Detection of Weak Unstable Predicates in Distributed Programs

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Abstract—This paper discusses detection of global predicates in a distributed program. Earlier algorithms for detection of global predicates proposed by Chandy and Lamport work only for stable predicates. A predicate is stable if it does not turn false once it becomes true. Our algorithms detect even unstable predicates, without excessive overhead. In the past, such predicates have been regarded as too difficult to detect. The predicates are specified by using a logic described formally in this paper. We discuss detection of weak conjunctive predicates that are formed by conjunction of predicates local to processes in the system. Our detection methods will detect whether such a predicate is true for any interleaving of events in the system, regardless of whether the predicate is stable. Also, any predicate that can be reduced to a set of weak conjunctive predicates is detectable. This class of predicates captures many global predicates that are of interest to a programmer. The message complexity of our algorithm is bounded by the number of messages used by the program. The main applications of our results are in debugging and testing of distributed programs. Our algorithms have been incorporated in a distributed debugger that runs on a network of Sun workstations in UNIX.

Index Terms— Unstable predicates, predicate detection, distributed algorithms, distributed debugging

I. Introduction

DISTRIBUTED program is one that runs on multiple processors connected by a communication network. The state of such a program is distributed across the network, and no process has access to the global state at any instant. Detection of a global predicate, i.e., a condition that depends on the state of multiple processes, is a fundamental problem in distributed computing. This problem arises in many contexts, such as designing, testing, and debugging of distributed programs.

A global predicate may be either stable or unstable. A stable predicate is one that never turns false once it becomes true. Some examples of stable predicates are deadlock and termination. Once a system has terminated, it will stay terminated. An unstable predicate is one without such a property. Its value may alternate between true and false. Chandy and Lamport [3] have given an elegant algorithm to detect stable predicates. Their

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algorithm is based on taking a consistent global snapshot of the system and checking whether the snapshot satisfies the global predicate. If the snapshot satisfies the stable predicate, then it can be inferred that the stable predicate is true at the end of the snapshot algorithm. Similarly, if the predicate is false for the snapshot, then it was also false at the beginning of the snapshot algorithm. By taking such snapshots [22] periodically, a stable property can be detected. Bouge [2] has extended this method for repeated snapshots. This approach does not work for an unstable predicate that may be true only between two snapshots, and not when the snapshot is taken. An entirely different approach is required for such predicates.

In this paper, we present an approach that detects a large class of unstable predicates. We begin by defining a logic that is used for specification of global predicates. Formulas in this logic are interpreted over a single run of a distributed program. A run of a distributed program generates a partial order of events, and there are many total orders consistent with this partial order. We call a formula strong if it is true for all total orders, and weak if there exists a total order for which it is true. We consider a special class of predicates defined in this logic in which a global state formula is either a disjunction or a conjunction of local predicates. Since disjunctive predicates can be detected simply by incorporating a local predicate detection mechanism at each process, we focus on conjunctive predicates. In this paper, we describe algorithms for detection of weak types of these predicates. Detection of strong predicates is discussed in [10].

Many of our detection algorithms use time stamp vectors as proposed by Fidge [6] and Mattern [17]. Each process detects its local predicate and records the time stamp associated with the event. These time stamps are sent to a checker process that uses these time stamps to decide whether the global predicate became true. We show that our method uses the optimal number of comparisons by providing an adversary argument. We also show that the checking process can be decentralized, making our algorithms useful even for large networks.

The algorithms presented in this paper have many applications. In debugging a distributed program, a programmer may specify a breakpoint on a condition using our logic and then detect whether the condition became true. Our algorithms can also be used for testing distributed programs. Any condition that must be true in a valid run of a distributed program may be specified, and then its occurrence can be verified. An important property of our algorithms is that they detect even those errors that may not manifest themselves in a particular execution, but may do so with different processing speeds. As an example,

consider a distributed mutual exclusion algorithm. In some run, it may be possible that two processes do not access a critical region, even if they both had permission to enter the critical region. Our algorithms will detect such a scenario under certain conditions described in this paper.

Cooper and Marzullo [5] and Haban and Weigel [11] also describe predicate detection, but they deal with general predicates. Detection of such predicates is intractable, because it involves a combinatorial explosion of the state space. For example, the algorithm proposed by Cooper and Marzullo [5] has complexity $O(k^n)$, where k is the maximum number of events that a monitored process has executed and n is the number of processes. The fundamental difference between our algorithm and their algorithm is that their algorithm explicitly checks all possible global states, whereas our algorithm does not. Miller and Choi [19] mainly discuss linked predicates. They do not discuss detection of conjunctive predicates (in our sense) that are most useful in distributed programs. Moreover, they do not make distinction between program messages and messages used by the detection algorithm. As a result, the linked predicate detected by Miller and Choi's algorithm may be true when the debugger is present, but may become false when it is removed. Our algorithms avoid this problem. Hurfin, Plouzeau, and Raynal [12] also discuss methods for detecting atomic sequences of predicates in distributed computations. Spezialetti and Kearns [23] discuss methods for recognizing event occurrences without taking snapshots. However, their approach is suitable only for monotonic events that are similar to stable properties. An overview of these and some other approaches can be found in [20].

This paper is organized as follows: Section II presents our logic for describing unstable predicates in a distributed program. It describes the notion of a distributed run, a global sequence, and the logic for specification of global predicates. Section III discusses a necessary and sufficient condition for detection of weak conjunctive predicates. It also shows that detection of weak conjunctive predicates is sufficient to detect any global predicate on a finite state program or any global predicate that can be written as a Boolean expression of local conditions. Section IV presents an algorithm for detection of a weak conjunctive predicate. Section V describes a technique to decentralize our algorithm. Section VI gives some details of an implementation of our algorithms in a distributed debugger. Finally, Section VII gives conclusions of this paper.

II. OUR MODEL

A. Distributed Run

We assume a loosely coupled message-passing system without any shared memory or a global clock. A distributed program consists of a set of n processes denoted by $\{P_1, P_2, \cdots, P_n\}$ communicating solely via asynchronous messages. In this paper, we are concerned with a single run r of a distributed program. Each process P_i in that run generates a single execution trace r[i] that is a finite sequence of *states* and *actions* that alternate, beginning with an initial state. The state of a process is defined by the value of all of its variables including its program counter. For example, the process P_i

generates the trace $s_{i,0}a_{i,0}s_{i,1}a_{i,1}\cdots a_{i,l-1}s_{i,l}$, where s_i 's are the local states and a_i 's are the local actions in the process P_i . There are three kinds of actions: internal, send, and receive. A send action, denoted by $send\langle i,j,\phi\rangle$, means the sending of a message ϕ from the process P_i to the process P_j . A receive action denoted by $receive(\langle i,j,\phi\rangle)$ means the receiving of a message ϕ from the process P_i by the process P_j . We assume in this paper that no messages are lost, altered, or spuriously introduced. We do not make any assumptions about the firstin, first-out (FIFO) nature of the channels. A $run\ r$ is a vector of traces with r[i] as the trace of the process P_i . From the reliability of messages we obtain the following equation:

$$receive(\langle i, j, \phi \rangle) \in r[j] \Leftrightarrow send(\langle i, j, \phi \rangle) \in r[i].$$

We also define a happened-before relation (denoted by →) between states similar to that of Lamport's happened-before relation between events.

Definition 1: The state s in the trace r[i] happened-before (\rightarrow) the state t in the trace r[j] if and only if one of the following conditions holds:

- 1) i = j, and s occurs before t in r[i].
- 2) The action following s is the send of a message, and the action before t is the reception of that message.
- 3) There exists a state u in one of the traces such that $s \to u$ and $u \to t$.

The relation \rightarrow is a partial order on the states of the processes in the system. As a result of rules 2) and 3) in the above definition, we say that there is a *message path* from state s to state t if $s \rightarrow t$ and they are in different processes. A run can be visualized as a valid error-free process time diagram [16].

Example 1: Consider the following distributed program:

```
Process P_1;

var x:integer initially 7;

begin

l_0: send(x) to P_2;

l_1: x:=x-1;

l_2: send(x) to P_{2i}

l_3:

end;

Process P_2;

var y,z:integer initially (0,0);

begin

m_0: receive(y) from P_1;

m_1: receive(z) from P_1;

m_2:
end;
```

Labels l_0, \dots, l_3 and m_0, \dots, m_2 denote possible values of program counters. A distributed run r is given by the following equation:

```
\begin{split} r[1] &= ((l_0,7), send(1,2,7), (l_1,7), internal,\\ (l_2,6), send(1,2,6), (l_3,6))\\ r[2] &= ((m_0,0,0), receive(1,2,7), (m_1,7,0),\\ receive(1,2,6), (m_2,7,6)). \end{split}
```

Another run r' can be constructed when two messages sent by the process P_1 are received in the reverse order:

```
\begin{split} r'[1] &= ((l_0,7), send(1,2,7), (l_1,7), internal,\\ (l_2,6), send(1,2,6), (l_3,6))\\ r'[2] &= ((m_0,0,0), receive(1,2,6), (m_1,6,0),\\ receive(1,2,7), (m_2,6,7)). \end{split}
```

B. Global Sequence

A run defines a partial order (\rightarrow) on the set of actions and states. For simplicity, we ignore actions from a run, and focus just on states in traces. Thus, r[i] denotes the sequence of states of P_i . In general, there are many total orders that are consistent with (or linearizations of) this partial order. A global sequence corresponds to a view of the run that could be obtained, given the existence of a global clock. Thus, a global sequence is a sequence of global states where a global state is a vector of local states. This definition of a global state is different from that of Chandy and Lamport, which includes states of the channels. In our model, a channel is just a set of all of those messages that have been sent, but not received yet. Because this set can be deduced from all of the local states, we do not require the state of channels to be explicitly included in the global state. We denote the set of global sequences consistent with a run r as linear(r). A global sequence g is a finite sequence of global states denoted as $g = g_0 g_1 \cdots g_l$, where g_k is a global state for 0 < k < l. Its suffix, starting with g_k (i.e., $g_k.g_{k+1}\cdots g_l$), is denoted by g^k . Clearly, if the observer restricts his attention to a single process P_i , then he would observe r[i] or a stutter of r[i]. A stutter of r[i] is a finite sequence where each state in r[i] may be repeated a finite number of times. The stutter arises because we have purposely avoided any reference to physical time. Let s || t mean that $s \not\to t \wedge t \not\to s$. Then, a global sequence of a run is defined as given below.

Definition 2: g is a global sequence of a run r (denoted by $g \in linear(r)$) if and only if the following constraints hold:

```
\begin{split} (S1): \ \forall i: \ g \ \text{restricted to} \ P_i = r[i] \ \text{(or a stutter of} \ r[i]) \\ (S2): \ \forall k: \ g_k[i] ||g_k[j], \ \text{where} \ g_k[i] \ \text{is the state of} \ P_i \ \text{in the} \\ \text{global state} \ g_k. \end{split}
```

Example 2: Some global sequences consistent with the run r in Example 1 are given below:

```
\begin{split} g &= [(l_0, 7, m_0, 0, 0), (l_1, 7, m_0, 0, 0), (l_2, 6, m_1, 7, 0), \\ (l_3, 6, m_1, 7, 0), (l_3, 6, m_2, 7, 6)] \\ h &= [(l_0, 7, m_0, 0, 0), (l_1, 7, m_0, 0, 0), (l_2, 6, m_0, 0, 0), \\ (l_3, 6, m_1, 7, 0), (l_3, 6, m_2, 7, 6)]. \end{split}
```

Our model of a distributed run and global sequences does not assume that the system computation can always be specified as some interleaving of local actions. The next global state of a global sequence may result from multiple independent local actions.

C. Logic Operators

There are three syntactic categories in our logic: bool, lin, and form. The syntax of our logic is as follows:

```
form ::= \underline{A}: \lim |\underline{E}: \lim \lim := \lim |\sin |\sin |\sin | \lim |\sin |\sin |\sin |\sin |\sin | \lim |\sin |\cos |\cos |\cos |\sin | bool bool ::= a predicate over a global system state.
```

A bool is a Boolean expression defined on a single global state of the system. Its value can be determined if the global state is known. For example, if the global state has (x=7,y=0), then the bool $(x \ge y)$ is true. Here x and y could be variables in different processes. A lin is a temporal formula defined over a global sequence. \diamond lin means that there exists a suffix of the global sequence such that lin is true for the suffix [21]. We use \square as the dual of \diamond . We have also introduced a binary operator (\hookrightarrow) to capture sequencing directly. $p \hookrightarrow q$ means that there exist suffixes g^i and g^j of the global sequence such that p is true of the suffix g^j , and i < j. A form is defined over a set of global sequences, and it is simply a lin qualified with the universal (A:) or the existential (E:) quantifier. Thus, the semantics of our logic are as follows:

```
iff
g \models bool
                                         g_0 \models bool
g \models \neg lin
                                 iff
                                          \neg(g \models lin)
g \models lin_1 \wedge lin_2
                                 iff
                                         g \models lin_1 \land g \models lin_2
g \models lin_1 \lor lin_2
                                 iff
                                          g \models lin_1 \lor g \models lin_2
g \models \Box lin
                                 iff
                                         \forall i: g^i \models lin
g\models \diamond lin
                                 iff
                                         \exists i: g^i \models lin
g \models lin_1 \hookrightarrow lin_2
                                iff
                                          \exists i, j : (i < j) \land g^i \models lin_1 \land
                                          g^j \models lin_2
r \models \underline{A} : lin
                                 iff
                                         \forall g:g \in linear(r):g \models lin
r \models \underline{E} : lin
                                 iff \exists g : g \in linear(r) : g \models lin.
```

 \underline{A} , and \underline{E} quantify over the set of global sequences that a distributed run may exhibit, given the trace for each process. \underline{A} :p means that the predicate p holds for all global sequences, and \underline{E} :p means that the predicate p holds for some global sequence. We call formulas starting with \underline{A} : strong formulas, and formulas starting with \underline{E} : weak formulas. The intuition behind the term strong is that a strong formula is true no matter how fast or slow the individual processes in the system execute. That is, it holds for all execution speeds that generate the same trace for an individual process. A weak formula is true if and only if there exists one global sequence in which it is true. In other words, the predicate can be made true by choosing appropriate execution speeds of various processors.

The difficulty of checking truthness of a global predicate arises from two sources. First, if there are n processes in the system, the total number of global sequences (in which a global state is not repeated) is exponential in n and the size of the traces. Second, the global state is distributed across the network during an actual run. Thus, detection of any general predicate in the above logic is not feasible in a distributed program. To avoid the problem of combinatorial explosion, we focus on detection of predicates belonging to a class that we believe captures a large subset of predicates interesting to a programmer. We use the word local to refer to a predicate or condition that involves the state of a single process in the system. Such a condition can be easily checked by the process itself. We detect predicates that are Boolean expressions of local predicates. Following are examples of the formulas detectable by our algorithms:

1) Suppose that we are developing a mutual exclusion algorithm. Let CS_i represent the local predicate that the process P_i is in a critical section. Then, the following formula detects any possibility of violation of mutual

exclusion for a particular run:

$$\underline{E}: \diamond (\mathbf{CS}_1 \wedge \mathbf{CS}_2).$$

2) In example 2, we can check if:

$$\underline{E}: \diamond (x=6) \wedge (P_2 \text{ at } m_0).$$

Note that the following expression:

$$\diamond(x=6) \land (P_2 \text{ at } m_0)$$

is not true for the global sequence g, but that it is true for the global sequence h. Our algorithm will detect the above predicate to be true for the run r, even though the global sequence executed may be g.

3) Assume that in a database application, serializability is enforced by using a two-phase locking scheme [15]. Further assume that there are two types of locks: read and write. Then, the following formula may be useful to identify an error in implementation:

$$\underline{E}: \diamond (P_1 \text{ has read lock}) \wedge (P_2 \text{ has write lock}).$$

III. WEAK CONJUNCTIVE PREDICATES

A weak conjunctive predicate (WCP) is true for a given run if and only if there exists a global sequence consistent with that run in which all conjuncts are true in some global state. Practically speaking, this type of predicate is most useful for bad or undesirable predicates (i.e., predicates that should never become true). In such cases, the programmer would like to know whenever it is possible that the bad predicate may become true. For example, consider the classical mutual exclusion situation. We may use a WCP to check whether the correctness criterion of never having two or more processes in their critical sections at the same time is met. We would want to detect that the predicate "process x is in its critical section and process y is in its critical section." It is important to observe that our algorithms will report the possibility of mutual exclusion violation even if it was not violated in the execution that happened. The detection will occur if and only if there exists a consistent cut in which all local predicates are true. Thus, our techniques detect errors that may be hidden in some run because of race conditions.

A. Importance of Weak Conjunctive Predicates

Conjunctive predicates form the most interesting class of predicates, because their detection is sufficient for detection of any global predicate that can be written as a Boolean expression of local predicates. This observation is shown below.

Lemma 1: Let p be any predicate constructed from local predicates using Boolean connectives. Then, $\underline{E}: \diamond p$ can be detected by using an algorithm that can detect $\underline{E}: \diamond q$, where q is a pure conjunction of local predicates.

Proof: We first write p in its disjunctive normal form. Thus, $\underline{E}: \diamond p = \underline{E}: \diamond (m_1 \vee \cdots \vee m_l)$, where each m_i is a pure conjunction of local predicates. Next we observe that the following condition exists:

$$\underline{E}: \diamond (m_1 \vee \cdots \vee m_l)$$

$$= \{ \text{ semantics of } \underline{E} \text{ and } \diamond \}$$

$$\exists g: \exists i: g^i \models (m_1 \vee m_2 \cdots \vee m_l)$$

$$= \{ \text{ semantics of } \vee \}$$

$$\exists g: \exists i: (g_i \models m_1 \vee g_i \models m_2 \vee \cdots g^i \models m_l)$$

$$= \{ \text{ distribute } \exists \text{ over } \vee \text{ twice} \}$$

$$\exists g: \exists i: g_i \models m_1 \vee \cdots \vee \exists g: \exists i: g_i \models m_l$$

$$= \{ \text{ semantics of } \underline{E} \text{ and } \diamond \}$$

$$\underline{E}: \diamond m_1 \vee \underline{E}: \diamond m_2 \vee \cdots \vee \underline{E} \diamond m_l.$$

Thus, the problem of detecting $\underline{E}: \diamond p$ is reduced to solving l problems of detecting $\underline{E}: \diamond q$, where q is a pure conjunction of local predicates.

Our approach is most useful when the global predicate can be written as a Boolean expression of local predicates. As an example, consider a distributed program in which x,y and z are in three different processes. Then, the following condition:

$$\underline{E}: \diamond even(x) \land ((y < 0) \lor (z > 6)),$$

can be rewritten as

$$\underline{E} : \diamond (even(x) \land (y < 0)) \lor \\ \underline{E} : \diamond (even(x) \land (z > 6)),$$

where each part is a weak conjunctive predicate.

We note that even if the global predicate is not a Boolean expression of local predicates, but is satisfied by only a finite number of possible global states, then it can again be rewritten as a disjunction of weak conjunctive predicates. For example, consider the predicate $\underline{E}: \diamond (x=y)$, where x and y are in different processes. (x=y) is not a *local* predicate, because it depends on both processes. If we know that x and y can only take values $\{0,1\}$, however, then the above expression can be rewritten as

$$E: \diamond ((x=0) \land (y=0)) \lor ((x=1) \land (y=1)).$$

This is equivalent to

$$(\underline{E}: \diamond (x=0) \land (y=0)) \lor (\underline{E}: \diamond (x=1) \land (y=1)).$$

Each of the disjuncts in this expression is a weak conjunctive predicate.

We observe that predicates of the form \underline{A} : $\Box bool$ can also be easily detected, because they are simply duals of \underline{E} : $\diamond bool$, which can be detected as shown in Section.

We have emphasized conjunctive predicates, but not disjunctive predicates, in this paper, because disjunctive predicates are quite simple to detect. To detect a disjunctive predicate \underline{E} : \diamond LP $_1 \lor LP<math>_2 \lor \cdots \lor LP_m$, it is sufficient for the process P_i to monitor LP $_i$. If any part of the process finds its local predicate true, then the disjunctive predicate is true.

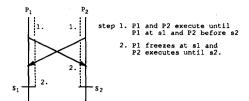


Fig. 1. Incomparable states producing a single global state.

B. Conditions for Weak Conjunctive Predicates

We use LP_i to denote a local predicate in the process P_i and $LP_i(s)$ to denote that the predicate LP_i is true in the state s. We say that $s \in r[i]$ if s occurs in the sequence r[i].

Our aim is to detect whether \underline{E} : $\diamond(\operatorname{LP}_1 \wedge \operatorname{LP}_2 \wedge \cdots \operatorname{LP}_m)$ holds for a given r. We can assume that $m \leq n$ because $\operatorname{LP}_i \wedge \operatorname{LP}_j$ is just another local predicate if LP_i and LP_j belong to the same process. We now present a theorem that states the necessary and sufficient conditions in order for a weak conjunctive predicate to hold.

Theorem 1: \underline{E} : $\diamond(LP_1 \wedge LP_2 \wedge \cdots LP_m)$ is true for a run r iff, for all $1 \leq i \leq m$, $\exists s_i \in r[i]$ such that LP_i is true in state s_i , and s_i and s_j are incomparable for $i \neq j$. That is, $r \models \underline{E}$: $\diamond(LP_1 \wedge LP_2 \wedge \cdots LP_m) \Leftrightarrow \exists s_1 \in r[1], s_2 \in r[2], \cdots, s_m \in r[m]$ such that $\langle \forall i: 1 \leq i \leq m: LP_i(s_i) \rangle \wedge \langle \forall i, j: 1 \leq i < j \leq m: (s_i||s_j) \rangle$.

Proof: First, assume that \underline{E} : $\diamond(\operatorname{LP}_1 \wedge \operatorname{LP}_2 \wedge \cdots \operatorname{LP}_m)$ is true for the run r. By definition, there is a global sequence $g \in \operatorname{linear}(r)$ that has a global state, g_* where all local predicates are true. We define $s_i = g_*[i]$ for all i. Clearly, $\forall i : \operatorname{LP}_i(s_i) \wedge (s_i \in r[i])$. Now consider any two distinct indices i and j between 1 and m. Since s_i and s_j correspond to the same global state, s_i and s_j must be incomparable by (S2). Therefore, $\exists s_1 \in r[1], s_2 \in r[2], \cdots, s_m \in r[m] : \langle \forall i : 1 \leq i \leq m : \operatorname{LP}_i(s_i) \rangle \wedge \langle \forall i, j : 1 \leq i \leq j \leq m : (s_i || s_j) \rangle$.

We prove the other direction (\Leftarrow) for m=2. The proof for the general case is similar. Assume that there exist states $s_1 \in r[1]$ $s_2 \in r[2]$ such that states s_1 and s_2 are incomparable, and $\operatorname{LP}_1(s_1) \wedge \operatorname{LP}_2(s_2)$. This implies that there is no message path from s_1 to s_2 or vice versa. Thus, any message received in or before s_2 could not have been sent after s_1 , and any message received in or before s_1 could not have been sent after s_2 as Fig. 1 illustrates. Thus, it is possible to construct the following execution (global sequence).

- 1) Let both processes execute consistently with the run r either until P_1 is at s_1 and P_2 is before s_2 , or until P_2 is at s_2 and P_1 is before s_1 . Assume, without loss of generality, that the former case holds.
- 2) Freeze P_1 at s_1 , and let P_2 execute until it is at s_2 . This is possible because there is no message sent after s_1 and received before s_2 .

We now have a global state $g_* = (s_1, s_2)$ such that both LP₁ and LP₂ are true in g_* .

IV. DETECTION OF WEAK CONJUNCTIVE PREDICATES

Theorem 1 shows that it is necessary and sufficient to find a set of incomparable states in which local predicates are true to

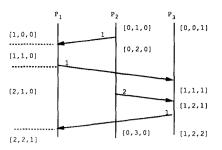


Fig. 2. Examples of lcm vectors.

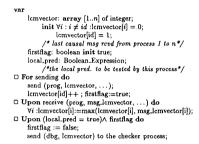


Fig. 3. Algorithm for weak conjunctive predicates: Nonchecker process P_{id} .

detect a weak conjunctive predicate. In this section, we present a centralized algorithm to do so. Later, we show how the algorithm can be decentralized. In this algorithm, one process serves as a checker. All other processes involved in WCP are referred to as nonchecker processes. These processes, shown in Fig. 3, check for local predicates.

Each nonchecker process keeps its own local last causal message vector (lcmvector) of time stamps. These time stamp vectors are slight modifications of the virtual time vectors proposed by [6], [17]. For the process P_i , lcmvector [i] $(i \neq j)$ is the message id of the most recent message from P_i (to anybody) that has a causal relationship to P_i . lcmvector [j]for the process P_i is the next message id that P_i will use. To maintain the lcmvector information, we require every process to include its lemvector in each program message it sends. Whenever a process receives a program message, it updates its own lemvector by taking the component-wise maximum of its lcmvector and the one contained in the message. Fig. 2 illustrates this by showing P_1 's lcmvector in each interval. Whenever the local predicate of a process becomes true for the first time since the most recently sent message (or the beginning of the trace), it generates a debug message containing its local time stamp vector and sends it to the checker process.

One of the reasons why the above algorithm is practical is that a process is not required to send its lcmvector every time that the local predicate is detected. A simple observation tells us that the lcmvector need not be sent if there has been no message activity since the last time the lcmvector was sent, because the lcmvector can change its value only when a message is sent or received. We now show that it is sufficient to send the lcmvector once after each message is sent, regardless of the number of messages received.

Let local(s) denote that the local predicate is true in state s. We define the predicate first(s) to be true iff the local predicate is true for the first time since the most recently sent message (or the beginning of the trace). We say that $wcp(s_1, s_2, \cdots, s_m)$ is true if $s_1, s_2, \cdots s_m$ are the states in different processes, making the wcp true (as in Theorem 1).

Theorem 2: $\exists s_1, \dots, s_m : wcp(s_1, s_2, \dots s_m) \Leftrightarrow \langle \exists s_1', \dots, s_m' : wcp(s_1', s_2', \dots, s_m') \land \forall i : 1 \leq i \leq m : first <math>(s_i') > .$

Proof: (\Leftarrow) is trivially true. We show (\Rightarrow). By symmetry, it is sufficient to prove the existence of s_1' such that $wcp(s_1', s_2, \cdots, s_m) \land first(s_1')$. We define s_1' as the first state in the trace of P_1 since the most recently sent message or the beginning of the trace such that $local(s_1')$ is true. As s_1 exists, we know that s_1' also exists. By our choice of s_1' , $first(s_1')$ is true. Our proof obligation is to show that $wcp(s_1', s_2, \cdots, s_m)$. It is sufficient to show that $s_1' || s_j$ for $2 \le j \le m$. For any s_j , $s_1 \not\rightarrow s_j$, and there is no message sent after s_1' and before s_1 . Therefore, $s_1' \not\rightarrow s_j$. Also, $s_j \not\rightarrow s_1'$; otherwise, $s_j \rightarrow s_1'$ and $s_1' \rightarrow s_1$ would imply that $s_j \rightarrow s_1$, a contradiction. Therefore, we conclude that $s_1' || s_j$ for any $2 \le j \le m$.

We now analyze the complexity of nonchecker processes. The space complexity is given by the array lcmvector and is O(n). The main time complexity is involved in detecting the local predicates, which is the same as for a sequential debugger. Additional time is required to maintain time vectors. This is O(n) for every receive of a message. In the worst case, one debug message is generated for each program message sent, so the worst-case message complexity is $O(m_s)$, where m_s is the number of program messages sent. In addition, program messages have to include time vectors.

We now give the algorithm for the checker process that detects the WCP by using the debug messages sent by other processes. The checker process has a separate queue for each process involved in the WCP. Incoming debug messages from processes are enqueued in the appropriate queue. We assume that the checker process gets its message from any process in FIFO order. Note that we do not require FIFO for the underlying computation. Only the detection algorithm needs to implement FIFO property for efficiency purposes. If the underlying communication is not FIFO, the checker processes can ensure that it receives messages from nonchecker processes in FIFO by using sequence numbers in messages.

The checker process applies the following definition to determine the order between two lcmvectors. For any two lcmvectors, u and v, u < v if and only if $(\forall i : u[i] \leq v[i]) \land (\exists j : u[j] < v[j])$. Furthermore, if we know the processes from which the vectors came, the comparison between two lcmvectors can be made in constant time. Let $Proc : \mathcal{N}^n \to \{1,2,\cdots,n\}$ map a lcmvector to the process to which it belongs. Then, the required computation to check whether the lcmvector u is less than the lcmvector v is as follows:

$$(u[\mathit{Proc}(u)] \leq v[\mathit{Proc}(u)]) \wedge (u[\mathit{Proc}(v)] < v[\mathit{Proc}(v)]). \tag{P1}$$

Lemma 2: Let s and t be states in processes P_i and P_j with lemvectors u and v, respectively. Then, $s \to t$ iff u < v.

Proof: $(s \to t) \Rightarrow (u < v)$. If $s \to t$, then there is a message path from s to t. Therefore, because P_j updates its lcmvector upon receipt of a message, and because this update is done by taking the component-wise maximum, we know that the following expression holds: $\forall k: u[k] \leq v[k]$. Furthermore, because v[j] is the next message id to be used by P_j , P_i could not have seen this value as $t \not \to s$. We therefore know that v[j] > u[j]. Hence, the following expression holds: $(\forall k: u[k] \leq v[k]) \land (u[j] < v[j])$ Thus, $(s \to t) \Rightarrow (u < v)$.

We now show that $\neg(s \to t) \Rightarrow \neg(u < v)$. First, $\neg(s \to t) \Leftrightarrow (t \to s) \lor (s||t)$. If $(t \to s)$, then (v < u) by the first part of this theorem. If (s||t)), then there is no message path from the state s to the state t, and vice versa. Hence, when P_i is at s and P_j is at t, the following expression holds:

- (1) (u[j] < v[j]), and
- $(2) \qquad (v[i] < u[i]).$

Therefore,
$$(s||t)$$
) $\Rightarrow \neg (u < v)$.

Thus, the task of the checker process is reduced to checking ordering between lcmvectors to determine the ordering between states. The following observation is critical for reducing the number of comparisons in the checker process.

Lemma 3: If the lcmvector at the head of one queue is less than the lcmvector at the head of any other queue, then the smaller lcmvector may be eliminated from further consideration in checking to see whether the WCP is satisfied.

Proof: In order for the WCP to be satisfied, we must find a set of lcmvectors, one from each queue, such that each is incomparable with all the others in the set. If the lcmvector at the head of one queue (q_i) is less than that at the head of another queue (q_j) , we know that it will be less than any other lcmvectors in q_j , because the queues are in increasing order from head to tail. Also, any later arrivals into q_j must be greater than that at the head of q_i . Hence, no entry in q_j will ever be incomparable with that at the head of q_i , so the head of q_i may be eliminated from further consideration in checking to see whether the WCP is satisfied.

The algorithm given in Fig. 4 is initiated whenever any new lcmvector is received. If the corresponding queue is nonempty, then it is simply inserted in the queue; otherwise, there exists a possibility that the conjunctive predicate may have become true. The algorithm checks for incomparable lcmvectors by comparing only the heads of queues. Moreover, it compares only those heads of the queues that have not been compared earlier. For this purpose, it uses the variable *changed*, which is the set of indices for which the head of the queues have been updated. The *while* loop maintains the following invariant expression:

(I)
$$\forall i, j \notin changed : \neg empty(q_i) \land \neg empty(q_j) \Rightarrow head(q_i)||head(q_j).$$

This is done by finding all of those elements that are lower than some other elements and including them in *changed*. This means that there cannot be two comparable elements in $\{1, 2, \cdots, m\}$ —*changed*. The loop terminates when *changed* is empty. At that point, if all queues are nonempty, then, by the invariant I, we can deduce that all of the heads are

```
var q_1 \dots q_m: queue of lcmvector; changed, newchanged: set of \{1,2,\dots,m\} \square Upon recv(elem) from P_k do insert\{q_k, \text{elm}\}; if (\text{head}(q_k) = \text{elem}) then begin changed: = \{k\}; while (\text{changed} \neq \phi) begin newchanged: = \{k\}; for i in changed, and j in \{1,2,\dots,m\} do if (\neg empty(q_1) \land \neg empty(q_1)) then begin if head(q_1) \land head(q_2) then newchanged:=newchanged \cup {i}; if head(q_2) \land head(q_2) then newchanged::=newchanged \cup {j}; end; /* if */ changed::= newchanged; for i in changed delethead(q_i); end; /* while */ if \forall i : \neg empty(q_i) then found:=true; end; /* if */
```

Fig. 4. Algorithm for weak conjunctive predicates: The checker process.

incomparable. Let there be m queues with at most p elements in any queue. The next theorem deals with the complexity of the algorithm shown in Fig. 4.

Theorem 3: The algorithm shown in Fig. 4 requires at most $O(m^2p)$ comparisons.

Proof: Let comp(k) denote the number of comparisons required in the kth iteration of the while loop. Let t denote the total number of iterations of the while loop. Then, the total number of comparisons equals $\sum_{k=1}^t comp(k)$. Let changed(k) represent the value of changed at the kth iteration. |changed(k)| for $k \geq 2$ represents the number of elements deleted in the k-1 iteration of the while loop. From the structure of the for-loops we get the result that comp(k) = O(m * |changed(k)|). Therefore, the total number of comparisons required are $\sum_{k=1}^t comp(k) = m * \sum_{k=1}^t |changed(k)| \leq m * pm = O(m^2p)$.

Theorem 4 proves that the complexity of the above problem is at least $\Omega(m^2p)$, thus showing that our algorithm is optimal [8].

Theorem 4: Any algorithm that determines whether there exists a set of incomparable vectors of size m in m chains of size at most p makes at least pm(m-1)/2 comparisons.

We first show it for the case when the size of each queue is exactly 1, i.e., p=1. The adversary will give to the algorithm a set in which either zero or exactly one pair of elements are comparable. The adversary also chooses to answer "incomparable" to first m(m-1)/2-1 questions. Thus, the algorithm cannot determine whether the set has a comparable pair, unless it asks about all of the pairs.

We now show the result for a general p. Let $q_i[k]$ denote the kth element in the queue q_i . The adversary will give the algorithm q_i 's with the following characteristic:

$$\forall i, j, k : q_i[k] < q_j[k+1].$$

Thus, the above problem reduces to p instances of the problem that checks whether any of the m elements are incomparable. If the algorithm does not completely solve one instance, then the adversary chooses that instance to show m queues consistent with all of its answers, but different in the final outcome. \square

V. DECENTRALIZATION OF THE DETECTION ALGORITHM

We now show techniques for decentralizing the above algorithm. From the property (P1), we can deduce that if a set of vectors S forms an antichain (i.e., if all pairs of vectors are incomparable), then the following expression holds:

$$\forall distinct \ s, t \in S : s[Proc(s)] > t[Proc(s)].$$
 (P2)

We denote this condition by the predicate inc(S). The following theorem shows that the process of checking inc(S) can be decomposed into that of checking it for smaller sets.

Theorem 5: Let S,T, and U be sets of lcmvectors such that $S=T\cup U$. Let max_X represent the lcmvector formed by taking the component-wise maximum of all vectors in the set X. Then, the following expression holds:

```
\begin{split} & inc(S)iffinc(T) \wedge inc(U) \wedge \\ & (\forall t \in T : max_T[\operatorname{Proc}(t)] > max_U[\operatorname{Proc}(t)]) \wedge \\ & (\forall u \in U : max_U[\operatorname{Proc}(u)] > max_T[\operatorname{Proc}(u)]). \end{split}
```

Proof: (\Rightarrow) inc(T) and inc(U) are clearly true because $T,U\subseteq S.$ We show that the following expression holds:

$$(\forall t \in T: \max_{T}[Proc(t)] > \max_{U}[Proc(t)]).$$

The other conjunct is proved in a similar fashion.

From (P2), we deduce that $\forall distinct\ s,t\in T: t[Proc(t)]>s[Proc(t)]$. This means that $\max_T[Proc(t)]=t[Proc(t)]$ by the definition of \max_T . From (P2), we also deduce that $\forall u\in U: t[Proc(t)]>u[Proc(t)]$. This means that $t[Proc(t)]>\max_U[Proc(t)]$ by the definition of \max_U . From the above two assertions, we conclude that $(\forall t\in T:\max_T[Proc(t)]>\max_U[Proc(t)])$.

(⇐) We will show that (P2) holds for S, i.e., that \forall distinct $s,t \in S$: s[Proc(s)] > t[Proc(s)]. If both s and t belong either to T and U, then the above is true from inc(T) and inc(U). Let us assume, without loss of generality, that $t \in T$ and $u \in U$. We need to show that t[Proc(t)] > u[Proc(t)] (the other part is proved similarly). From inc(T), we conclude that $\max_T[Proc(t)] = t[Proc(t)]$. Now, from $\max_T[Proc(t)] > \max_U[Proc(t)]$ we conclude that t[Proc(t)] > u[Proc(t)].

By using the above theorem and the notions of a hierarchy, the algorithm for checking WCP can be decentralized as follows. We may divide the set of processes into two groups. The group checker process checks for WCP within its group. On finding one, it sends the maximum of all lcmvectors to a higher process in the hierarchy. This process checks the last two conjuncts of the above theorem. Clearly, the above argument can be generalized to a hierarchy of any depth.

Example 6: Consider a distributed program with four processes. Let the lcmvectors corresponding to these processes be $S = \{(4, 4, 6, 2), (3, 6, 4, 1), (3, 5, 7, 2), (2, 5, 4, 3)\}$. Now, instead of checking whether the entire set consists of incomparable vectors, we divide it into two subsets: $T = \{(4, 4, 6, 2), (3, 6, 4, 1)\}$, and $U = \{(3, 5, 7, 2), (2, 5, 4, 3)\}$. We check that each one of them is incomparable. This computation can be done by group checker processes. Group processes send $\max_{T} = (4, 6, 6, 2)$ and

 $\max_{U} = (3, 5, 7, 3)$ to the higher-level process. This process can check that \max_T is strictly greater than \max_U in the first two components and \max_U is strictly greater than \max_T in the last two components. Hence, by Theorem 5, all vectors in the set S are pairwise incomparable.

VI. IMPLEMENTATION: UTDDB

The main applications of our results are in debugging and testing of distributed programs. We have incorporated our algorithms into the distributed debugger called the University of Texas Distributed Debugger (UTDDB) [14]. The online debugger is able to detect global states or sequences of global states in a distributed computation. UTDDB consists of two types of processes: coordinator type and monitor type. There exists only one coordinator process, but the number of monitor processes is the same as the number of application processes in the underlying distributed computation.

The coordinator process serves as the checker process for WCP and as the user-interface of UTDDB to the programmer. It accepts input from the programmer, such as distributed predicates to be detected. It also reports to the programmer whether the predicate is detected.

Monitor processes are hidden from the programmer. Each of the monitor processes detects local predicates defined within the domain of the application process that it is monitoring. This is done by single-stepping the program. After each step, the monitor examines the address space of the application process to check whether any of the simple predicates in its list are true. It is also responsible for implementing algorithms described as a nonchecker process in Section IV. In particular, it maintains the vector clock mechanism.

In a distributed debugger, the delays between the occurrence of a predicate, its detection, and halting of the program may be substantial. Thus, when the program is finally halted, it may no longer be in a state in which the programmer is interested. Therefore, for the weak conjunctive predicate, UTDDB gives the programmer the option of rolling back the distributed computation to a consistent global state where the predicate is true. The coordinator uses the set of time stamps that detected the WCP predicate to calculate this global state, which it then sends to all the monitors. As the application processes execute, they record incoming events to a file. So, when a monitor receives a message telling it to roll back an application process, the monitor restarts the application process and replays the recorded events until the process reaches a local state that is part of the global state where the weak conjunctive predicate is true. Such a restart assumes that the only nondeterminism in the program is due to reordering of messages.

Our algorithms are also used in a trace analyzer (another part of UTDDB) for distributed programs [4]. Our analyzer monitors a distributed program and gathers enough information to form a distributed run as described in Section II. This approach reduces the probe effect that the distributed program may experience if the detection is carried out while the program was in execution. The user can then ask UTDDB whether any predicate expressed in a subset of the logic described in this paper ever became true. We are currently extending these algorithms for detection of sequences of global predicates [1], [9], [25], and relational global predicates [24].

VII. CONCLUSION

We have discussed detection of global predicates in a distributed program. Earlier algorithms for detection of global predicates proposed by Chandy and Lamport work only for stable predicates. Our algorithms detect even unstable predicates with reasonable time, space, and message complexity.

Our experience with these algorithms has been extremely encouraging. In the current implementation, the main overhead is in the local monitor process for checking local predicates. By providing special hardware support, even this overhead can be reduced. For example, most architectures provide special hardware support such as breakpoint traps if certain location is accessed. This feature can be used to make detection of local predicates of the form (program at line x) very efficient.

We believe that algorithms presented in this paper should be part of every distributed debugger, because they incur low overhead, and are quite useful in identifying errors in the

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