PROBABILITY

Random experiment

Favourable cases, exhaustive cases

Types of events : Mutually exclusive events, independent events, dependent events

Probability of an event

$$= \frac{\text{number of favourable cases to the event}}{\text{total number of exhaustive cases}}$$

Axioms on probability :

1) If E is an event, then probability of event $E = P(E)$ has the bounds 0 and 1.

   i.e., $0 \le P(E) \le 1$

2) If E is an impossible event, then $P(E) = 0$
3) If E is a certain event, then $P(E) = 1$
4) If E is an event, then $P(\bar{E}) = 1 - P(E)$
5) If $E_1, E_2, \ldots, E_n$ are mutually exclusive events, then

   $P(E_1 \cup E_2 \cup \ldots \cup E_n) = P(E_1) + P(E_2) + \ldots + P(E_n).$

Addition theorem on probability :

If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B).$

Conditional probability :

$P(A/B) = P(A \text{ given } B)$
= Probability (the event A after the event B has already happened)

Multiplication theorem on probability :

$$P(AB) = P(A \cap B) = \begin{cases} P(A)P(B/A) \\ P(B)P(A/B) \end{cases}$$

Note :

1. If A and B are independent events, then $P(AB) = P(A \cap B) = P(A)P(B)$
2. If $E_1, E_2, \ldots, E_n$ are independent events, then
   $P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1 E_2 \ldots E_n) = P(E_1)P(E_2)\ldots P(E_n).$

Baye's theorem :

If $E_1, E_2, \ldots, E_n$ are mutually exclusive events of sample space S and X is the subset of union of E_i's, then

$$P(E_i / X) = \frac{P(X / E_i)P(E_i)}{\sum_{i=1}^{n} P(X / E_i)P(E_i)}$$

Problems :

1. Four cards are drawn from a pack of cards. Find the probability that
   (i) all are diamonds
   (ii) there is one card of each suit
   (iii) there two spades and two hearts
   (iv) all cards having distinct numbers

2. Twelve balls are distributed at random among 3 boxes. What is the probability that the first box will contain 3 balls?

3. Out of $2n+1$ tickets consequently numbered, 3 tickets are drawn at random. Find the chance that numbers on the drawn tickets are in arithmetic progression.

4. A and B throw a pair of dice alternatively. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins the game of throwing dice, then find the probability of A's chance of winning.

5. A doctor has decided to prescribe two new drugs 'A' and 'B' to 200 heart patients as follows: 50 get drug A, 50 get drug B and 100 get both the drugs. The 200 patients were chosen so that each had an 80% chance of having a heart attack if given neither drug. Drug A reduces the probability of heart attack by 35%, drug B reduces the probability of heart attack by 20% and the two drugs, when given together, work independently. If a randomly selected patient has a heart attack, what is the probability that the patient was given drug A, drug B and both drugs?

6. The completion of a construction project depends on whether the carpenters and plumbers working on the project will go on strike. The probabilities of delay are 100%, 80%, 40% and 5% if both go on strike, carpenters alone go on strike, plumbers alone go strike and neither of them strikes respectively. Also there is 60% chance that plumbers strike if carpenters strike and if plumbers go on strike there is 30% chance that carpenters would follow. It is known that the chance for the plumbers strike is 10%.

   (a) Determine the probability of delay in completion of the project.

   (b) If there is a delay in completion of the project, determine

      (i) Probability that both carpenters and plumbers strike.

      (ii) Probability that carpenters strike and plumbers do not.

      (iii) Probability of carpenters strike.