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Problem Set IV - Solution

4.1 (cf. MAS-COLELL, p.302, 9.B.7)

Consider the finite horizon bilateral bargaining game, but instead of assuming that players discount future payoffs, assume that it costs $c < v$ to make an offer. (Only the player making an offer incurs this cost, and players who have made offers incur this cost even if no agreement is ultimately reached.) What is the (unique) SPNE of this alternative model?

Solution: Consider the last period of the game. The offering player will offer $(v, 0)$ and the offered player will accept. In the second to last period, the offering player will offer the other player a share that will make him indifferent between accepting now and rejecting and obtaining v in the next period. With the given cost c for making an offer, the offered player in the second to the last period will accept any offer that gives him at least $v - c$. Similary, we can show that the offering player in the third to last period offers $(v, 0)$.
If T is odd, then player 1 will be the last player to make an offer. Thus, he will offer $(v, 0)$ in every period in which he makes offers and accept an offer if he obtains at least $v - c$. Player 2 will offer $(v - c, c)$ in every period in which she makes offers and accept if she obtains a non-negative payoff. The result is that in the first period player 1 will offer $(v, 0)$ and player 2 will accept.
If T is even, then player 2 will be the last player to make an offer. Thus she will offer $(0, v)$ in every period in which she makes offers and accept an offer if she obtains at least $v - c$. Player 1 will offer $(c, v - c)$ in every period in which he makes offers and accept if he obtains a non-negative payoff. The result is that in the first period player 1 will offer $(c, v - c)$ and player 2 will accept.

4.2 (cf. MAS-COLELL, p.302, 9.B.9)

Consider a game in which the following simultaneous-move game is played twice:

		Player 2		
		b_1	b_2	b_3
	a_1	10,10	2,12	0,13
Player 1	a_2	12,2	5,5	0,0
	a_3	13,0	0,0	1,1

The players observe the actions chosen in the first play of the game prior to the second play. What are the pure strategy subgame perfect Nash equilibria of this game?

Solution: The pure strategy NE of the one-shot game are (a_2, b_2) and (a_3, b_3). Thus any SPNE involves playing either of these in the second period. Thus, playing either of these strategies in both periods constitutes a SPNE. Additionally, the players could use them in any combination in the two periods. This results in the following two classes of SPNE (a total of four SPNE):

1) Player 1 plays a_i and player 2 plays b_i in both periods, $i \in \{2, 3\}$.
2) Player 1 plays a_i in the first period and a_j in the second period; player 2 plays b_i in the first period and b_j in the second period, $i, j \in \{1, 2\}$ and $i \neq j$.

However, there exist more SPNE in this game. The reason is that player 1 (or 2) can punish the other player by playing a_3 (or b_3 respectively) in the second period, if the other player did not cooperate in the first period. (Note that this can only happen because there are more than one NE in the second stage.) This gives rise to two more classes of SPNE each of them containing three more SPNE.
The three SPNE of the first class are:

3) Fix $i \in \{1, 2, 3\}$.
Player 1's strategy: Play a_i in period 1; Play a_2 in period 2 if player 2 played b_1 in period 1, otherwise play a_3.
Player 2's strategy: Play b_1 in period 1. Play b_2 in period 2 if player 1 played a_i in period 1, otherwise play b_3.

The three SPNE of the second class are:

4) Fix $i \in \{1, 2, 3\}$.
Player 2's strategy: Play b_i in period 1; Play b_2 in period 2 if player 1 played a_1 in period 1, otherwise play b_3.
Player 1's strategy: Play a_1 in period 1; Play a_2 in period 2 if player 2 played b_i in period 1, otherwise play a_3.

To check that each of the 6 SPNE described by these classes are indeed a SPNE, note that by deviating a player looses 4 in the second period (no discounting) and no player can gain more than 3 in any of the described strategy profiles.

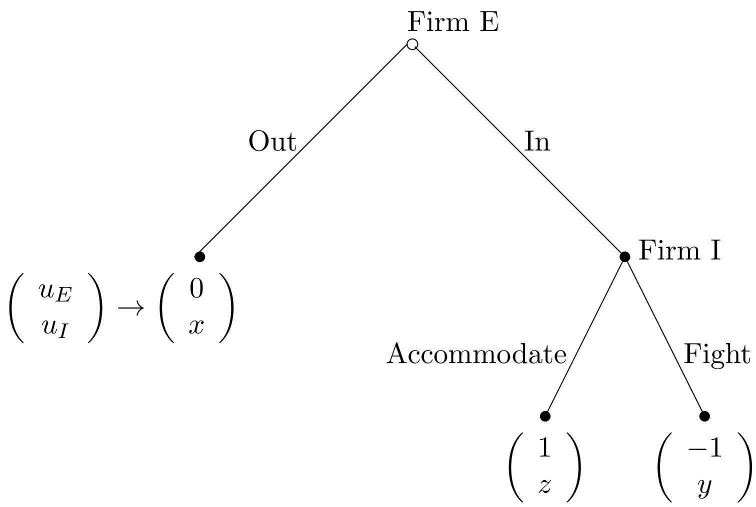
4.3 (cf. MAS-COLELL, p.303, 9.B.14)

At time 0, an incumbent firm (firm I) is already in the widget market, and a potential entrant (firm E) is considering entry. In order to enter, firm E must incur a cost of $K > 0$. Firm E's only opportunity to enter is at time 0. There are three production periods. In any period in which both firms are active in the market, the game below is played. Firm E moves first, deciding whether to stay in or exit the market. If it stays in, firm I decides whether to fight (the upper payoff is for firm E). Once firm E plays out, it is out of the market forever; firm E earns zero in any period during which it is out of the market, and I earns x. The discount factor for both firms is δ.
Asumme that

(A.1) $x > z > y$.

(A.2) $y + \delta x > (1 + \delta)z$.

(A.3) $1 + \delta > K$.



a) What is the (unique) subgame perfect Nash equilibrium of this game?

Solution: The extensive form of the game is depicted in figure 9.B.14(a) (the letter ’„d “is used instead of δ). Simple backward induction leads to the unique SPNE which is shown by arrows in the figure: Firm E enters at $t = 0$, and always plays In thereafter. Firm I plays Accommodate for all $t = 1, 2, 3$.

b) Suppose now that firm E faces a financial constraint. In particular, if firm I fights *once* against firm E (in any period), firm E will be forced out of the market from that point on. Now what is the (unique) subgame perfect Nash equilibrium of this game? (If the answer depends on the values of the parameters beyond the three assumptions, indicate how.)

Solution: The extensive form of the modified game is depicted in figure 9.B.14(b). Using Backward induction, firm I will always play Accomodate in period $t = 3$, and therefore if $t = 3$ is reached, firm E will play In. This causes firm I to choose Fight in $t = 2$ since this causes firm E to exit the market, and due to condition A.2 we have that for firm I:

$$z + \delta y + \delta^2 x = z + \delta(y + \delta x) > z + \delta(1 + \delta)z = z + \delta z + \delta^2 z.$$

This causes firm E to choose Out in $t = 2$. Working backward we get that at $t = 1$, firm I chooses Accommodate and firm E chooses In. However, the choice of firm E at $t = 0$ depends on the value of k. If $k > 1$ then firm E will choose not to enter, and if $k < 1$ then E will enter. For $k = 1$ both are part of the (unique) continuation SPNE, so there are two SPNE in this case.

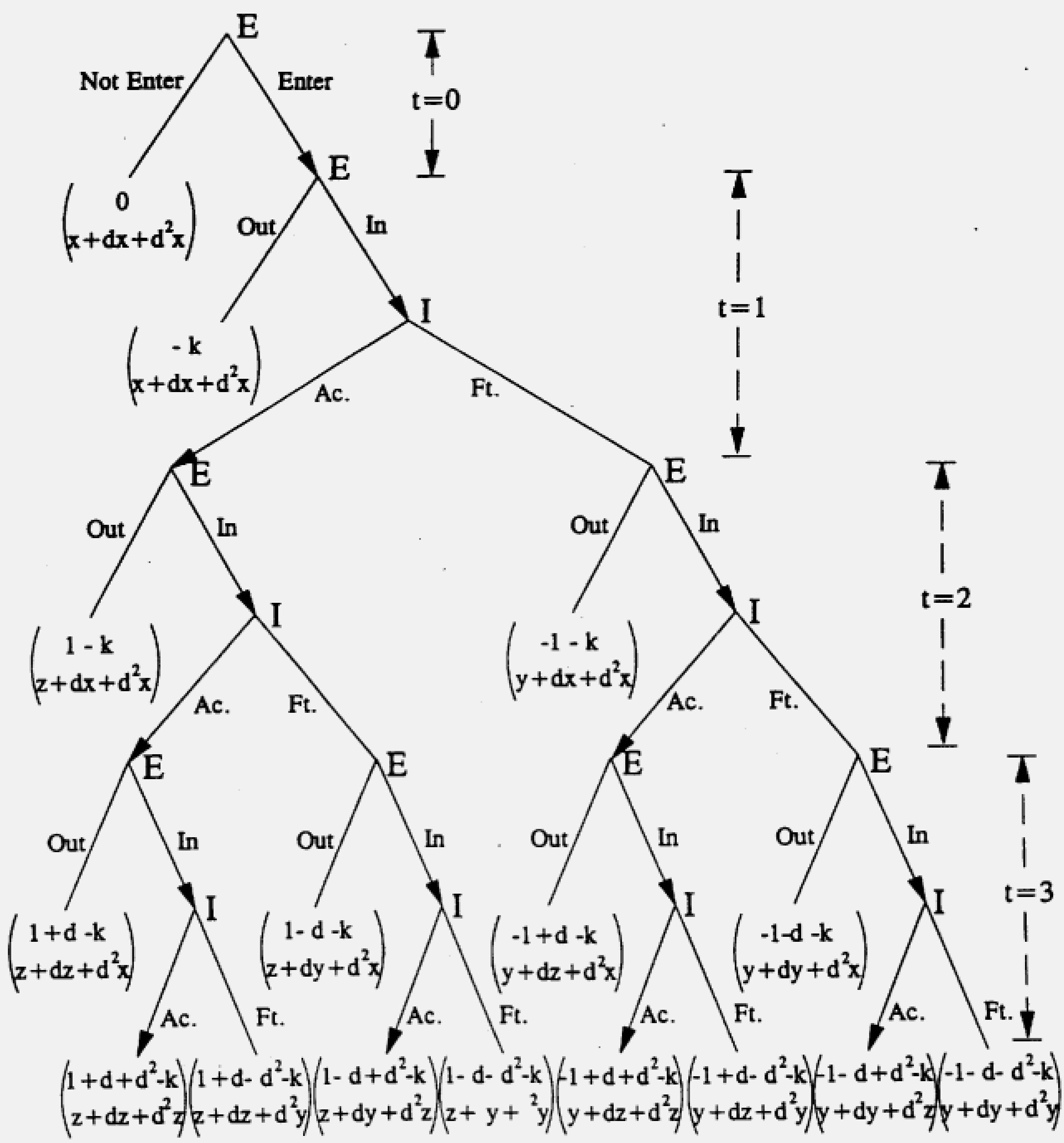


Figure 9.B.14(a)

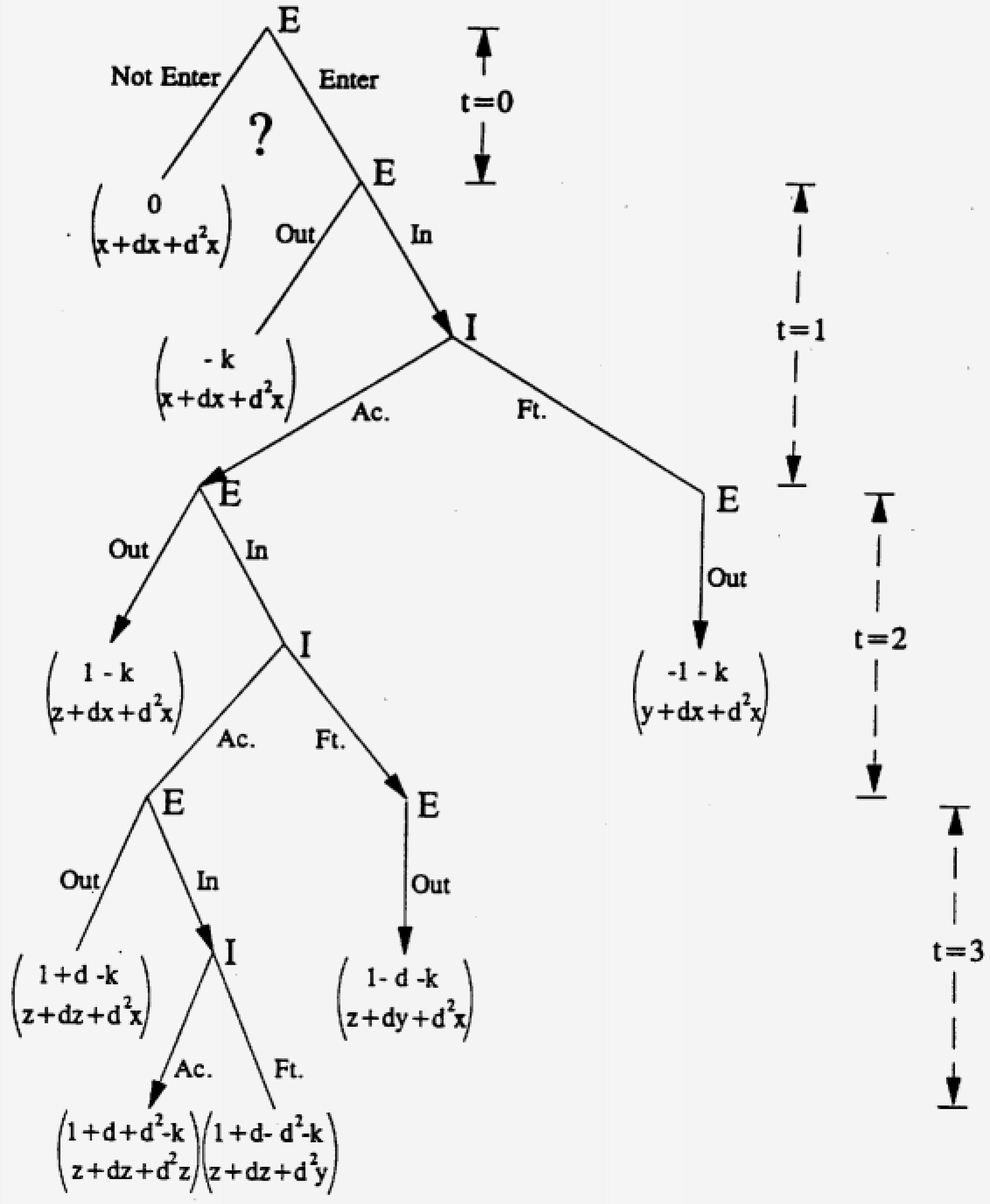


Figure 9.B.14(b)