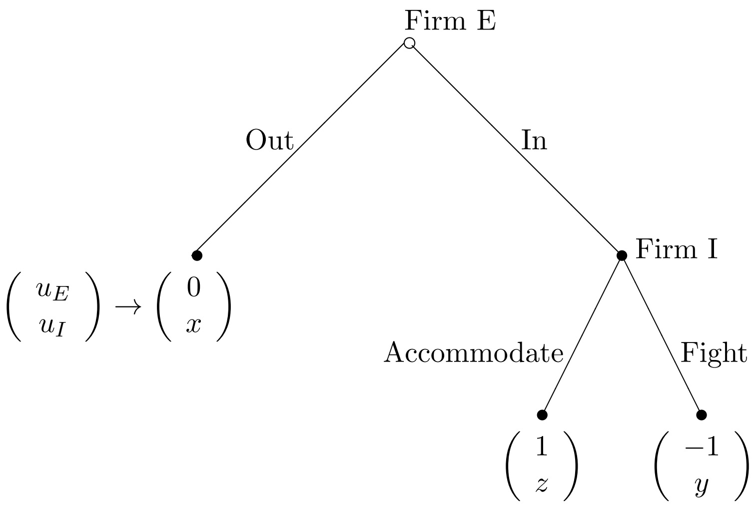
4.3 (cf. MAS-COLELL, p.303, 9.B.14)

At time 0, an incumbent firm (firm I) is already in the widget market, and a potential entrant (firm E) is considering entry. In order to enter, firm E must incur a cost of $K > 0$. Firm E's only opportunity to enter is at time 0. There are three producction periods. In any period in which both firms are active in the market, the game below is played. Firm E moves first, deciding whether to stay in or exit the market. If it stays in, firm I decides whether to fight (the upper payoff is for firm E). Once firm E plays out, it is out of the market forever; firm E earns zero in any period during which it is out of the market, and I earns x. The discount factor for both firms is δ.
Asumme that

(A.1) $x > z > y$.

(A.2) $y + \delta x > (1 + \delta)z$.

(A.3) $1 + \delta > K$.



a) What is the (unique) subgame perfect Nash equilibrium of this game?

b) Suppose now that firm E faces a financial constraint. In particular, if firm I fights *once* against firm E (in any period), firm E will be forced out of the market from that point on. Now what is the (unique) subgame perfect Nash equilibrium of this game? (If the answer depends on the values of the parameters beyond the three assumptions, indicate how.)

Enjoy!

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