

# Economics and Behaviour

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Wintersemester 2015

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# Content of teaching

The course covers topics from behavioral economics with regard to contents and methods. In addition, the students gain insight into the design of economic experiments. Furthermore, the students will become acquainted with reading and critically evaluating current research papers in the field of behavioral economics.

## **Prerequisites**

None. Recommendations: Basic knowledge of microeconomics and statistics are recommended. A background in game theory is helpful, but not absolutely necessary.

## **Aim**

The students gain insight into fundamental topics in behavioral economics; get to know different research methods in the field of behavioral economics; learn to critically evaluate experimental designs; get introduced to current research papers in behavioral economics; become acquainted with the technical terminology in English.

## **Bibliography**

- Kahnemann, Daniel: Thinking, Fast and Slow. Farrar, Straus and Giroux, 2011.
- Ariely, Dan: Predictably irrational. New York: Harper Collins, 2008.
- Ariely, Dan: The Upside of Irrationality. New York: HarperCollins, 2011.

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# 1 Gametheorie

## Introduction

In analysing game theoretical situations we distinguish between the judging

- **prescriptive** - means containing an indication of approval or disapproval
- **normative** - means relating to our model

or the kind of equilibrium

	<b>complete information</b>	<b>incomplete information</b>
<b>static games</b>	Nash-Equilibrium	Bayesian-Nash-Equilibrium
<b>dynamic games</b>	Perfect Nash-Equilibrium	Perfect Bayesian-Nash-Equilibrium

## 1.1 Games in strategic form and the Nash-Equilibrium

A **game in strategic form** is completely characterised by  $\{N, S, u\}$  where

- (1)  $N$  is the number of players and for player  $n$  would that mean  $n \in \{1, \dots, N\}$ .
- (2) For each player we have a set of *pure* strategies  $S$ .
- (3) For each player  $n \in \{1, \dots, N\}$  we have an expected utility function  $u : S \rightarrow \mathbb{R}$

For two players ( $P1$  and  $P2$ ) and two possible signals ( $a$  and  $b$ ) we could use the matrix form, where the  $u_i(x, y)$  is the utility function for Player  $i$  for signal  $x$  for  $P1$  and  $y$  for  $P2$  with  $x, y \in \{a, b\}$

$P1 / P2$	<b>a</b>	<b>b</b>
<b>a</b>	$(u_1(a, a), u_2(a, a))$	$(u_1(a, b), u_2(a, b))$
<b>b</b>	$(u_1(b, a), u_2(b, a))$	$(u_1(b, b), u_2(b, b))$

We call a **set of strategies** in a game a complete plan of actions for each situation in the game.

### Example 1.1 (Prisoner's Dilemma)

The **Prisoner's Dilemma** accrues e.g. in the following situation:

two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge. They hope to get both sentenced to a year in prison on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either to: betray the other by testifying that the other committed the crime, or to cooperate with the other by remaining silent. The offer is:

- If A and B each betray the other, each of them serves 6 years in prison
- If A betrays B but B remains silent, A will be set free and B will serve 9 years in prison (and vice versa)
- If A and B both remain silent, both of them will only serve 1 year in prison (on the lesser charge)

P1 / P2	defects	cooperates
defects	$(-6, -6)$	$(0, -9)$
cooperates	$(-9, 0)$	$(-1, -1)$

Other Interpretations for the Prisoner's Dilemma

- Collusion on prices
- Investing in human capital vs. arming for a war
- Buying a SUV vs. a smaller car

**Definition 1.2 (Strict dominance)**

A strategy  $s_i''$  is **strictly dominated** if and only if there exists another strategy  $s_i'$  such that

$$u(s_i', s_{-i}) \geq u(s_i'', s_{-i}) \quad \forall s_{-i} \in S_i$$

In the **Prisoner's Dilemma** *cooperate* is strictly dominated by *defect*. Simply the elimination of strictly dominated strategies leads to the prediction of (*defects, defects*).

**Example 1.3**

Iterated elimination of strictly dominated strategies leads to

- for Player 2:  $l$  strictly dominates  $r$
- after having eliminated  $r$  we can further eliminate  $d$ , since  $d$  is then strictly dominated by  $u$

Important to notice is that here, the prediction we derived relies immensely on the rationality of all players.

**Definition 1.4 (A strategy profile)**

We call a vector  $S = (S_1, \dots, S_N)$  of dimension  $N$  that specifies a strategy for every player in the game a **strategy profile**.

**Definition 1.5 (Nash-Equilibrium)**

A informal definition for a **Nash-Equilibrium** would be that it is the mutual best response for every player, therefore a strategy profile in which no player can do better by unilaterally changing their strategy.

Defining it formally would mean: a strategy profile  $x^* \in S$  is a Nash-Equilibrium if no unilateral deviation in strategy by any single player is profitable for that player, that is

$$\forall i \in \{1, \dots, N\}, x_i \in S_i : \quad u_i(x_i^*, x_{-i}^*) \geq u_i(x_i, x_{-i}^*)$$

**Example 1.6 (Battle of the sexes)**

The next example is a two-player coordination game.

Imagine a couple that agreed to meet this evening, but both individually cannot recall if they will be attending the opera or a football match. The husband would most of all like to go to the football game. The wife would like to go to the opera. Both would prefer to go to the same place rather than different ones.

Hence the **Battle of sexes** in strategic form could look something like:

M / F	football	opera
football	(1, 2)	(0, 0)
opera	(0, 0)	(2, 1)

The two Nash-Equilibriums in this game are  $(opera, opera)$  and  $(football, football)$  since

$$u_i(opera, opera) \geq u_i(football, opera) \quad \forall i \in \{1, 2\}$$

**Example 1.7 (The Beauty-Contest)**

missing in my notes.

**1.1.1 Presentations**

todo

**Example 1.8 (Guessing-Game / Beauty-Contest)**

The **Guessing-Game** is a game with at least two players where the sequel can be described as following

- $n \geq 2$  players
- Every player guesses a number  $b_i \in \{0, 1, 2, \dots, 100\}$
- Goal is to guess  $b_i$  as close as possible to  $\frac{p}{n} \cdot \sum_{i=1}^n b_i = p \cdot \varnothing$ ,  $p \in (0, 1)$
- The best guess (closes to  $p \cdot \varnothing$ ) wins, in case of a tie a random device that is 'fair' decides who win the price  $P > 0$

**1. Question:** Is  $(0, \dots, 0)$  a Nash-Equilibrium?

**Answer:** Yes. Assume all bidders except for bidder  $i$  bid 0.

- if bidder  $i$  bids 0 expected win equals  $\frac{1}{n}P$
- if bidder  $i$  bids something above 0 his expected profit is going to be 0 as 0 is closer to  $p \cdot \varnothing$  then the bet  $b > 0$  of player  $i$ :

$$p \cdot \varnothing = p \frac{(n-1)0 + 1b}{n} = p \frac{b}{n}$$

$$d\left(b, p \frac{b}{n}\right) > d\left(p \frac{b}{n}, 0\right) \iff \left|b - p \frac{b}{n}\right| > \left|p \frac{b}{n}\right|$$

and since  $b > p \cdot \varnothing$  we can simplify this further

$$b > (p \cdot b) \cdot \frac{2}{n} \text{ is true because of } n \geq 2 \text{ and } p < 1.$$

**2. Question:** is  $(0, \dots, 0)$  the unique Nash-Equilibrium here?

**Answer:** Yes, since:

$$b_i^* \leq \frac{1}{2} \frac{\sum_{j \neq i} b_j^*}{n-1}$$

$$\Rightarrow \sum_{i=1}^n b_i^* \leq \frac{1}{2} \frac{\sum_{i=1}^n \sum_{j \neq i} b_j^*}{n-1} = \frac{1}{2} \frac{(n-1) \sum_{j=1}^n b_j^*}{n-1}$$

$$= \frac{1}{2} \sum_{j=1}^n b_j^*$$

$$\iff \sum_{i=1}^n b_i^* \leq \frac{1}{2} \sum_{i=1}^n b_i^*$$

therefore  $(0, \dots, 0)$  is the only Nash-Equilibrium here.

**3. Question:** is  $(0, \dots, 0)$  also a strictly dominant strategy?

**Answer:** No. Looking e.g. at the counterexample:

50 players. 48 of them bid the number 100, bidder 49 bids 0 then the optimal strategy for player 50 is to bid 97.

Therefore 0 is not the best answer and cannot be a strictly dominant strategy.

**Example 1.9**

something is missing in my notes here



## 1.2 Subgame-Perfect Nash-Equilibrium

The Dictator-Game is nicely analyzed by Christoph Engel in his book *Dictator-Games: A meta study* (2011). In the following part we'd like to look at a modification of the Dictator-Game:

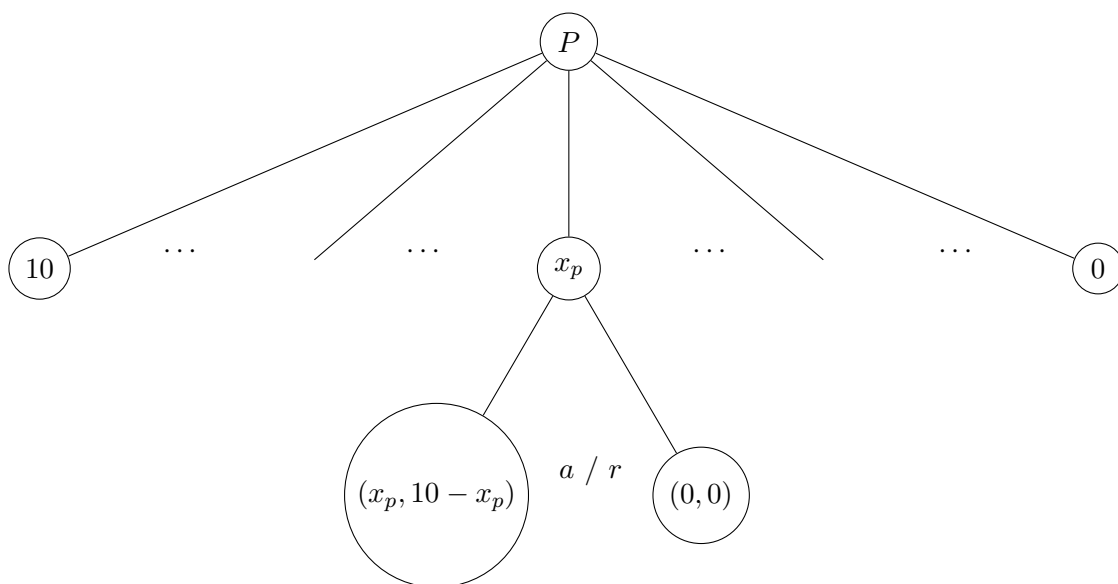
### Example 1.10 (Ultimatum-Game)

The Ultimatum-Game is a dynamic game under complete information.

We look at two players in two stages. The first player (the proposer (P)) receives a sum of money ( $M = 10$ ) and proposes how to divide the sum between himself ( $x_p$ ), where  $x_p \in \{0, 1, 2, \dots, 10\}$ , and another player ( $10 - x_p$ ). The second player (the responder (R)) chooses to either accept or reject this proposal. If the second player accepts, the money is split according to the proposal. If the second player rejects, neither player receives any money.

Lets sum this up again:

- P proposes split up  $(x_p, 10 - x_p)$
- R accepts or rejects
  - If R accepts ( $a$ ), proposal becomes implemented. P receives  $x_p$  and R  $10 - x_p$
  - If R rejects ( $r$ ), the whole money gets destroyed.



A strategy set in this game would have to look like

- Proposer sets a  $x_p$
- Reciever decides for *any*  $x_p$  that might come up if he'd accept or reject that offer.

( strategy needs to specify a complete action plan. )

**1. Question:** Can the outcome  $(5, 5)$  be stabilized as a Nash-Equilibrium outcome?

**Answer:** Yes. Say P proposes  $x_p = 5$  and the strategy set for R is defined by accepting for any value of  $x_p \leq 5$  and rejecting the offer for values larger then 5.

In this situation  $(5, 5)$  would be stabilized as a Nash-Equilibrium.

**2. Question:** Is there another Nash-Equilibrium that stabilizes the (5, 5) outcome?

**Answer:** Yes. If P again proposes  $x_p = 5$  and the strategy set for R is defined by accepting for only  $x_p = 5$  and rejecting for any other case, so  $x_p \neq 5$ .

**3. Question:** Can (0, 10) be stabilized as a Nash-Equilibrium?

**Answer:** Yes. We set the strategy for P as  $x_p = 0$  and for R demand accepting for  $x_p = 0$  and rejecting for any other case, meaning for  $x_p \geq 1$ .

As we can see the Nash-Equilibrium can lead to an infinite amount of outcomes some of them even with implausible threats. We'd therefore would like to refine this kind of equilibrium which leads to the (subgame) perfect Nash-Equilibrium.

### Definition 1.11 (Subgame)

A subgame is any part of a game that meets the following criteria:

- It has a single initial node that is the only member of that node's information set (i.e. the initial node is in a singleton information set).
- If a node is contained in the subgame then so are all of its successors
- If a node in a particular information set is in the subgame then all members of that information set belong to the subgame.

### Definition 1.12 ((Subgame-)Perfect Nash-Equilibrium)

A strategy profile is a Subgame-Perfect Nash-Equilibrium if it represents a Nash equilibrium of every subgame of the original game. Informally, this means that if the players played any smaller game that consisted of only one part of the larger game and their behavior represents a Nash equilibrium of that smaller game, then their behavior is a subgame perfect equilibrium of the larger game.

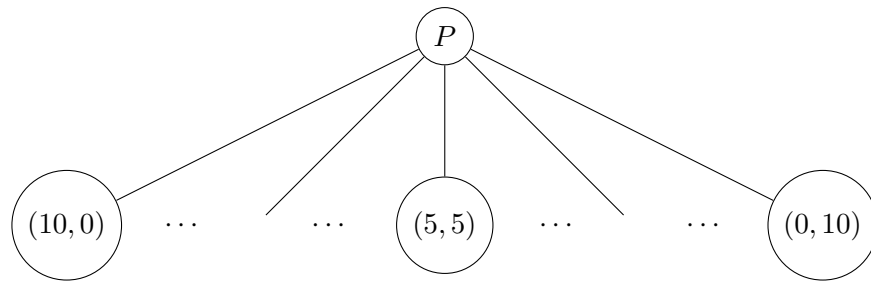
How to find a Perfect Nash-Equilibrium:

- (1) Define (one set of) optimal actions for the last subgame
- (2) Replace that decision nodes with the respective outcome
- (3) Repeat 1) and 2) until the first decision node.

### Example 1.13 (Sequel to the Ultimatum-Game)

Searching for the Perfect Nash-Equilibrium in this case leads to:

- (1) Defining the optimal actions
  - In the case  $P$  chooses  $x_p = 10$ , then  $R$  receives  $10 - x_p = 0$  and he is indifferent between refusing and accepting. Let's assume for now he'd accept in this case.
  - In all other cases, meaning  $x_p \in [0, 10)$ , would  $R$  receive  $10 - x_p > 0$ . Therefore he would accept the offer in all cases.
- (2) Now we can reduce the game to the following game tree



- (3) In the reduced game we have already reached the first node and can now analyze the situation for Nash-Equilibriums. Here  $P$  chooses  $x_p = 10$  since it results in his highest utility.

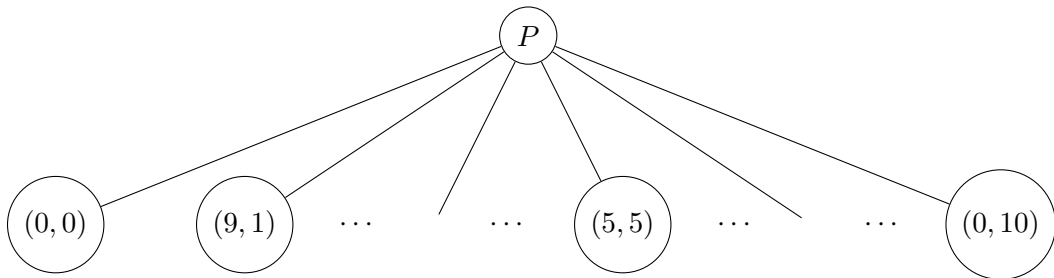
$\Rightarrow P$  plays  $x_p = 10$  together with  $R$  always accepts constitutes a Subgame-Perfect Nash-Equilibrium.

If we'd look for another equilibrium it yields

- (1) the optimal actions

- Again, in all cases  $x_p \in [0, 10)$  would  $R$  receive  $10 - x_p > 0$ . Therefore he would accept the offer in all cases.
- If he now would refuse to the offer  $x_p = 10$  it would be plausible therefore subgame perfect, since he is indifferent between both choices.

- (2) the tree changes only slightly:



$\Rightarrow P$  plays  $x_p = 9$  together with  $R$  always accepts if  $x_p < 10$  and refuses for  $x_p = 10$  constitutes also a Subgame-Perfect Nash-Equilibrium.

### 1.2.1 Presentations

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