Directions. Here are some questions to help you prepare for the individual portion of the competition. We will be testing conceptual and procedural understanding of computer number systems, computer operations, functions, data structures, algorithms, and reading pseudocode. The problems below are compiled from an array (programming joke) of sources and should serve as good practice for the day-of. We recommend using books or online resources to brush up on topics you don't know. We will try posting a solution key as soon as possible.

Problem 1. Which is *not* true about system buses?

a. The control bus can be read and written to by CPU, memory, and I/O
b. Unlike the data bus, the control bus is bidirectional
c. USB and PCI are commonly used expansion buses
d. Only one device can read/write to the data bus at a given time

Problem 2. What does URL stand for?

a. Universal Registry Lookup
b. Uniform Resource Locator
c. Universal Resource Link
d. Uniform Registered Link

Problem 3. RGB color values are represented using hex values from 0 to 255 in the format #[R][G][B]. For instance #00 00 00 represents black and #FF FF FF represents white. What does #00 FF FF represent?

a. Navy blue
b. Cyan
c. Magenta
d. Green

Problem 4. Which of the following is correct regarding stacks and queues?

a. A stack is vertical and a queue is horizontal
b. Popping an element from a stack returns the last element pushed to the stack
c. Pushing an element to a queue removes the first element in the queue
d. There is no difference between stacks and queues

Problem 5. Given the head of a singly linked-list with more than three elements, the code to to remove the third element in the list is

a. head.next = head.next.next; head.previous = null;
b. list[1] = null;
c. head.next.next = head.next.next.next;
d. head.next.next.data = null;

Problem 6. Find f(8) given f(0) = 3, f(1) = 4, and f(x) = f(x - 1) + f(x - 2) when x > 1.

Problem 7. $f(x, y) = f((x + a)^2, (y - 4)^2) + b$ when $x \leq y$ and $x + y$ otherwise. If a > 0, f(2, 3) = 19, and f(9, 9) = 148, find a and b.

Problem 8. What is $717_8 - 1011_2$, expressed in decimal?

Problem 9. What is the probability that a decimal number from 0-100 has exactly four 1s in its binary representation?

Problem 10. What is the first number greater than 0 for which the number and its double can be represented in hex using only letters?

Problem 11. Find the smallest possible value of n, where n > 7 and where the sum of the digits in n_8 = the product of the digits in n_{16}.

Problem 12. If an unconnected graph has 20 edges, it must have at least how many vertices?

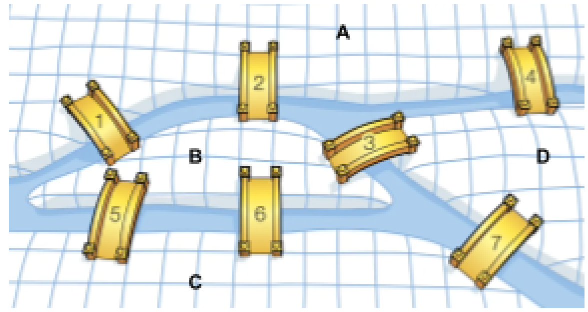
Problem 13. What is the difference between the time complexities of a depth-first search and a breadth-first searc of a binary tree with 4 vertices?

Problem 14. Trace the pseudocode below. What is the final value of TEMP?

```
String A = "BANANAS", T = ""
int TEMP = 0
for int j = len(A) - 1 to 0 step -1
      T = T + A[j]
next
for int j = 0 to len(A) - 1
      if A[j] == T[j] then TEMP += 1
next
String GOOD = A[1:5], BAD = T[1:5]
if GOOD == BAD then
      TEMP += 5
else TEMP /= 2
END IF
```

§15 Bridges of Königsberg

In 1736, Leonhard Euler posed a question that would go on to lay the foundations for graph theory. In the old Prussian city of Königsberg, one might find themselves walking up and down the seven bridges connecting various parts of the town otherwise separated by a river. Euler, always the innovator, wondered whether it was possible to travel back and forth between sections A-D by traversing each of the bridges 1-7 *exactly once*.



Euler realized that the picture of Königsberg can be represented as a graph G = (V, E) where V is the set of vertices (sections A-D) and E is the set of edges represented by unordered pairs (bridges 1-7).

Problem 15.1. Draw the graph G. What is the highest degree present in any vertex of this graph?

Adjacency matrices are a helpful tool to help us further analyze graphs.

Problem 15.2. Write out the adjacency matrix for G, labeling the rows and columns with their corresponding vertices.

Problem 15.3. Notice that A is symmetrical, meaning $A^T = A$. What does this say about the directionality of G? How is it involved in defining the set E?

> **Remark.** The transpose of a matrix M is represented by M^T, where each element $M^T_{ab} = M_{ba}$. In other words, the matrix is flipped over its diagonal, and the rows become columns.

For the following problems, a path of length n between X and Y is defined as a way to get from X to Y by crossing a bridge exactly n times. You are allowed to cross the same bridge more than once, and the order of crossing distinct bridges *does* matter.

Problem 15.4. What is the probability that a randomly selected path from the set paths of length 2 between any two distinct vertices in G starts at B and ends at C?

Problem 15.5. How many paths of length 3 are there between A and A? How many paths of length 3 are there between D and D?

Eponymously, an Euler path is defined as a path that starts at any vertex and hits every vertex *at least* once by traversing every edge *exactly* once. Euler would eventually prove that this graph had no Euler path, and it was therefore impossible to cross each bridge exactly once.

Problem 15.6. The reason this graph has no Euler path is because no vertex has an even degree. Why does it matter whether the graph has at least one vertex that is even?

Problem 15.7. Give a solution to the Königsberg problem if bridge 7 is removed.