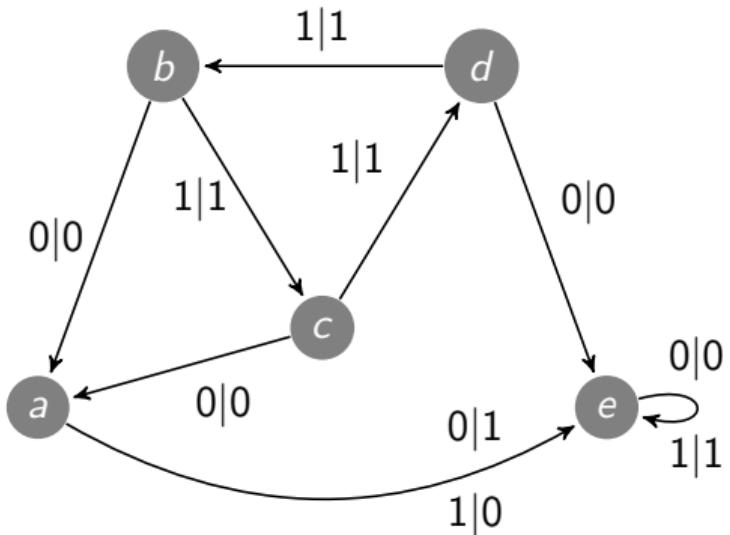


Decision problems in automaton (semi)groups

Marie Coutureel IRIF

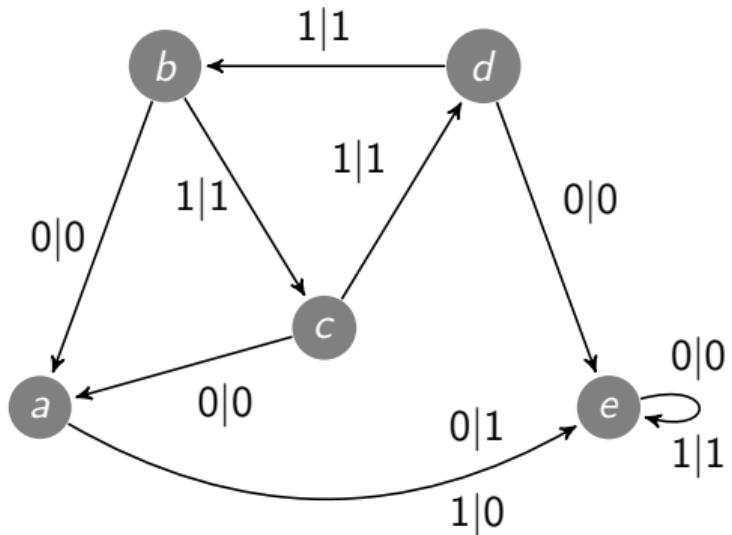
November 27, 2025

Motivation



Grigorchuk automaton

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Grigorchuk automaton

Decision problems

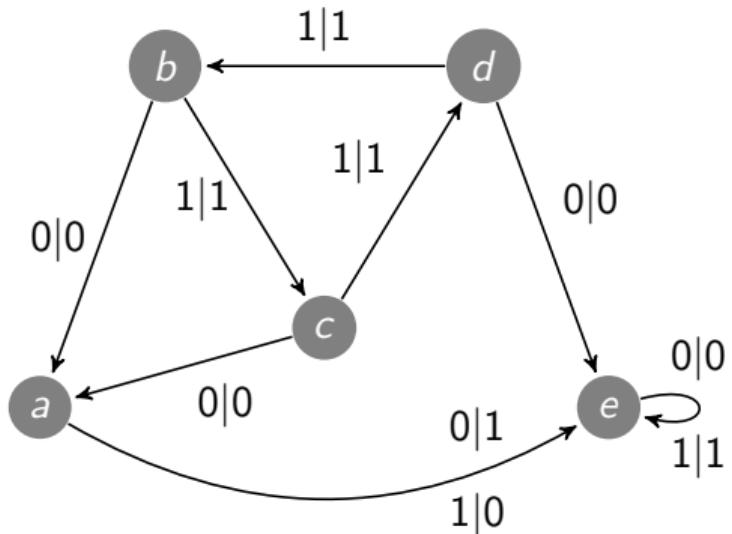
for automata semigroups

decidable
word

?
transitivity

undecidable
order
finiteness
liberty

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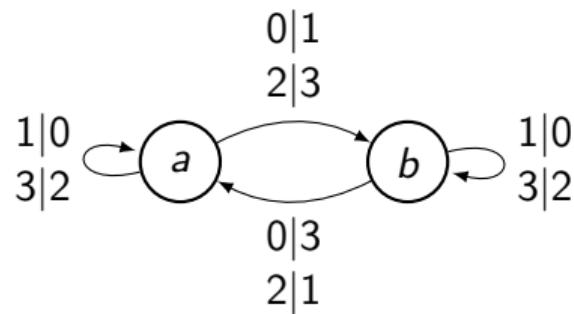
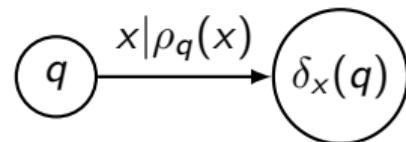
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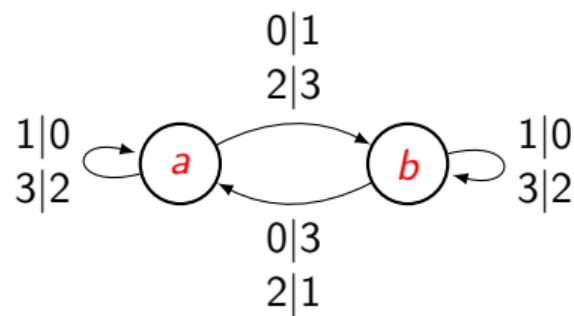
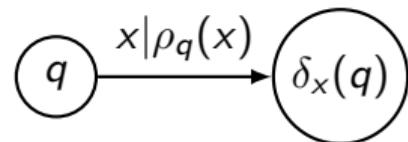
Mealy Automata

$$\mathcal{M} = (Q, \Sigma, \delta, \rho)$$



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Inversibility

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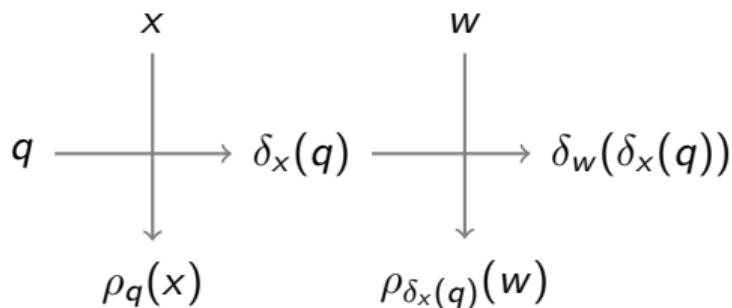


Actions and generated (semi)groups

$$\mathcal{M} = (Q, \Sigma, \delta, \rho)$$

$$\rho_q : \Sigma^* \longrightarrow \Sigma^*$$

$$\rho_q(xw) = \rho_q(x)\rho_{\delta_x(q)}(w)$$

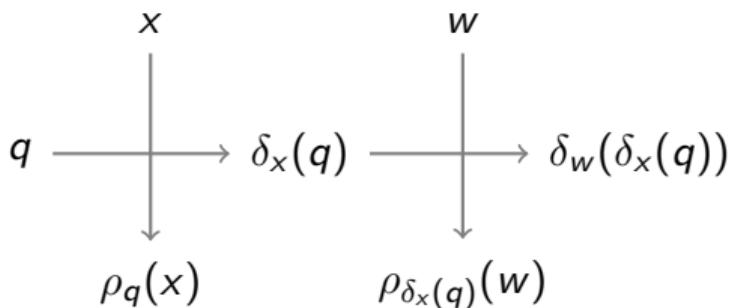


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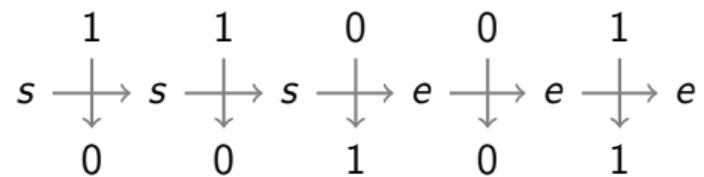
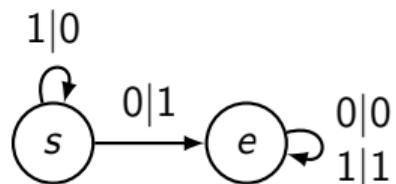
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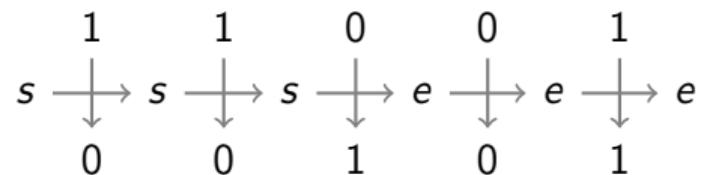
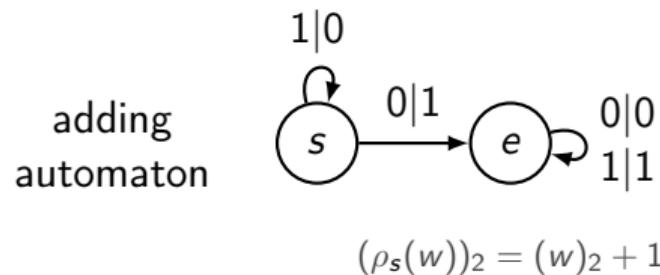


We consider the *semigroup* $\langle \mathcal{M} \rangle_+$ generated by $\{\rho_q, q \in Q\}$.

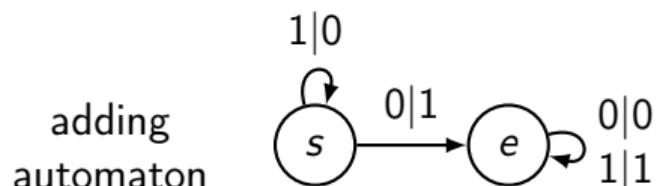
The adding automaton



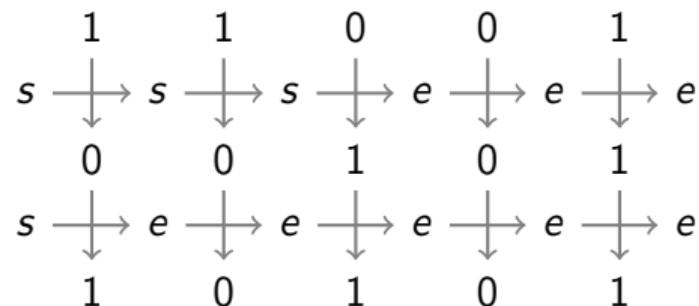
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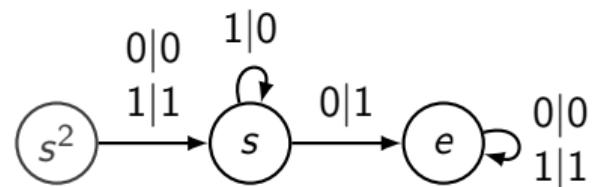
The adding automaton



$$\begin{aligned}(\rho_s(w))_2 &= (w)_2 + 1 \\ (\rho_{s^2}(w))_2 &= (w)_2 + 2\end{aligned}$$

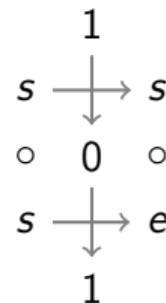


The adding automaton

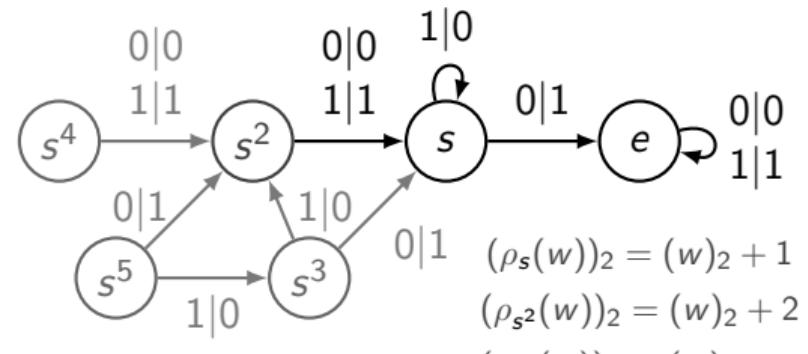


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The adding automaton



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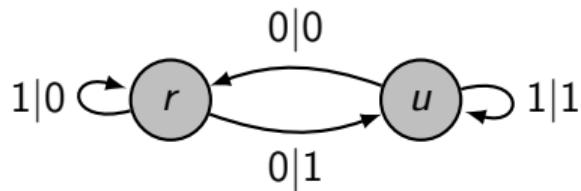
$$(\rho_{s^2}(w))_2 = (w)_2 + 2$$

$$(\rho_{s^n}(w))_2 = (w)_2 + n$$

$$(\rho_{s^{-n}}(w))_2 = (w)_2 - n$$

$$\langle s \rangle \cong (\mathbb{Z}, +)$$

The Lamplighter automaton



$$\langle a, t | a^2, [t^n a t^{-n}, t^m a t^{-m}], n, m \in \mathbb{Z} \rangle$$



Motivation

Decision problems

for automata semigroups

decidable	?	undecidable
word	transitivity	order finiteness liberty

for automata groups

decidable	?	undecidable
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Mealy automaton and automatic sequences

Theorem [Grigorchuk, Savchuk] (2020)

A 1-Lip map from Σ^* to Σ^* could be generated by a Mealy automaton if and only if the reduced coefficients of its Van de Put series constitute a $|\Sigma|$ -automatic sequence.

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PHD : Algorithmics of automata (semi)groups : dynamic and combinatoric approaches.

Under the supervision of Matthieu Picantin and Olivier Carton.