Computational Assignment 2
MEGN 471: Heat Transfer - Spring 2023

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Introduction

Conjugate heat transfer was used in this study to determine an optimal fin array density on a 30W heat chip. The tested density would use 1x1, 3x3, 6x6, 9x9, and 12x12 aluminum fin arrays of length 15m and diameter of 2mm. The primary methods for computational analysis included CFD in Solidworks at varying fluid velocities, as well as MATLAB code to test a base scenario with the convection coefficient h_{avg} = 150 W/m^{2}K.

Methodology

As stated in the introduction, two main methods were employed to calculate the variation in max temperature v. fin count. The first method used computational fluid dynamics (CFD) with instructions for system setup found here. This method utilized air as its working fluid, 6061 aluminum alloy as the material for the heat sink, and silicon as the material for the CPU chip. The initial temperature of the system was set at 20°C with a fluid velocity of 8 m/s passing through the fin array in the x-direction, and the heat generation rate in the CPU chip was set to 15 W as only half of the array and chip assembly was modeled to decrease model complexity and lower processing time. The maximum temperature at the base of the array and the location of the max temperature in the array from the CFD analysis were found and compared against the results of the basic fin theory calculations.

The second method used an analytical approach to solve for the maximum temperature within each fin configuration. In this method, the team used the general equation for heat rejection from a fin array in Equation 1.

$$P_{chip} = q_{fin}N_{fins} + h_{avg}\theta_b A_b$$

Equation 1: *Heat transfer equation for fin array with N fins*

In this equation, P_{chip} is known and equal to 30W. The $q_{fin}N_{fins}$ term represents the total heat dissipated by the fins (W), where q_{fin} is the heat transfer rate from an individual fin (W), and N_{fins} is the number of fins in the fin array (ie. 12x12 array would be 144 fins). Note that q_{fin} is unknown and needs to be solved for using Table 3.4 from Fundamentals of Heat Transfer (in Appendix). Given the system configuration and the unknowns in the heat transfer equation, Case A (A.3.77) can be used to solve for q_{fin} shown in Equation 2.

$$q_{fin} = M\frac{sinh(mL) + (\frac{h}{mk})cosh(mL)}{cosh(mL) + (\frac{h}{mk})sinh(mL)}$$

Equation 2: *Heat transfer for single fin in A 3.77*

In Equation 2, $M = \sqrt{hPkA_c}\theta_b$, and $m = \sqrt{hP/kA_c}$. Provided that all of these values are known (see Appendix for list of values) aside from θ_b, Equation 2 can be substituted into Equation 1 for q_{fin} as a function of θ_b. Now, referencing the updated Equation 1 (Equation 3 below), there is one equation and one unknown - as A_b can be calculated as the prime area (m^2), which is the area of the non-fin surface exposed to fluid.

$$P_{chip} = \left(\sqrt{hPkA_c}\theta_b \frac{sinh(mL) + (\frac{h}{mk})cosh(mL)}{cosh(mL) + (\frac{h}{mk})sinh(mL)}\right) * N_{fins} + h_{avg}\theta_b A_b$$

Equation 3: *Heat transfer for single fin in A 3.77*

This means the equation can be solved by MATLAB or by hand to calculate θ_b, which is the temperature at the base of a fin minus the atmospheric temperature ($\theta_b = T_b - T_\infty$). Provided the team is looking for the hottest temperature within the fin, the θ_b equation can be rearranged to show that $T_b = \theta_b + T_\infty$ (°C) where both variables are now known. The base temperature T_b is the hottest temperature within a given fin provided that fins conduct heat along their primary axis (lengthwise) from base to tip. This analytical solution can be applied to multiple fin configurations for the problem, provided that the number of fins is given. Therefore, the methodology was written as a function and called in a for loop for each fin configuration in the study. It is important to note that some parameters within the function will change given the number of fins, such as the prime area A_b. This is accounted for in the code as part of the Equation 1 calculation.

Results and Discussion

The primary results that the CFD analysis found were the maximum temperature at the base of the fin array and the location of the maximum temperature in the system. Increasing the number of fins in the array exhibits an inverse relationship between the number of fins and maximum temperature. Figure 1 shows the temperature distribution across a 1x1 array:

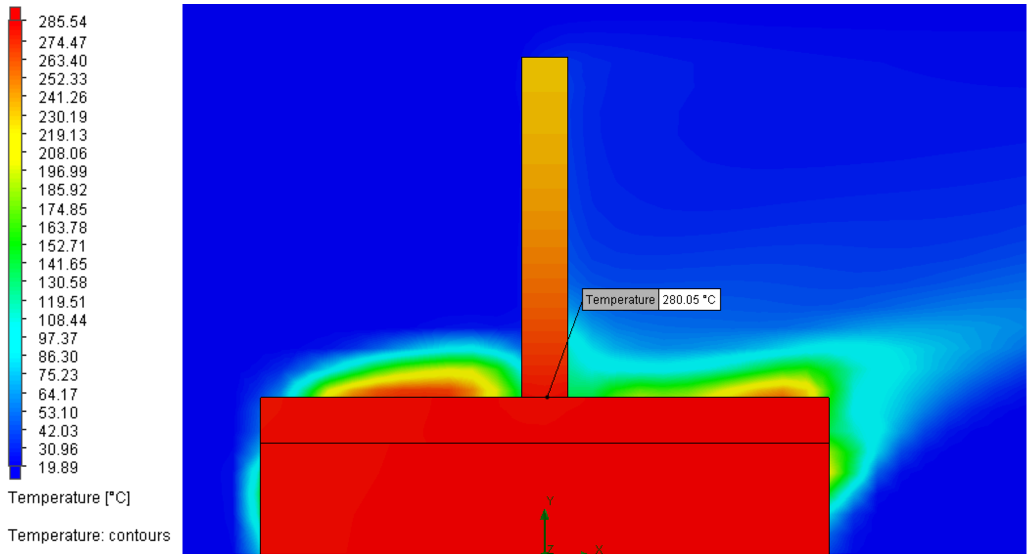

Figure 1: *1x1 Fin Array Temperature Profile*

The maximum temperature in this system is 285.54°C which occurs along the bottom edge of the CPU. The fin base temperature is slightly lower at 280.05°C. This temperature will be used in comparison against the MATLAB results so it is labeled above. Additionally, the fin tip temperature is also labeled.

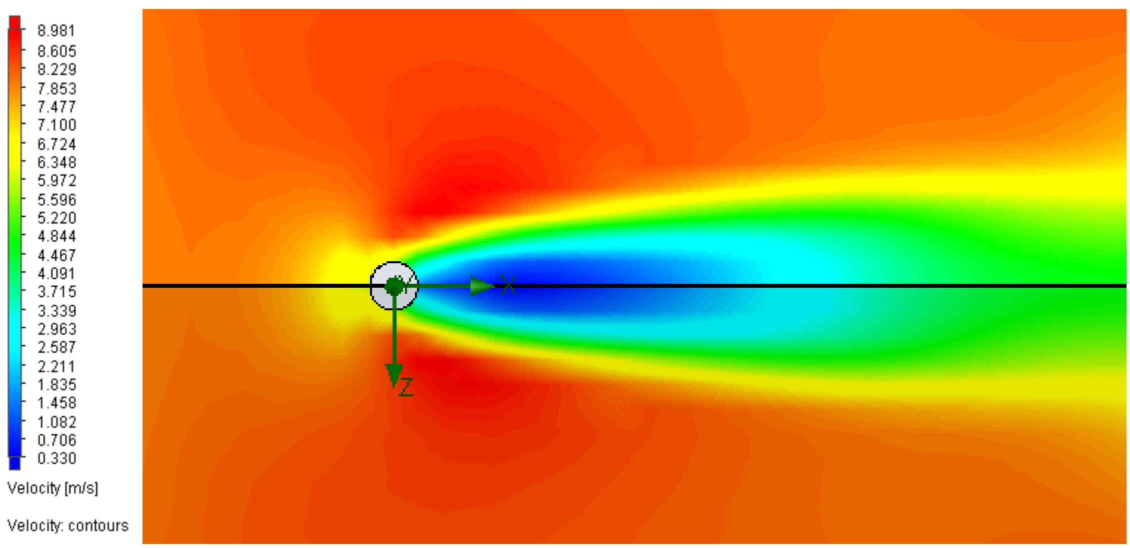

Figure 2: *1x1 Fin Array Velocity Profile*

Figure 2 illustrates the velocity of the air as it travels through a 1x1 array. An area of low-velocity results in the wake of the fin.

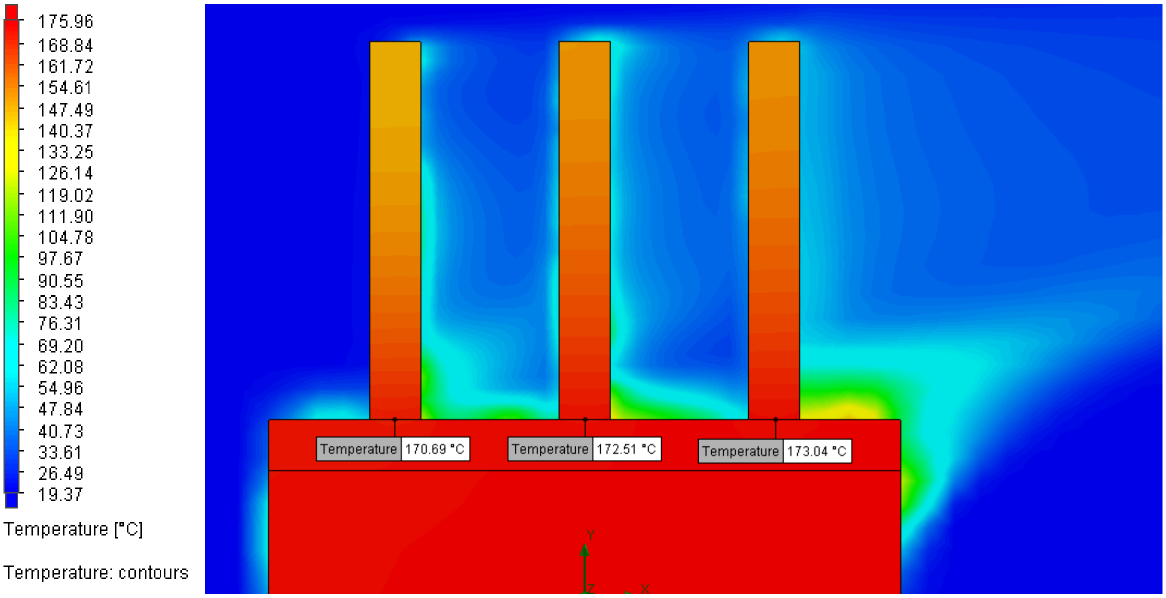

Figure 3: *3x3 Fin Array Temperature Profile*

The maximum temperature found in the 3x3 system is 175.96°C with the average fin base temperature being 172.08°C. This is significantly lower than the 1x1 array which is expected due to the improved heat dispersion provided by the additional fins.

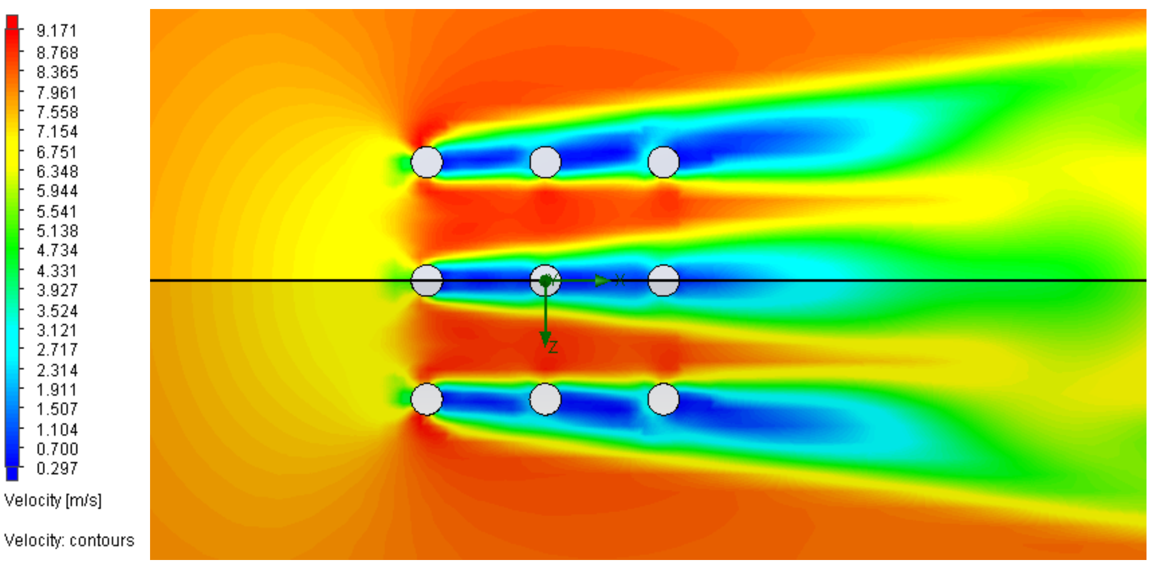

Figure 4: *3x3 Fin Array Velocity Profile*

In the 3x3 array, lines of higher velocity air (compared to the 1x1 case) can be observed between the fins. This facilitates convection further both through the increased air speed and the resulting turbulence of the multiple fins but the trailing edge fins do not have the same benefits as they meet the lower velocity wake coming off of the leading fins.

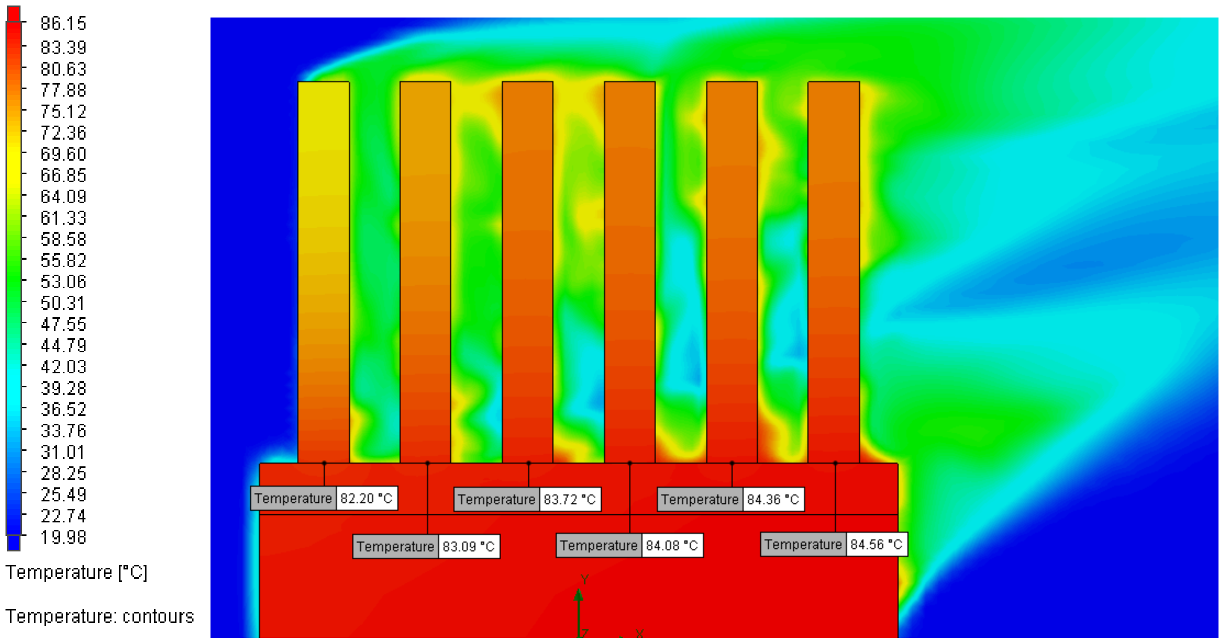

Figure 5: *6x6 Fin Array Temperature Profile*

The maximum temperature found in the 6x6 system is 86.15°C with the average fin base temperature being 83.7°C. This is even lower than the 3x3 array and follows the trend of lowering temperature as fins are added. The leading edge fin (to the left) sees the lowest temperature distribution as it meets the cooling air whilst the others contend with already heated air from the fins around them.

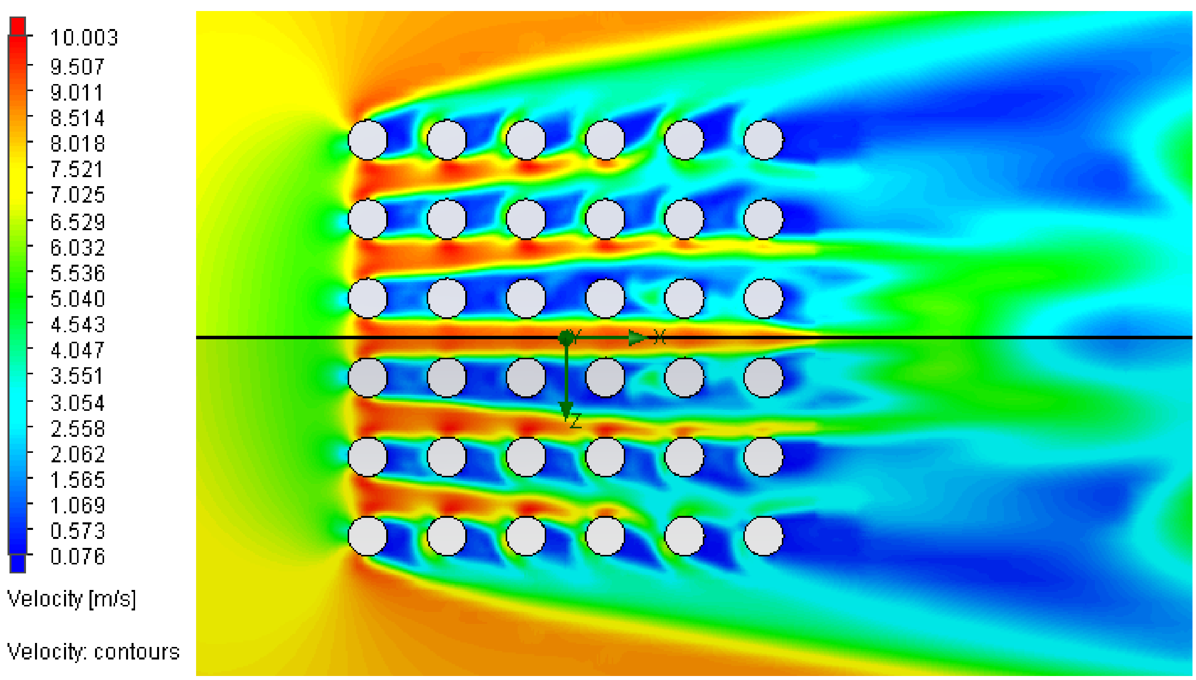

Figure 6: *6x6 Fin Array Velocity Profile*

The leading edge of the array exhibits the same behavior as the 3x3 array with higher air speed resulting between the fins. In this case, however, some resulting lateral flow (upwards and downwards in the above figure) causes the streams to drop in velocity which begins to hurt the convection of the fins significantly.

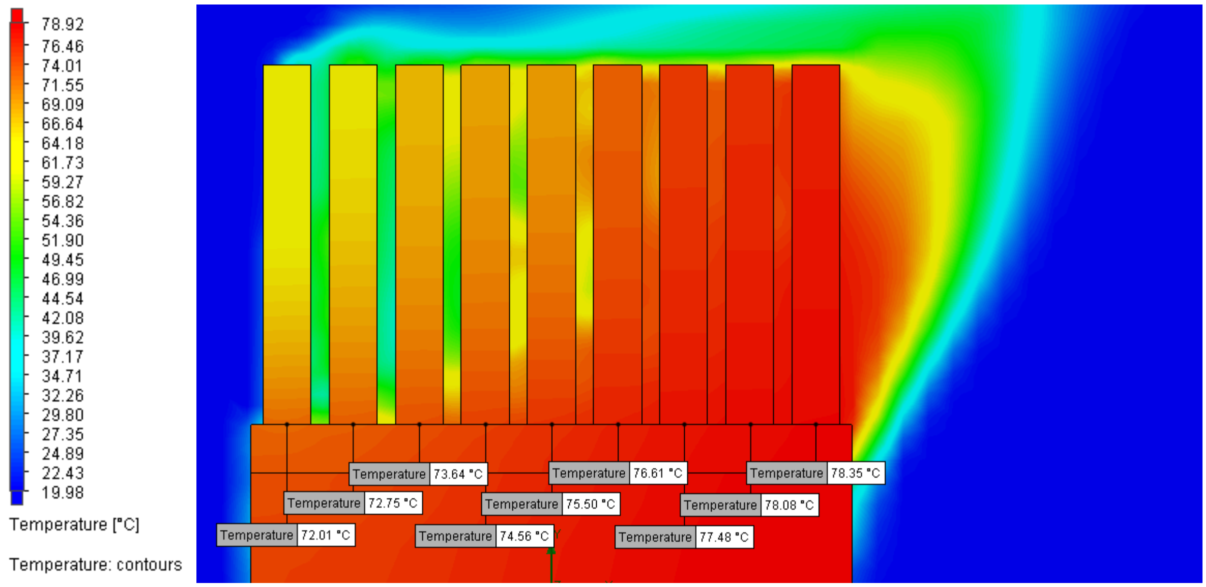

Figure 7: *9x9 Fin Array Temperature Profile*

The maximum temperature found in the 9x9 system is 78.92°C with the average fin base temperature being 75.44°C. This is still lower than the 6x6 array but the temperature decrease is beginning to become less apparent indicating that the addition of more fins is not helping with heat dispersion as much as before. The first few leading edge fins are lower in temperature not only because of their immediate contact with the cooling air but also the lessened heat flow through each fin.

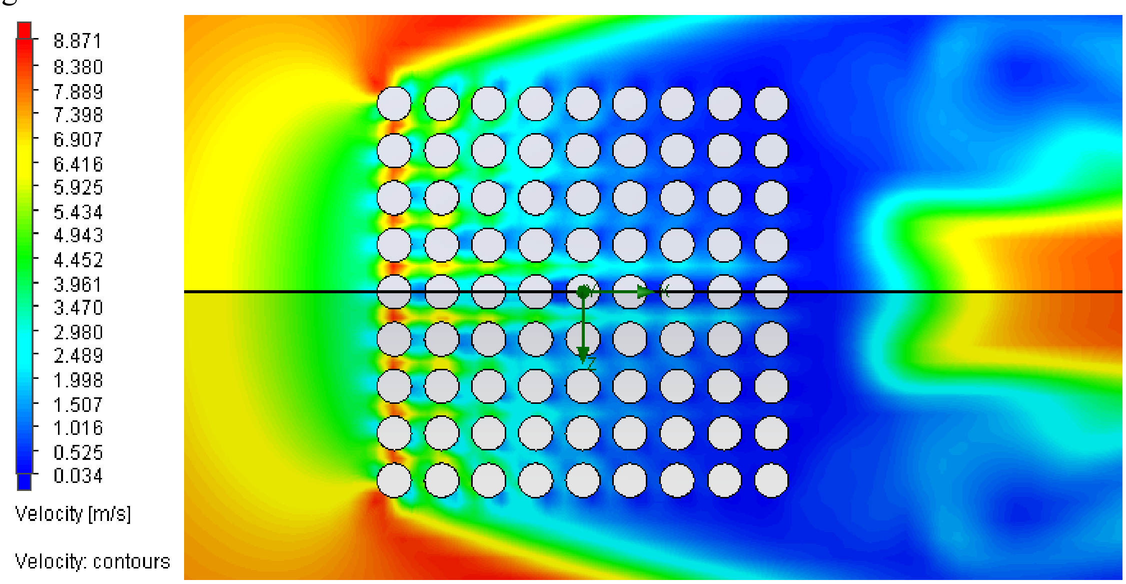

Figure 8: *9x9 Fin Array Velocity Profile*

The 9x9 fin array compacts the fins too tightly to allow free/accelerated fluid flow through the array. This results in poor convection on the interior fins and majorly hinders the heat transfer efficiency of the array.

These two methods vary in where their maximum temperature location is measured. In the CFD model, the very base of the circuit chip is the hottest point in the simulation whilst the temperature data from MATLAB is positioned at the fin base. As the number of fins increases, the location of the maximum temperature in the CFD model is observed to shift toward the side of the array that is experiencing lower air speeds. In the MATLAB analysis, the location of the maximum temperature does not change. There are minor differences between the chip and fin base temperature in SolidWorks, so probes were placed to acquire average fin base temperatures from the simulations. This is not necessarily required due to the small temperature delta between the two points on the CFD model but, for the sake of having a direct comparison, it is useful to have this data regardless. By using the average fin temperatures from the simulation, it is possible to avoid conflicts when comparing evidence. However, certain design criteria could cause the location of the temperature to become relevant, such as if the CPU chip was required to stay below a certain temperature threshold. In this case, the MATLAB analysis may underestimate the maximum temperature in the CPU even if the average temperatures from both analyses are the same.

Comparing the values found in the CFD study to the analytical predictions in MATLAB - plotted in Figure 9 - it is evident that the two results of the studies diverge most severely on the edge cases (1 fin and 12x12 fins). This change in result can be attributed to variations in the convective coefficient (h), in the CFD simulations, whereas in the MATLAB study h was assumed to be a constant at h_{avg}. This change most likely explains the close analytical accuracy at the median fin count of 6x6.

Aside from this trend, another interesting delineation between the analysis and CFD was the eventual increase of max temp in the fin array as the final count increased. As discussed earlier, increasing the number of fins past a critical point (between 10x10 and 12x12 array) results in an increased measured temperature. This change is due to a decrease in velocity moving past the fins as fin density increases - this effectively decreases h and rescues the array's convection capacity. Conversely, MATLAB does not account for the change, and instead only models the convection relationship as a function of area exposed. The result is a parabolic type shape for the CFD method and an exponential shape for the MATLAB method.

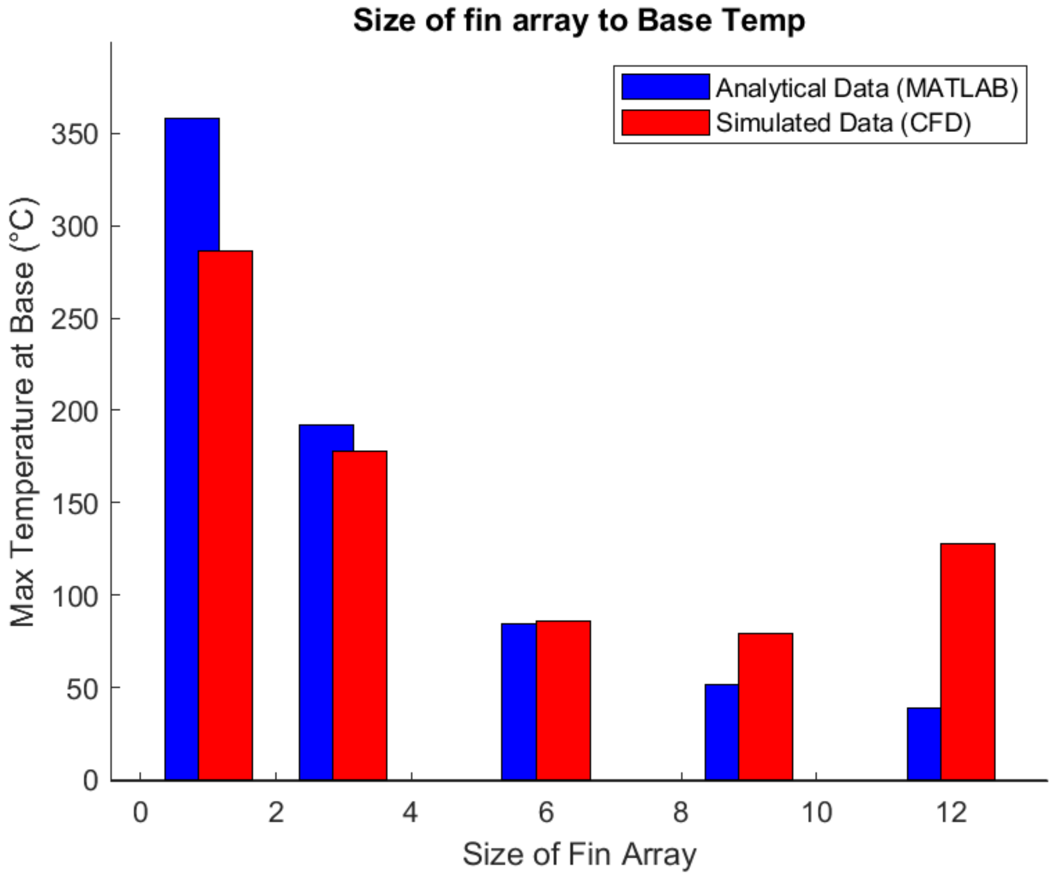

Figure 9: *MATLAB v. CFD max temperature per fin array*

Conclusion

In conclusion, the array that was found to be the most effective at reducing the temperature of the chip differed based on the analytical approach taken. Basic fin theory concludes that the 12-fin array is the most effective while SolidWorks simulation concludes that the 9-fin array is the most effective. The assumptions used in the calculations for the basic fin theory, such as a constant heat transfer coefficient, do not accurately describe the arrays on the end cases (1 fin and 12x12 fins). In these scenarios, the value of h_{avg} used in the MATLAB code differs too greatly from the true varying value of h, resulting in the discrepancy seen in Figure 9. The SolidWorks simulation takes into account the changing value of h by simulating changing flow conditions throughout the array which allows it to not only to achieve more accurate results for all the arrays.

Group Contribution

Michael Allen:

Michael wrote the MATLAB code to calculate and plot the data used in this study and wrote the methodology section.

Cullen Hirstius:

Cullen ran the computational fluid dynamics for this study and wrote parts of the results/discussion section.

Hayden Payne:

Hayden wrote part of the methodology, results and discussion, and conclusion sections.

Appendix A

Figures generated by MATLAB. Source code posted on GitHub:
https://github.com/MC-Meesh/Computational_Heat_Transfer_2

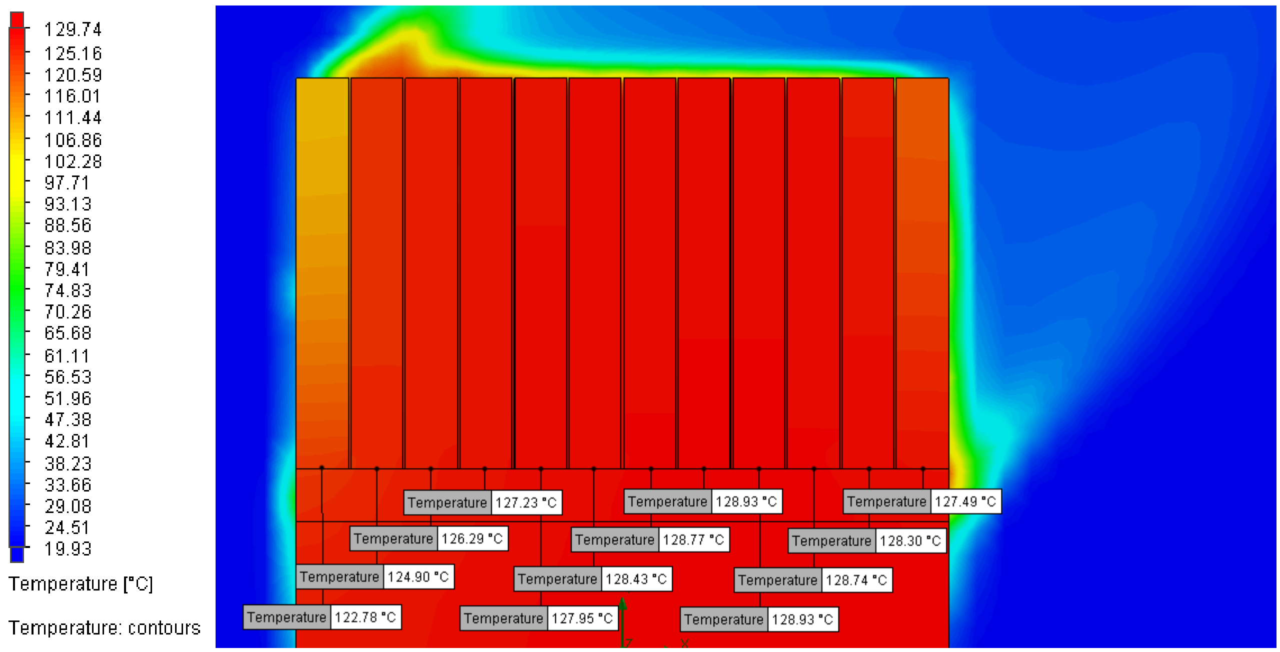

Figure A1: *12x12 Fin Array Temperature Profile*

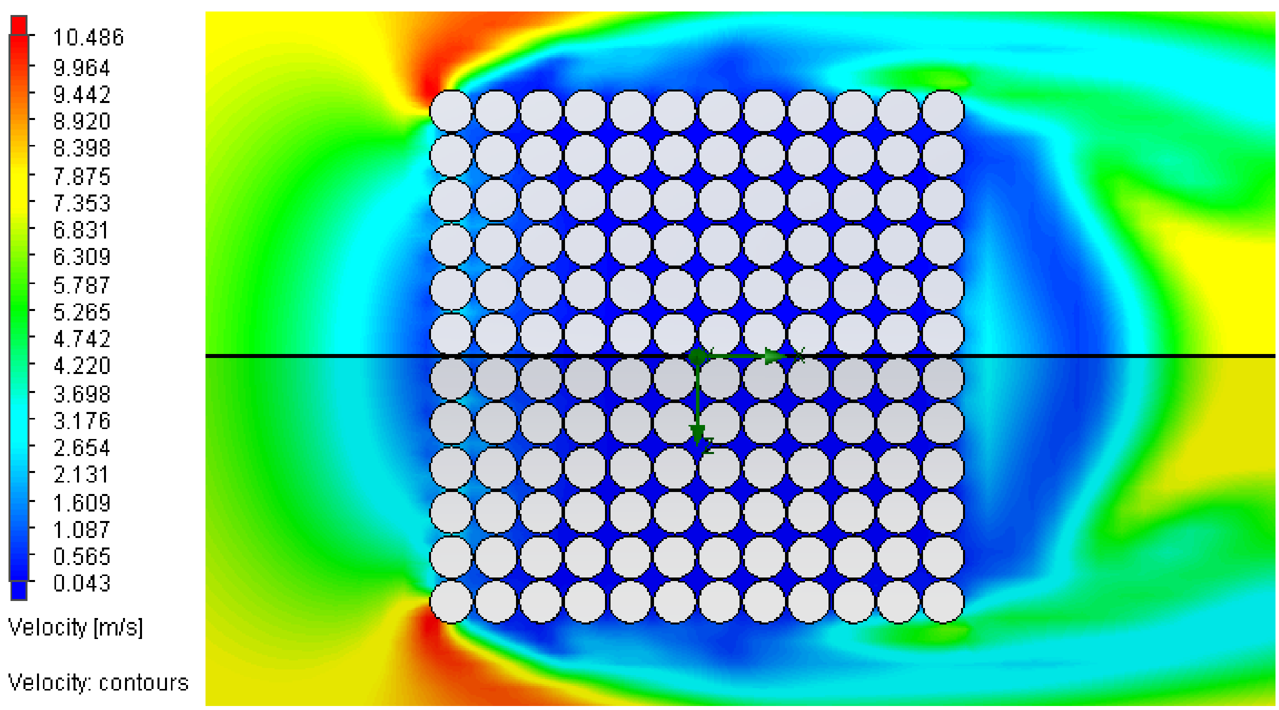

Figure A2: *12x12 Fin Array Velocity Profile*

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q	
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.75)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.77)
B	Adiabatic: $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	(3.80)	$M \tanh mL$	(3.81)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.82)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.83)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	(3.84)	M	(3.85)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

Table A1: *Table 3.4 from "Fundamentals of Heat Transfer"*

Parameter	Value	Units
P_{chip}	30	W
h_{avg}	125	$W/m^2 \cdot K$
k_{fin}	200	$W/m \cdot K$
$T_{ambient}$	20	°C
Length/width of chip	25	mm
Length of a fin	15	mm
Diameter of a fin	2	mm

Table A2: *Table of Given Parameters*

- P_{chip} is the power dissipated from the chip to the fin array (also known as q_{total})
- h_{avg} is the average conduction coefficient, impacting the rate of heat transfer from the surface of the fin array to surroundings
- k_{fin} is the thermal conductivity of a given fin, useful in determining the heat transfer rate (q_{fin}) through a fin
- $T_{ambient}$ is the atmospheric temperature of the room
- Length/Width$_{chip}$ represents the side dimensions of the chip, useful in calculating the base area A_b
- Length of the fin is how far a given fin protrudes from the base of the fin array, useful in calculating fin properties
- Diameter of the fin is the distance across the cylindrical fin, useful in calculating fin properties and areas