Prinzipien der komplexen FM-Synthese

Matthias Kemmer

Gliederung

- Parallelschaltung
- Kaskadenschaltung
- Feedbackschaltung
- Native Instruments' FM8

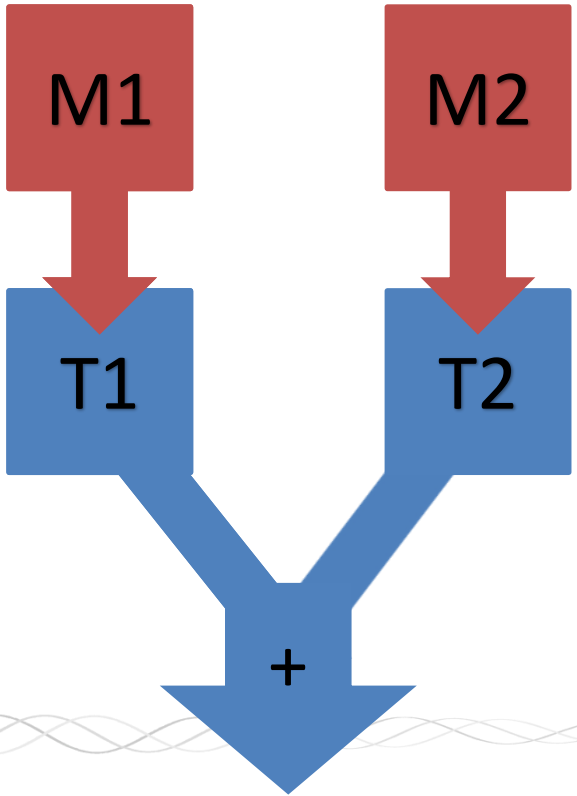
Parallelschaltung

2 Träger,
jeweils eigener Modulator

Pro Träger-Modulator-Paar gilt:

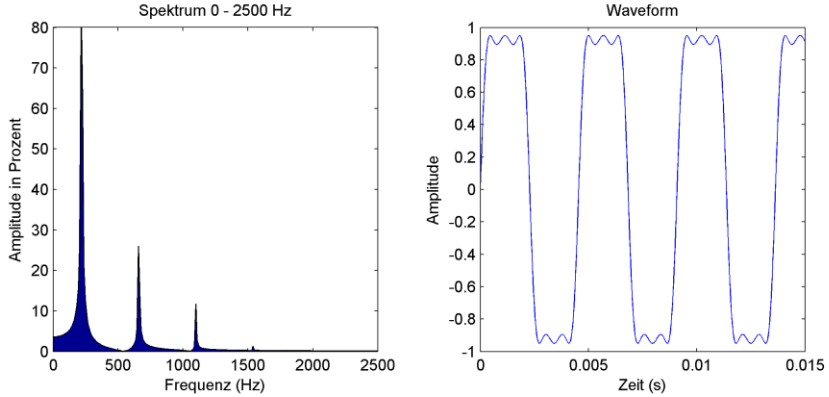
$$\sin(w_c t + I \sin(w_m t))$$

$$= \sum_{n=-\infty}^{\infty} J_n(I) \sin(w_c t + n w_m t)$$

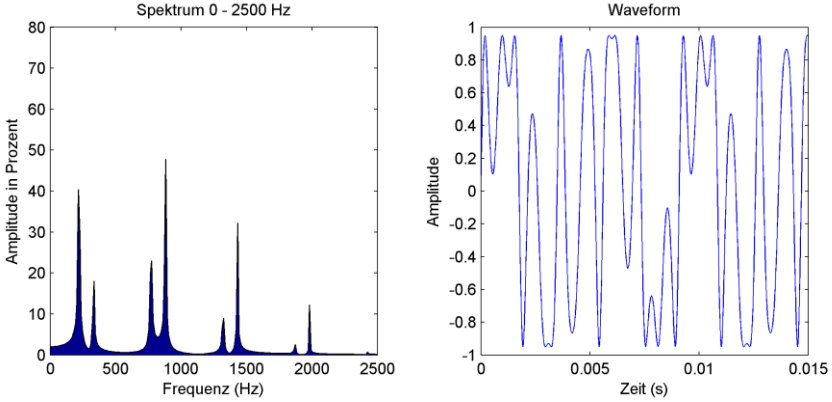


Einzel- vs. Parallelschaltung

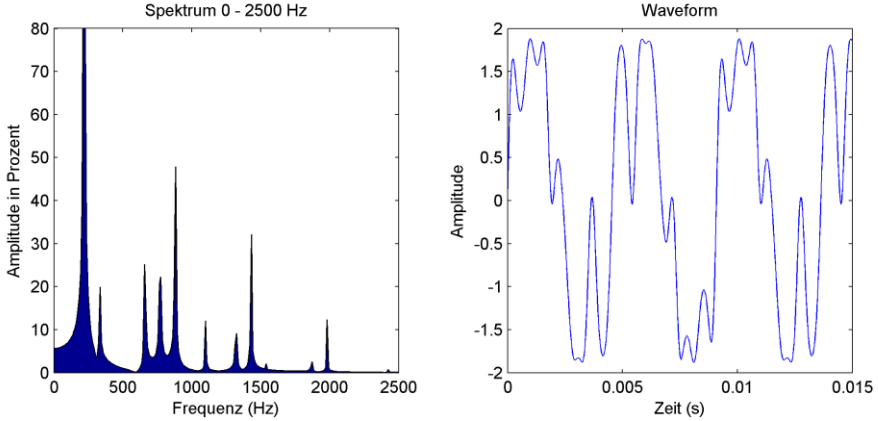
Trägerfreq. = 220; Modulatorfreq. = 440; Modindex = 1; Samplerate = 44100



Trägerfreq. = 330; Modulatorfreq. = 550; Modindex = 2; Samplerate = 44100



Parallele Träger (220,330) mit je eigenem Modulator (1-440,2-550)



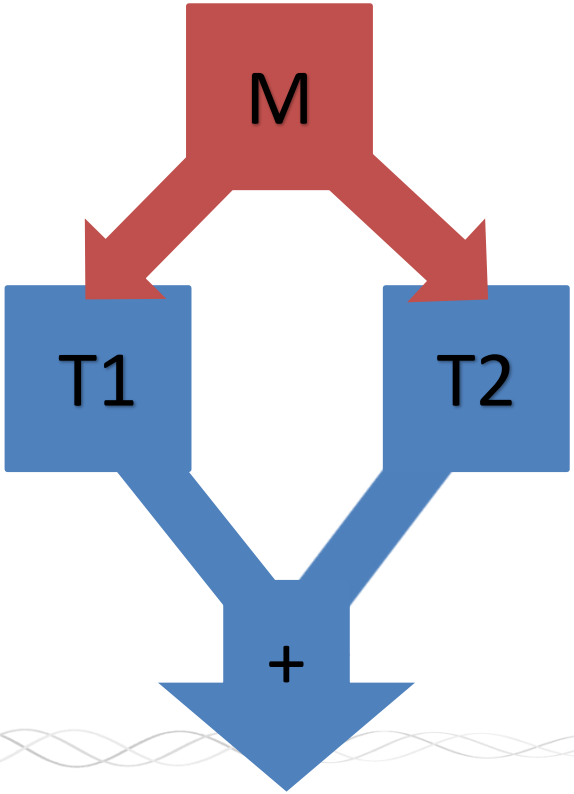
Parallelschaltung

2 Träger, **selber** Modulator

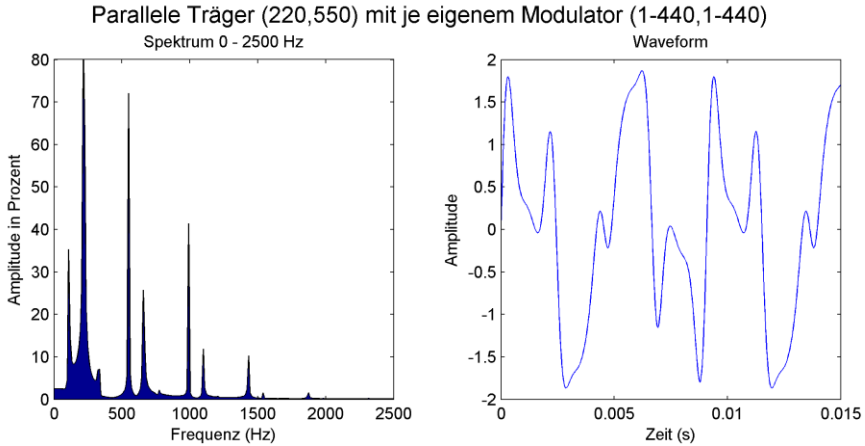
Pro Träger-Modulator-Paar gilt:

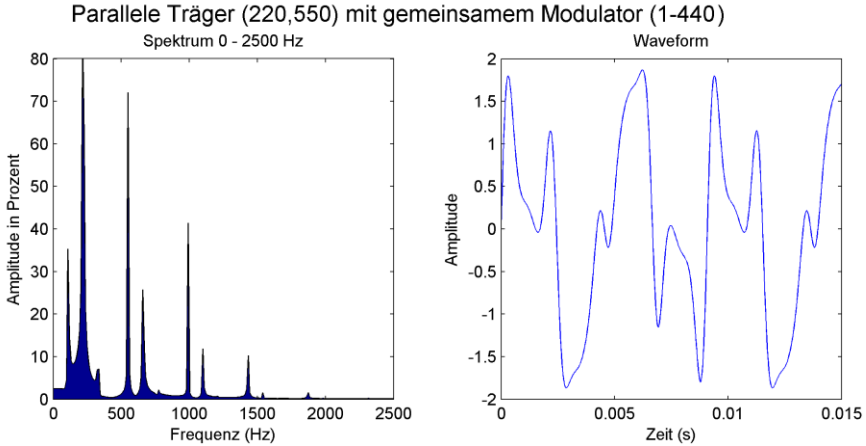
$$\sin(w_c t + I \sin(w_m t))$$

$$= \sum_{n=-\infty}^{\infty} J_n(I) \sin(w_c t + n w_m t)$$



2 Träger mit 2 Modulatoren mit gleichen Einstellungen vs. 2 Träger mit demselben Modulator



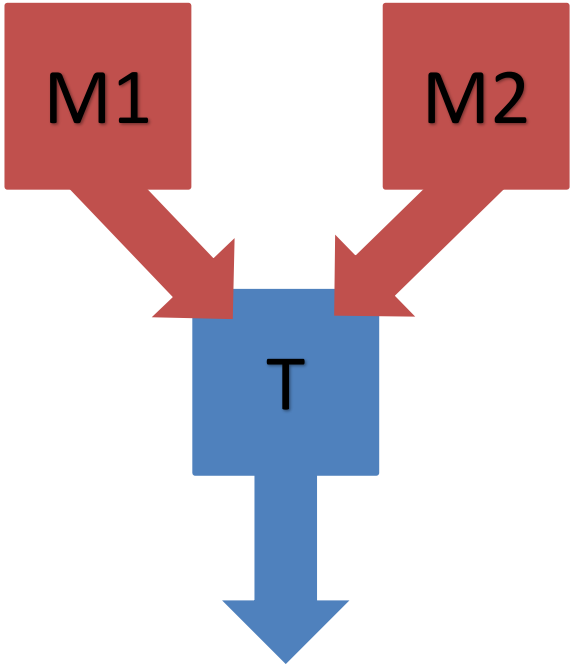


Parallelschaltung

1 Träger,
2 unabhängige Modulatoren

$$f_{FMparallel}(t) = \sin(w_c t + I_1 \sin(w_{m1} t) + I_2 \sin(w_{m2} t))$$

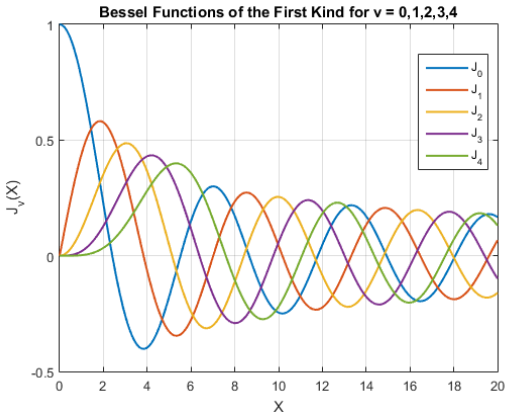
$$= \sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_i(I_1) J_k(I_2) \sin(w_c t + i w_{m1} t + k w_{m2} t)$$



Additionsterme der Seitenbänder bei 2 parallelen Modulatoren

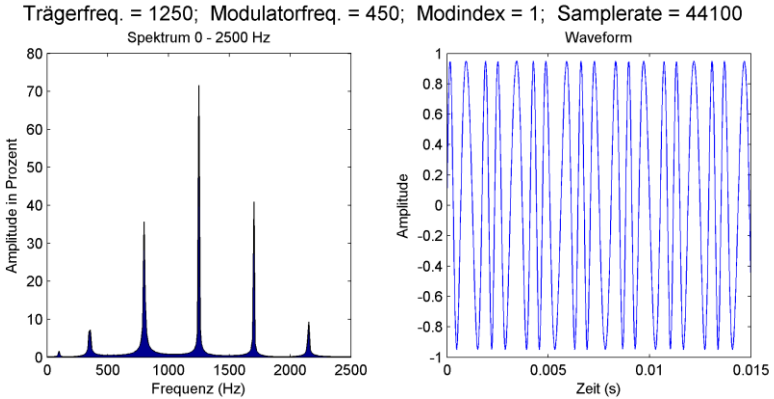
$$
\begin{aligned}
&\sin(w_c t + I_1 \sin(w_{m1} t) + I_2 \sin(w_{m2} t)) \\
&\quad = J_0(I_1) J_0(I_2) \sin(w_c t) \\
&\quad + J_0(I_1) J_1(I_2) \sin(w_c t + w_{m2} t) - J_0(I_1) J_1(I_2) \sin(w_c t - w_{m2} t) \\
&\quad + J_0(I_1) J_2(I_2) \sin(w_c t + 2w_{m2} t) + J_0(I_1) J_1(I_2) \sin(w_c t - 2w_{m2} t) \\
&\quad + J_1(I_1) J_0(I_2) \sin(w_c t + w_{m1} t) - J_1(I_1) J_0(I_2) \sin(w_c t - w_{m1} t) \\
&\quad + J_1(I_1) J_1(I_2) \sin(w_c t + w_{m1} t + w_{m2} t) - J_1(I_1) J_1(I_2) \sin(w_c t + w_{m1} t - w_{m2} t) \\
&\quad - J_1(I_1) J_1(I_2) \sin(w_c t - w_{m1} t + w_{m2} t) + J_1(I_1) J_1(I_2) \sin(w_c t - w_{m1} t - w_{m2} t) \\
&\quad + J_1(I_1) J_2(I_2) \sin(w_c t + w_{m1} t + 2w_{m2} t) + J_1(I_1) J_2(I_2) \sin(w_c t + w_{m1} t - 2w_{m2} t) \\
&\quad - J_1(I_1) J_2(I_2) \sin(w_c t - w_{m1} t + 2w_{m2} t) - J_1(I_1) J_2(I_2) \sin(w_c t - w_{m1} t - 2w_{m2} t) \\
&\quad + J_2(I_1) J_0(I_2) \sin(w_c t + 2w_{m1} t) + J_2(I_1) J_0(I_2) \sin(w_c t - 2w_{m1} t) \\
&\quad + J_2(I_1) J_1(I_2) \sin(w_c t + 2w_{m1} t + w_{m2} t) + J_2(I_1) J_1(I_2) \sin(w_c t + 2w_{m1} t - w_{m2} t) \\
&\quad + J_2(I_1) J_1(I_2) \sin(w_c t - 2w_{m1} t + w_{m2} t) - J_2(I_1) J_1(I_2) \sin(w_c t - 2w_{m1} t - w_{m2} t) \\
&\quad + J_2(I_1) J_2(I_2) \sin(2w_c t + 2w_{m1} t + 2w_{m2} t) + J_2(I_1) J_2(I_2) \sin(2w_c t + 2w_{m1} t - 2w_{m2} t) \\
&\quad + J_2(I_1) J_2(I_2) \sin(2w_c t - 2w_{m1} t + 2w_{m2} t) + J_2(I_1) J_2(I_2) \sin(2w_c t - 2w_{m1} t - 2w_{m2} t) \\
&\quad + \ldots
\end{aligned}
$$

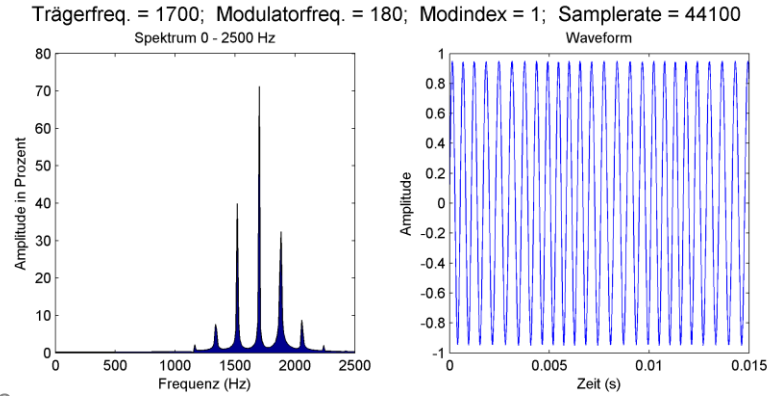
$$J_{-n}(x) = (-1)^n J_n(x)$$

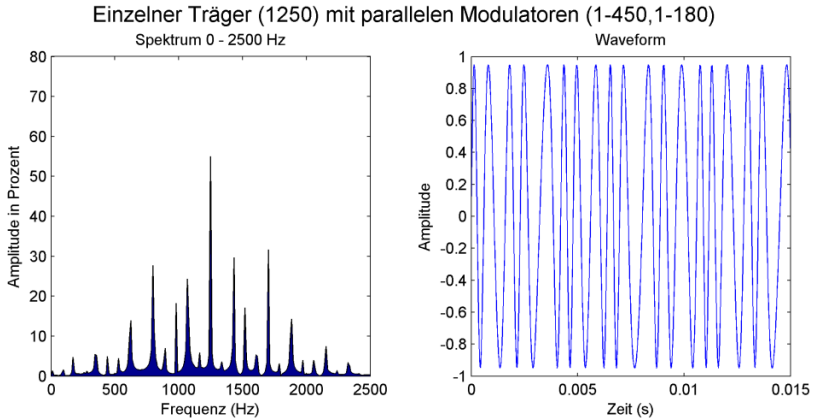


Bildquelle:
http://de.mathworks.com/help/matlab/ref/besselj.html

Spektrum der Parallelschaltung für 1 T, 2 M



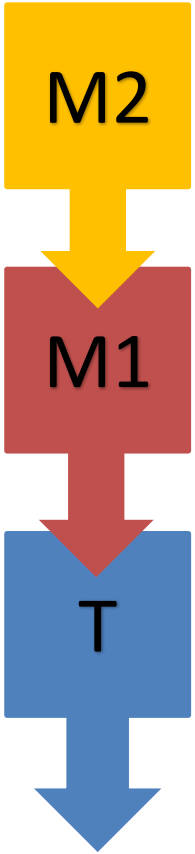




Kaskadenschaltung

$$f_{FMkaskade}(t) = \sin(w_c t + I_1 \sin(w_{m1} t + I_2 \sin(w_{m2} t)))$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_n(I_1) J_k(\mathbf{n} I_2) \sin(w_c t + n w_{m1} t + k w_{m2} t)$$



Formel Spektrum Parallelschaltung vs. Formel Spektrum Kaskadenschaltung

Parallel:

$$\sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_i(I_1) J_k(I_2) \sin(w_c t + i w_{m1} t + k w_{m2} t)$$

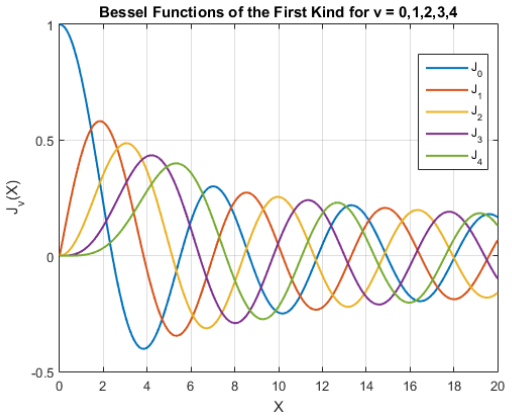
Kaskade:

$$\sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} J_n(I_1) J_k(\mathbf{n} I_2) \sin(w_c t + n w_{m1} t + k w_{m2} t)$$

Additionsterme der Seitenbänder bei 2 Modulatoren in Reihe

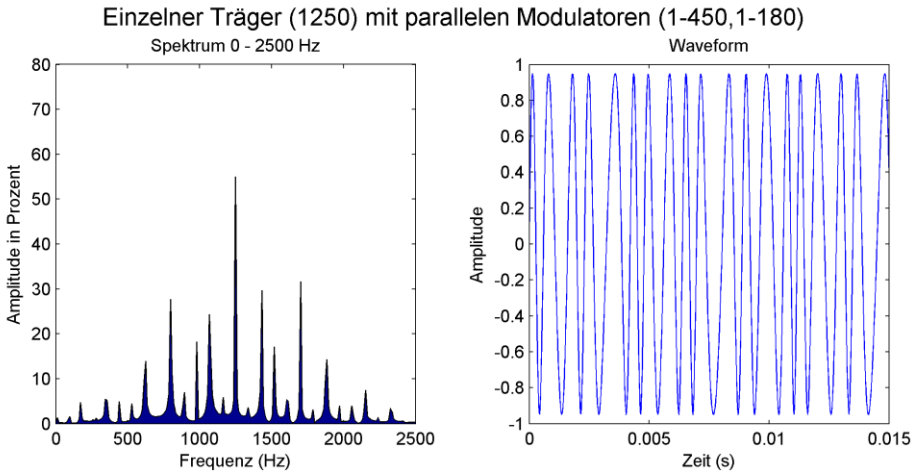
$$
\begin{aligned}
&\sin(w_c t + I_1 \sin(w_{m1} t) + I_2 \sin(w_{m2} t)) \\
&\quad = J_0(I_1) J_0(\mathbf{0} I_2) \sin(w_c t) \\
&\quad + J_0(I_1) J_1(\mathbf{0} I_2) \sin(w_c t + w_{m2} t) - J_0(I_1) J_1(I_2) \sin(w_c t - w_{m2} t) \\
&\quad + J_0(I_1) J_2(\mathbf{0} I_2) \sin(w_c t + 2 w_{m2} t) + J_0(I_1) J_1(I_2) \sin(w_c t - 2 w_{m2} t) \\
&\quad + J_1(I_1) J_0(\mathbf{1} I_2) \sin(w_c t + w_{m1} t) - J_1(I_1) J_0(I_2) \sin(w_c t - w_{m1} t) \\
&\quad + J_1(I_1) J_1(\mathbf{1} I_2) \sin(w_c t + w_{m1} t + w_{m2} t) - J_1(I_1) J_1(I_2) \sin(w_c t + w_{m1} t - w_{m2} t) \\
&\quad - J_1(I_1) J_1(\mathbf{1} I_2) \sin(w_c t - w_{m1} t + w_{m2} t) + J_1(I_1) J_1(I_2) \sin(w_c t - w_{m1} t - w_{m2} t) \\
&\quad + J_1(I_1) J_2(\mathbf{1} I_2) \sin(w_c t + w_{m1} t + 2 w_{m2} t) + J_1(I_1) J_2(I_2) \sin(w_c t + w_{m1} t - 2 w_{m2} t) \\
&\quad - J_1(I_1) J_2(\mathbf{1} I_2) \sin(w_c t - w_{m1} t + 2 w_{m2} t) - J_1(I_1) J_2(I_2) \sin(w_c t - w_{m1} t - 2 w_{m2} t) \\
&\quad + J_2(I_1) J_0(\mathbf{2} I_2) \sin(w_c t + 2 w_{m1} t) + J_2(I_1) J_0(I_2) \sin(w_c t - 2 w_{m1} t) \\
&\quad + J_2(I_1) J_1(\mathbf{2} I_2) \sin(w_c t + 2 w_{m1} t + w_{m2} t) + J_2(I_1) J_1(I_2) \sin(w_c t + 2 w_{m1} t - w_{m2} t) \\
&\quad + J_2(I_1) J_1(\mathbf{2} I_2) \sin(w_c t - 2 w_{m1} t + w_{m2} t) - J_2(I_1) J_1(I_2) \sin(w_c t - 2 w_{m1} t - w_{m2} t) \\
&\quad + J_2(I_1) J_2(\mathbf{2} I_2) \sin(2 w_c t + 2 w_{m1} t + 2 w_{m2} t) + J_2(I_1) J_2(I_2) \sin(2 w_c t + 2 w_{m1} t - 2 w_{m2} t) \\
&\quad + J_2(I_1) J_2(\mathbf{2} I_2) \sin(2 w_c t - 2 w_{m1} t + 2 w_{m2} t) + J_2(I_1) J_2(I_2) \sin(2 w_c t - 2 w_{m1} t - 2 w_{m2} t) \\
&\quad + \ldots
\end{aligned}
$$

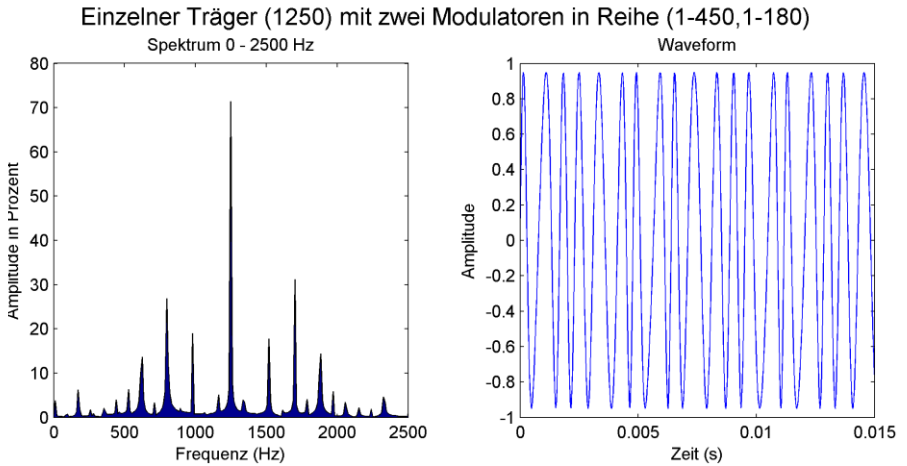
$$J_{-n}(x) = (-1)^n J_n(x)$$



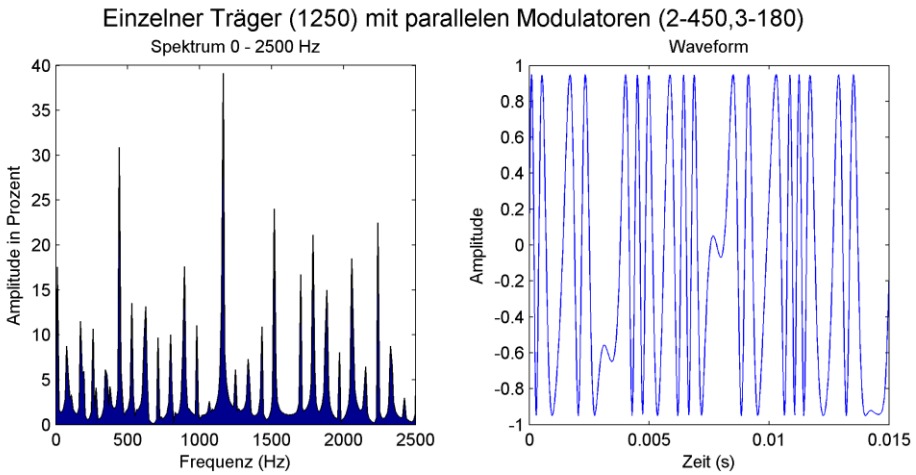
Bildquelle:
http://de.mathworks.com/help/matlab/ref/besselj.html

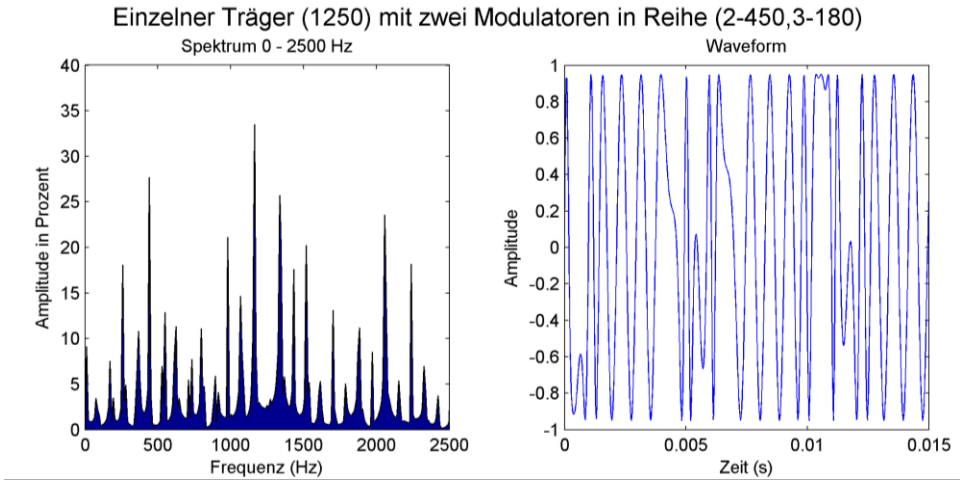
Vergleich: Parallel- und Kaskadenschaltung





Vergleich mit Modulationsindizes 2 und 3

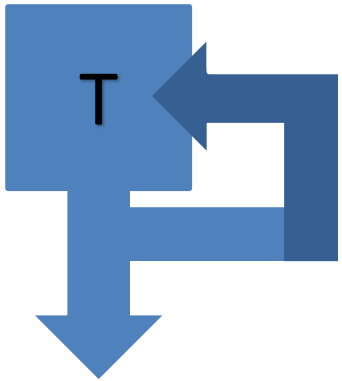




Feedbackschaltung

$$f_{FMfeedback}(t_n) = sin(w_c t_n + I f_{FMfeedback}(t_{n-1}))$$

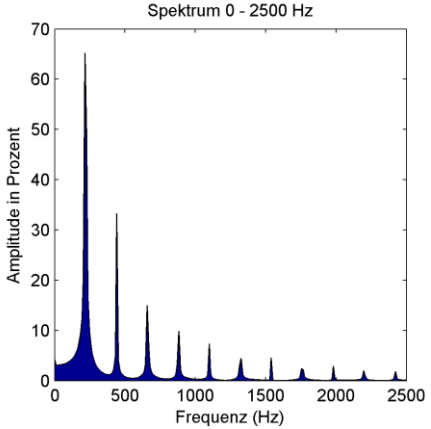
$$\text{mit} \quad f_{FMfeedback}(t_0) = 0$$

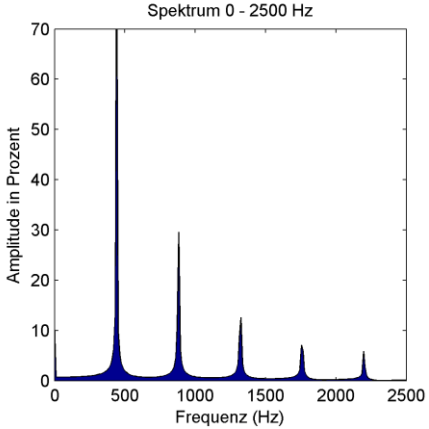


$$f_{FMfeedback}(t) = \sum_{n=1}^{\infty} \frac{2}{nI} J_n(nI) \sin(n w_c t)$$

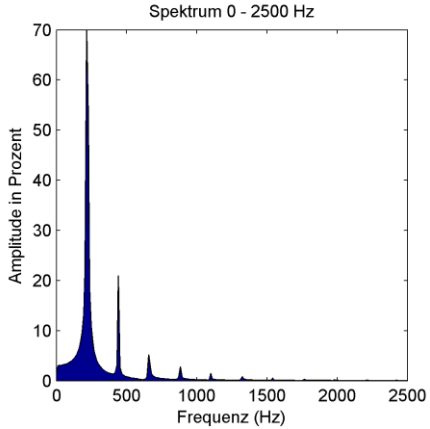
Spektrum der Feedbackschaltung

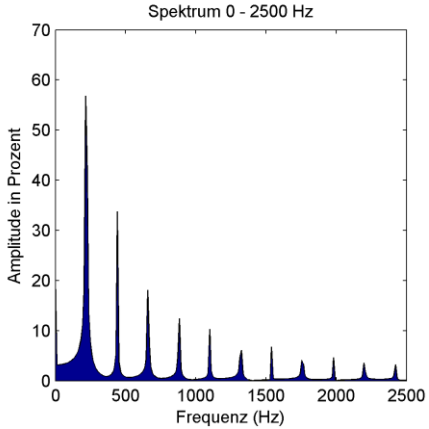
Träger mit Frequenz 220 und 440 rückgekoppelt mit Modulationsindices 1 und 1





Träger mit Frequenz 220 und 220 rückgekoppelt mit Modulationsindices 0,5 und 1,5





Praxisbeispiel: Modulationsmatrix FM8

Fazit

- Komplexe FM-Synthese ist extrem mächtig
- Resultierendes Spektrum schwer abschätzbar
- Durch moderne Synths wie FM8 *Trial & Error* einfach

Quellen

[AS64] M. Abramowitz and I.A. Stegun. *Handbook of Mathematical Functions: With Formulas, Graphs, and Mathematical Tables.* Applied mathematics series. Dover Publications, 1964. ISBN 0-486-61272-4.

[CB86] John M. Chowning and David Bristow. *FM Theory & Applications - By Musicians for Musicians.* Yamaha Music Foundation, 1986. ISBN 4-636-17482-8.

[Cho73] John M. Chowning. The synthesis of complex audio spectra by means of frequency modulation. *Journal of the Audio Engineering Society*, pages 526–534, 1973.

[Sch] B. Schottstaedt. Introduction to fm. https://ccrma.stanford.edu/software/snd/snd/fm.html.

[Sch77] B. Schottstaedt. The simulation of natural instrument tones using frequency modulation with a complex modulating wave. *Computer Music Journal*, 1(4):pp. 46–50, 1977.

Vielen Dank für eure Aufmerksamkeit!