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Problem Set III - Solution to 3.2

3.2 (cf. MAS-COLELL, p.262, 8.D.4)
Consider a bargaining situation in which two individuals are considering undertaking a business venture that will earn them 100 dollars in profit, but they must agree on how to split the 100 dollars. Bargaining works as follows: The two individuals each make a demand simultaneously. If their demands sum to more than 100 dollars, they fail to agree, and each gets nothing. If their demand sums to less than 100 dollars, they do the project, each gets his demand, and the rest goes to charity.

a) What are each player's strictly dominated strategies?

If player i demands $y \geq 100$, then any strategy of player j with $x \geq 0$ is payoff equivalent. Therefore, there exists no strictly dominated strategy.

b) What are each player's weakly dominated strategies?

Any strategy demanding (strictly) more than 100 dollars is weakly dominated.

Case 1: Player 2 demands $y \geq 100$.
Then any strategy of player 1 with $x \geq 0$ is payoff equivalent.

Case 2: Player 2 demands $0 \leq y < 100$.
Then player 1 could demand $x = 100 - y$ and would obtain a payoff of $100 - y$. Demanding $x > 100$ will give player 1 a payoff of 0.

Therefore, any strategy demanding more than 100 dollars is weakly dominated.

c) What are the pure strategy Nash equilibria of this game?

Claim: Any pair $(x, 100 - x)$ with $0 \leq x \leq 100$ is a pure strategy NE of this game.

Suppose, player 1 demands x with $0 \leq x \leq 100$. If player 2 demands $y = 100 - x$, his payoff will equal $100 - x \geq 0$. If player 2 demands $y > 100 - x$, the demands sum to more than 100 dollars and both players get 0. If player 2 demands $0 \leq y < 100 - x$, he will obtain his demand and therefore be worse off than if he would have demanded $100 - x$. Thus, if player 1 demands x, player 2's best response is to demand $y = 100 - x$. Similarly if player 2 demands $100 - x$, player 1's best response is to demand x.

Claim: Any pair (x, y) with $x, y \geq 100$ is a pure strategy NE of this game.

Whenever a player i demands 100 or more, individual j obtains a payoff of 0. Hence all strategies of j are payoff equivalent. In particular, all strategies of j are best-responses to a demand of 100 or more of individual i. Switching the roles of i and j yields the claim. Note that this is a NE in weakly dominated strategies.