Advanced Game Thoerie

Prof. Dr. Clemens Puppe

(inoffizielles Skript)

Wintersemester 2016/17

Karlsruher Institut für Technologie

Inhaltsverzeichnis

Kapitel 1

Noncooperative Games

1.1 Basic Elements of Noncooperative Games

Definition: *A* ***game*** *is a formal representation of a situation in which a number of individuals interact in a setting of strategic interdependence.*

- *The players: Who is involved?*
- *The rules: Who moves when? What do they know when they move? What can they do?*
- *The outcomes: For each possible set of actions by the players, what is the outcome of the game?*
- *The payoffs: What are the players' preferences over the possible outcomes?*

Example (of simultaneous move games)**:**

a) Matching Pennies

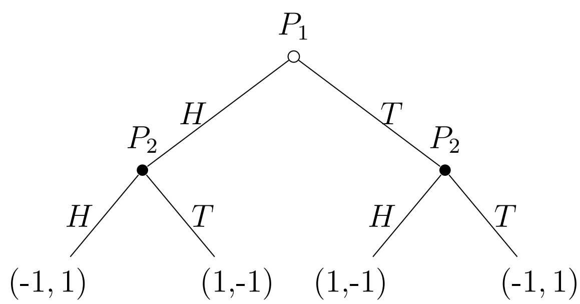
		Player 2	
		Heads	Tails
Player 1	Heads	$-1, 1$	$1, -1$
	Tails	$1, -1$	$-1, 1$

b) Meeting in New York

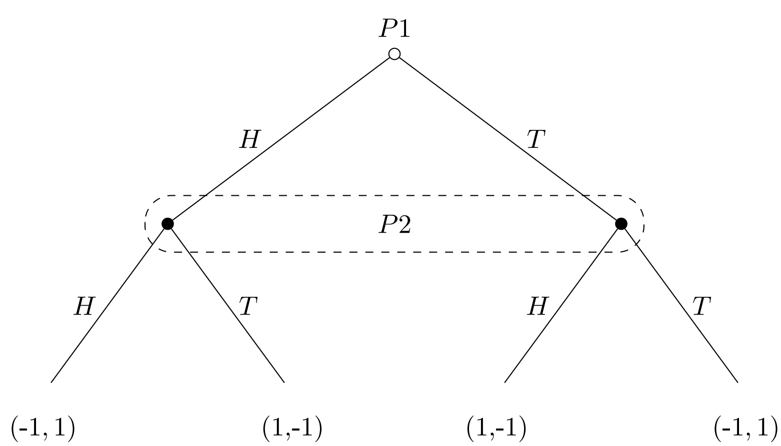
		Player 2	
		Empire State	Grand Central
Player 1	Empire State	$100, 100$	$0, 0$
	Grand Central	$0, 0$	$100, 100$

c) Examples of (simple) dynamic games

Prisoner's Dilemma in Extensive-form



d) Matching Pennies Version C



Definition (Information)**:**

a) ***Information Set:*** *A player doesn't know which of the nodes in the information set she is actually at. Therefore, at any decision node in a player's information set, there must be the same possible actions.*

b) ***Perfect Information:*** *A game is said to be of perfect information if each information set contains a single decision node. Otherwise, it is a game of* ***imperfect information****.*

Definition (Extensive Form Game)**:** *A game in* ***extensive form*** *consists of:*

(i) *A finite set of nodes $\mathcal{X}$, a finite set of possible actions $\mathcal{A}$, and a finite set of players $\{1, \ldots, l\}$.*

(ii) *A funktion $p\colon \mathcal{X} \to \{\mathcal{X} \cup \emptyset\}$ specifying a single immediate predecessor of each node x; $p(x) \in \mathcal{X}$ expect for one element x_0, the* ***initial node****. The immediate* ***successor node*** *of x are $s(x) = p^{-1}(x)$.*
To have a tree structure, a predecessor can never be a successor and vice versa. The set of ***terminal nodes*** *$T = \{x \in \mathcal{X}\colon s(x) = \emptyset\}$. All other nodes $X \setminus T$ are* ***decision nodes****.*

(iii) *A function $\alpha\colon \mathcal{X} \setminus \{x_0\} \to \mathcal{A}$ giving the action that leads to any non-initial node x from its immediate predecessor $p(x)$ with $x', x'' \in s(x); x' \neq x'' \Rightarrow \alpha(x') \neq \alpha(x'')$. The set of choices at decision node x is $c(x) = \{a \in \mathcal{A}\colon a = \alpha(x')$ for some $x' \in s(x)\}$.*

(iv) *A collection of information sets $\mathcal{H}$, and a function $H\colon \mathcal{X} \to \mathcal{H}$ assigning each decision node x to an information set $H(x) \in \mathcal{H}$ with $c(x) = c(x')$ if $H(x) = H(x')$. The choices available at information set H can be written as*

$$C(H) = \{a \in \mathcal{A}\colon a \in c(x) \text{ for } x \in H\}.$$

(v) *A function* $\iota\colon \mathcal{H} \to \{0,1,\dots,l\}$ *assigning a player to each information set (*$i = 0$ *'nature').*

The collection of player i's information set is denoted by

$$\mathcal{H}_i = \{H \in \mathcal{H}\colon i = \iota(H)\}.$$

(vi) *A function* $\rho\colon \mathcal{H}_0 \times \mathcal{A} \to [0,1]$ *assigning a probability to each action of nature with* $\rho(H,a) = 0$ *if* $a \notin C(H)$ *und* $\sum_{a\in C(H)} \rho(H,a) = 1$ *for all* $H \in \mathcal{H}_0$.

(vii) *A collection of payoff function* $u = \{u_1(\cdot),\dots,u_l(\cdot)\}$, *where* $u_i\colon T \to \mathbb{R}$.

A game in extensive form: $\Gamma_E = \{\mathcal{X}, \mathcal{A}, I, p(\cdot), \alpha(\cdot), \mathcal{H}, H(\cdot), \iota(\cdot), \rho(\cdot), u\}$.

Comment: Restrictions of this definition:

a) Finite set of actions

b) Finite number of moves

c) Finite number of players

Definition (Strategy)**:** *Let* $\mathcal{H}_i$ *denote the collection of player i's information sets,* $\mathcal{A}$ *the set of possible actions in the game, and* $C(H) \subset \mathcal{A}$ *the set of actions possible at information set* H*. A* ***strategy*** *for player* i *is a function* $s_i\colon \mathcal{H}_i \to \mathcal{A}$ *such that* $s_i(H) \in C(H)$ *for all* $H \in \mathcal{H}_i$.

Definition (Normal Form Representation)**:** *For a game with* I *players, the* ***normal form representation*** Γ_N *specifies for each player* i *a set of strategies* $\mathcal{S}_i$ *(with* $s_i \in \mathcal{S}_i$*) and a payoff function* $u_i(s_1,\dots,s_l)$*, formally*

$$\Gamma_N = [I, \{S_i\}, \{u_i(\cdot)\}].$$

Definition:

a) *$s_i\colon \mathcal{H}_i \to \mathcal{A}$ describes deterministic choices at each $H \in \mathcal{H}_i$ and is called a* ***pure strategy***

b) *a* ***mixed strategy*** *is a probability distribution over all pure strategies $\sigma_i\colon \mathcal{S}_i \to [0,1]$, with $\sigma_i(s_i) \geq 0$ and $\sum_{s_i \in \mathcal{S}_i} \sigma_i(s_i) = 1$.*

c) *player i's set of possible mixed strategies can be associated with the points of the simplex $\Delta(\mathcal{S}_i)$, called the* ***mixed extension*** *of $\mathcal{S}_i$.*

d) *since we assume that individuals are expected utility maximisers, player i's utility of a profile of mixed strategies $\sigma = (\sigma_i, \ldots, \sigma_l)$ is given by*

$$u_i(\sigma) = \sum_{s \in \mathcal{S}} [\sigma_1(s_1) \cdot \sigma_2(s_2) \cdot \ldots \cdot \sigma_l(s_l)] \cdot u_i(s),$$

where $s = (s_1, \ldots, s_l)$.

Definition (Behaviour Strategy)**:** *Given an extensive form game Γ_E, a* ***behaviour strategy*** *for player i specifies for every information set $h \in \mathcal{H}_i$ and action $a \in C(H)$, a probability $\lambda_i(a, H) \geq 0$, with*

$$\sum_{a \in C(H)} \lambda_i(a, H) = 1 \text{ for all } H \in \mathcal{H}_i.$$

Definition (Perfect Recall)**:** *A player has* ***perfect recall*** *if he doesn't „forget" what she once knew, including her own actions.*

Theorem 1.1: *If Γ_E is an extensive form game with perfect recall, then for any mixed strategy there is an outcome equivalent behaviour strategy and vice versa.*

1.2 Rationalisable Strategies

1.3 Nash Equilibrium

1.4 Subgame Perfection in Dynamic Games

1.5 Excercises

Kapitel 2

Kooperative Spiele

2.1 Der Kern

2.2 Der Shapley-Wert

2.3 Einfache Spiele

2.4 Konvexe Spiele

2.5 Übungen

Kapitel 3

Evolutionäre Spieltheorie