Mechanism and Market Design

Markets: Trade

Third part of the lecture held by Prof. Szech. By bilateral trades you often find a compromise in a trade that can be profitable for both parties (see movie clip).

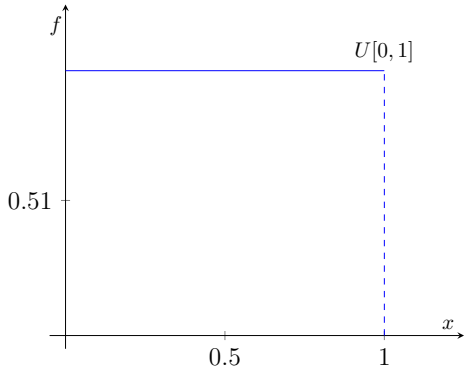
The Importance of Bilateral Trade

For the game theoretical analysis we need some basics:

Setting:

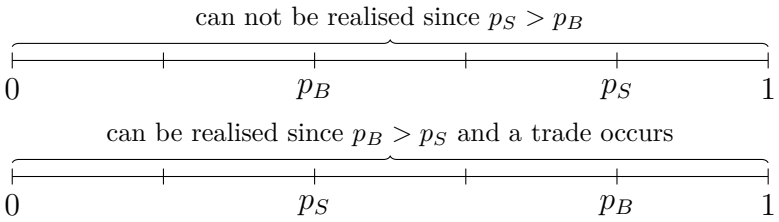
- 1 Buyer (B), 1 Seller (S)
- S ca produce an object of cost c
- B likes the object v much
- v is private info to B, common info is only $\tilde{v} \sim U[0,1]$
- c is private information to S, and again we assume $\tilde{c} \sim U[0,1]$ for B.
- Trade at price p will lead to profits:
  - $\pi_S = p - c$ for S
  - $\pi_B = v - p$ for B

For review, the uniform distribution: $U[0,1]$:



Mechanism - Double Auction (V. Smith):

a) B and S will simultaneously state prices p_S, p_B



b) Trade occurs if and only if $p_B \geq p_S$. Trading price $p = \frac{p_B + p_S}{2}$.

Fixed Price Equilibrium: Q: Can we find a BNE such that $p = \frac{1}{2}$ is achieved? A Bayes-Nash-Equilibrium that satisfies this condition if for example:

$$p_S(c) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq \frac{1}{2} \\ c & \text{if } c > \frac{1}{2} \end{cases}, \quad p_B(v) = \begin{cases} \frac{1}{2} & \text{if } v \geq \frac{1}{2} \\ v & \text{if } v < \frac{1}{2} \end{cases}$$

hence, this is a Fixed Price Equilibrium with fixed price $p = p_S = p_B = \frac{1}{2}$.

Proposition: For $x \in (0,1)$, the following strategies specify a fixed price (Bayesian) Nash-Equilibrium.

$$p_B(v) = \begin{cases} x, & \text{if } v \geq x \\ 0, & \text{if } v < x \end{cases}, \quad p_B(c) = \begin{cases} x, & \text{if } c \leq x \\ 1, & \text{if } c > x \end{cases}$$

Proof:

Considering the buyer:

a) Case 1: $v < x$:

action	profit
$p_B \in [0, x)$	$\pi = 0$ (no trade)
$p_B \in [x, 1]$	$\pi < 0$ (trade can happen)

b) Case 2: $v \geq x$:

action	profit
$p_B = [0, x)$	$\pi = 0$ (no trade)
$p_B = x$	$\pi \geq 0$ (trade can happen)
$p_B = (x, 1]$	

Considering the seller: (task 1)

depict (task 2):

a) a

b) When will trades occur according to the potential Equilibrium specified?

□

Exam

a) Theorie question, tasks in lecture are more or less the essentials - calculate and understand theory behind

b) Read paper, state in a few words what the paper is about - what do you think about the paper, extend improve or criticise the paper - what do you like, don't like about the paper or what is important