Normal distribution → Normal r.v is a continuous r.v.

Let us suppose that the normal r.v X has mean μ and SD σ

Here mean μ and SD σ are the parameters of the normal distribution.

The p.d.f $f(x)$ for the normal variable X is given by

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

Mode of Normal Distribution

$$f'(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \left[-\frac{2(x-\mu)}{2\sigma^2} \right]$$

$$f'(x) = 0 \Rightarrow x = \mu$$

$$f''(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \left[e^{\frac{-(x-\mu)^2}{2\sigma^2}} \left(-\frac{(x-\mu)}{\sigma^2} \right)^2 + e^{\frac{-(x-\mu)^2}{2\sigma^2}} \left(\frac{-1}{\sigma^2} \right) \right]$$

$$f''(\mu) = \frac{1}{\sqrt{2\pi}\,\sigma} \left(-\frac{1}{\sigma^2} \right) < 0$$

$\Rightarrow$ $x = \mu$ is the mode of the normal distribution

Median of the normal distribution

If M is the median of the normal distribution, then we have

$$\int_{-\infty}^{M} f(x)\,dx = \int_{M}^{\infty} f(x)\,dx = \frac{1}{2}$$

we have $\displaystyle\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = 1$ —(1)

$$dz = \frac{dx}{\sigma}$$

Substitute $z = \frac{x-\mu}{\sigma}$ then (1) can be written as

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-z^2}{2}} \sigma\,dz = 2\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 1 \Rightarrow \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} dz = \frac{1}{2} \quad \text{—(2)}$$

we have $\displaystyle\int_{M}^{\infty} f(x)\,dx = \frac{1}{2} \Rightarrow \int_{M}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{2}$ —(3)

Substitute $z = \frac{x-\mu}{\sigma}$ then (3) can be written as

$$dz = \frac{dx}{\sigma}$$

$$\int_{\frac{M-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-z^2}{2}} \sigma\,dz = \int_{\frac{M-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = \frac{1}{2} \quad \text{—(4)}$$

$x \to M$, $z \to \frac{M-\mu}{\sigma}$; $x \to \infty$, $z \to \infty$

comparing (2) and (4)

$$\frac{M-\mu}{\sigma} = 0 \Rightarrow \boxed{M = \mu}$$

In Normal distribution $\boxed{\text{mean} = \text{mode} = \text{median} = \mu}$

Mean deviation of the normal distribution

Mean deviation $= E(|X-\mu|) = \int_{-\infty}^{\infty} |x-\mu| \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$ (5)

Substitute $z = \frac{x-\mu}{\sigma}$ then (5) can be written as

Mean deviation $= \int_{-\infty}^{\infty} |\sigma z| \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz = \frac{2\sigma}{\sqrt{2\pi}} \int_0^{\infty} z e^{-\frac{z^2}{2}} dz$

$\Rightarrow \sqrt{\frac{2}{\pi}}\sigma \left[-e^{-\frac{z^2}{2}}\right]_{z=0}^{\infty} = \sqrt{\frac{2}{\pi}}\sigma = 0.7979\sigma \simeq 0.8\sigma$

In normal distribution

$P(a \le X \le b) = \int_a^b f(x)\,dx = \int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

we know $\boxed{P(-\infty < X < \infty) = 1}$

$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$

Standard Normal variable

A normal variable X is said to be a standard normal variable if its mean is zero and standard deviation is equal to 1

Let $z = \frac{X-\mu}{\sigma}$

Here mean of z is zero and s.d of z is 1.

Therefore. $z = \frac{X-\mu}{\sigma}$ is a standard normal variable.

Proof

$E(z) = $ mean of z

$= E\left(\frac{X-\mu}{\sigma}\right)$

$= \frac{1}{\sigma} E(X-\mu)$

$= \frac{1}{\sigma}[E(x) - \mu]$

$= \frac{1}{\sigma}[\mu - \mu] = 0$

$\text{Var } Z = E(z^2) - [E(z)]^2$

$= E(z^2) - 0$

$= E\left[\left(\frac{X-\mu}{\sigma}\right)^2\right] = \frac{1}{\sigma^2}\left[E((X-\mu)^2)\right]$

$\Downarrow$ σ^2 by basic definition

$\boxed{\text{Var } X = E[X-}$

$\Rightarrow \frac{\sigma^2}{\sigma^2} = 1$

$\boxed{SD = 1}$

To find $P(a \le X \le b)$

Let $z_1 = \frac{a-\mu}{\sigma}$ and $z_2 = \frac{b-\mu}{\sigma}$

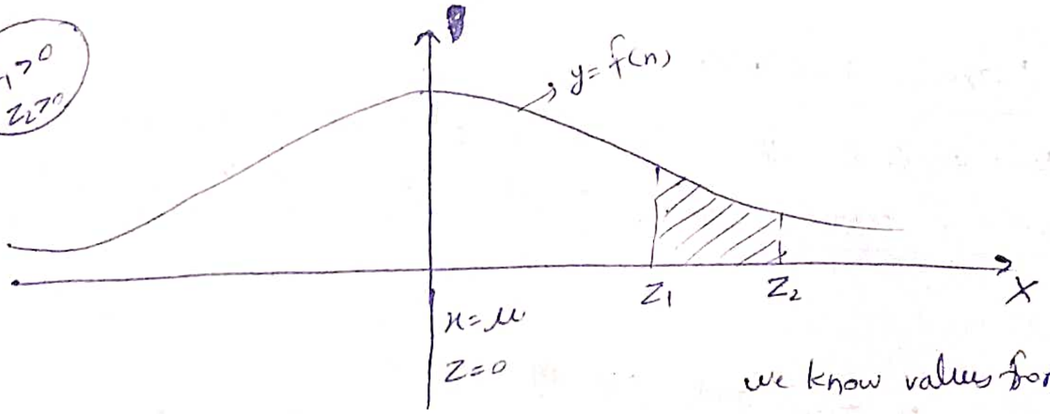
$$P(a \le X \le b) = \int_a^b \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = P(z_1 \le Z \le z_2)$$

Here the values of $\int_0^{z_0} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$, where $z_0 > 0$ (for various values of z_0) are available in the form of a table.

$$P(a \le X \le b) = P(z_1 \le Z \le z_2)$$

$$= \begin{cases} P(0 \le Z \le z_2) - P(0 \le Z \le z_1), & \text{if } z_1, z_2 > 0 \\ P(0 \le Z \le -z_1) + P(0 \le Z \le z_2), & \text{if } z_1 < 0,\ z_2 > 0 \\ P(0 \le Z \le -z_1) - P(0 \le Z \le -z_2), & \text{if } z_1, z_2 < 0 \end{cases}$$

Case $z_1 > 0$, $z_2 > 0$

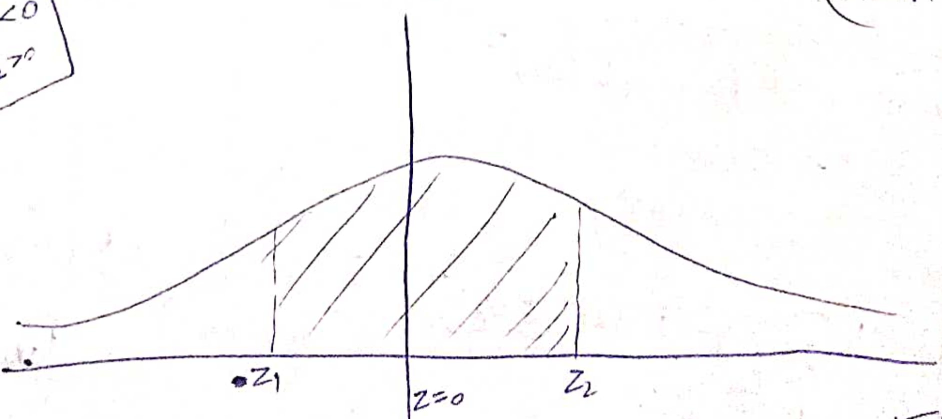


$\int_0^{z_2} f(x)dx$

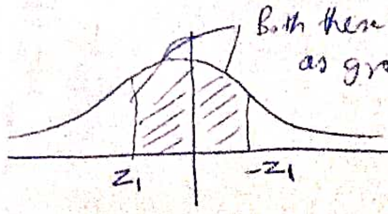
we know values from 0 to z_2 and 0 to z_1 $\int_0^{z_1} f(x)dx$

(from Area table)

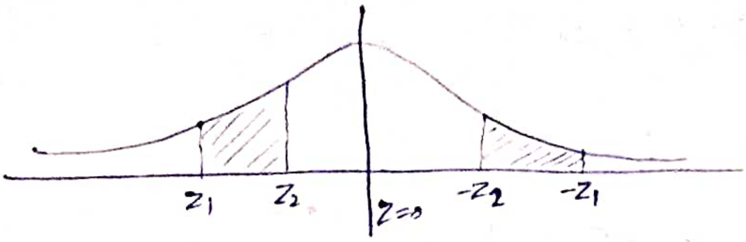
Case $z_1 < 0$, $z_2 > 0$



Both these are equal as graph is symmetry



Case $Z_1<0$
$Z_2<0$



Note → A binomial variable X with parameters n and p is referred as a normal variable if n is very large and neither p nor q is small, i.e if $n \geq 30$ and $p, q > 0.05$, then corresponding binomial variable X is referred as a normal variable.

Q1 Given that random variable X has binomial distribution with $n=50$ and $p=0.25$ find

(i) $P(X>10)$ (ii) $P(X<18)$ (iii) $P(X>21)$ (iv) $P(9<X<14)$

Soln So here as we can see $n \geq 30$ and $p, q > 0.05$

as $\boxed{n=50}$ $\boxed{p=0.25}$ $\boxed{q=0.75}$

So X referred as Normal variable

$\boxed{\text{Mean} = \mu = np}$ $\boxed{SD = \sigma = \sqrt{npq}}$

$\boxed{\mu = 12.5}$ $\boxed{\sigma = 2.6516 \simeq 2.65}$

(i) $P(X>10) = P(Z>-0.94)$

$Z = \dfrac{X-\mu}{\sigma} = \dfrac{X-12.5}{2.65}$

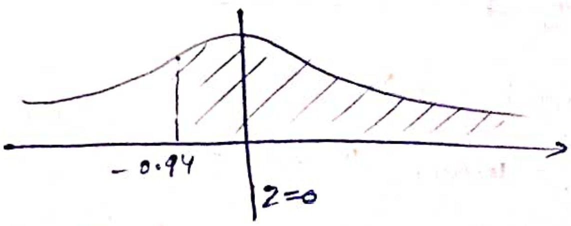
Here $X=10$

$\dfrac{10-12.5}{2.65} = \dfrac{-2.5}{2.65} = -0.94$

$$\boxed{\begin{array}{l} X>10 \\ X-\mu > 10-\mu \\ \dfrac{X-\mu}{\sigma} > \dfrac{10-\mu}{\sigma} \end{array}}$$

$Z > \dfrac{10-12.5}{2.65} \Rightarrow \boxed{Z>-0.94}$

$\Rightarrow P(-0.94<Z<0) + P(0<Z<\infty)$



$\Rightarrow P(-0.94<Z<0) + P(0<Z<\infty)$

$\Rightarrow P(0<Z<0.94) + P(0<Z<\infty)$

$\Rightarrow 0.3264 + \dfrac{1}{2} = 0.8264$

(ii) $P(X<18) = P\left(\frac{X-\mu}{\sigma} < \frac{18-\mu}{\sigma}\right)$

$= P(Z < 2.08)$

$= P(-\infty<Z<0) + P(0<Z<2.08)$

$= 0.5 + 0.4812 = 0.9812$

$Z = \frac{X-\mu}{\sigma}$

$= \frac{18-12.5}{2.64}$

$= 2.0754$

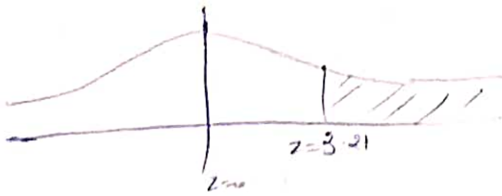
$\simeq 2.08$

(iii) $P(X>21) = P\left(\frac{X-\mu}{\sigma} > \frac{21-\mu}{\sigma}\right) = P(Z>3.21)$

$z = \frac{X-\mu}{\sigma}$

$= \frac{21-12.5}{2.64}$

$= 3.2075$

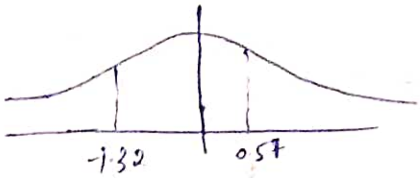


$\Rightarrow P(0<Z<\infty) - P(0<Z<3.21)$

$\Rightarrow 0.5 - 0.49994 = 0.0006$

(iv) $P(9<X<14) = P\left(\frac{9-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{14-\mu}{\sigma}\right)$

$\Rightarrow P(-1.32<Z<0.57)$



$\Rightarrow P(0<Z<1.32) + P(0<Z<0.57)$

$\Rightarrow 0.4066 + 0.2157$

$\Rightarrow 0.6223$

Q2 If X is a normal variable with mean 12 and s.d 4, then

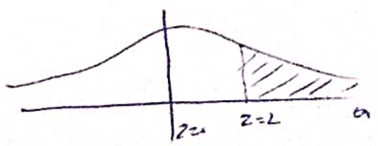
(a) find the probability of (i) $X \geq 20$ (ii) $0 \leq X \leq 12$

(b) find x_1, if $P(X>x_1) = 0.24$

(c) Find x_0 and x_1 when $P(x_0<X<x_1) = 0.5$ and $P(X>x_1) = 0.25$

Sol^n $P(X \geq 20) = P\left(\frac{X-\mu}{\sigma} \geq \frac{20-\mu}{\sigma}\right) = P(Z \geq 2)$

$\boxed{\frac{20-\mu}{\sigma} \quad \frac{20-12}{4} = \frac{8}{4}}$



$\Rightarrow P(0<Z<\infty) - P(0<Z<2)$

$\Rightarrow 0.5 - 0.4772$

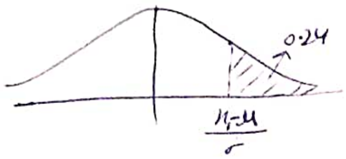
$\Rightarrow$

(ii) $P(0 \le X \le 12) = P\left(\frac{0-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{12-\mu}{\sigma}\right) = P(-3 \le Z \le 0)$

$\Rightarrow P(0 \le Z \le 3) = 0.4987$

(b) $P(X > x_1) = 0.24$

$P\left(\frac{X-\mu}{\sigma} > \frac{x_1-\mu}{\sigma}\right) = 0.24 \Rightarrow P\left(Z > \frac{x_1-12}{4}\right) = 0.24$



$P(0<Z<\infty) - P\left(0<Z<\frac{x_1-12}{4}\right) = 0.24$

$0.5 - 0.24 = P\left(0<Z<\frac{x_1-12}{4}\right)$

$0.26 = P\left(0<Z<\frac{x_1-12}{4}\right)$

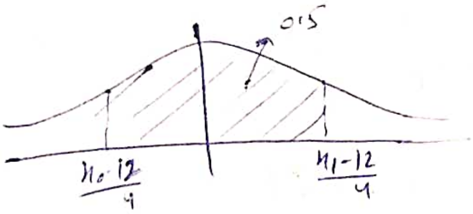
By using table values

$\frac{x_1-12}{4} = 0.705$

$\boxed{x_1 = 0.704 \times 4 + 12}$

(iii) $P(x_0 < X < x_1) = 0.5$

$P\left(\frac{x_0-12}{4} < Z < \frac{x_1-12}{4}\right) = 0.5$



$P(X > x_1) = 0.25$

$P\left(\frac{X-\mu}{\sigma} > \frac{x_1-\mu}{\sigma}\right) = 0.25$

$P\left(Z > \frac{x_1-12}{4}\right) = 0.25$

$P(0<Z<\infty) - P\left(Z > \frac{x_1-12}{4}\right) = 0.25$

$0.5 - 0.25 = P\left(Z > \frac{x_1-12}{4}\right)$

$P\left(Z - \frac{x_1-12}{4}\right) = 0.25$ —(2)

$P\left(\frac{x_0-12}{4} < Z < 0\right) + P\left(0<Z<\frac{x_1-12}{4}\right) = 0.5$

$P\left(\frac{x_0-12}{4} < Z < 0\right) + 0.25 = 0.5$

$P\left(\frac{x_0-12}{4} < Z < 0\right) = 0.25$ —(1)

from (1) and (2)

$-\left(\frac{x_0-12}{4}\right) = +\left(\frac{x_1-12}{4}\right) = 0.675$

Q If X is a normal variable with mean = 30 and S.D 5 find Prob that $|x-30| \ge 5$

Soln $P(|X-30| \ge 5) = 1 - P(|X-30| \le 5) \Rightarrow 1 - P(-5 \le x-30 \le 5)$

$\Rightarrow 1 - P(25 \le x \le 35)$

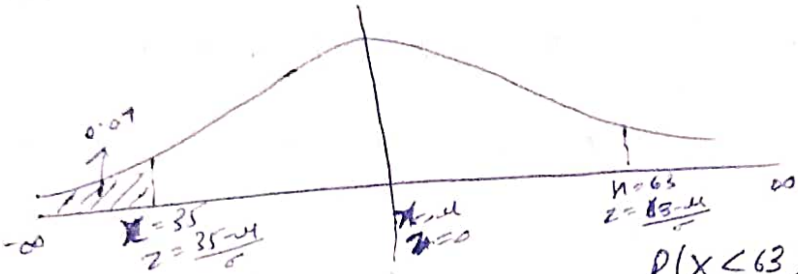
$\Rightarrow 1 - P\left(\frac{25-30}{5} \le Z \le \frac{35-30}{5}\right)$

$\Rightarrow 1 - P(-1 \le Z \le 1)$

$\Rightarrow 1 - [2P(0 \le Z \le 1)]$

$\Rightarrow 1 - 2 \times 0.3413 \Rightarrow$

Q In a distribution exactly normal, 7% of the items are under 35 and 89% of the items are under 63. What are the mean and SD of distribution?



$P(X<63) = 89\% = 0.89$

$P(X<35) = 7\% = 0.07$

Since 0.07 is less than 0.5 so 0.07 must be on left side

$P(-\infty < X < 35) = 0.07$ and we know $P(-\infty < X < \mu) = 0.5$ so it must be on left side

By both of these we can say $35 < \mu$

$P(-\infty < X < 63) = 0.89$ and we know $(P(-\infty < X < \mu) = 0.5)$

By both of these we can say that $63 > \mu$

$P(-\infty X < 63) \Rightarrow P(-\infty < X < \mu) + P(\mu < X < 63) = 0.89$

$\Rightarrow P\left(-\infty < Z < \frac{\mu-\mu}{\sigma}\right) + P\left(\frac{\mu-\mu}{\sigma} < Z < \frac{63-\mu}{\sigma}\right) = 0.89$

$\Rightarrow P(-\infty < Z < 0) + P\left(0 < Z < \frac{63-\mu}{\sigma}\right) = 0.89$

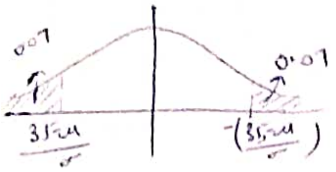
$\Rightarrow 0.5 + P\left(0 < Z < \frac{63-\mu}{\sigma}\right) = 0.89$

$\Rightarrow P\left(0 < Z < \frac{63-\mu}{\sigma}\right) = 0.89 - 0.5 = 0.39$ — (1)

$P(-\infty < X < 35) \Rightarrow P\left(-\infty < \frac{X-\mu}{\sigma} < \frac{35-\mu}{\sigma}\right) = 0.07$

$\Rightarrow P\left(-\infty < Z < \frac{35-\mu}{\sigma}\right) = 0.07$

$\Rightarrow P\left(\frac{35-\mu}{\sigma} < Z < \infty\right) = 0.07$



~~$P(Z \geq 35)$~~

$P(-\infty < z < \frac{35-\mu}{\sigma}) = 0.07$

$P(z \geq -(\frac{35-\mu}{\sigma})) = 0.07$

$P(0 < z < \infty) - P(0 < z < -(\frac{35-\mu}{\sigma})) = 0.07$

$0.5 - 0.07 = P(0 < z < -(\frac{35-\mu}{\sigma}))$

$0.43 = P(0 < z < -(\frac{35-\mu}{\sigma}))$

By using table values $\boxed{-(\frac{35-\mu}{\sigma}) = 1.475}$ -A

from (1)

$P(0 < z < \frac{63-\mu}{\sigma}) = 0.39$

By using table values $\boxed{\frac{63-\mu}{\sigma} = 1.225}$ -B

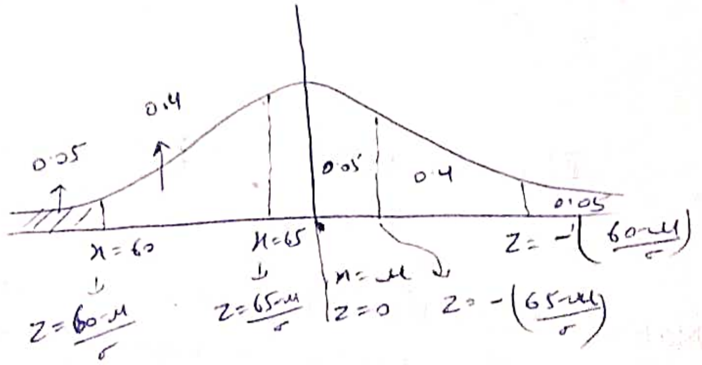
Solving A and B we get μ and σ

Q5 Of a large group of people 5% are under 60 inches in height and the height of the 40% of the people lie between 60 and 65 inches. Find the mean and S.d of the distribution.

Soln $X \rightarrow$ height of the people

$P(X < 60) = 5\% = 0.05$ $\quad P(60 < X < 65) = \frac{40}{100} = 0.4$

$P(0 \leq X < 60) = 0.05$



$P(0 < z < -(\frac{65-\mu}{\sigma})) = 0.05$

$P(0 < z < -(\frac{60-\mu}{\sigma})) = 0.45$

from table we get values for $-(\frac{65-\mu}{\sigma})$ and $-(\frac{60-\mu}{\sigma})$

Q6

Sol: Estimation I → N.V with $\mu = 325$, $\sigma = 60$
Estimation II → N.V with $\mu = 300$, $\sigma = 50$
X = Sales in the quarter

(i) $P(X > 350$ w.r.t Estimation-I$)$ $\boxed{Z = \frac{X-\mu}{\sigma}}$

Here we use μ and σ of Estimation ①

$$P\left(Z > \frac{350-\mu}{\sigma}\right) = P\left(Z > \frac{350-325}{60}\right) = P\left(Z > \frac{25}{60}\right) = P(Z > 0.41)$$

$$\Rightarrow P(0 < Z < \infty) - P(0 < Z < 0.41)$$

$$\Rightarrow 0.5 - 0.1591 =$$

(ii) $P(X > 350$ w.r.t Estimation-II$)$

Here we'll use μ and σ of Estimation ②

$$P\left(Z > \frac{350-\mu}{\sigma}\right) = P\left(Z > \frac{350-300}{50}\right) = P(Z > 1)$$

$$\Rightarrow P(0 < Z < \infty) - P(0 < Z < 1)$$

$$\Rightarrow 0.5 - 0.3413$$

(iii) $P(\text{Estimation-I} / X > 350) = \dfrac{P(X > 350 \text{ w.r.t } E\text{-I})\, P(E\text{-I})}{P(X > 350)}$ → using Baye's Theorem

$$P(E\text{-I}) = P(E\text{-II}) = 0.5$$

$$P(X > 350) = P\left(\frac{X > 350}{E\text{-I}}\right) P(E\text{-I}) + P\left(\frac{X > 350}{E\text{-II}}\right) P(E\text{-II})$$ → Total probability

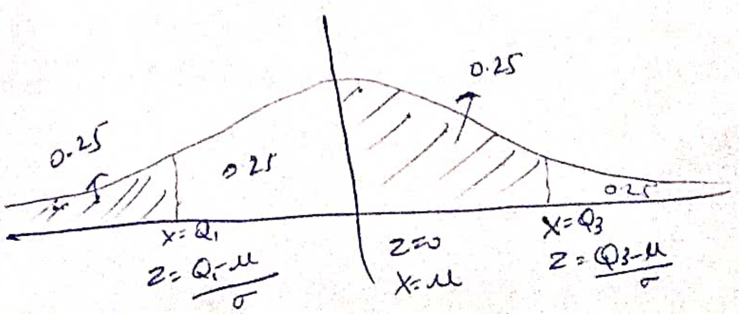
(iv) $P(E\text{-II} / X > 350) = \dfrac{P(X > 350 \text{ w.r.t } E\text{-II})\, P(E\text{-II})}{P(X > 350)}$

Q Show that for Normal distribution the quartile deviation mean deviation and standard deviation are approximately in the ratio 10 : 12 : 15

Soln QD : MD : S.D = 10 : 12 : 15

$\boxed{MD = \frac{4}{5} SD}$ → in the derivation we have seen this

$$Q.D = \frac{Q_3 - Q_1}{2}$$ where Q_1 = 1st quartile, Q_3 = 3rd Quartile



$$\boxed{-\left(\frac{Q_1-\mu}{\sigma}\right)=\left(\frac{Q_3-\mu}{\sigma}\right)}$$

$$P\left(0<z<\frac{Q_3-\mu}{\sigma}\right)=0.25$$

from table

$$\frac{Q_3-\mu}{\sigma}=0.675$$

$$Q_3=\mu+0.675\sigma$$

$$-\left(\frac{Q_1-\mu}{\sigma}\right)=0.675$$

$$-Q_1+\mu=0.675\sigma$$

$$Q_1=\mu-0.675\sigma$$

$$Q.D=\frac{Q_3-Q_1}{2}=\frac{2\times 0.675\sigma}{2}$$

$$\boxed{Q.D=0.675\sigma}$$

$Q.D : M.D : SD$

$0.675\sigma : \frac{4}{5}\sigma : \sigma \quad = 0.675 : \frac{4}{5} : 1$

multiply by 15

$10.125 : 12 : 15 \quad \simeq \boxed{10 : 12 : 15}$

Q8 Fit the Normal distribution to the following data:

X Class interval	60-65	65-70	70-75	75-80	80-85	85-90	90-95	95-100
Frequency	3	21	150	335	326	135	26	4

these are observed frequencies

Soln

Fitting of data ⇒ finding of theoritical frequencies

Theoritical frequency of X = (probability of X) × total frequency

for Normal distribution ⇒ parameters are mean μ, S.D σ

class intervals

C.I	freq (f_i)	midvalue of C.I
60-65	3	62.5
65-70	21	67.5
70-75	150	72.5
75-80	335	77.5
80-85	326	82.5
85-90	135	87.5
90-95	26	92.5
95-100	4	97.5
	1000	

total frequency Σf_i

$$\mu = \text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$=$$

$$= 79.945$$

$$S.D = \sqrt{var}$$

$$var = \sigma^2 = \frac{1}{\Sigma f_i}\Sigma f_i x_i^2 - \Sigma(\text{mean})^2$$

$$= 29.65$$

$$\sigma = SD = \sqrt{var} = 5.445$$

$P(60 < X < 65) = P\left(\frac{60-\mu}{\sigma} < Z < \frac{65-\mu}{\sigma}\right)$

and so on

frequency (X lies between 60 and 65) = $P(60 < X < 65) \times 1000 =$
for every class interval we have to do these steps.

Exponential distribution :- A r.v X is said to follow exponential distribution with parameter $\theta > 0$ if its p.d.f is given by

$$f(x) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

Characteristics of Exponential distribution i.e mean and variance
let's find these

M.G.F for exponential distribution about origin

$$M_0(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x)\,dx = \int_0^\infty e^{tx}\,\theta e^{-\theta x}\,dx = \int_0^\infty \theta e^{(t-\theta)x}\,dx$$

$$\Rightarrow \left[\frac{\theta e^{(t-\theta)x}}{t-\theta}\right]_{x=0}^{\infty} = \frac{\theta}{\theta - t} = \frac{1}{1-\frac{t}{\theta}} = 1 + \frac{t}{\theta} + \frac{t^2}{\theta^2} + \frac{t^3}{\theta^3} + \cdots$$

$$\Downarrow$$

$$\left(1-\frac{t}{\theta}\right)^{-1}$$

Here we take $t=0$, $e^{-\infty} = \frac{1}{e^\infty} = 0$

mean $= \frac{d}{dt} M_0(t)\Big|_{t=0}$

Mean and Variance of exponential distribution

$$\text{Mean} = E(X) = \frac{d}{dt} M_0(t)\Big|_{t=0} = \frac{1}{\theta}$$

$$E(X^2) = \frac{d^2}{dt^2} M_0(t)\Big|_{t=0} = \frac{2}{\theta^2}$$

$$\text{Variance} = E(X^2) - [E(X)]^2 = \frac{2}{\theta^2} - \frac{1}{\theta^2} = \frac{1}{\theta^2}$$

Important result on exponential distribution

Show that the exponential distribution "lacks memory" i.e if X follows an exponential distribution then for every constant $a \geq 0$, it has $P(Y \leq x / X \geq a) = P(X \leq x)$ for all x, where $Y = X - a$.

Proof The pdf for the exponential distribution with parameter θ is given by

$$f(x) = \theta e^{-\theta x}, \quad \theta > 0, \; x \geq 0$$

we have

$$\boxed{P(Y \leq x / X \geq a)} = \frac{P(Y \leq x \cap X \geq a)}{P(X \geq a)} \quad \text{— (1)}$$

$$\Rightarrow \boxed{P(Y \leq x \cap X \geq a)} = P(X - a \leq x \cap X \geq a) \quad \text{since } (Y = X - a)$$

$$\Rightarrow P(X \leq x + a \cap X \geq a) = P(a \leq X \leq a + x)$$

$$\Rightarrow \int_a^{a+x} \theta e^{-\theta x} dx = \left[\frac{\theta e^{-\theta x}}{-\theta}\right]_a^{a+x} = -\left[e^{-\theta(a+x)} - e^{-\theta a}\right]$$

$$\Rightarrow e^{-\theta a} - e^{-\theta(a+x)} = e^{-\theta a}\left[1 - e^{-\theta x}\right]$$

$$\boxed{P(X \geq a)} = \int_a^{\infty} \theta e^{-\theta x} dx = \left[\frac{\theta e^{-\theta x}}{-\theta}\right]_a^{\infty} = -\left[e^{-\theta x}\right]_a^{\infty} = -\left[e^{-\infty} - e^{-\theta a}\right]$$

$$\Rightarrow \boxed{e^{-\theta a}}$$

∴ from (1)

$$P(Y \leq x / X \geq a) = \frac{e^{-\theta a}\left[1 - e^{-\theta x}\right]}{e^{-\theta a}} = 1 - e^{-\theta x} \quad \text{— (A)}$$

∴

$$P(X \leq x) = \int_0^x f(x) dx = \int_0^x \theta e^{-\theta x} dx = \left[\frac{\theta e^{-\theta x}}{-\theta}\right]_0^x = -\left[e^{-\theta x} - 1\right]$$

$$\Rightarrow 1 - e^{-\theta x} \quad \text{— (B)}$$

from (A) and (B)

$$P(Y \leq x / X \geq a) = P(X \leq x)$$

P(6 ≤ X < 6.7) - P(...

Hypergeometric Distribution → When the population is finite and sampling is done without replacement, so that the events are stochastically dependent (although random), we obtain hypergeometric distribution.

Note → Here population means number of items in sample space.

Consider an urn with N balls. M of which are white and N−M are non white. Suppose that we draw a sample of n balls from the urn at random without replacement. then the probability of getting k white balls out of n (k ≤ n) drawn balls is given by

$$\frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}} \quad \text{where } \binom{M}{k} = {}^{M}C_{k}$$

Definition → A r.v X is said to follow hypergeometric distribution if it assume only non-negative values and its p.d.f is given by

$$p(x) = P(X=x) = h(x, N, M, n) = \begin{cases} \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} & x = 0, 1, 2, \ldots n \\ 0 \quad , & \text{otherwise} \end{cases}$$

Hypergeometric distribution approximated as a binomial distribution:

Hypergeometric distribution tends to a binomial distribution if

$N \to \infty$ and $\frac{M}{N} \to p$ (mean $\frac{M}{N} < 1$)

Problems A taxi cab has 12 Maruti swift cars and 8 Tata Vista cars. If 5 of these cars in the shop are in repair then find the prob that

(i)

Sol^n (i) $\dfrac{{}^{12}C_3 \times {}^{8}C_2}{{}^{20}C_5}$ (ii) $P(x=3) + P(x=4) + P(x=5)$

$$\frac{{}^{12}C_3 \times {}^{8}C_2}{{}^{20}C_5} + \frac{{}^{12}C_4 \times {}^{8}C_1}{{}^{20}C_5} + \frac{{}^{12}C_5 \times {}^{8}C_0}{{}^{20}C_5}$$

(iii) P(All are maruti) + P(all are Tatavista)

$$\frac{{}^{12}C_5 \times {}^{8}C_0}{{}^{20}C_5} + \frac{{}^{12}C_0 \times {}^{8}C_5}{{}^{20}C_5}$$

Q2 200 students of firstyears MCA ------- probability that $R \geq 3$.

Soln Here we are discussing about batch.
So in a batch either a student will be resident or not
So it is the case of Binomial distribution

$$p = \frac{M}{N} = \frac{40}{200} = \frac{1}{5} = 0.2$$

M = resident students because we are talking about residents every time

$\boxed{n = 10}$ $\boxed{p = 0.2}$

$p(R)$ = prob that there are R residential student in a batch

$$p(R) = {}^{n}C_R\, p^R q^{n-R} = {}^{10}C_R (0.2)^R (0.8)^{10-R}$$

$$P(R \geq 3) = 1 - P(R<3) = 1 - \left[{}^{10}C_0 (0.2)^0 (0.8)^{10} + {}^{10}C_1 (0.2)^1 (0.8)^9 + {}^{10}C_2 (0.2)^2 (0.8)^8\right]$$

Q3 Find the prob that the income tax official will catch 3 income tax returns with the illegimate deductions, if the official randomly selects 5 returns among from 12 returns of which 6 returns contains illegimate deductions

12

6 legimitate 6 illegimitate

$$\boxed{\text{Req Prob} = \frac{{}^{6}C_3 \times {}^{6}C_2}{{}^{12}C_5}}$$