Computational Assignment 4

MEGN 471: Heat Transfer - Spring 2023

Michael Allen, Cullen Hirstius, Hayden Payne

5/3/2023

Introduction

Convective heat transfer analysis was used to determine the optimal storage orientation for beer cans given different can dimensions based on varying heat transfer coefficient (h) values. Cans can either be stored vertically or horizontally in a fridge with controlled air temperature and no forced convection. Can height varies from 5 *cm* to 30 *cm* and diameter varies from 2 *cm* to 10 *cm* in the horizontal orientation. The primary method for computational analysis was MATLAB code to compile equations and calculate results.

Methodology

To calculate the convective heat transfer coefficient, the team used the following constant properties:

Parameter	Value	Meaning
T_s	300 K	Can surface temperature
T_∞	277 K	Fluid temperature
T_f	288.5 K	Film temperature
α	$20.982*10^{-6}$ m^2/s	Thermal diffusivity
ρ	1.125 kg/m^3	Density
μ	$178.85*10^{-7}$ $N*s/m^2$	Dynamic viscosity
k	$14.867*10^{-6}$ W/m*K	Kinematic viscosity
Pr	0.71	Prandtl number
β	$3.466*10^{-3}$ 1/K	Volumetric thermal expansion coefficient
g	9.81 m/s^2	Acceleration due to gravity

Table 1: *constant fluid properties for the system*

These constants were used in the following set of equations to calculate the Grashof number (Gr), Nusselt number (Nu), and convective heat transfer coefficient given varying height values. The equations for the Rayleigh number (*Ra*) and $Check_v$ can be used to check the bounds of validity for using the *Nu* equation in this scenario.

$$Gr_v = \frac{(g*\beta*(T_s - T_\infty)*H^3)}{v^2}\ ; \qquad Ra_v = Gr_v * Pr\ ; \qquad Check_v = \frac{D}{H} - \frac{35}{Gr_v^{\ 0.25}};$$

$$Nu_v = \frac{4}{3} * (\frac{Gr_v}{4})^{1/4} * \frac{0.75*Pr^{1/2}}{(0.609+1.221*Pr^{1/2}+1.238*Pr)^{1/4}}\ ; \qquad h_v = \frac{Nu_v*k}{H}\ ;$$

Equations 1-5: *set of equations used for calculating convective heat transfer coefficient for cans in vertical orientation*

Equations 6-9 below are required to calculate *Gr*, *Nu*, *Ra*, and convective heat transfer coefficient for cans stacked in a horizontal orientation. The relationship between the fluid properties and the value of the convective heat transfer coefficient depend on the untiless parameters *Gr* and *Ra*.

$$Gr_h = \frac{(g*\beta*(T_s - T_\infty)*D^3)}{v^2}\ ; \qquad Ra_h = Gr_h * Pr\ ; \qquad Nu_h = 0.6 + (\frac{0.387*Ra_h^{\ 1/6}}{(1+(0.559/Pr)^{9/16})^{8/27}})^2\ ;$$

$$h_h = \frac{Nu_h*k}{D}\ ;$$

Equations 6-9: *set of equations used for calculating convective heat transfer coefficient for cans in horizontal orientation*

For vertically oriented cans, the characteristic length along which heat transfer occurs is the height of the can. For horizontally oriented cans, the characteristic length is the diameter of the can. In both of these scenarios, the respective characteristic length is parallel with gravity and thus the length over which natural convection occurs.

Calculating *Ra* in both of these scenarios checks the bounds of validity regarding turbulent and laminar flow. In the vertical orientation, *Ra* varies from roughly $2*10^4$ to $6.7*10^7$. Assuming that an *Ra* value of around 10^9 indicates a transition from laminar to turbulent flow, the air flow in the vertical configuration can be approximated as laminar. The *Ra* values calculated for the horizontal configuration vary from $2*10^4$ to $2.51*10^6$, indicating that the air flow in this system can be also approximated as linear. This, however, does not mean that the results calculated using the given equation for *Nu* will provide accurate results. The value given by the $Check_v$ should be positive for a given configuration of diameter and height in order to use equation 4. Some values for $Check_v$ are slightly negative and thus may not yield accurate results when using this equation, namely when the aspect ratio of can height to diameter was close to one. The values of *h* calculated given these dimensions may be inaccurate.

Results and Discussion

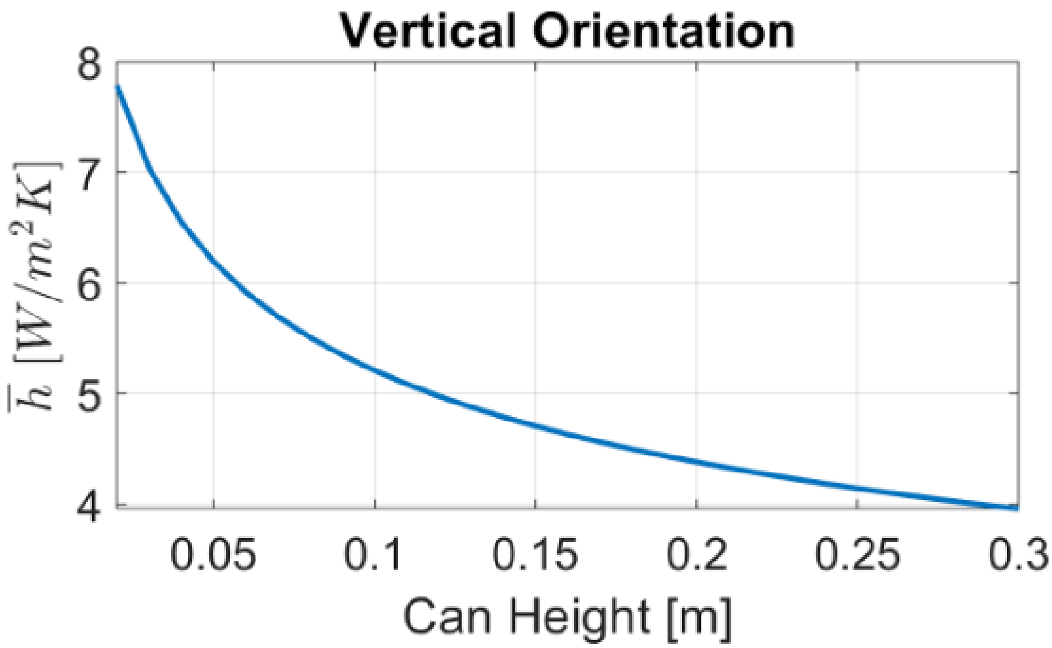

Figure 1: *Convection Coefficient vs Can Height in Vertical Orientation*

Figure 1 above shows how changing the can height in the vertical orientation changes the convection coefficient. By increasing the height, the convection coefficient decreases. This is due to the heated air rising against the can's surface which results in worse heat transfer (lower temperature delta and more developed flow) towards the top of the can.

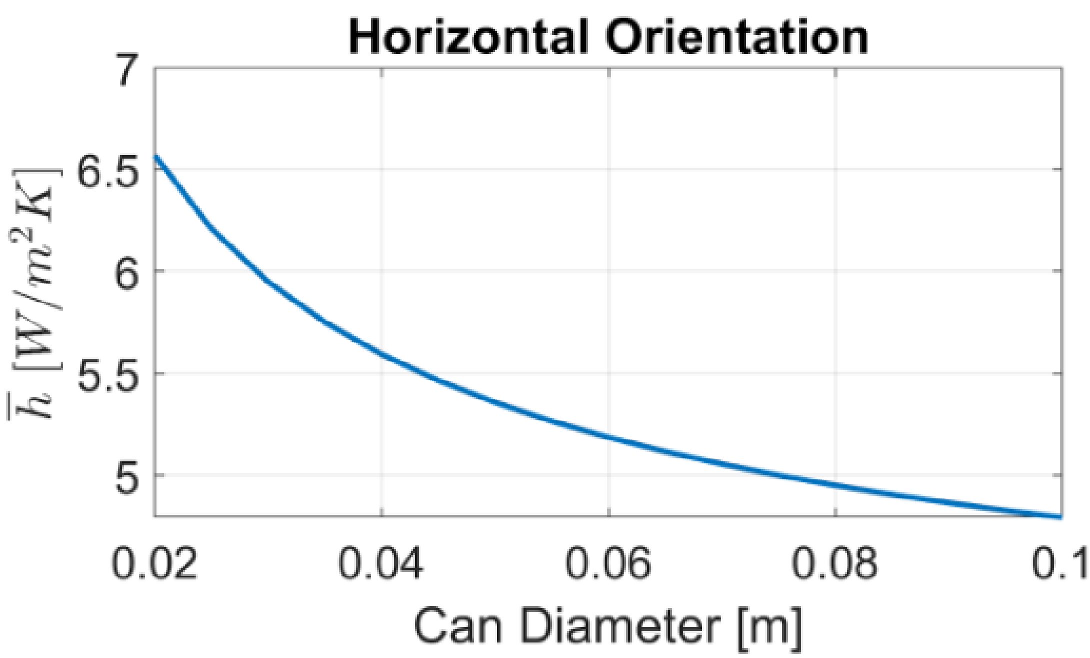

Figure 2: *Convection Coefficient vs Can Diameter in Horizontal Orientation*

Figure 2 shows how changing the can diameter in the horizontal orientation changes the convection coefficient. By increasing the diameter of the can, the convection coefficient decreases. This is due to the larger can further hindering the air flow from the downward facing section which results in poor heat transfer around the lower side up towards the midplane of the can (due to a lower temperature delta and higher flow development in the region).

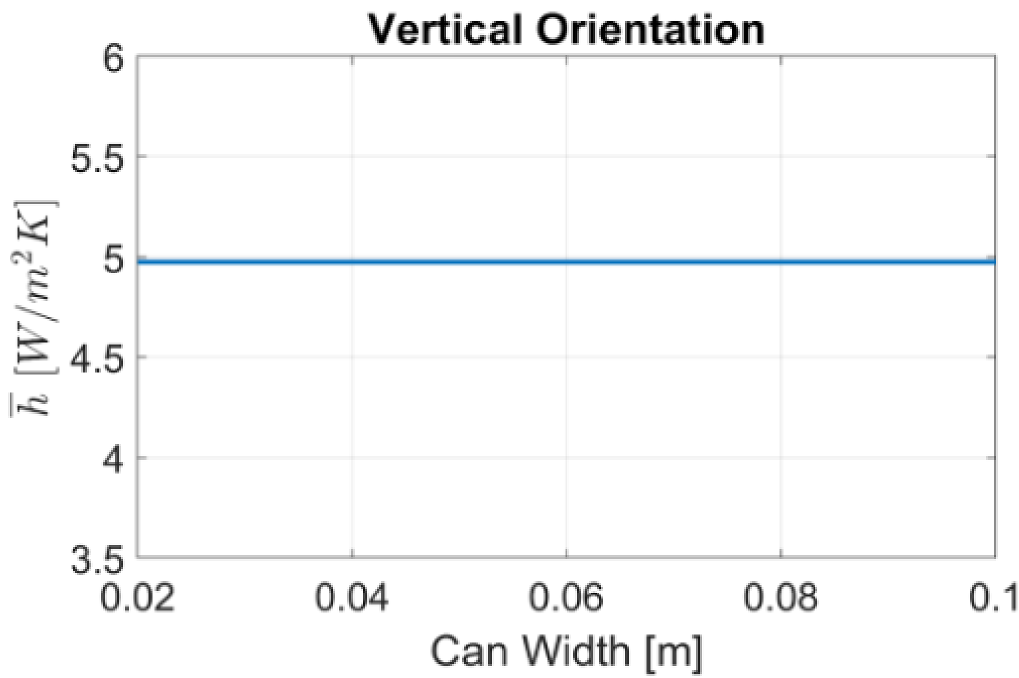

Figure 3: *Convection Coefficient vs Can Diameter in Vertical Orientation*

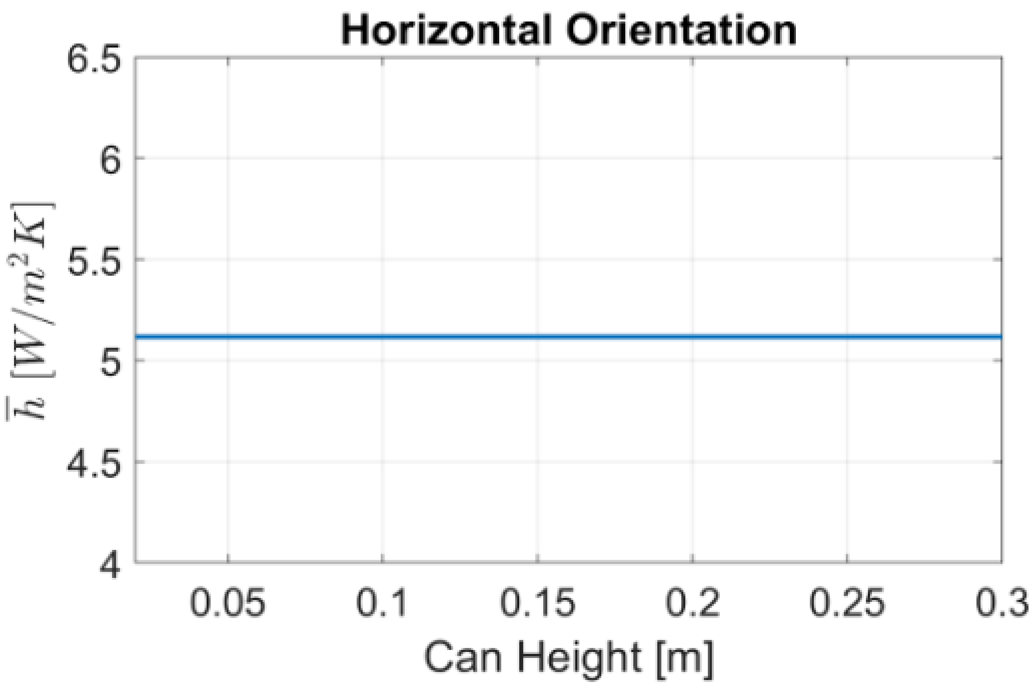

Figure 4: *Convection Coefficient vs Can Height in Horizontal Orientation*

By changing the diameter in the vertical orientation, the cross sectional shape that the convection currents are forming on is not changing. Basically, despite the can growing in size, the shape of the 2-dimensional convection current at a vertical cross section on the can will remain the same (resembling natural convection on one side of a vertical plate). The result is that the heat flux from the convection will not change. Likewise, for the horizontal can, changing the height/length will not change the geometry of the convection current. No matter how long the can is, the currents will look the same at all cross sections taken along the can's length. Therefore, adding more length will not change the heat flux.

Container Type (D/H, mm)	Vertical h (W/m^2*K)	Horizontal h (W/m^2*K)
Beer Car (65/120)	4.975	5.117
Wine Bottle (80/280)	4.025	4.951
Tall Boy (65/150)	4.705	5.117
Seltzer (55/160)	4.630	5.266

Table 2: *Convection Coefficient Values for Different Containers and Orientations*

The table above contains the average convection coefficient data for the beer can in the vertical and horizontal positions. From these values, it is plain to see that storing the cans horizontally will result in faster cooling of the beer due to the higher convection coefficient. This trend of horizontal orientations being better for cooling continues for all the other containers, however the largest impact is seen for the wine bottle. This is most likely due to the extremely tall nature of the bottle which lends itself to poor convection in the vertical orientation. By turning it horizontally, the poor convection from its large height is mitigated (but the diameter is still relatively large so the coefficient is still lower than all the other containers).

By increasing the characteristic length of the cylinder, the convection coefficient will decrease. This can easily be explained using the vertical can; by increasing the height of the can, the convection current will maintain contact with the can for a longer period of time during its ascent. As the fluid ascends, the temperature profile develops further resulting in worse heat transfer as it approaches the top. The result is a lower average convection coefficient as the worse heat transfer negatively impacts the average values resulting in them decreasing. The same goes for the horizontal scenario, but the convection currents around a cylinder are more complex than those along a flat plate.

Conclusion

Natural convection can be majorly impacted by the aspect ratios of the shapes it occurs on. By changing the diameter and height of a cylinder, the average convection coefficient can change by a significant amount; up to 25% in some cases. Additionally, the orientation of the cylinders matters as it changes how natural convection acts on the cylinder. This causes there to be a difference in the convection coefficient between a vertically and horizontally placed can, with the later having a higher coefficient. As such, orienting cans horizontally in a fridge will cause them to cool faster than if they are stored vertically (although they may roll out when the door is opened).

Group Contribution

Michael Allen:

Michael wrote the MATLAB code to calculate and plot the data used in this study.

Cullen Hirstius:

Cullen wrote the results/discussion and conclusion sections.

Hayden Payne:

Hayden wrote the introduction and methodology sections.

Appendix

Figures generated by MATLAB. Source code posted on GitHub:
https://github.com/Player-II/Computational_Heat_Transfer_4