Computational Assignment 1

MEGN 471: Heat Transfer - Spring 2023

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Introduction

As an introduction to computational heat transfer analysis, the team observed a scenario in which a 2 m ×1 m window would be placed into a building wall. The scope of the system analyzed was only the window itself, neglecting heat loss by surroundings (walls, sealant, and window frame). The conditions for the experiment included a hot internal temperature (T_i = 21 °C) with a constant convection coefficient (h_i), and a cold outside temperature (T_o = -2 °C) with constant irradiation from the sun (G). Experimental variables included an outside convection coefficient (h_0), number of panes in window (N), and window geometry (2m^2 vs .5m^2). Other pertinent values, such as thermal radiation loss, (E), thermal conductivity for glass (k_g), and air (k_a) are assumed to be constant or calculated at runtime of the code. The objective of analyzing these conditions is to determine how they impact heat flux (energy lost from inside) and in turn the cost of heating a home per window during the winter months.

Methodology

Observing the system setup, it can be noted that no heat transfer is considered for the surroundings or internal temperature gradients. As such, the analysis was able to be effectively modeled as a 1D system for heat transfer. This system is depicted below in Figure 1, where a baseline case of 2 window panes was used (this scenario will be the standard for calculations and diagrams in the report).

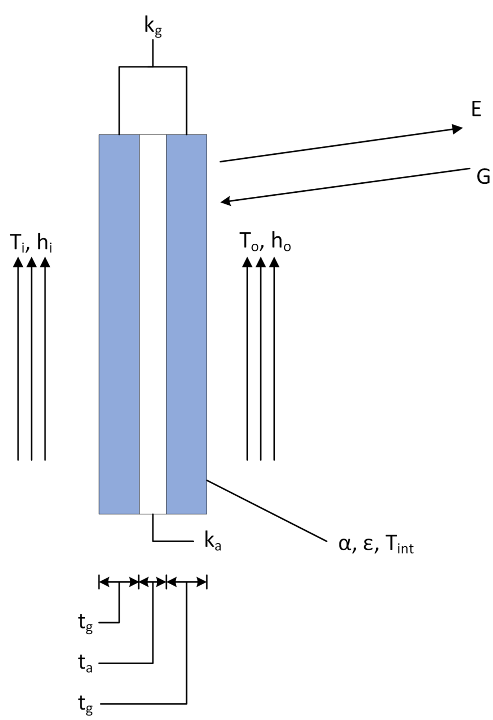

Figure 1: Side View of Window Configuration with Two Panes (N = 2)

Provided the 1D nature of the problem, a thermal circuit diagram can be used to efficiently depict and calculate modes of heat transfer as shown in Figure 2.

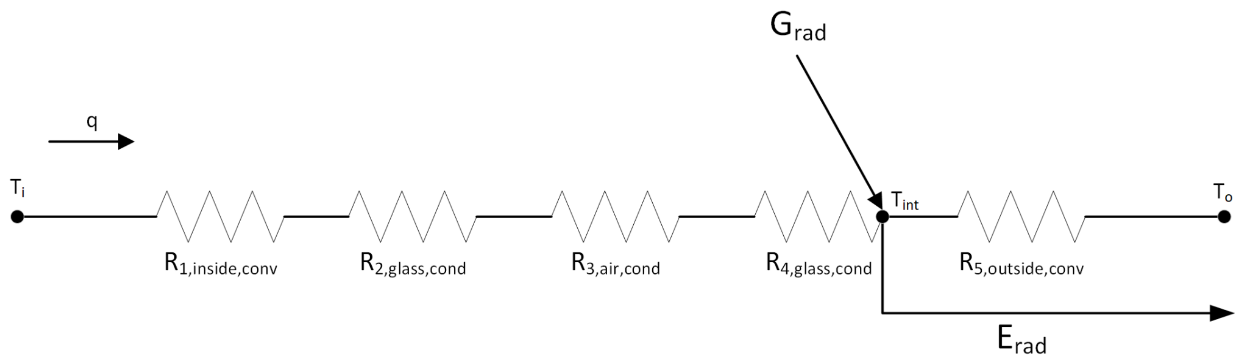

Figure 2: Thermal Circuit Diagram for Window with Two Panes (N = 2)

Provided the simplified diagram, solving for key values such as T_{int} can be done using a system of equations derived from energy balance and thermal resistance equations. The thermal resistances of convection and conduction can be determined given Equation 1 and Equation 2 presented below.

$$R_{conv} = \frac{1}{hA}$$

Equation 1: Thermal Resistance Equation for Convection

$$R_{cond} = \frac{t}{kA}$$

Equation 2: Thermal Resistance Equation for Conduction

Equation 1 can be applied to find the thermal resistance for convection given h, the heat transfer coefficient, and A, the cross-sectional area of the glass across which heat is transferred. The value of h varied based on whether the convection was occurring inside or outside, with the heat transfer coefficient inside the room being 10 W/m^2 and the heat transfer coefficient outside the room ranging from 10 to 100 W/m^2. The value of A is equal to 2 m^2. Equation 2 can be used to find the thermal resistance of conduction in each individual pane of glass as well as the air gaps between panes given t, the thickness of each pane of glass, k, the thermal conductivity of glass, and A, the cross-sectional area of the glass across which heat is transferred. The thickness of the glass is given as 0.007 m, the thermal conductivity of glass is given as 1.5 W/m*K, and the area is 2 m^2 as it was in Equation 1. The thickness of the air gaps between the panes is also 0.007 m, the area is 2 m^2, and the coefficient of thermal conductivity for air is 0.0245 W/m*K. The thermal resistance found in a single glass pane can be multiplied by the number of panes and added to the sum of the thermal resistance from the air gaps to calculate the total cumulative thermal resistance in the window due to conduction. The thermal resistances for convection and conduction can then be used in the overall heat transfer equation for the system to solve for the surface temperature of the outermost pane of glass.

Results and Discussion

The primary two variables of this simulation are the number of glass panes and air pockets and the outdoor convection coefficient. Changing these two variables has a direct impact on the heat transfer rate from inside the house to the outdoors.

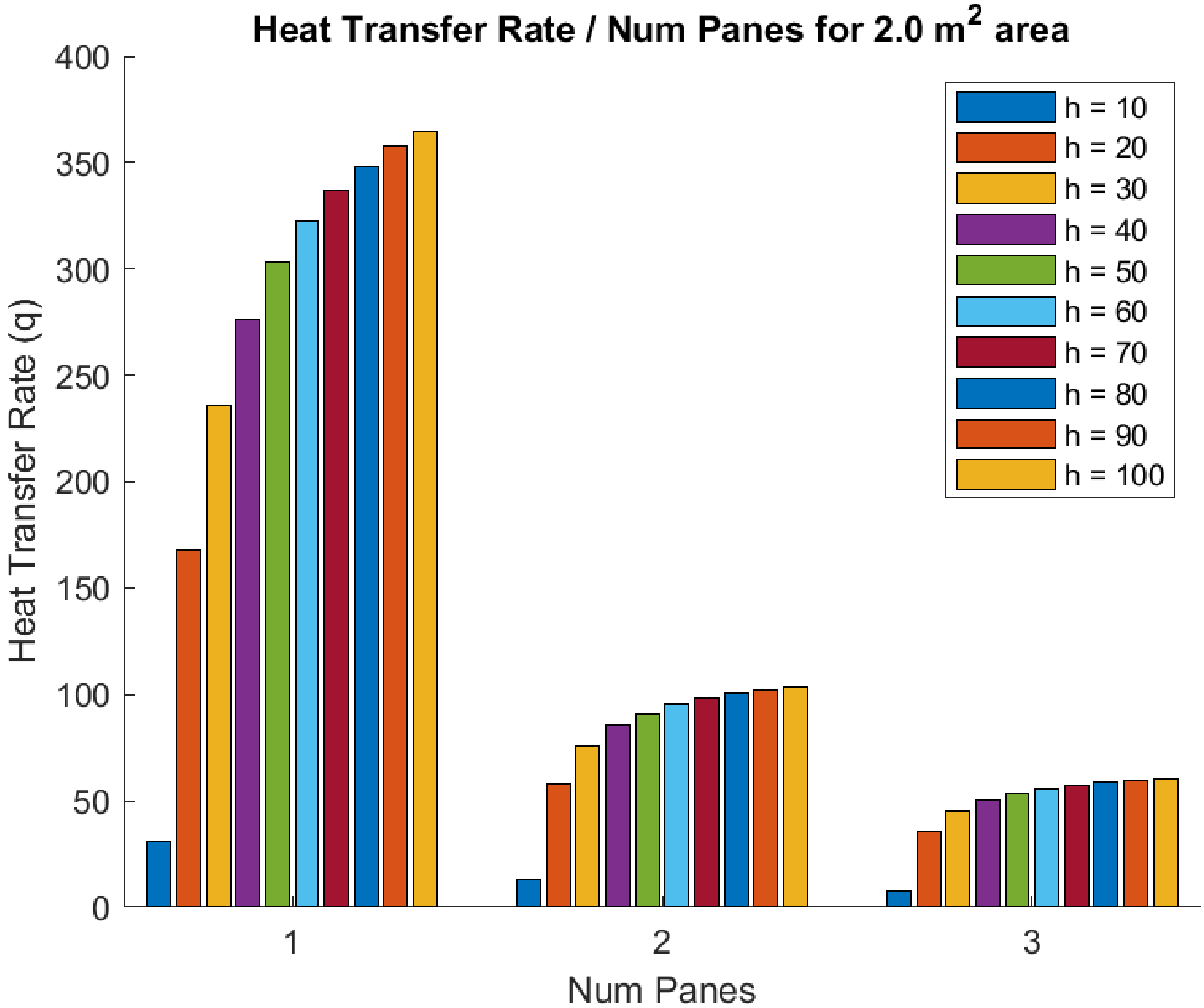

Figure 3: Heat Transfer Rate (W) with varied Pane Numbers and Convection Coefficients (h)

The figure above shows that the heat transfer rate decreases as the pane number increases and the convection coefficient decreases. The impact of the panes follows the basic concept of thermal resistance and insulation. By adding panes together, not only does the overall glass thickness increase (which would improve the insulation of the window), but layers of air are added between the panes. Air is a good insulator, as can be seen by its low conduction coefficient of .0245 W/m*K, so the addition of air layers provides a large increase to the thermal resistance of the system. The result is a lower overall temperature gradient/slope through the window which is indicative of a lower heat transfer rate.

The impact of the outdoor convection coefficient is also shown on Figure 3. As can be seen from the single pane data, the heat transfer rate increases as the outdoor convection coefficient increases. This would be straight forward if the interface temperature was held constant. However, in this case, the increased convection heat transfer rate that would result from this change must be balanced with the internal convection and conduction heat transfer rate so the temperature gradient across the window must adjust to balance the equations. In this case, the interface temperature will drop so that both equations converge, but due to the greater convection coefficient value outdoors, the decrease in interface temperature will lower the outdoor convection heat transfer rate equation at a greater rate compared to the indoors equation. This will cause the equation to favor a higher general heat transfer rate through the window whilst also resulting in the aforementioned interface temperature drop which can be seen in the figure below. In sum, as the convection coefficient increases the interface temperature asymptotically approaches the outdoor temperature (T_o).

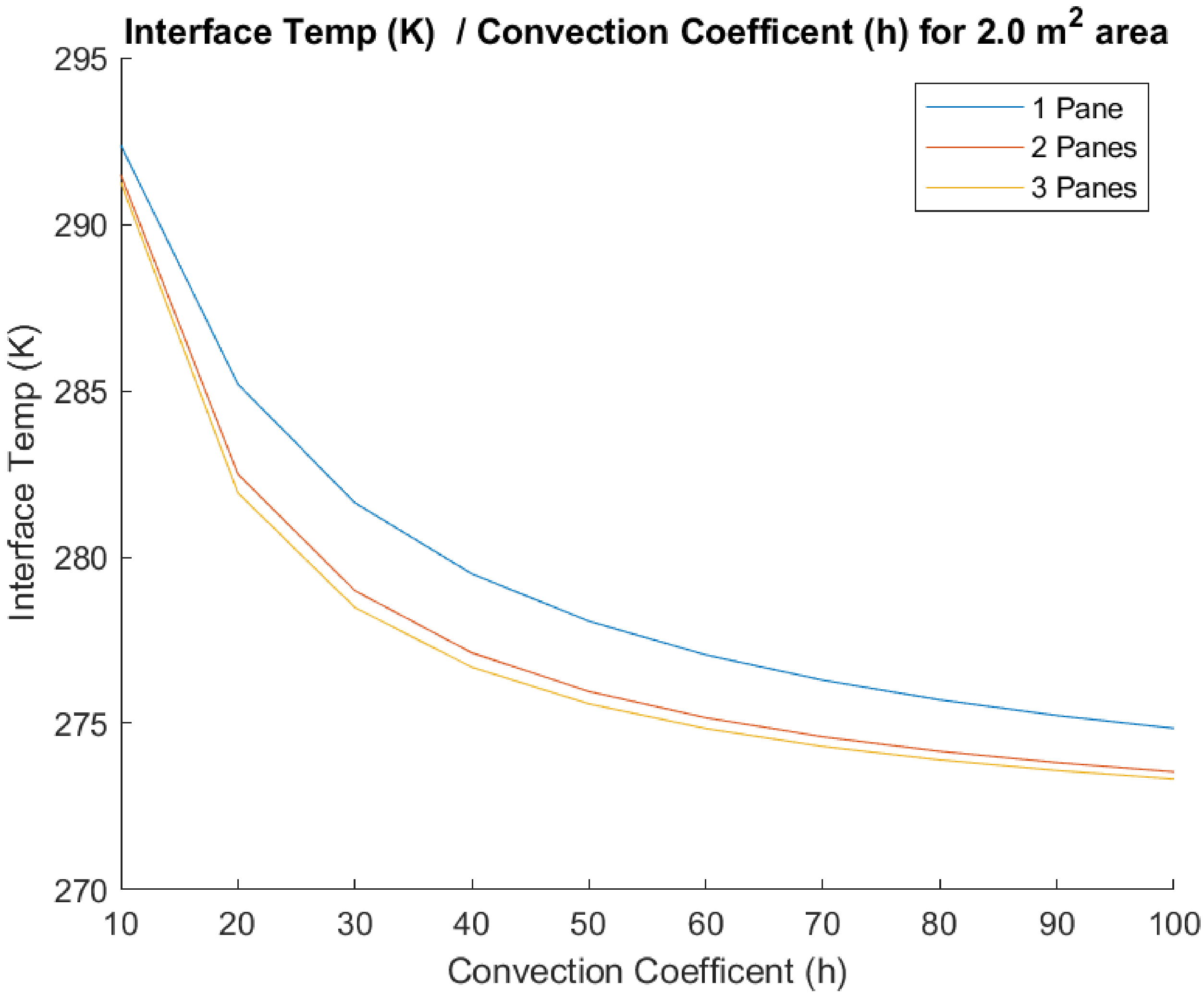

Figure 4: Interface Temperature vs Convection Coefficient

The cost analysis of the window takes into account the heating costs due to heat losses as well as the capital costs associated with the installation of multiple panes. The results are shown in the figure below.

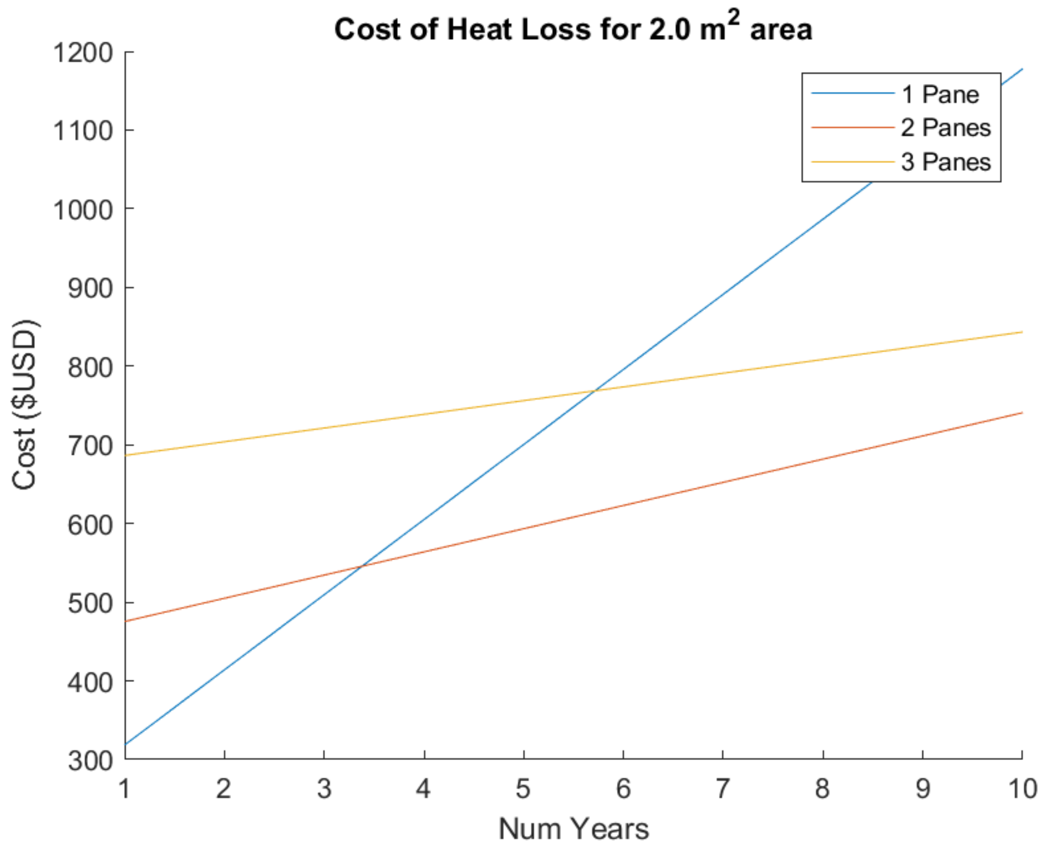

Figure 5: Summed Cost of Each Pane Solution vs Years of Use

This data shows that a single pane solution is viable for approximately the first 3 years but afterwards a 2 pane solution is the best option. This shift is due to the doubled capital cost of installing two panes finally being overcome by the heat loss savings associated with a double pane and air gap window. The 3 pane solution is unable to overcome its capital costs in the 10 year time frame as the additional heat loss that it prevents over the 2 pane solution is not significant enough to save money. This would change if the time frame was extended as the slope of the 3 pane graph is less than that of the 2 pane graph. The essential takeaway from this data is that the addition of an air gap is significant in increasing the thermal resistance of the system but following additions may not be necessary in the short term due to the compounding capital costs associated with adding multiple panes.

This system had a combination of convection and thermal radiation losses (and gains) on its outward facing side. The figure below shows the ratio of these two losses.

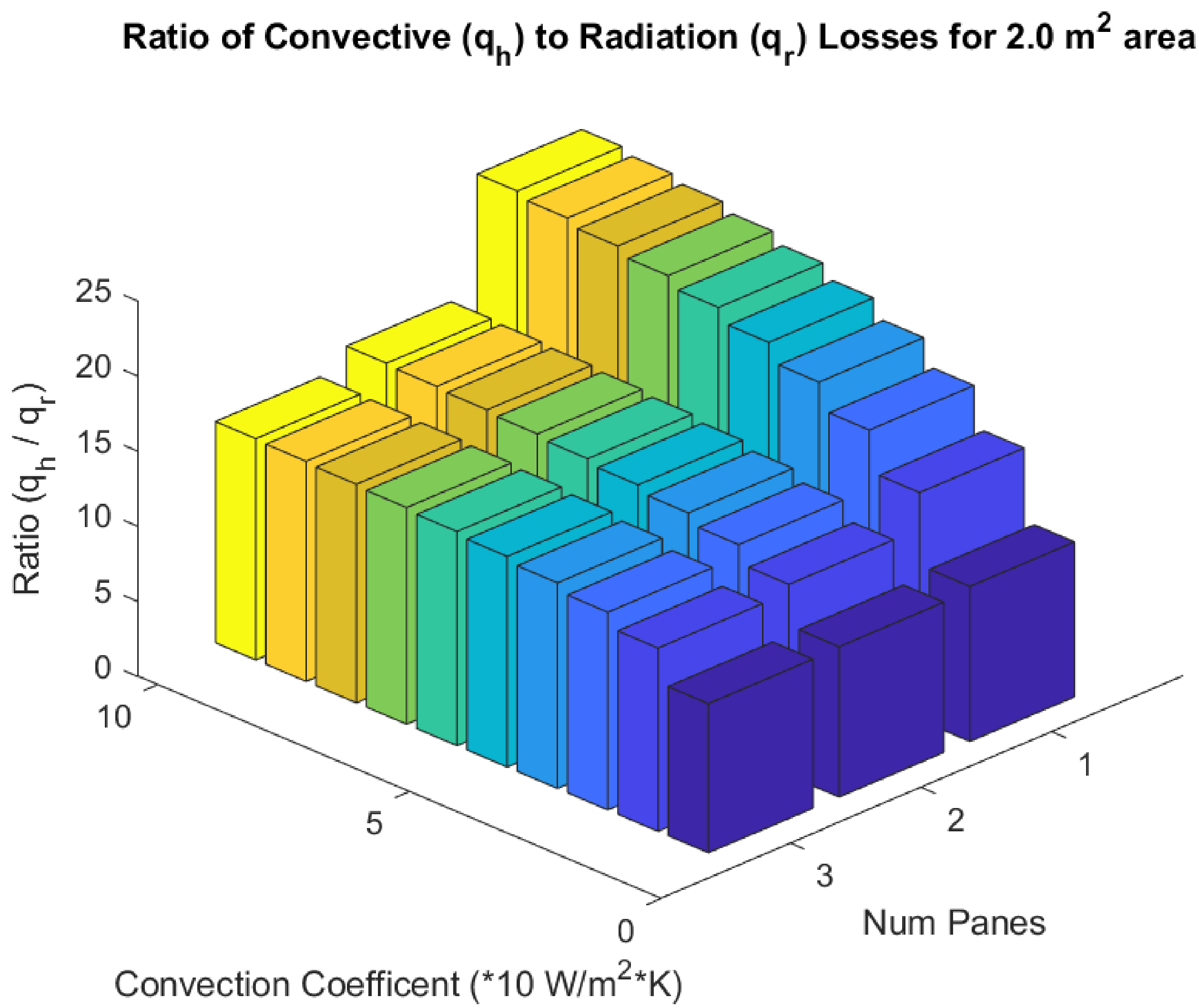

Figure 6: Ratio of Convection over Radiation Heat Losses

This figure shows that no matter the number of panes or convection coefficient, the convection heat losses significantly outweigh the radiation heat losses. This is due to the radiation heat loss being dependent upon the interface temperature being high. As was established earlier, the interface temperature is at or below room temperature so the radiation losses are going to be very small compared to the convection heat losses. This makes modeling radiation essentially irrelevant as the magnitude of losses through the radiation pathway can be assumed as effectively zero.

The process of shrinking the window works to change the overall heat transfer of the system. The flux of the system should remain close to the same but the shrunk area will result in less heat loss to the outdoors. The result is a shift in the economics of the window design which can be seen in the figure below.

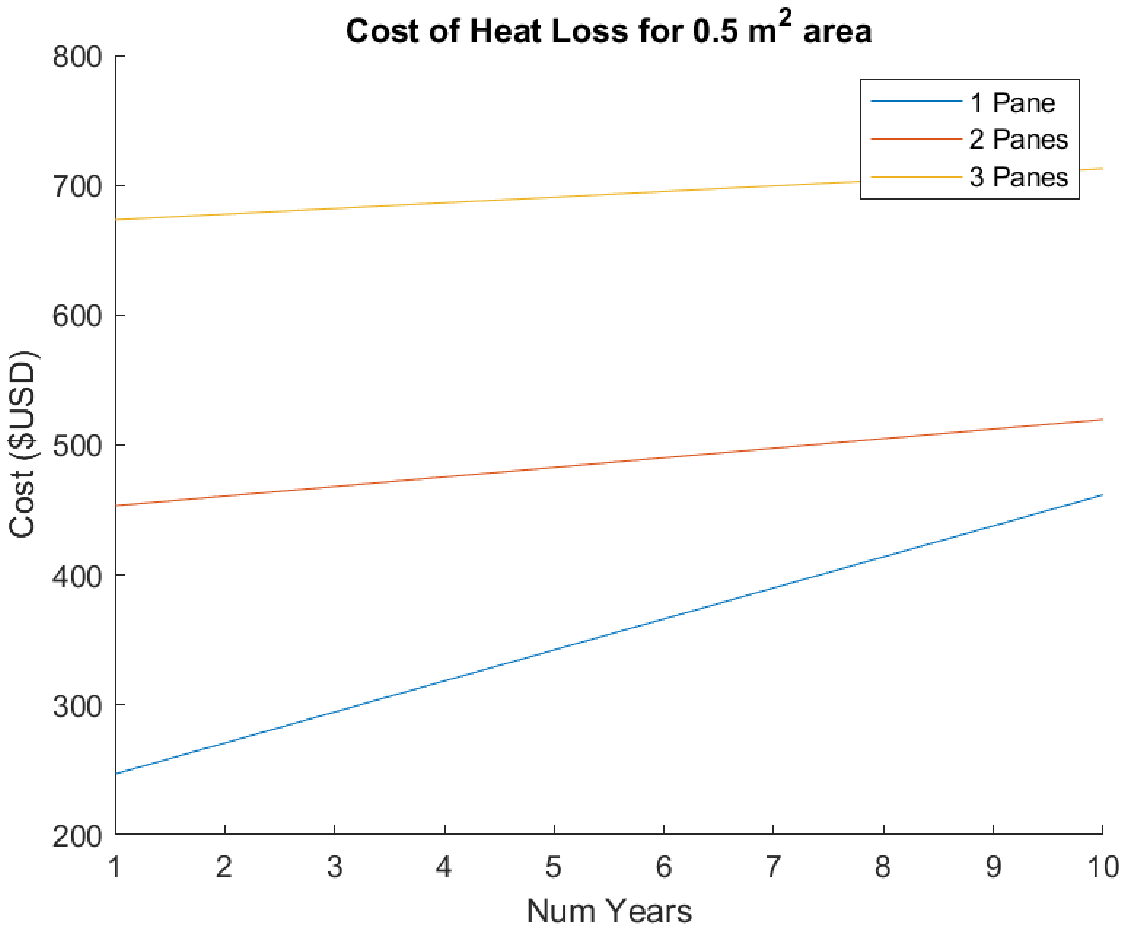

Figure 7: Summed Cost of Each Solution for Smaller Window

Due to the lower heat transfer rates of the smaller window, the single pane solution is far more viable (assuming the cost per pane is still the same). This is due to the heat loss costs being lower as a result of the shrunk area making the capital cost associated with the 2+ pane solution take longer to overcome.

Conclusion

In conclusion, increasing the number of panes of glass lowers the heat flux through the window in an exponentially decaying relationship. Due to this relationship, the amount of heat that escapes through a window with three panes is lower than that of a window with two panes, but is not low enough to compensate for the cost of adding an additional pane when considered over a 10-year timeframe. When considered over a timeframe less than three years, however, a single-pane window provides a better return on investment. Furthermore, varying the convection coefficient has a logarithmic impact on interface temperature and heat flux through the system. This implies that annual costs could be substantially reduced if the convection coefficient is able to be reduced below 40 W/m^2*K, but will not rise significantly if the convection coefficient increases past this point.

Group Contribution

Michael Allen:

Michael wrote the MATLAB code to calculate and plot the data used in this study, and also wrote the introduction and part of the methodology.

Cullen Hirstius:

Cullen provided the descriptive figures used in the study in addition to direction regarding calculations. He also wrote the results and discussion section.

Hayden Payne:

Hayden wrote the conclusion and part of the methodology in addition to assisting in MATLAB debugging and calculation/system setup.

Appendix A

Figures generated by MATLAB. Source code posted on GitHub:
https://github.com/MC-Meesh/Computational_Heat_Transfer_1

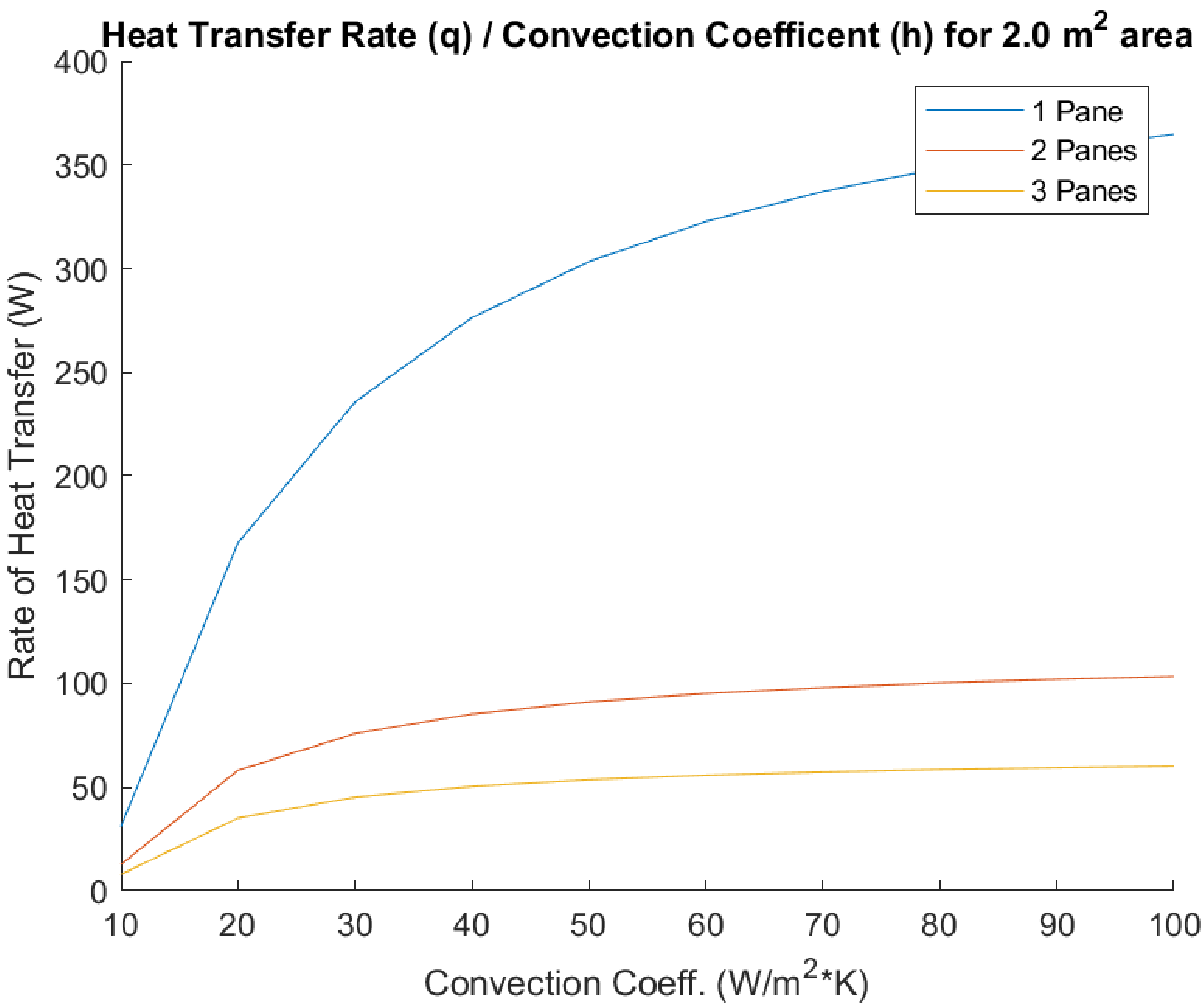

Figure A1: Heat Transfer Rate vs Convection Coefficient

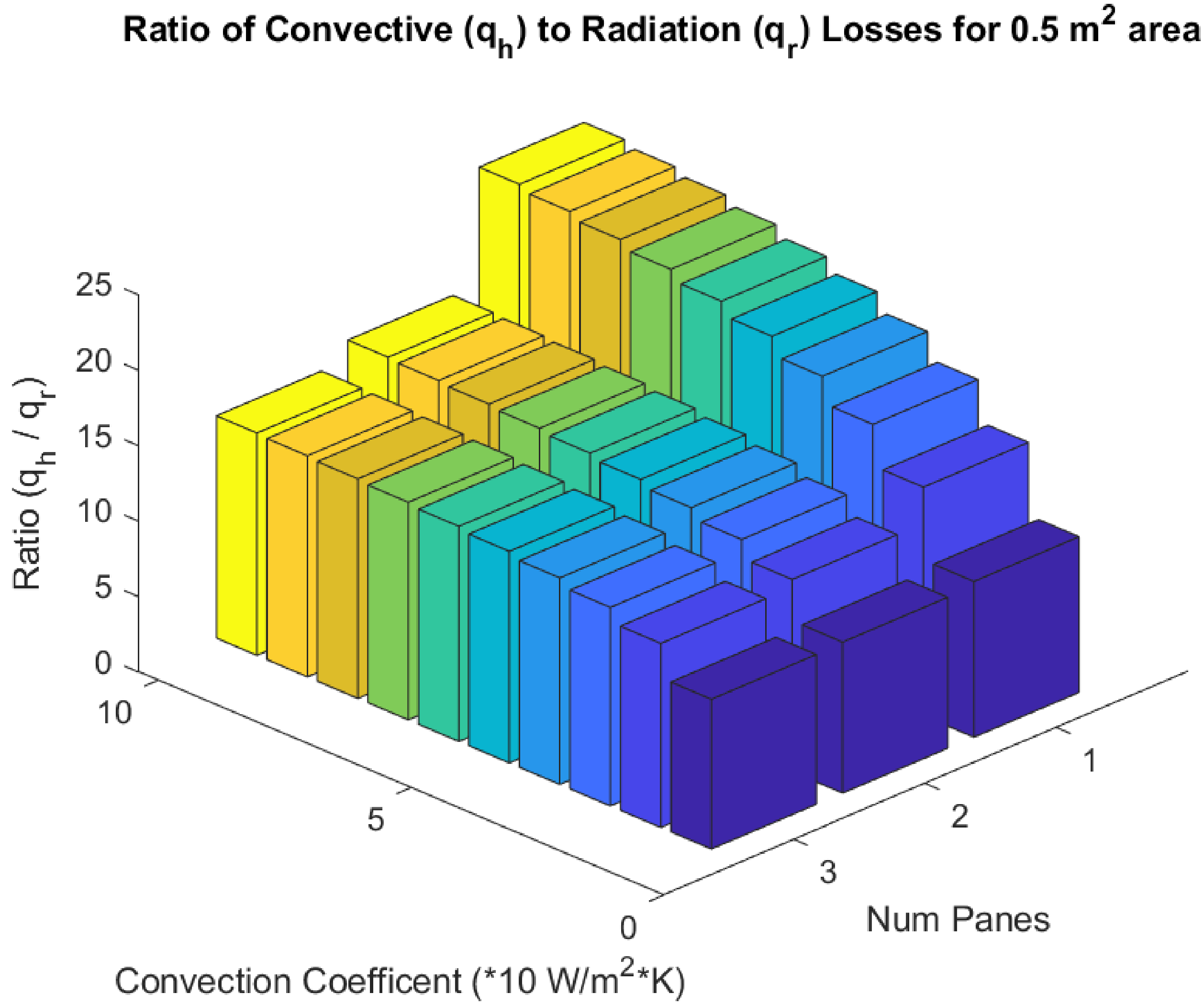

Figure A2: Ratio of Losses due to Convection Versus Losses due to Radiation for Smaller Area

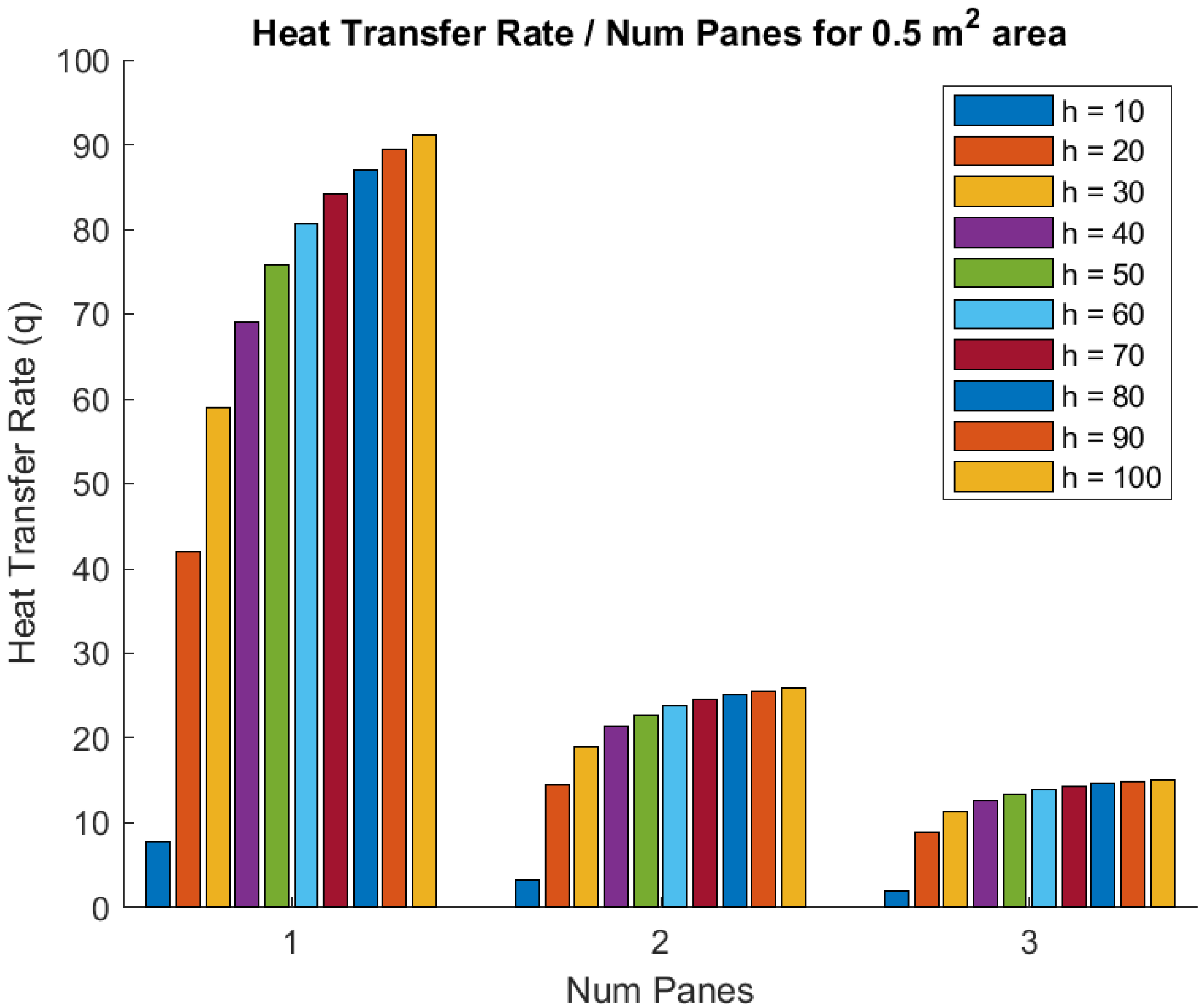

Figure A3: Small Area Heat Transfer Rate (W) with Varied Pane Numbers and Convection Coefficients (h)

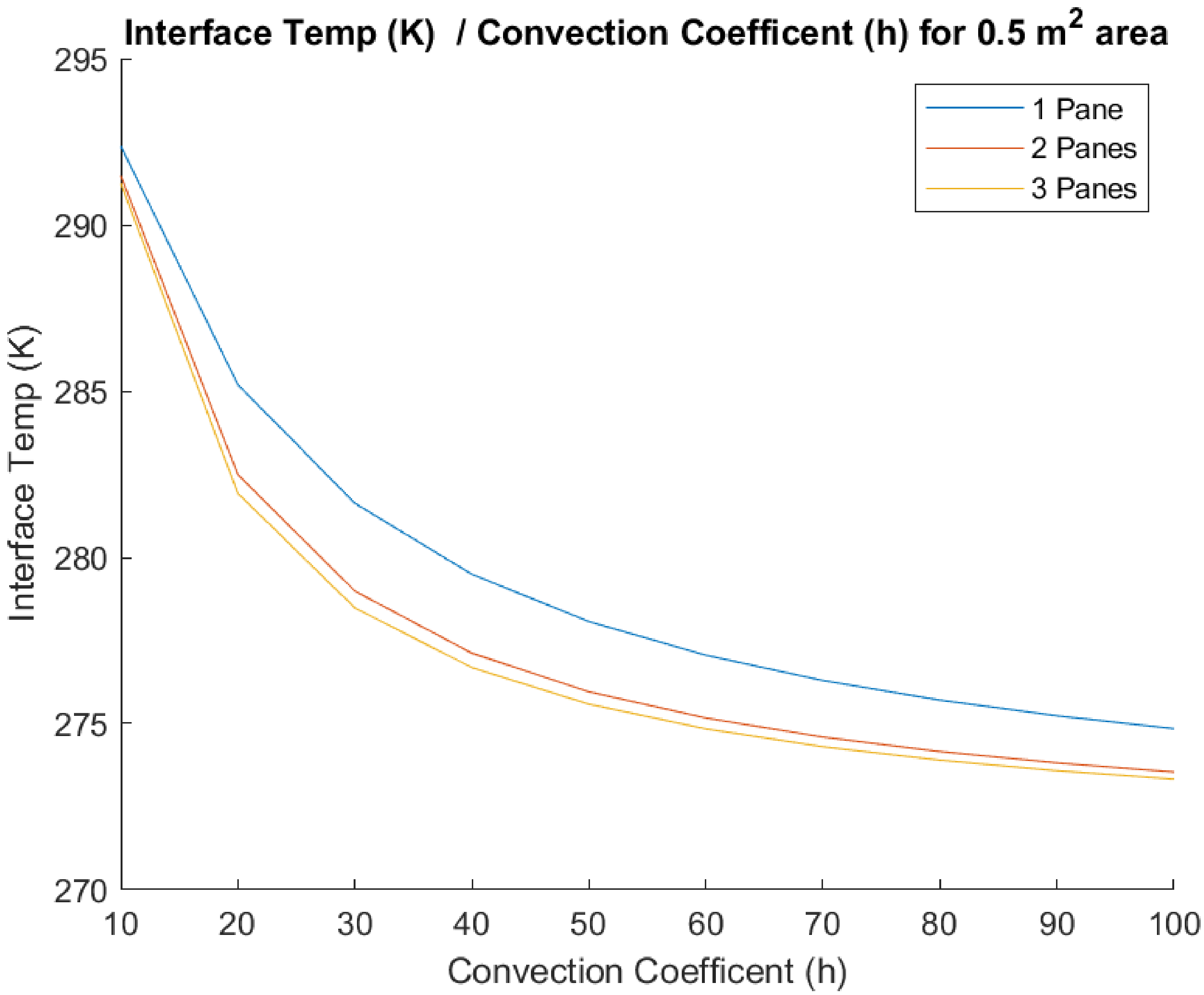

Figure A4: Interface Temperature vs Convection Coefficient for Smaller Area

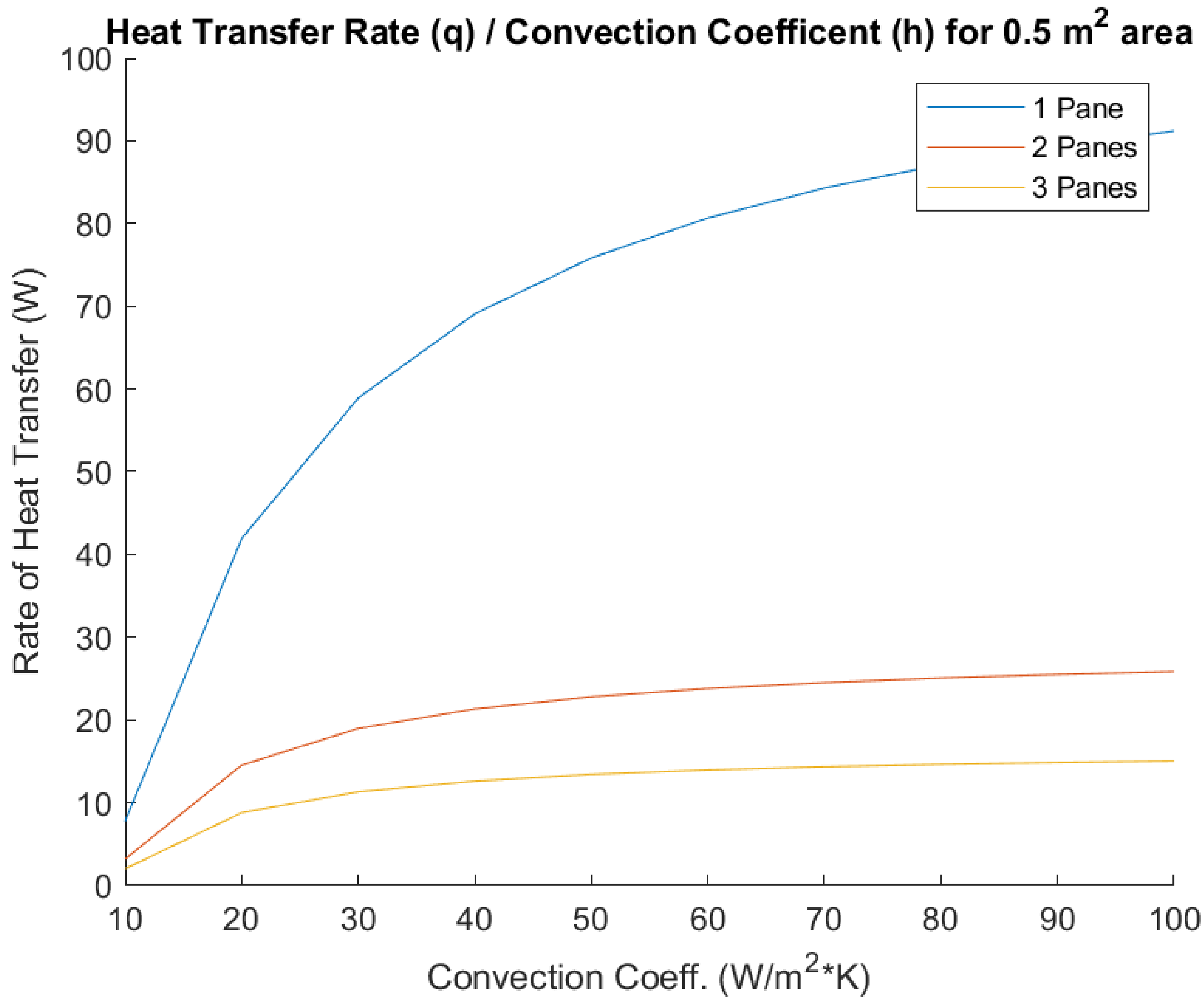

Figure A5: Heat Transfer Rate vs Convection Coefficient for Smaller Area