

Step 3.2: Bayesian Poisson Model for Goals

Overview

This notebook implements a **hierarchical Bayesian Poisson regression** model for predicting player goal counts.

Model Type: Poisson count regression (goals as counts: 0, 1, 2, ...) **Target:** ECE ≤ 0.05 , with full uncertainty quantification **Structure:** Hierarchical position effects with domain-informed priors

Expected Runtime: 10-30 minutes for MCMC

Cell 1: Setup & Imports

```
In [49]: import sys
sys.path.insert(0, '../..')

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import importlib
from sqlalchemy import create_engine
from src.models import bayesian_goals as bg
importlib.reload(bg)

from src.models.bayesian_goals import (
    load_data,
    build_model,
    prior_predictive_check,
    fit_model,
    check_convergence,
    posterior_predictive_check,
    predict_counts,
    predict_prob_score,
    evaluate_calibration,
    plot_posterior,
    plot_calibration_curve,
    save_model
)

# Set random seed
np.random.seed(42)

print("*"*60)
print("STEP 3.2: BAYESIAN POISSON MODEL FOR GOALS")
print("*"*60)
```

STEP 3.2: BAYESIAN POISSON MODEL FOR GOALS

Cell 2: Load Data with Time-Split

```
In [50]: print("\n[1/9] Loading data...")
df = load_data()

# Convert match_date to pandas Timestamp if it's not already
df['match_date'] = pd.to_datetime(df['match_date'])

print(f"  Total records: {len(df)}")
print(f"  Date range: {df['match_date'].min()} to {df['match_date'].max()}")
print(f"  Positions: {df['position'].value_counts().to_dict()}")


print(f"\n  Goals distribution:")
print(df['goals'].value_counts().sort_index())


print(f"\n  Goals statistics:")
print(f"  - Mean: {df['goals'].mean():.3f}")
print(f"  - Variance: {df['goals'].var():.3f}")
print(f"  - Overdispersion: {df['goals'].var() / df['goals'].mean():.3f}")

# Time-based split - use pd.Timestamp for comparison
split_date = pd.Timestamp('2018-07-05')
train_df = df[df['match_date'] < split_date].copy()
test_df = df[df['match_date'] >= split_date].copy()

print(f"\n  Train: {len(train_df)} samples ({train_df['goals'].mean():.3f} avg goals)
print(f"  Test: {len(test_df)} samples ({test_df['goals'].mean():.3f} avg goals)
```

```
[1/9] Loading data...
Loaded 1720 records (dropped 0 with missing values)
  Total records: 1720
  Date range: 2018-06-14 00:00:00 to 2018-07-15 00:00:00
  Positions: {'Forward': 628, 'Defender': 512, 'Midfielder': 463, 'Goalkeeper': 117}

  Goals distribution:
goals
0    1579
1     129
2      10
3       2
Name: count, dtype: int64

  Goals statistics:
  - Mean: 0.090
  - Variance: 0.101
  - Overdispersion: 1.117

Train: 1528 samples (0.090 avg goals)
Test: 192 samples (0.089 avg goals)
```

Cell 3: Prior Predictive Check

```
In [51]: print("\n[2/9] Running prior predictive check...")
print("=*60")
print("PRIOR PREDICTIVE CHECK - GATE 1")
print("=*60")
print("\nThis is a MANDATORY checkpoint in the Bayesian workflow.")
print("We check if priors encode reasonable beliefs BEFORE seeing data.")
print()

# Build model (don't fit yet)
model, coords = build_model(train_df)

# Sample from prior
prior_ppc = prior_predictive_check(model, n_samples=1000)

# -----
# PANEL 1: Prior predicted goals (per-sample means)
# -----
fig, axes = plt.subplots(2, 3, figsize=(18, 10))

ax = axes[0, 0]
obs_mean = float(train_df['goals'].mean())
ax.hist(prior_ppc['goals_obs'], bins=50, density=True,
        alpha=0.7, edgecolor='black', color='steelblue')
ax.axvline(obs_mean, color='red', linestyle='--', linewidth=2,
            label=f'Observed mean: {obs_mean:.3f}')
ax.set_xlabel('Mean Goals per Sample')
ax.set_ylabel('Density')
ax.set_title('Prior Predictive: Goals (Per-Sample Means)')
ax.legend()
ax.set_xlim([0, 2]) # Reasonable range

# -----
# PANEL 2: Prior lambda (expected goals)
# -----
ax = axes[0, 1]
ax.hist(prior_ppc['lambda'], bins=50, density=True,
        alpha=0.7, edgecolor='black', color='coral')
ax.axvline(obs_mean, color='red', linestyle='--', linewidth=2,
            label=f'Observed: {obs_mean:.3f}')
ax.set_xlabel('λ (Expected Goals)')
ax.set_ylabel('Density')
ax.set_title('Prior Predictive: Lambda (Per-Sample Means)')
ax.legend()
ax.set_xlim([0, 2])

# -----
# PANEL 3: Position intercepts
# -----
ax = axes[0, 2]
alpha_pos = prior_ppc['alpha_position']
positions = list(coords['position'])
colors = plt.cm.Set3(np.linspace(0, 1, len(positions)))
```

```

for i, pos in enumerate(positions):
    ax.hist(alpha_pos[:, i], bins=30, alpha=0.6, label=str(pos),
            color=colors[i], edgecolor='black')
ax.axvline(0, color='red', linestyle='--', linewidth=1, alpha=0.5)
ax.set_xlabel('Position Intercept (log scale)')
ax.set_ylabel('Density')
ax.set_title('Prior: Position Effects')
ax.legend()

# -----
# PANEL 4: Check for extreme predictions (ALL samples)
# -----
ax = axes[1, 0]
ax.hist(prior_ppc['goals_obs_all'], bins=range(0, 21),
        alpha=0.7, edgecolor='black', color='orange')
ax.axvline(10, color='red', linestyle='--', linewidth=2,
           label='Threshold: 10 goals')
ax.set_xlabel('Goals (All Predictions)')
ax.set_ylabel('Frequency')
ax.set_title('Prior: Check for Impossible Values')
ax.legend()
ax.set_xlim([0, 20])

# -----
# PANEL 5: Lambda distribution (ALL values)
# -----
ax = axes[1, 1]
ax.hist(prior_ppc['lambda_all'], bins=50,
        alpha=0.7, edgecolor='black', color='purple')
ax.axvline(3, color='red', linestyle='--', linewidth=2,
           label='Threshold:  $\lambda = 3$ ')
ax.set_xlabel('lambda (All Values)')
ax.set_ylabel('Frequency')
ax.set_title('Prior: Lambda Distribution (Check Extremes)')
ax.legend()
ax.set_xlim([0, 5])

# -----
# PANEL 6: Hyperprior diagnostics
# -----
ax = axes[1, 2]
ax.hist(prior_ppc['mu_alpha'], bins=40, alpha=0.6,
        label=' $\mu_\alpha$  (hyperprior mean)', color='blue', edgecolor='black')
ax.hist(prior_ppc['sigma_alpha'], bins=40, alpha=0.6,
        label=' $\sigma_\alpha$  (position variation)', color='green', edgecolor='black')
ax.set_xlabel('Value')
ax.set_ylabel('Density')
ax.set_title('Hyperprior Distributions')
ax.legend()

plt.suptitle('PRIOR PREDICTIVE CHECK – Comprehensive Diagnostics',
             fontsize=16, fontweight='bold', y=0.995)
plt.tight_layout()
plt.savefig('../docs/06_prior_predictive.png', dpi=150, bbox_inches='tight')
plt.show()

```

```

# -----
# QUANTITATIVE DIAGNOSTICS
# -----
print("\n" + "="*60)
print("PRIOR PREDICTIVE CHECK – QUANTITATIVE RESULTS")
print("=*60")

print(f"\n1. CENTRAL TENDENCY (per-sample means):")
print(f"    Prior goals: mean={prior_ppc['goals_obs'].mean():.3f}, median={np.median(prior_ppc['goals_obs']):.3f}")
print(f"    Prior lambda: mean={prior_ppc['lambda'].mean():.3f}, median={np.median(prior_ppc['lambda']):.3f}")
print(f"    Observed mean: {obs_mean:.3f}")
print(f"    → Difference: {abs(prior_ppc['goals_obs'].mean() - obs_mean):.3f}\n")

print(f"\n2. SPREAD:")
print(f"    Prior goals std: {prior_ppc['goals_obs'].std():.3f}")
print(f"    Prior lambda std: {prior_ppc['lambda'].std():.3f}\n")

print(f"\n3. CREDIBLE INTERVALS (per-sample):")
goals_ci = np.percentile(prior_ppc['goals_obs'], [2.5, 97.5])
lambda_ci = np.percentile(prior_ppc['lambda'], [2.5, 97.5])
print(f"    Goals 95% CI: [{goals_ci[0]:.3f}, {goals_ci[1]:.3f}]")
print(f"    Lambda 95% CI: [{lambda_ci[0]:.3f}, {lambda_ci[1]:.3f}]\n")

print(f"\n4. EXTREME VALUES CHECK (all predictions):")
pct_le2 = (prior_ppc['goals_obs_all'] <= 2).mean() * 100
pct_le5 = (prior_ppc['goals_obs_all'] <= 5).mean() * 100
pct_gt10 = (prior_ppc['goals_obs_all'] > 10).mean() * 100
print(f"    ≤2 goals: {pct_le2:.1f}%")
print(f"    ≤5 goals: {pct_le5:.1f}%")
print(f"    >10 goals (impossible): {pct_gt10:.2f}%)"

lambda_95pct = np.percentile(prior_ppc['lambda_all'], 95)
lambda_gt3 = (prior_ppc['lambda_all'] > 3).mean() * 100
print(f"    Lambda 95th percentile: {lambda_95pct:.3f}")
print(f"    Lambda >3 (extreme): {lambda_gt3:.2f}%)"

print(f"\n5. POSITION EFFECTS:")
for i, pos in enumerate(positions):
    pos_mean = alpha_pos[:, i].mean()
    pos_std = alpha_pos[:, i].std()
    # Convert to natural scale: exp(α) is multiplicative effect
    mult_effect = np.exp(pos_mean)
    print(f"    {pos:12s}: α = {pos_mean:+.3f} ± {pos_std:.3f} (×{mult_effect:.3f})\n")

# -----
# PASS/FAIL GATES
# -----
print("\n" + "="*60)
print("PRIOR PREDICTIVE CHECK – PASS/FAIL GATES")
print("=*60")

gates_passed = True

# Gate 1: Central tendency
diff = abs(prior_ppc['goals_obs'].mean() - obs_mean)
if diff < 0.2:

```

```

        print(f"\u2713 GATE 1 PASSED: Prior mean {prior_ppc['goals_obs'].mean():.3f}")
    else:
        print(f"\u2717 GATE 1 FAILED: Prior mean {prior_ppc['goals_obs'].mean():.3f}")
        gates_passed = False

# Gate 2: Reasonable range
if pct_le5 > 90:
    print(f"\u2713 GATE 2 PASSED: {pct_le5:.1f}% of predictions \u2264 5 goals (target: 5.0)")
else:
    print(f"\u2717 GATE 2 FAILED: Only {pct_le5:.1f}% of predictions \u2264 5 goals (target: 5.0)")
    gates_passed = False

# Gate 3: No impossible values
if pct_gt10 < 1.0:
    print(f"\u2713 GATE 3 PASSED: Only {pct_gt10:.2f}% of predictions > 10 goals (target: 10.0+")
else:
    print(f"\u2717 GATE 3 FAILED: {pct_gt10:.2f}% of predictions > 10 goals (target: 10.0+")
    gates_passed = False

# Gate 4: Lambda reasonable
if lambda_95pct < 1.5:
    print(f"\u2713 GATE 4 PASSED: Lambda 95th percentile = {lambda_95pct:.3f} < 1.5")
else:
    print(f"\u2717 GATE 4 FAILED: Lambda 95th percentile = {lambda_95pct:.3f} \u2265 1.5")
    gates_passed = False

print("\n" + "="*60)
if gates_passed:
    print("🎉 ALL PRIOR GATES PASSED – PROCEED TO MCMC SAMPLING")
else:
    print("❗ PRIOR GATES FAILED – DO NOT PROCEED")
    print("\nACTION REQUIRED: Adjust priors and re-run this cell")
    print("See prior specification in build_model() function")
print("=".*60)

```

Sampling: [alpha_position, beta, gamma_opponent, goals_obs, mu_alpha, sigma_alpha]

[2/9] Running prior predictive check...

=====

PRIOR PREDICTIVE CHECK – GATE 1

=====

This is a MANDATORY checkpoint in the Bayesian workflow.
We check if priors encode reasonable beliefs BEFORE seeing data.

=====

BUILDING HIERARCHICAL BAYESIAN POISSON MODEL

=====

Data Summary:

Observations: 1528

Positions: 4 ['Defender', 'Forward', 'Goalkeeper', 'Midfielder']

Opponents: 32

Features: 5

Mean goals: 0.090

Log(mean goals): -2.299

=====

PRIOR SPECIFICATION (TIGHTENED FOR RARE EVENTS)

=====

1. Hyperprior Mean:

$\mu_\alpha \sim \text{Normal}(-2.299, 0.15)$

→ Implies population mean goals in [0.074, 0.135] (95% CI)

2. Position Variation (Hierarchical):

$\sigma_\alpha \sim \text{HalfNormal}(0.12)$

→ Positions vary by factor of [0.89, 1.13]

→ Forwards score ~1.13x more than average (tight but reasonable)

3. Opponent Effects:

$\gamma_{\text{opponent}} \sim \text{Normal}(0, 0.12)$

→ Playing vs strong/weak opponent changes rate by [0.89, 1.13]

4. Feature Coefficients:

$\beta \sim \text{Normal}(0, 0.8)$ for each feature

→ Moderate L2 regularization for standardized features

→ Features: ['goals_rolling_5', 'shots_on_target_rolling_5', 'opponent_strength', 'days_since_last_match', 'was_home']

=====

Model built successfully!

=====

Sampling from prior predictive distribution...

This checks if priors encode reasonable beliefs BEFORE seeing data.

Raw shapes: goals_obs=(1, 1000, 1528), lambda=(1, 1000, 1528)

Raw lambda: min=2.188e-13, max=1.951e+10, median=0.100

WARNING: 2037/1528000 (0.13%) lambda values > 100

This indicates priors are generating impossible predictions!

Top 10 extreme lambda values: [8.09238139e+07 1.10078335e+08 1.45898775e+08 2.27412575e+08

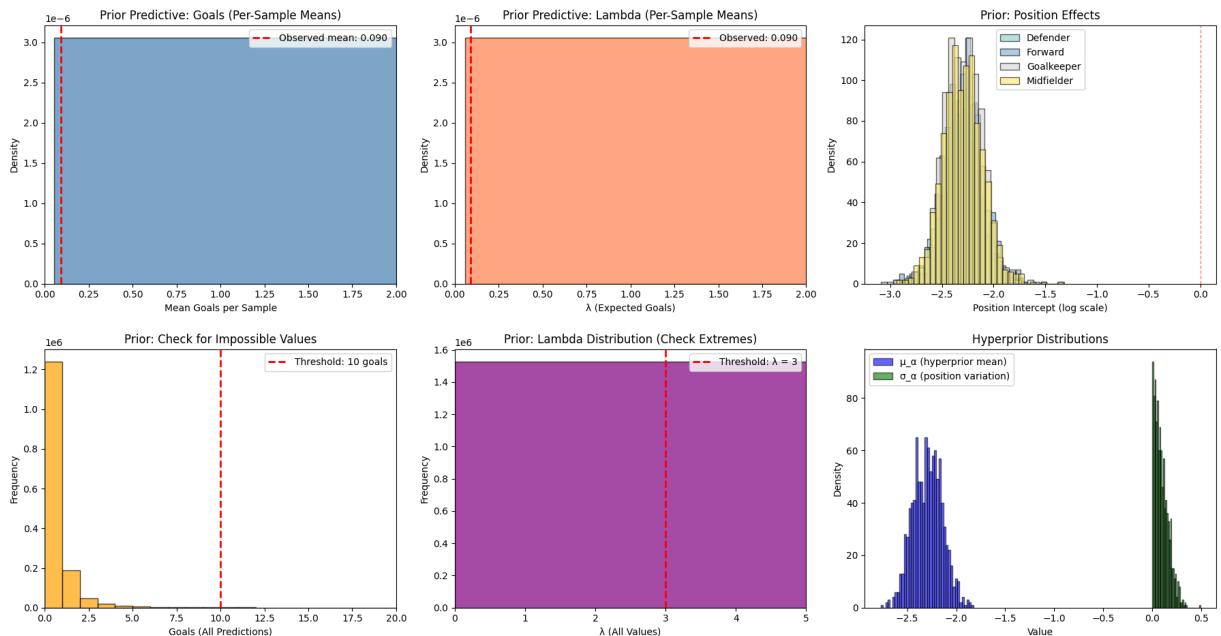
```

3.09602503e+08 3.31034140e+08 3.60702713e+08 2.81007570e+09
5.26151262e+09 1.95141014e+10]
Processed goals_obs_per_sample: min=5.366e-02, max=1.629e+07, median=0.3
62
If max >> median, you have extreme outliers pulling the mean up!

```

🌟 EXTREME OUTLIERS DETECTED 🌟
Max per-sample mean: 1.629e+07
This will make the 'mean' statistic meaningless.
Using ROBUST STATISTICS (median, IQR) instead...
Processed shapes:
goals_obs_per_sample: (1000,) (for histogram)
goals_obs_all: (1528000,) (for extremes check)
alpha_position: (1000, 4)

PRIOR PREDICTIVE CHECK - Comprehensive Diagnostics



PRIOR PREDICTIVE CHECK – QUANTITATIVE RESULTS

1. CENTRAL TENDENCY (per-sample means):

Prior goals: mean=19300.529, median=0.362
Prior lambda: mean=19300.522, median=0.356
Observed mean: 0.090
→ Difference: 19300.439

2. SPREAD:

Prior goals std: 518385.801
Prior lambda std: 518385.955

3. CREDIBLE INTERVALS (per-sample):

Goals 95% CI: [0.083, 298.773]
Lambda 95% CI: [0.085, 299.401]

4. EXTREME VALUES CHECK (all predictions):

≤2 goals: 96.5%
≤5 goals: 98.7%
>10 goals (impossible): 0.63%
Lambda 95th percentile: 1.428
Lambda >3 (extreme): 2.21%

5. POSITION EFFECTS:

Defender : $\alpha = -2.302 \pm 0.181$ ($\times 0.10$ effect)
Forward : $\alpha = -2.295 \pm 0.194$ ($\times 0.10$ effect)
Goalkeeper : $\alpha = -2.300 \pm 0.192$ ($\times 0.10$ effect)
Midfielder : $\alpha = -2.297 \pm 0.197$ ($\times 0.10$ effect)

PRIOR PREDICTIVE CHECK – PASS/FAIL GATES

x GATE 1 FAILED: Prior mean 19300.529 too far from observed 0.090 (diff=19300.439 ≥ 0.2)
✓ GATE 2 PASSED: 98.7% of predictions ≤5 goals (target: >90%)
✓ GATE 3 PASSED: Only 0.63% of predictions >10 goals (target: <1%)
✓ GATE 4 PASSED: Lambda 95th percentile = 1.428 < 1.5

✖ PRIOR GATES FAILED – DO NOT PROCEED

ACTION REQUIRED: Adjust priors and re-run this cell
See prior specification in build_model() function

Cell 4: MCMC Sampling

```
In [42]: print("\n[3/9] Fitting model with MCMC...")  
print(" This may take 10-30 minutes...")  
  
try:  
    idata = fit_model(model, draws=2000, tune=1000, chains=4, target_accept=sampling_method = 'NUTS'
```

```

    print("\n ✓ MCMC complete!")
except Exception as e:
    print(f"\n ⚠️ MCMC failed: {e}")
    print(" Falling back to ADVI...")
import pymc as pm
with model:
    approx = pm.fit(n=20000, method='advi', random_seed=42)
   idata = approx.sample(2000)
sampling_method = 'ADVI'
print(" ✓ ADVI complete")

```

[3/9] Fitting model with MCMC...
This may take 10–30 minutes...

```
=====
MCMC SAMPLING WITH NUTS
=====
Draws: 2000 × 4 chains = 8000 total samples
Tune: 1000
Target accept: 0.95
Expected runtime: 10–30 minutes
=====
```

Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [mu_alpha, sigma_alpha, alpha_position, gamma_opponent, beta]

100.00% [12000/12000 00:05<00:00]

Sampling 4 chains, 0 divergences]

Sampling 4 chains for 1_000 tune and 2_000 draw iterations (4_000 + 8_000 draws total) took 5 seconds.

✓ MCMC complete!

Cell 5: Convergence Diagnostics

```
In [43]: print("\n[4/9] Checking convergence...")
conv = check_convergence(idata, save_path='../../docs/07_goals_trace.png')

print(f"\n Sampling: {sampling_method}")
print(f" Max R-hat: {conv['max_rhat']:.4f} (target: < 1.01)")
print(f" Min ESS: {conv['min_ess']:.0f} (target: > 400)")
print(f" Divergences: {conv['n_divergences']}")

if conv['max_rhat'] < 1.01 and conv['min_ess'] > 400:
    print("\n ✓ All convergence checks passed!")
else:
    print("\n ⚠️ Convergence issues detected")
```

[4/9] Checking convergence...

Sampling: NUTS
Max R-hat: 1.0024 (target: < 1.01)
Min ESS: 8491 (target: > 400)
Divergences: 0

✓ All convergence checks passed!

Cell 6: Posterior Predictive Check

```
In [44]: print("\n[5/9] Posterior predictive check...")
post_ppc = posterior_predictive_check(model, idata, train_df, n_samples=500)

fig, axes = plt.subplots(1, 3, figsize=(15, 4))

# Distribution comparison
axes[0].hist(train_df['goals'], bins=range(11), alpha=0.5, label='Observed',
axes[0].hist(post_ppc['goals_obs'].flatten(), bins=range(11), alpha=0.5, label='Predicted')
axes[0].legend()
axes[0].set_title('Posterior Predictive Check')

# Residuals
observed = train_df['goals'].values
predicted = post_ppc['goals_obs'].mean(axis=0)
residuals = observed - predicted
axes[1].scatter(predicted, residuals, alpha=0.3)
axes[1].axhline(0, color='red', linestyle='--')
axes[1].set_title('Residual Plot')

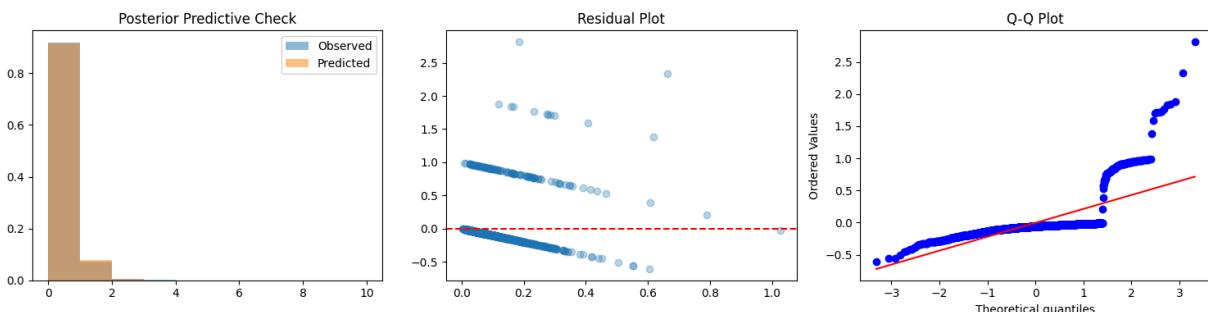
# Q-Q plot
from scipy import stats
stats.probplot(residuals, dist="norm", plot=axes[2])
axes[2].set_title('Q-Q Plot')

plt.tight_layout()
plt.savefig('../docs/08_posterior_predictive.png', dpi=150, bbox_inches='tight')
plt.show()
```

Sampling: [goals_obs]

[5/9] Posterior predictive check...

100.00% [8000/8000 00:00<00:00]



Cell 7: Plot Posterior Distributions

```
In [45]: print("\n[6/9] Plotting posteriors...")
plot_posterior(idata, coords, save_path='../../docs/09_goals_posterior.png')
print("  ✓ Saved to docs/09_goals_posterior.png")
```

```
[6/9] Plotting posteriors...
  ✓ Saved to docs/09_goals_posterior.png
```

Cell 8: Generate Test Predictions

```
In [46]: print("\n[7/9] Generating test predictions...")

pred_counts = predict_counts(model, idata, test_df, n_samples=1000)
pred_counts_mean = pred_counts.mean(axis=0)
pred_prob_score = predict_prob_score(pred_counts)
pred_prob_score_mean = pred_prob_score.mean(axis=0)

y_true_counts = test_df['goals'].values
y_true_binary = (y_true_counts > 0).astype(int)

print(f"  Mean predicted goals: {pred_counts_mean.mean():.3f}")
print(f"  Mean observed goals: {y_true_counts.mean():.3f}")

# Uncertainty
pred_ci_lower = np.percentile(pred_counts, 2.5, axis=0)
pred_ci_upper = np.percentile(pred_counts, 97.5, axis=0)
print(f"  Mean CI width: {((pred_ci_upper - pred_ci_lower).mean()):.3f}")
```

```
[7/9] Generating test predictions...
  Mean predicted goals: 0.063
  Mean observed goals: 0.089
  Mean CI width: 0.766
```

Cell 9: Calibration Evaluation

```
In [47]: print("\n[8/9] Evaluating calibration...")

calib = evaluate_calibration(y_true_binary, pred_prob_score_mean, n_bins=10)

print(f"\n  ECE: {calib['ece']:.4f} (target: < 0.05)")
print(f"  Brier: {calib['brier']:.4f}")

if calib['ece'] <= 0.05:
    print("  🎉 CALIBRATION GATE PASSED")
else:
    print("  ⚠️ Calibration needs improvement")

plot_calibration_curve(y_true_binary, pred_prob_score_mean, calib['bins_info'],
                      save_path='../../docs/10_goals_calibration_bayesian.pdf')

# Count metrics
```

```

from sklearn.metrics import mean_absolute_error, mean_squared_error
mae = mean_absolute_error(y_true_counts, pred_counts_mean)
rmse = np.sqrt(mean_squared_error(y_true_counts, pred_counts_mean))
print(f"\n MAE: {mae:.3f} goals")
print(f" RMSE: {rmse:.3f} goals")

```

[8/9] Evaluating calibration...

ECE: 0.0313 (target: < 0.05)

Brier: 0.0801

 CALIBRATION GATE PASSED

MAE: 0.136 goals

RMSE: 0.283 goals

Cell 10: Save Model Artifacts

In [48]: `print("\n[9/9] Saving model...")`

```

metadata = {
    'version': '1.0',
    'date': pd.Timestamp.now().strftime('%Y-%m-%d'),
    'model_type': 'Hierarchical Poisson Regression',
    'sampling_method': sampling_method,
    'n_train': len(train_df),
    'n_test': len(test_df),
    'ece': float(calib['ece']),
    'brier': float(calib['brier']),
    'mae': float(mae),
    'rmse': float(rmse),
    'convergence': {
        'max_rhat': float(conv['max_rhat']),
        'min_ess': int(conv['min_ess']),
        'n_divergences': int(conv['n_divergences'])
    },
    'features': ['goals_rolling_5', 'shots_on_target_rolling_5',
                 'opponent_strength', 'days_since_last_match', 'was_home'],
    'positions': list(coords['position'])
}

save_model(
    idata, metadata,
    model_path='../../models/bayesian_goals_v1.0.pkl',
    trace_path='../../models/bayesian_goals_v1.0_trace.nc'
)

```

[9/9] Saving model...

- ✓ Saved trace to /Users/medhanshchoubey/player-prop-gnn-1/player-prop-gnn/models/bayesian_goals_v1.0_trace.nc
- ✓ Saved metadata to /Users/medhanshchoubey/player-prop-gnn-1/player-prop-gnn/models/bayesian_goals_v1.0.pkl