Computational Assignment 3
MEGN 471: Heat Transfer - Spring 2023

Michael Allen, Cullen Hirstius, Hayden Payne
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Introduction

Radiative heat transfer analysis was used to determine the optimal camp stove and pot configuration given three separate scenarios regarding a protective shroud to reflect heat between the stovetop and the pot. Scenario 1 involves only the stove and pot with a gap in-between. Scenario 2 replaces this gap with an opaque, diffuse, and gray cylindrical aluminum "shroud." In scenario 3 this shroud is made of a material coated in black paint. The tested scenarios each analyze the effect that changing the distance between the stove and pot from 10 *mm* to 300 *mm* has on the view factor from the stove to the pot ($F_{stove\text{-}pot}$) and the heat transfer rate (q_{pot}). The primary method for computational analysis was MATLAB code to compile equations and calculate results.

Methodology

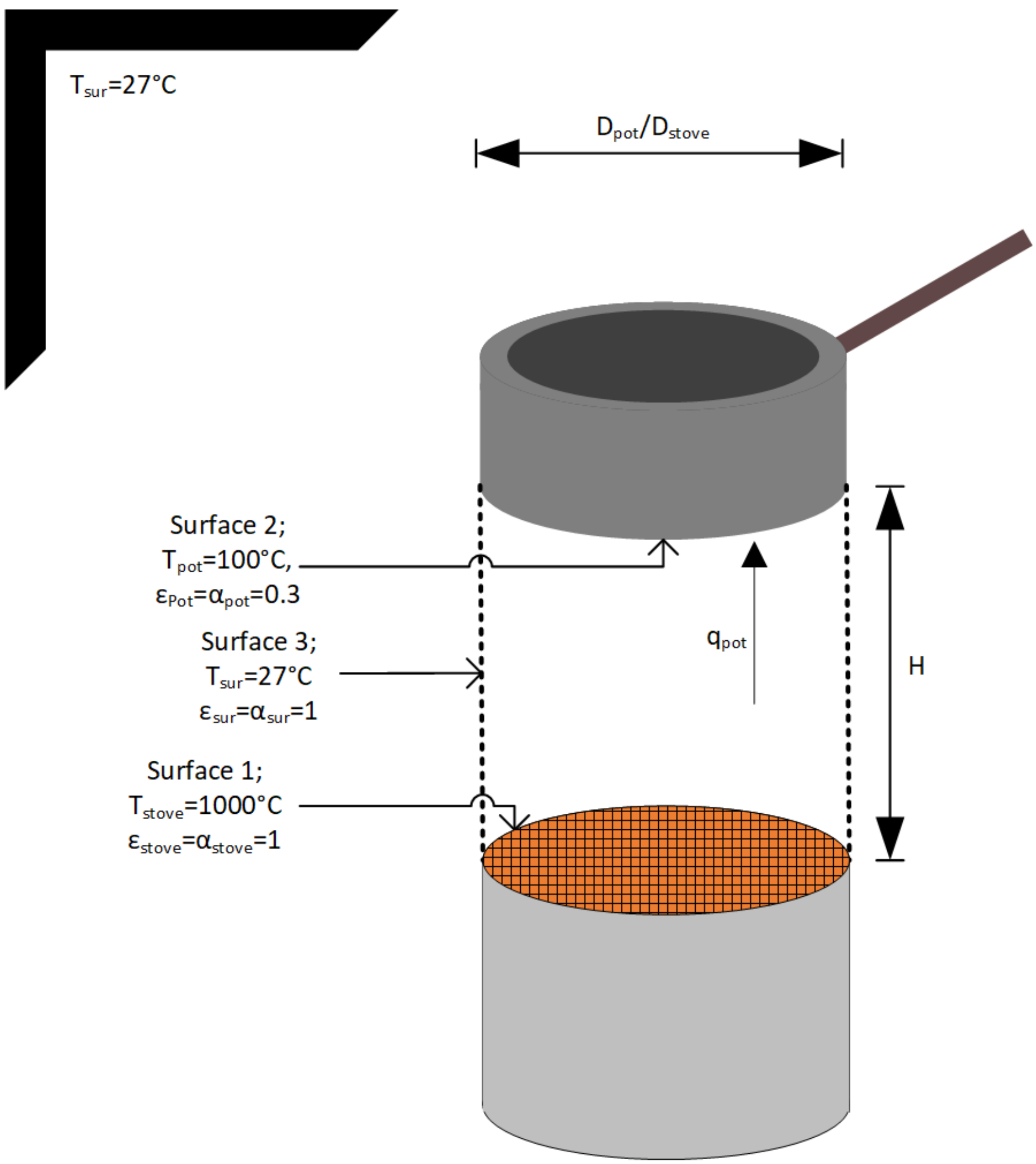


Figure 1: *Drawing of Scenario 1*

To calculate the radiative heat transfer to pot, the team used the following general parameters:

Surface	Parameter	Value	Meaning
Stove (Surface 1)	T_{stove}	1273 K	Temperature of stovetop
	D_{stove}	200mm	Diameter of stovetop
	ε_{stove}	1	Emissivity of stovetop (blackbody)
Pot (Surface 2)	T_{pot}	373 K	Temperature of pot
	D_{pot}	200mm	Diameter of pot
	ε_{pot}	0.3	Emissivity of pot
Other	T_{surr}	300 K	Surrounding temperature
	H	10mm - 300mm	Distance between stovetop and pot

Table 1: *General scenario parameters*

Additionally, the following parameters varied between each of the three scenarios:

Surface	Parameter	Value	Meaning
Virtual Surface (Surface 3)	T_{surr}	300 K	Temperature of surroundings and virtual surface
	$D_{surface}$	200mm	Diameter of virtual surface
	$\varepsilon_{surface}$	1	Emissivity of virtual surface(blackbody)
	$A_{surface}$	$\pi H(\frac{D_{surface}}{2})^2$	Area of virtual surface

Table 2: *Scenario 1, Surface 3 parameters*

Surface	Parameter	Value	Meaning
Aluminum	T_{surr}	300 K	Temperature of surroundings and

(Surface 3)			foil
	D_{foil}	200mm	Diameter of foil
	ε_{foil}	0.1	Emissivity of foil
	A_{foil}	$\pi H(\frac{D_{foil}}{2})^2$	Area of foil

Table 3: *Scenario 3, Surface 3 parameters*

Surface	Parameter	Value	Meaning
Painted Surface (Surface 3)	T_{surr}	300 K	Temperature of surroundings and shroud
	D_{shroud}	200mm	Diameter of shroud
	ε_{shroud}	0.9	Emissivity of shroud
	A_{shroud}	$\pi H(\frac{D_{shroud}}{2})^2$	Area of protective shroud

Table 4: *Scenario 3, Surface 3 parameters*

Note that given the diameter of each surface/shroud is the same, the area will be the same for the same H when tested. Given these parameters, the team can evaluate the view factors between each surface in the model, including a virtual surface/shroud that acts between the pot and stove shown in Figure 1. The view factors were calculated using a combination of first-order equations and reciprocity principles. The set of equations for finding the view factors from Surface 1 to Surface 2 are shown below:

$$R_j = \frac{D_{pot}}{2H}; \quad R_i = \frac{D_{pot}}{2H}; \quad S = 1 + \frac{1+R_j^2}{R_i^2}; \quad F_{ij} = \frac{1}{2}\left(S - \left(S^2 - 4\left(\frac{R_j}{R_i}\right)^2\right)^{\frac{1}{2}}\right)$$

Equations 1-4: *Set of equations for calculating the view factor between stovetop and pot*

The remaining view factors could be calculated using reciprocity, such as the case of finding the view factor from the pot to the stove calculated as $F_{pot-stove} = (\frac{A_{stove}}{A_{pot}})F_{stove-pot}$. Note that in this equation the area of the pot and the stove are equal, meaning that the view factors are the same: $F_{ij} = F_{pot-stove} = F_{stove-pot}$. The view factor from the stove or pot to the virtual surface/shroud could be calculated by finding the remaining percentage of radiation that does not hit the opposite surface, eg: $F_{pot-shroud} = F_{stove-shroud} = 1 - F_{ij}$. This works provided the pot and stovetop are both flat surfaces, meaning they do not absorb any of their own radiation and thus 100% of the radiation is emitted outwards towards either the opposite surface or the surroundings. In sum, this resulted in the following 9 equations for each of the scenarios:

Stove View Factors:

$$F_{stove-pot} = F_{ij} = \frac{1}{2}(S - (S^2 - 4(\frac{R_j}{R_i})^2)^{\frac{1}{2}});$$

$$F_{stove-stove} = 0;$$

$$F_{stove-surr} = 1 - F_{stove-pot};$$

Pot View Factors:

$$F_{pot-pot} = 0;$$

$$F_{pot-stove} = F_{ij} = \frac{1}{2}(S - (S^2 - 4(\frac{R_j}{R_i})^2)^{\frac{1}{2}}) = (\frac{A_{stove}}{A_{pot}})F_{stove-pot};$$

$$F_{pot-surr} = 1 - F_{pot-stove};$$

Virtual Surface/Shroud View Factors:

$$F_{shroud-pot} = (\frac{A_{pot}}{A_{shroud}})F_{pot-shroud};$$

$$F_{shroud-stove} = (\frac{A_{shroud}}{A_{stove}})F_{stove-shroud};$$

$$F_{shroud-shroud} = 1 - F_{shroud-stove} - F_{shroud-pot};$$

Equations 5-13: *Defining equations for the view factors of each surface*

Given these view factors, setting up a system of equations for each surface's heat transfer enables the definition of heat transfer into the pot for the varying scenarios:

<u>Scenario 1:</u>

Surface 1 (Stove):

$$J_1 = \sigma T_{stove}^{4}$$

Surface 2 (Pot):

$$q_{pot} = \frac{(\sigma T_{pot}^{4} - J_2)\varepsilon_{pot} A_{pot}}{(1-\varepsilon_{pot})} = A_{pot}(F_{pot-stove}(J_2 - J_1) + F_{pot-surface}(J_2 - J_3))$$

Surface 3 (Virtual Surface):

$$J_3 = \sigma T_{surr}^{4}$$

Equations 14-16: *Heat transfer equations for Scenario 1*

<u>Scenario 2:</u>

Surface 1 (Stove):

$$J_1 = \sigma T_{stove}^4$$

Surface 2 (Pot):

$$q_{pot} = \frac{(\sigma T_{pot}^4 - J_2)\varepsilon_{pot} A_{pot}}{(1-\varepsilon_{pot})} = A_{pot}(F_{pot-stove}(J_2 - J_1) + F_{pot-foil}(J_2 - J_3))$$

Surface 3 (Aluminum shroud):

$$q_{foil} = \frac{(\sigma T_{surr}^4 - J_2)\varepsilon_{foil} A_{foil}}{(1-\varepsilon_{foil})} = A_{foil}(F_{foil-stove}(J_3 - J_1) + F_{foil-pot}(J_3 - J_2))$$

Equations 17-19: *Heat transfer equations for Scenario 2*

<u>Scenario 3:</u>

Surface 1 (Stove):

$$J_1 = \sigma T_{stove}^4$$

Surface 2 (Pot):

$$q_{pot} = \frac{(\sigma T_{pot}^4 - J_2)\varepsilon_{pot} A_{pot}}{(1-\varepsilon_{pot})} = A_{pot}(F_{pot-stove}(J_2 - J_1) + F_{pot-surface}(J_2 - J_3))$$

Surface 3 (Painted shroud):

$$q_{foil} = \frac{(\sigma T_{surr}^4 - J_2)\varepsilon_{shroud} A_{shroud}}{(1-\varepsilon_{shroud})} = A_{shroud}(F_{shroud-stove}(J_3 - J_1) + F_{shroud-pot}(J_3 - J_2))$$

Equations 20-22: *Heat transfer equations for Scenario 3*

Solving this system of equations in each scenario for radiosity enabled the team to calculate the heat transfer to the pot given the change in height (impacts view from stove to pot), and therefore create definitive results. Full symbol definitions can be found in the appendix.

Results and Discussion

Radiosities (W/m^2)	Scenario 1	Scenario 2	Scenario 3
J_1 (Stove)	$1.489 * 10^5$	$1.489 * 10^5$	$1.489 * 10^5$
J_2 (Pot)	$4.034 * 10^4$	$8.2621 * 10^4$	$4.2995 * 10^4$
J_3 (Surroundings or Shroud)	459.27	$9.8191 * 10^4$	$6.5952 * 10^3$

Table 5: *Surface Radiosity Values at 100 mm in Height*

Table 5 above shows how the radiosity values of each surface vary between the three scenarios. In this case, scenario 1 has a constant radiosity as it is only dependent upon its own temperature due to it being a blackbody, a feature that is held constant between every scenario. J_2 and J_3, however, are impacted by the changing of surface 3's properties. In particular, the addition of a shroud (scenarios 2 and 3) causes the radiosity out of surface 3 to increase massively when compared to scenario 1. This is due to scenario 1 being a virtual surface with no reflectivity. In the shrouded cases, both materials reflect some of the radiation they receive (mostly coming from the stove flame) which drives up their radiosity values by a significant amount. Some of this reflected radiation reaches surface 2 of the pot. As a result, the higher reflectivity of the aluminum causes the pot to receive and reflect even more radiation resulting in a much higher radiosity value when compared to the open system and the painted shroud (which only gives a marginal increase in radiosity due to its low reflectivity).

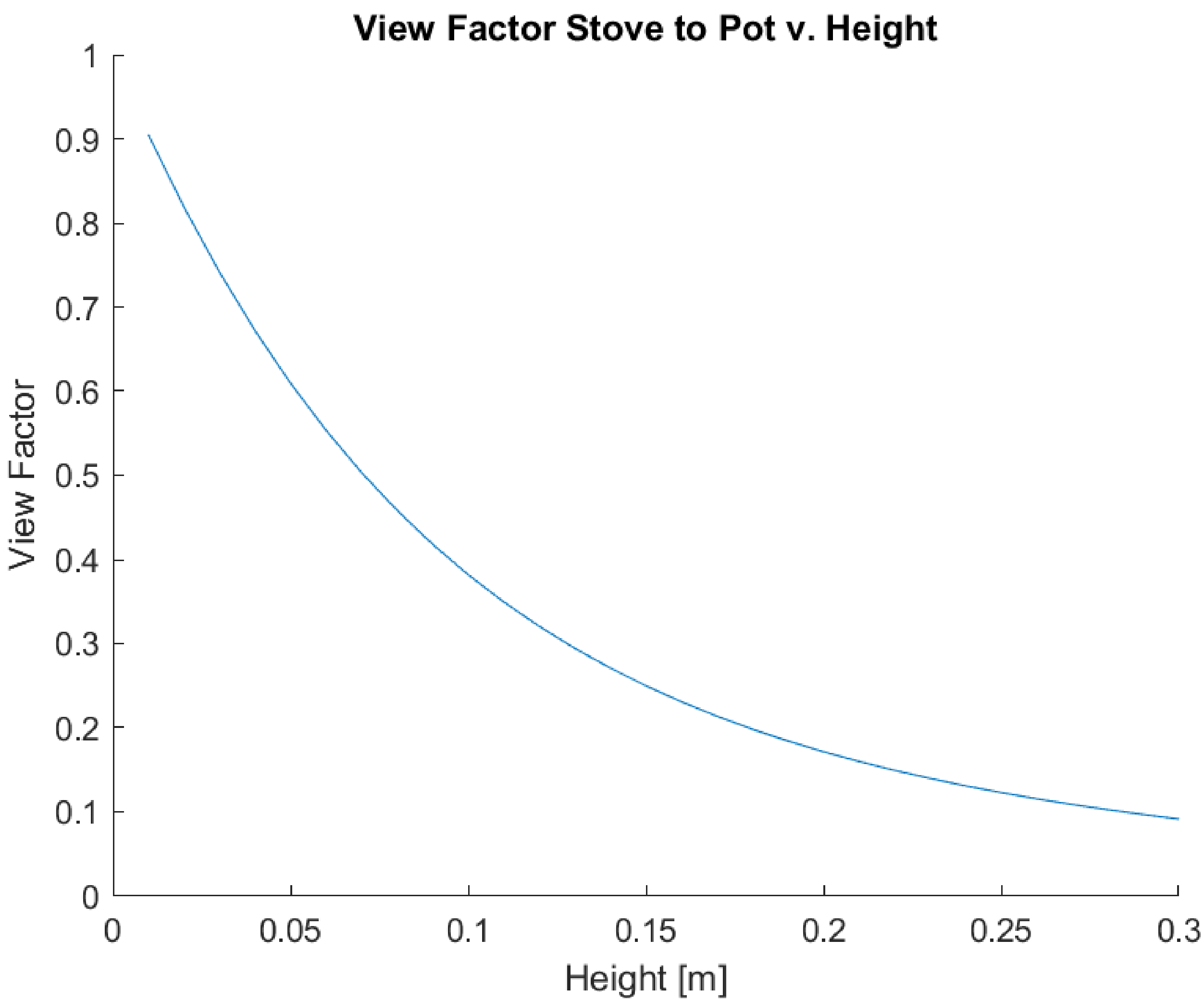


Figure 2: *Stove to Pot View Factor with Varied Height*

As can be seen in Figure 1, the view factor from the stove to the pot decreases as the height/separation between the two increases. This is mathematically supported by Equations 1-4 (shown above) and makes sense from an analytical perspective. Naturally, as the pot is moved further away and the gap to the surroundings/shroud grows, the amount of radiation directly reaching the pot from the stovetop will decrease. This is independent of the shroud material as the ability of the material to absorb, emit, etc. is irrelevant in calculating the view factor which is entirely driven by geometric conditions. As the geometry of the system does not change between the scenarios, the view factor will remain the same.

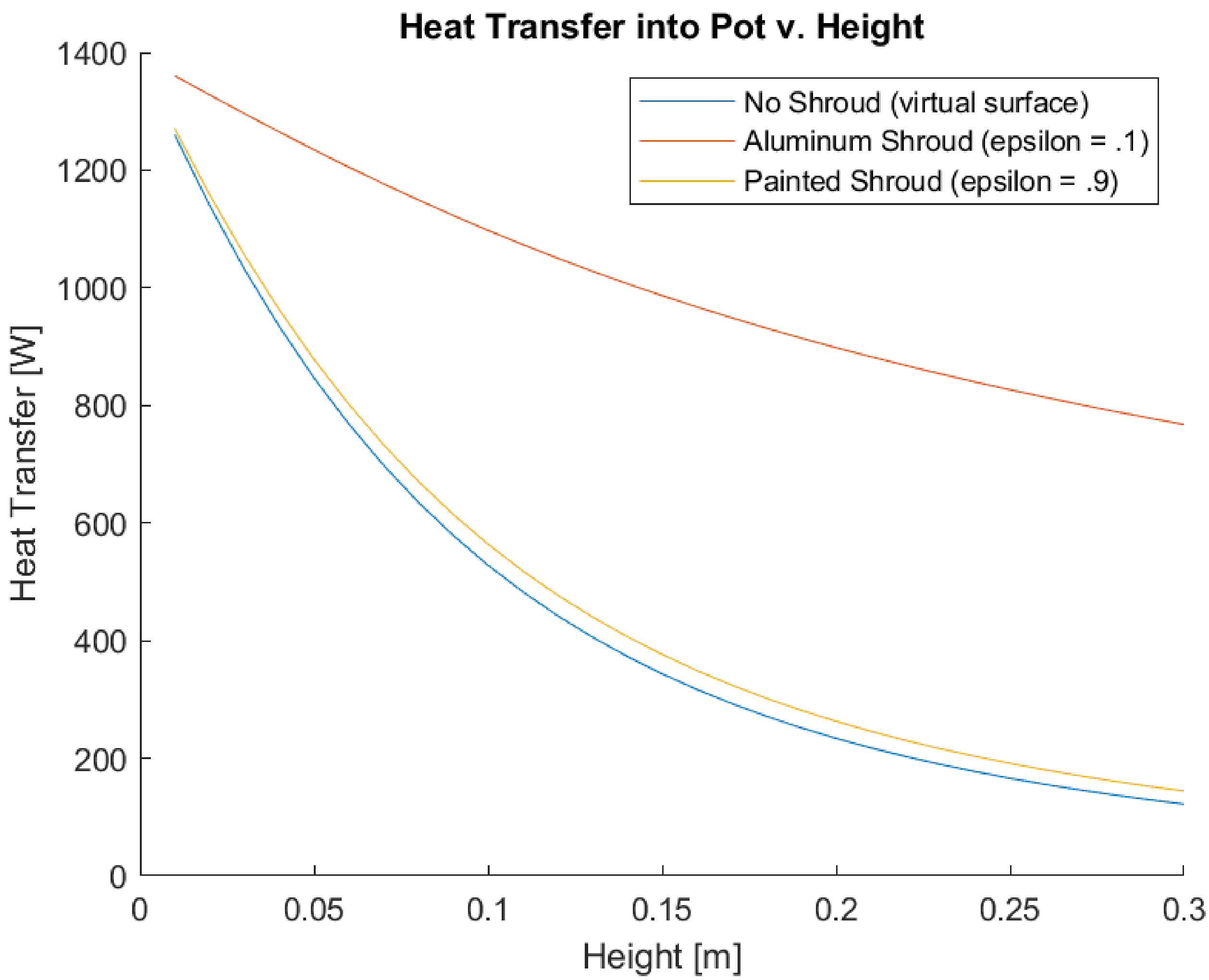


Figure 3: *Pot Received Heat with Varied Height*

Figure 2 shows the relationship between the heat received by the pot and separation between the pot and the stove. As the separation increases, the heat transfer decreases. This is due to the view factor from the stove to the pot decreasing and the view factor to the shroud/surroundings increasing resulting in more heat lost to the shroud/surroundings instead of entering the pot. This relationship is shown further in Figure 1 earlier in the report. The varying of shroud material properties shows that the aluminum shroud is the most effective and with the painted shroud being slightly better than having no shroud. This is due to the changing emissivity/absorptivity of the materials. As the shroud and surroundings are considered gray, diffuse, and opaque, the only variable adjusted when changing materials is alpha/epsilon (and rho by extension). For the aluminum, the alpha and epsilon are very low at 0.1, leaving rho to be 0.9 as indicated by the following equation (tau is zero due to the material being opaque):

$$\alpha + \tau + \rho = 1$$

Equation 23: *Relationship between Absorptivity, Transmissivity, and Reflectivity*

The result is that most of the heat that hits the shroud from the stove will reach the pot at some point as it is reflected off the aluminum instead of absorbed. By contrast, the painted shroud with an epsilon/alpha of 0.9 will have a reflectivity of 0.1 which indicates that most of the heat hitting the shroud will be absorbed instead of reflected back out into the enclosure. This creates the massive gap in heat transfer to the pot which can be seen in Figure 2. Additionally, the small gap between the painted shroud and the open surroundings can be explained by the epsilon/alpha of the open system being 1, which is very similar to the 0.9 of the shroud. Thus, they perform similarly to each other when only the enclosure radiation is taken into account.

When selecting the optimal camping stove setup, the aluminum foil shroud appears to be the best option by a significant margin due to its much larger radiosity value (to the pot specifically). The high reflectivity of the foil is a very large help in this scenario and helps to make a good cooking setup with minimal losses to the surrounding area. However, when viewing the results of the painted shroud and open system, there isn't much of a difference, which makes choosing between the two seem difficult. This issue can be easily resolved by considering convection on the system. In the open system, convection will impact the bottom of the pot, the stovetop, and any natural convective flow within the cooking area that may have been facilitating heat transfer. By contrast, the painted shroud will only have convection on its outside area, maintaining the critical temperatures and flow fields within the system. Thus, the painted shroud is likely a better candidate due to its ability to protect the system from wind and facilitate more effective convection.

Conclusion

In conclusion, the optimal camp stove and pot configuration is that which is outlined in scenario 2. The aluminum shroud greatly increases the amount of radiation that is reflected toward the pot and thus the amount of heat that the pot absorbs. Additionally, placing the pot closer to the stove increases the amount of heat transferred into the pot due to the increasing view factor between the stove and the pot. The black shroud is likely the second best option despite having similar results as scenario 1 due to the omission of convection from this analysis.

Group Contribution

Michael Allen:

Michael wrote the MATLAB code to calculate and plot the data used in this study and wrote the methodology section.

Cullen Hirstius:

Cullen made the scenario 1 drawing and wrote the results/discussion section.

Hayden Payne:

Wrote the introduction and conclusion and compiled the appendix.

Appendix

Figures generated by MATLAB. Source code posted on GitHub:
https://github.com/MC-Meesh/Computational_Heat_Transfer_3

Table 13.2 View Factors for Three-Dimensional Geometries [4]

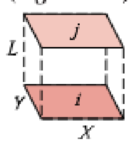
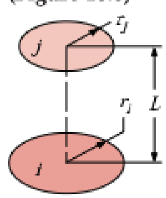
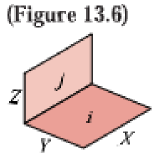
Geometry	Relation
Aligned Parallel Rectangles (Figure 13.4) 	$\bar{X} = X/L,\ \bar{Y} = Y/L$ $F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{ \ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y} \right\}$
Coaxial Parallel Disks (Figure 13.5) 	$R_i = r_i/L,\ R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$
Perpendicular Rectangles with a Common Edge (Figure 13.6) 	$H = Z/X,\ W = Y/X$ $F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$

***Figure A1**: Table 13.2 from "Fundamentals of Heat Transfer"*

- α is the absorptivity of a surface measured from 0 to 1
- A is the area of each associated surface such as A_{foi} for the area of the foil shroud
- D is the diameter of a circular surface
- ε is the emissivity of a surface measured from 0 to 1
- F_{ij} is the view factor between two surfaces i and j
- H is the distance between the stove and the pot and varies from 10 *mm* to 300 *mm*
- J is the radiosity from a given surface measured in W/m^2, useful when calculating net radiation
  q is the heat transfer into a surface measured in W
- R is the radius of a component of the cooking setup, useful when calculating areas of surfaces or view factors
- ρ is the reflectivity of a surface measured from 0 to 1
- S is a placeholder used when substituting terms into the view factor equations for coaxial parallel disks

- σ is the Stefan-Boltzmann constant $\sigma = 5.67*10^{-8}$ W/m^{2}K
- τ is the transmissivity of a surface measured from 0 to 1
- T is the temperature of a given surface in K