Propositional Logic

The phrase propositional logic is composed of two words

Proposition Logic

What is logic?

① Logic is the science of reasoning. It helps us to understand and reason about different mathematical statements.

② With rules of logic, we would be able to think about mathematical statements and finally we would be able to prove or disprove those mathematical statements precisely.

 # purpose of Logic → is to construct valid arguments (or proofs)

Once we prove a mathematical statement is TRUE then we call it a Theorem. and this is the basis of whole mathematics

what is proposition?

Proposition is a declarative sentence (a sentence that is declaring a fact or stating an argument) which can be either TRUE or FALSE but not both.

eg
① Delhi is capital of India
② Water froze this morning.
③ $1+1=2$

Sentence which are not propositions
① What time is it?
② $x+1=2$

 # Propositional Logic → area of logic that studies ways of joining and/or modifying propositions to form more complicated propositions and it also studies the logical relationships and properties derived from these combined/altered propositions.

eg
statement1 - "Adam is good in playing football"
Statement2 - "Adam is good in playing football and this time he is representing his college at National level." → joining two proposition using logical connective

eg statement1 - "I enjoy watching television."
Statement2 - "It is not the case that I enjoy watching television."

modifying the statement using negation

Fact → Propositional logic is sometimes called as "sentential logic" or "statement logic".

why do we need Compound Propositions?

because most of the mathematical statements are constructed by combining one or more than one propositions. As simple as that!

Propositional variables: These helps us to reduce burden by writing long statments again and again.

p = Adam is good in playing football.
q = this time he is representing his college at National level

$p \wedge q$ ← p and q

Variables that are used to represent propositions are called propositional variables.

Logical Operators → There are 6 logical operators that we will focus on:

1. Negation
2. Conjunction
3. Disjunction
4. Exclusive OR
5. Implication (Conditional)
6. Biconditional

① Negation → let p be a proposition. $\sim p$ is called negation of p which ~~satisfy~~ simply states that "It is not the case that p".
if p is true then $\sim p$ is false. if p is false then $\sim p$ is true.

eg let p be a proposition - "Adam and Eve lived together for many years"
then $\sim p$ will be - "It is not the case that Adam and Eve lived together for many years."

or

"Adam and Eve haven't live together for many years"

p	$\sim p$
T	F
F	T

② Conjunction → Let p and q be two propositions. Conjunction of p and q is denoted by $p \wedge q$ where both p and q are true then only compound proposition $p \wedge q$ is true

Ex " 12 is divisible by 3 and 3 is prime number" ⇒ True
T T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$\boxed{\text{Imp Note → Sometime we use 'but' instead of 'and'}}$

Ex- "12 is divisible by 3 but 3 is prime number."

③ Disjunction → Let p and q be two propositions. Disjunction of p and q is denoted by $p \vee q$ when both p and q are false then only compound proposition $p \vee q$ is false.

eg " 16-4 = 10 or 4 is an even number" = True
F T

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

④ Exclusive-OR

Consider the following statement:
"In order to get a job in this multinational company, experience with $\underbrace{\text{C++}}_{p}$ or $\underbrace{\text{java}}_{q}$ is mandatory

or ↓ inclusive OR (Disjunction)

experience with C++ ✓
" " java ✓
both ✓

p or q or both

Consider the following statement
"When you buy a car from XYZ company, you get $\underbrace{\text{\$2500 cashback}}_{p}$ or $\underbrace{\text{accessories}}_{q}$ worth \$2500

or ↓ Exclusive OR (XOR)

cashback ✓
accessories ✓
both ✗

$\boxed{\text{p or q but not both}}$

Definition → Let p and q be two propositions. The exclusive OR of p and q (denoted by $p \oplus q$) is a proposition that simply means exactly one of p and q will be true but both cannot be true.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

⑤ Implication (Conditional) → Let p and q be propositions. The propositi "if p then q" denoted by $p \to q$ is called implication or conditional statemen

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is called hypothesis (or premise) and q is called conclusion (or consequence)

$$\boxed{p \to q \equiv \sim p \vee q}$$

Ex → "If you try hard for your exam, then you will succeed"

p = you tried hard for your exam.

q = you succeed.

Case 1: "You tried hard for your exam" and "you succeed"

p = true, q = true

Compound proposition $p \to q$ is true

Case 2: "You tried hard for you exam" but "you failed"

p = true, q = false

Compound proposition $p \to q$ is false

Case 3: "You haven't tried hard for your exam" and "you succeeded".

p = false, q = true

Compound proposition $p \to q$ is True why?

because you can make the compound proposition false only when you satisfy the first condition itself i.e p. If that itself not satisfied then we cannot make compound proposition false. Not false means true.

Case 4: "You haven't tried hard for your exam" and "You failed".
p = False   q = False

Compound proposition $p \to q$ is True.

Example-2 "If you have connection with seniors, then you will get promoted".

T	T	= T
T	F	= F
F	T or F	= T

Example-3 "If you get 100% marks on final exam, then you will be awarded a Trophy".

T	F	= F
F	T or F	= T

* **Remember** → If p is false then it doesn't matter what will be the truth value of q, $p \to q$ is always TRUE

Practice

① If 1+1=3, then dogs can fly → True
F F

② If 1+1=2, then dogs can fly → False
T F

③ If monkey can fly, then 1+1=3 → True
F F

④ If 1+1=2, then 2+2=5 → False
T F

Different ways to represent conditional statements:

"if p then q"
"p implies q"
"q when p"
"q whenever p"
"q follows from p"

"p only if q"
"q is necessary for p"
"p is sufficient for q"
"q unless ~p"
→ very important

Implication, Converse, contrapositive and Inverse

Implication or conditional statement : $p \to q$

Converse : $q \to p$

Contrapositive : $\sim q \to \sim p$

Inverse : $\sim p \to \sim q$

Example → "If it rains today then I will stay at home"

Converse → "If i will stay at home then it rains today."

Contrapositive → "If i will not stay at home, then it does not rain today

Inverse → "If it does not rain today then i will not stay at home."

Facts
① Implication and contrapositive both are equivalent
② Converse and inverse both are equivalent
③ Neither converse nor inverse is equivalent to Implication.

Truth Table

p	q	$\sim p$	$\sim q$	$p\to q$	$q\to p$	$\sim q\to\sim p$	$\sim p\to\sim q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	F	T	F
F	F	T	T	T	T	T	T

Biconditional Operator

Let p and q be two propositions The biconditional statement of the form $p \leftrightarrow q$ is the proposition "p if and only if q".

$p \leftrightarrow q$ is true whenever the truth values of p and q are same

p	q	$p\leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q How "p if and only if q" make sense?

Sol "p if and only if q" composed of two statements "p if q" and "p only if q".

"p only if q" = if p then q and

"p if q" = if q then p

So $\boxed{p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

Representations: ① "p is neccessary and sufficient for q and vice-versa"
② if p then q, and coversely
③ p iff q

Example : let p be a proposition "You get promoted" and
let q be a proposition "You have connection".

then $p \leftrightarrow q$ is the statement: "You get promoted if and only if you have connections".

Precedence of logical operators

Precedence of operators helps us to decide which operator will get evaluated first in a complicated looking compound proposition.

For example $p \to q \wedge \sim p$

$(p \to [q \wedge (\sim p)])$

Let p = true q = false

$p \to q \wedge \sim p$:

$T \quad \underbrace{F \quad F}_{F} = F$

operators	Names	Precedence
$\neg$ or $\sim$	Negation	1
$\wedge$	Conjunction	2
$\vee$	disjunction	3
$\to$	Implication	4
$\leftrightarrow$	Biconditional	5

Problem Construct the truth table for compound proposition below:

$$\underbrace{p \to \underbrace{\sim q \wedge \sim p}_{r}}_{K} \leftrightarrow \sim q$$

p	q	~p	~q	r	p→r=K	K↔~q
T	T	F	F	F	F	T
T	F	F	T	F	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

Tautology Implication ($\Rightarrow$)

- "P tautologically implies Q" or $p \Rightarrow q$, if q is true whenever p is true.
- If $p \Rightarrow q$ then $p \to q$ is a tautology.
- Examples : ① $p \Rightarrow (p \text{ or } q)$
  ② $(p \wedge q) \Rightarrow (p \leftrightarrow q)$
  ③ $(p \wedge q) \Rightarrow q$

$p \Rightarrow (p \text{ or } q)$

p	q	$p \vee q$	$p \Rightarrow (p \text{ or } q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T ✓

Equivalence ($\Leftrightarrow$) ($\equiv$)

- If p and q are any two propositions are $p \Leftrightarrow q$ if p and q have same truth values.
- Ex: $p \to q \Leftrightarrow (\sim p \text{ or } q)$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$(p \to q) \Leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

- If there are two formulas F_1 and F_2 and they are equivalent means.
  $\boxed{F_1} \leftrightarrow \boxed{F_2}$ is tautology
- means if $F_1 \leftrightarrow F_2$ is tautology then $\boxed{F_1 \equiv F_2 \text{ or } F_1 \Leftrightarrow F_2}$

Note → If biconditional of two formulas is tautology then F_1 and F_2 are equivalent.

Translate english sentences into Logical Expressions

Reasons
1. Removes ambiguity
2. Easy manipulation
3. Able to solve puzzles

Example: "You are not allowed to watch adult movies if your age is less than 18 years or you have no age proof."

Step ① Find the connectives which are connecting the two propositions together
Step ② Rename the propositions

Let q = "You are allowed to watch adult movies"
r = "Your age is less than 18 years
s = "You have age proof"

$$\boxed{(r \lor \sim s) \to \sim q}$$

Problem: Are there system specifications consistent:

"The system is in multiuser state if and only if it is operating normally."
"If the system is operating normally, then the kernel is functioning".
"The kernel is not functioning or the system is in interrupt mode."
"If the system is not in multiuser state, then it is in interrupt mode"
"The kernel is not in interrupt mode."

Soln Let p = "The system is in multiuser state"
q = "The system is operating normally
r = "The kernel is functioning."
s = "The system is in interrupt mode."

1. $p \leftrightarrow q$
2. $q \to r$
3. $\sim r \lor s$
4. $\sim p \to s$
5. $\sim s$

Note Consistent means assigning truth values to propositional variables in such a way that finally we would be able to make all specifications "true".

Tip → Always start from the statement which involves least number of propositions.

So ⑤ $\sim s$ is true only when $\boxed{s = \text{False}}$

Now ③ $\sim r \vee s$ is true only when $\boxed{r = \text{false}}$

② $q \to r$ is true only when $\boxed{q = \text{False}}$

① $p \leftrightarrow q$ is true only when $\boxed{p = \text{False}}$

from all these

④ $\sim p \to s$ this will become false so we are not able to make all of these true so then specifications are not consistent.

Tautology, Contradiction, Contingency and Satisfiability

Tautology: Compound proposition which is always TRUE.

Ex

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Contradiction: Compound proposition which is always FALSE.

Ex

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Contingency → Compound proposition which is sometimes true and sometimes false.

Ex

p	q	$p \wedge q$
T	F	F
T	T	T
F	F	F
F	T	F

Satisfiability → A compound proposition is satisfiable if there is atleast one TRUE result in its truth table

Unsatisfiability → not even a single true result in its truth table.
we can say contradiction is unsatisfiable.

Valid → A compound proposition is valid when it is a tautology.

Invalid → when it is either contradiction or contingency.

Important take aways: ① Tautology is always satisfiable but satisfiable is not always tautology.

② Invalid not only mean a compound proposition is always FALSE. If a compound proposition is sometimes TRUE and sometimes FALSE, then also it is said to be invalid.

Summary:

Tautology	Contradiction	Contingency
always true	always False	Sometimes TRUE or False
valid	invalid	invalid
Satisfiable	unsatisfiable	Satisfiable

Logical Equivalences

Definition → The compound propositions p and q are said to logically equivalence if $p \leftrightarrow q$ is a Tautology. Logical Equivalence is denoted by $\equiv$ or $\Leftrightarrow$

Example: $p \wedge T \equiv p$

p	T	$p \wedge T$
T	T	T
F	T	F

Most common and famous Logical Equivalences:

1. Identity law (a) $p \wedge T \equiv p$ (b) $p \vee F \equiv p$
2. Domination Laws (a) $p \vee T \equiv T$ (b) $p \wedge F \equiv F$
3. Idempotent laws (a) $p \vee p \equiv p$ (b) $p \wedge p \equiv p$
4. Double negation law $\sim(\sim p) \equiv p$
5. Commutative laws (a) $p \vee q \equiv q \vee p$ (b) $p \wedge q \equiv q \wedge p$
6. Associative laws (a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ (b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
7. Distributive laws (a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (b) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$
8. Demorgan's law (a) $\sim(p \wedge q) \equiv \sim p \vee \sim q$ (b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$
9. Absorption law (a) $p \vee (p \wedge q) \equiv p$ (b) $p \wedge (p \vee q) \equiv p$
10. Negation laws (a) $p \vee \sim p \equiv T$ (b) $p \wedge \sim p \equiv F$

Logical equivalences involving conditional statements

1. $p \to q \equiv \sim p \vee q$ (imp)
2. $p \to q \equiv \sim q \to \sim p$
3. $p \vee q \equiv \sim p \to q$
4. $p \wedge q \equiv \sim(q \to \sim p)$
5. $\sim(p \to q) \equiv p \wedge \sim q$
6. $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

   $\to (\sim p \vee q) \wedge (\sim p \vee r)$

   $\to (\sim p) \vee (q \wedge r)$

   $\to p \to (q \wedge r)$
7. $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
8. $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
9. $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Logical Equivalences involving biconditionals

① $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

② $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

Proof $p \leftrightarrow q = (p \to q) \wedge (q \to p)$

$= (\sim q \to \sim p) \wedge (\sim p \to \sim q)$

$= (\sim p \to \sim q) \wedge (\sim q \to \sim p)$

$= \sim p \leftrightarrow \sim q$

③ $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

$\Rightarrow (p \to q) \wedge (q \to p)$

$(a+b)(c+d)$

$ac + ad + bc + bd$

$\Rightarrow (\sim p \vee q) \wedge (\sim q \vee p)$

$\Rightarrow (\sim p \wedge \sim q) \vee \underbrace{(\sim p \wedge p)}_{F} \vee \underbrace{(q \wedge \sim q)}_{F} \vee (q \wedge p)$

$\Rightarrow (\sim p \wedge \sim q) \vee (q \wedge p)$

④ $\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

Rules of Inference in Propositional Logic
(Basic Terminology and Examples)

Premise → is a proposition on the basis of which we would be able to draw a conclusion. You can think of premise as an evidence or assumption. Therefore, initially we assume something is true and on the basis of that assumption, we draw some conclusion.

Conclusion → is a proposition that is reached from the given set of premises. You can think of it as the result of the assumptions that we made in an argument.

$\boxed{\text{if premise then conclusion}}$ if premise is true then conclusion must be true

Argument → Sequence of statements that ends with a conclusion

OR

it is a set of one or more premises and a conclusion.

Valid argument → An argument is said to be valid if and only if it is not possible to make all premises true and a conclusion false.

Example of an argument:

P_1 → "If I love cat then I love Dog." (p = I love cat, q = I love Dog) $\quad p \to q$

P_2 → "I love Cat." $\quad p$

C → Therefore, "I love dog". $\quad \therefore q$

$$\boxed{((p \to q) \wedge p) \to q}$$

Here first we'll assume all premises are true then we'll check if there is any way by which conclusion can be false. If it is possible to make conclusion false then argument is invalid else argument is valid.

$$\overset{T}{p} \to \overset{T}{q} \quad T \text{ (assume)}$$

(this must be true only then $p \to q$ is true)

$$p \quad T \text{ (assume)}$$

$$\therefore \underset{T}{q} \to \text{so conclusion is also true}$$

Hence this is valid argument

Ex 2 $P_1 \to$ "If I love cat then I love dog." $\quad p \to q$

$P_2 \to$ "I love dog." $\quad q$

$C \to$ "Therefore, I love cat" $\quad \therefore p$

$$\begin{array}{ll} p \to q & T \text{ (assume)} \\ q & T \text{ (assume)} \\ \hline \therefore p\ (F) & \end{array}$$

Here we can see p can be true or false in both cases $p \to q$ is true so when p is false our conclusion becomes false so invalid argument.

Definition and Types of Inference Rules

Rules of Inference: are the templates for constructing valid arguments

Inference → deriving conclusion from evidences (premises)

Types of Inference Rules

① **Modus Ponens:**

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array} \quad \text{OR} \quad [(p \to q) \wedge p] \to q$$

② **Modus Tollens:**

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p \end{array} \quad \text{OR} \quad [(p \to q) \wedge \sim q] \to \sim p$$

③ **Hypothetical Syllogism:**

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array} \quad \text{OR} \quad [(p \to q) \wedge (q \to r)] \to (p \to r)$$

try to make $p \to r$ false, by taking p as true and r as false, we can observe that we are not able to make premises $p \to q$ and $q \to r$ True so

$$\begin{array}{ll} p \to q & T \text{ (assume)} \\ q \to r & T \text{ (assume)} \\ \hline p \to r & \Rightarrow \text{True} \\ T \quad\ T & \end{array}$$

So valid argument

④ Disjunctive Syllogism

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array} \quad \text{OR} \quad [(p \vee q) \wedge (\sim p)] \rightarrow q$$

⑤ Addition

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array} \quad \text{OR} \quad p \rightarrow (p \vee q)$$

⑥ Simplification

$$\begin{array}{l} p \wedge q \\ \hline \therefore p \end{array} \quad \text{OR} \quad \begin{array}{l} p \wedge q \\ \hline \therefore q \end{array} \quad \text{OR} \quad (p \wedge q) \rightarrow p \ \text{ OR } \ (p \wedge q) \rightarrow q$$

⑦ Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array} \quad \text{OR} \quad [(p) \wedge (q)] \rightarrow (p \wedge q)$$

⑧ Resolution

$$\begin{array}{l} p \vee q \\ \sim p \vee r \\ \hline \therefore q \vee r \end{array} \quad \text{OR} \quad [(p \vee q) \wedge (\sim p \vee r)] \rightarrow (q \vee r)$$

How to build arguments using rules of inference

Ex①

Premises : "Ram works hard" "If Ram works hard, then he is a dull boy", and "If Ram is a dull boy, then he will not get the job".

Conclusion : "Ram will not get the job"

Let H = Ram works hard , D = Ram is dull boy

J = Ram will get the job

$$\begin{array}{l} H \\ H \rightarrow D \\ D \rightarrow \sim J \\ \hline \therefore \sim J \end{array}$$

$$\begin{array}{l} H \\ H \rightarrow D \\ \hline D \end{array} \quad \text{(by modus ponens)}$$

$$\begin{array}{l} D \\ D \rightarrow \sim J \\ \hline \sim J \end{array} \quad \text{(by modus ponens)}$$

$$\begin{array}{l} H \rightarrow D \\ D \rightarrow \sim J \\ \hline H \rightarrow \sim J \end{array} \quad \text{(Hyp. Syllogism)}$$

$$\begin{array}{l} H \rightarrow \sim J \\ H \\ \hline \sim J \end{array} \quad \text{(Modus Ponens)}$$

Ex2 Premises — "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on." "If the sailing race is held, then the trophy will be awarded" and "The trophy was not awarded."

Conclusion "It rained"

Let R = It rains, F = it is foggy, S = the sailing race will be held
D = Lifesaving demonstration will go on, T = trophy will be awarded.

$(\sim R \vee \sim F) \to S \wedge D$

$S \to T$

$\sim T$

$\therefore R$

$$\begin{array}{l} S \to T \\ \sim T \\ \hline \therefore \sim S \end{array}$$ Modus Tollens

addition Rule $\Rightarrow \sim S \vee \sim D = \sim(S \wedge D)$

$$\underbrace{(\sim R \vee \sim F)}_{p} \to \underbrace{S \wedge D}_{q}$$

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \sim p \end{array}$$

$= \sim(\sim R \vee \sim F)$

$= R \wedge F$ (simplification Rule)

$= R$

Note Two more inference rules are

(9) $(p \to q) \wedge (r \to s)$

$p \vee r$

$q \vee s$

Conjunctive dilemma

(10) $(p \to q) \wedge (r \to s)$

$\sim q \vee \sim s$

$\sim p \vee \sim r$

disjunctive dilemma

Checking the validity of the argument

Problem Show that the following argument is valid. If today is Tuesday, then I have a test in Mathematics or Economics. If my Economics Professor is sick, then I will not have a test in Economics. Today is Tuesday and my Economics professor is sick. Therefore I have a test in Mathematics.

Solution T = Today is tuesday M = I have a test in Mathematics

E = I have a test in Economics S = My Economics Professor is sick

$P_1 \to \quad T \to M \vee E \quad T$

$P_2 \to \quad S \to \sim E \quad T$

$P_3 \to \quad T \wedge S \quad T$

$C \to \quad \therefore M \quad F$

#
There are multiple methods to check the validity of an argument

① Rule of inference

② Another method is fastest (according to this we need to make all premises true and conclusion false) ⇒ this implies that argument is invalid. But if we would not be able to make all the premises true and conclusion false then the argument is 100% valid

for P_3 to be true S and T both must be true

If S is true then for P_2 to be true $\sim E$ must be true

i.e E must be false

Now $P_1 \to \overset{T}{T} \to \overset{F}{M} \vee \overset{F}{E}$ $T \to F = F$

For P_1 to be true M must be true then only P_1 can be true

Here, we are not able to make premise 1 true

So this argument is valid argument

Rules of Inference in Proposition logic (Solved Problem-1)

Problem: Consider the following logical inferences

I1 : If it rains then cricket match will not be played
• The ~~chect~~ cricket match was played

Inference: There was no rain

I2: If it rains then the cricket match will not be played
It did not rain

Inference: The cricket match was played.

Which of the following is TRUE?

(a) Both I1 and I2 are correct inferences.
(b) I1 is correct but I2 is not a correct inference
(c) I1 is not correct but I2 is correct inference.
(d) Both I1 and I2 are not correct inferences.

Soln Let R = it rains and C = cricket match was played.

(I1)

$$\begin{array}{l} R \rightarrow \sim C \\ \underline{C} \\ \therefore \sim R \end{array}$$

$R \rightarrow \sim C \equiv C \rightarrow \sim R$

$$\begin{array}{l} C \rightarrow \sim R \\ \underline{C} \\ \therefore \sim R \end{array}$$ (by modus ponens)

we know that
$p \rightarrow q \equiv \sim q \rightarrow \sim p$
implication contrapositive

So this is valid argument

Also we can check by another method

$P_1: \overset{T}{R} \rightarrow \overset{F}{\sim C}$ T (assume)
$P_2: \underline{C_T}$ T (assume)
$C: \therefore \sim R.$ F (assume)

(P1) $(T \rightarrow F = F)$

By this premise 1 can't be true so this is valid argument

(as R is false so whether C is true of false result will always be true)

(I2) $P_1: \overset{F}{R} \rightarrow \sim C$ T
$P_2: \underline{\sim R}$ T
$\therefore C$ F

as we can see ~~...~~ P_1 and P_2 ~~...~~ be true when conclusion is false so we can say this is an invalid argument.

Problems on Inference

Determine the following arguments are valid or not

(1) $p \to q$

$q \to r$

$r \to s$

C: $p \to s$

$P_1 : p \to q$

$P_2 : q \to r$

$P_3 : r \to s$

$$\boxed{\text{By Hypothetical Syllogism}}$$

$$\begin{array}{ll} P_1: & p \to q \\ P_2 & q \to r \\ \hline & p \to r \\ P_3 & r \to s \\ \hline & p \to s \end{array}$$

this is required conclusion so valid argument.

(2) $r \to s$

$\sim s$

$\sim r$ $\boxed{\text{By modus tollens}}$ Valid argument

(3) $r \to s$

$p \to q$

$r \vee p$

$s \vee q$

$P_1 : r \to s$

$P_2 : p \to q$

$P_3 : r \vee p$

~~$r \vee q \equiv \sim r \to q$ or $q \vee r \equiv \sim q \to r$~~

~~$P_3: \sim q \to r$, $P_1: r \to s$, $\sim q \to s \equiv \sim s \to q$, $P_2: p \to q$~~

~~$\sim q \to s \equiv \sim s \to q$ contrapositive~~

$r \vee p \equiv \sim r \to p$

or

$p \vee r \equiv \sim p \to r$

Also we can do it by using conjunctive dilemma

$$\begin{array}{ll} P_3: & \sim p \to r \\ P_1: & r \to s \\ \hline & \sim p \to s \\ & \equiv \\ & \sim s \to p \\ P_2 & p \to q \\ \hline & \sim s \to q \\ & \equiv \\ & s \vee q \end{array}$$

required conclusion so valid argument

$\sim p \to s \equiv \sim s \to p$ contrapositive

$\sim s \to q \equiv s \vee q$

④ $p \to (r \to s)$

$\sim r \to \sim p$

p

---

s

$P_1: p \to (r \to s)$

$P_2: \sim r \to \sim p$

$P_3: p$

P_1, P_3

$P_1: p \to (r \to s)$

$P_3: p$

$P_4: r \to s$

$\left(\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array} \right)$ modus ponens

$P_2: \sim r \to \sim p \equiv p \to r$ (contrapositive)

$P_2: p \to r$

$r \to s$

$P_5: p \to s$

$P_5: p \to s$

$P_1: p$

$P_6: s$

So valid argument

⑤ $P_1: \sim r \to (s \to \sim t)$

$P_2: \sim r \vee w$

$P_3: \sim p \to s$

$P_4: \sim w$

$t \to p$

$P_2 \quad \sim r \vee w \equiv r \to w$

$P_2: r \to w$

$P_4: \sim w$

$\sim r$ (Modus tollens)

$P_1: \sim r \to (s \to \sim t)$

$\sim r$

$s \to \sim t$ (modus ponens)

$P_3: \sim p \to s$

$s \to \sim t$

$\sim p \to \sim t$ (Hypothetical syllogism)

$|||$ (contrapositive)

$t \to p$ → required conclusion

So valid argument

⑥ $P_1: p$

$P_2: p \to q$

$P_3: q \to r$

---

r

$P_2: p \to q$

$P_1: p$

---

q (modus ponens)

$P_3 \quad q \to r$

---

r (modus ponens)

→ required conclusion

So valid argument

⑦ $P_1: \sim p$

$P_2: p \to q$

$P_3: q \to r$

$\overline{\qquad\qquad}$

$\sim r$

$P_2: p \to q$

$P_3: q \to r$ (hypothetical syllogism)

$\overline{p \to r}$

$P_1 \sim p$

→ we can't conclude here anything so invalid argument

⑧ $P_1: (p \wedge q) \to \sim t$

$P_2: w \vee r$

$P_3: w \to p$

$P_4: r \to q$

$C: (w \vee r) \to \sim t$

$\dfrac{p \wedge q}{\therefore p}$ (simplification)

$P_6: p \to \sim t$

$P_2: w \vee r \equiv r \vee w \equiv \sim r \to w$

$P_2: \sim r \to w$

$P_3: w \to p$

$P_5: \sim r \to p$

$P_6: p \to \sim t$

$\sim r \to \sim t$

→ we are not able to conclude above conclusion so this is invalid argument.

⑨ $p \to (Q \to R)$

$p \wedge q$

$\therefore R$

$p \to (Q \to R) \equiv \sim p \vee (Q \to R) \equiv \sim p \vee (\sim Q \vee R)$

$\equiv (\sim p \vee \sim Q) \vee (\sim p \vee R)$

$\equiv \sim p \vee \sim Q \vee R$

$= \sim (p \wedge Q) \vee R$

$= (p \wedge Q) \to R$

$p \to (Q \to R) \equiv (p \wedge Q) \to R$

$(p \wedge Q)$ (Modus Ponens)

$\therefore R$

So valid argument

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1. $P \to (R \to S)$
2. $\sim R \to \sim P$
3. P

$\therefore S$

1. $P \to (R \to S)$

$\boxed{\sim R \to \sim P \equiv P \to R}$ (contrapositive)

$\Downarrow$ 2

2, 3

$$\frac{\begin{array}{c} P \to R \\ P \end{array}}{R} \text{ (M.P)}$$

4 - R

1, 3

$$\frac{\begin{array}{c} P \to (R \to S) \\ P \end{array}}{R \to S} \text{ (M.P)}$$

5 - $R \to S$

4, 5

$$\frac{\begin{array}{c} R \to S \\ R \end{array}}{S}$$

Required conclusion

So valid argument.