A Math-Heuristic Approach for Two Echelon Vendor Managed Inventory Routing Problem

Mehmet Can Aydın · Utku Yavuz · Yarkın Akçıl
Alp O. Özver · Çağla D. Özmen · Aslı Erol

Department of Industrial Engineering, Özyegin University, İstanbul, Turkey

Abstract

This paper studies a two-echelon vendor managed inventory routing problem of a honey packager company that delivers packaged products from a single facility to multiple retailers and customers. The objective is minimizing the total distribution cost while satisfying customers' demand through retailers. The problem is complex because it combines inventory management with routing decisions. We propose a mathematical optimization model and develop a three-step clustering-based math-heuristic algorithm to solve the problem since commercial solvers fail to provide high-quality solutions within a given time limit. The algorithm yields (on average) 16% improvement compared to objective value performances of the commercial solver.

Keywords: Vendor managed inventory, inventory routing problem, math-heuristics, clustering

1. Introduction

Supply chain optimization has become a common practice for large-scale companies in their pursuit of optimizing cost related elements. Many companies have utilized scientific methods in their exercises, using models and algorithms to optimize their global supply chains. These exercises can be generalized in forms of combinatorial optimization problems, such as the travelling salesman problem (TSP) and it's more general form that was first introduced in 1959 by George Dantzig and John Ramser, the vehicle routing problem (VRP).

The inventory routing problem (IRP) is a variant of the vehicle routing problem, where the priority is on the vehicles routing as well as the inventory management. The inventory routing problem is defined as follows: A vehicle fleet of fixed capacities serve customers of fixed demand by the period from the central depot. Customers must be assigned vehicles and vehicles are routed so that the total route cost is minimized. These homogeneous vehicles deliver a single product to multiple customers depending on their consumption rate/orders. The route costs depend on the distance between customers or between customers and the depot [10]. Determining and managing the optimum inventory amount are equally important as the routing process, where the inventory holding costs are significant and meeting customers demands perfectly are of paramount importance [11].

The inventory routing problem separates from the vehicle routing problem, where the previous decisions affect upcoming ones and these decisions are made over a planning horizon [6]. Furthermore, the vehicle routes, delivery amounts and the starting and ending points of the vehicles can be tracked while also constructing feasible and efficient routes with demand dependant delivery volumes to each customer, not exceeding the inventory capacity for both warehouse(s) and vehicle(s) and avoiding product shortage over the designated periods [6]. This way, vendor(s) are dependent on the customer(s) demand and consumption rate for the replenishment amount, which routes to use, which customers to serve and which vehicles to use without being dependent on the central depot. In networks with vendor managed inventory (VMI) replenishment system where the supplier observes the inventory levels of its customers and satisfies their demand accordingly. The vendor managed inventory enables the transportation resources to be distributed uniformly. This enables for a higher efficiency and optimized transportation costs [12].

Initially introduced by Dantzig and Ramser, the vehicle routing problem has many iterations and forms as it is one of the most well-known and thoroughly studied problems in the literature, such as the inventory routing problem. The aim of the inventory routing problem is to determine the best possible combinatorial route for the vehicles while also determining the ideal inventory amounts. Due to it's NP-Hard nature, a number of solution methods have also been introduced. Earlier studies done on inventory routing problem were mainly based on single period models and [8] can be acknowledged as the first pioneers of inventory routing problem. The problem mentioned in [8] is a single day inventory routing problem with a limited amount of inventory. The solution proposed includes modified vehicle routing problem heuristics formulated as nonlinear integer programming which aims to minimize inventory holding, shortage and transportation costs while determining routes of each vehicle. In order to understand how to solve the inventory routing problem better we must first explore the solutions proposed to the vehicle routing problem. One of these studies uses arcs and clusters to solve the vehicle routing problem. [9] transforms the vehicle routing problem into a capacitated arc routing problem (CARP) and proposes an exact algorithm to solve the routing problem in and out of the clusters, treating the inside of the clusters as a generalized traveling salesman problem determining the least-cost throughout the circuit. There are studies that use density based clustering methods rather than distance dependent greedy clustering . [7] states that green vehicle routing problem (G-VRP) is formulated as a mixed integer linear program. The problem includes the Modified Clarke and Wright Savings heuristic and the density-based clustering algorithm (DBCA). DBCA separates the vehicle routing problem into routing and clustering. The main point of DBCA is that for each vertex of a cluster, a specific radius (e), at least the minimum number of adjacent corners (minpts) must be contained. There are also studies that cluster data points using local optimization algorithms which locally search for the optimum solution and generally do not achieve global optimum solution. [4] studied two-echelon vehicle routing problem (2E-VRP) and they solved their problem with a clustering algorithm. Their aim was to minimize the total transportation cost. In this article, first-level vehicle routing is made between depot and satellites, and second-level vehicle routing is made between satellites and customers. Consequently, the final solution is made by solving the first and second level capacitated vehicle routing problems. There are many extensions of the vehicle routing problem such as the two-echelon vehicle routing problem. [13] studies two-echelon vehicle routing problem which is an extension of a classic vehicle routing problem. Their transportation network has two levels. In the first level it is the connection from the depot to intermediate depots. In the continuation, the second level represents the connection between intermediate depots and the customers with infinite time horizons. There are different preferences when it comes to determining the heuristic approach to the vehicle routing problem and some studies prefer a math-heuristic approach.

[3] proposes clustering-based heuristics for the two-echelon vehicle routing problem. They work in two stages. The first phase attempts to provide a decent workable solution using a clustering algorithm, while the second phase aims to develop it. They use a separating technique throughout, breaking the problem down into two routing sub-problems, one for each step. The direct shipping criteria assign a customer to its closest satellite in euclidean distance, which is the basis for the initial clustering. Finally, the experiments show that clustering-based meta-heuristics perform well and that a two-echelon structure will minimize delivery costs dramatically. [5] studied multi-depot heterogeneous flat vehicle routing problems with time windows. This is accomplished by

identifying a small number of feasible clusters, each enclosing a number of customer locations then calculating average travel distances and times between any two of them. The vendor managed inventory system is the type of inventory management system where the vendor checks the buyer's inventory levels and makes resupply decisions. The aim of vendor managed inventory is to create a mutually advantageous arrangement in which both vendor and customers can control the availability and flow of items more easily and precisely. [14] proposes a model for an inventory system that corresponds to the vendor managed inventory system. The objective of the model is to minimize total inventory cost for the distributor while stock-out is allowed. The inventory routing problem consists of inventory management and vehicle routing. The inventory routing problem emerges in the context of vendor managed inventory, in which a supplier decides on product completion for its customers. inventory routing problem problems are tried to be solved by looking at more than one parameter. In order to solve these problems, numerous solution approaches were proposed.[2] stated the integrated inventory management and vehicle scheduling in several variants of inventory routing problem. A variety of ways to solve these issues have been presented. The inventory routing problem models can be distinguished by their associated parameters, which have various properties. These include but are not limited to retailers' consumption rate, stable or variable rate; when the planning horizon is finite or infinite; when demands are assumed deterministic or stochastic. [2] addresses an inventory routing problem model planning horizon that is finite. There are multiple solution approaches for inventory routing problem and genetic algorithm (GA) is one of them.[1] stated that a depot has an adequate supply of inventory that can satisfy all demand throughout the finite time horizon inventory routing problem with multiple fleet sizes. Maximum Level(ML) policy is used as an inventory policy. To solve their problem, they sed a genetic algorithm which is one of the heuristic algorithms. To solve inventory routing problem, some studies prefer a hybrid genetic algorithm when a heuristic approach must be determined. [11] studied an inventory routing problem that has a single assembly plant and multiple unique suppliers that supply distinct products. The study has been made based on a many-to-one structure. The demand is deterministic for the product, and time-varying. A hybrid genetic algorithm that considers both inventory and transportation costs has been proposed to solve the problem. The number of capacitated vehicles is assumed to be infinite to better allow flexibility. Later, the exact number of vehicles will be determined. Inventory routing problems can be formulated using mixed integer linear programming since it is a widely used effective approach for solving large and complex problems. [15] claimed a two-echelon inventory-routing problem (2E-IRP) which is formulated as a mixed-integer linear programming (MILP). It is considered as a single depot problem with a finite time horizon which is represented as t. Since the products have an expiration date, they are sorted in the inventory accordingly. Products are stored at the depot with a certain capacity or since they are perishable products, they are disposed of as waste. [12] proposed an inventory routing problem with the vendor managed inventory system that produced products distributed from a single manufacturer to multiple retailers. To determine replenishment times and quantities and vehicle routes, they proposed a genetic algorithm to solve the problem. A mixed integer linear programming model is constructed for the VMIRPL. The objective of the model is to maximize supply chain profits for the distributor while lost sales are allowed.

In this paper, we consider an inventory routing problem where a fleet of vehicles deliver products to customers from the production center through retailers. The aim of this paper is to design a two-echelon inventory routing system solution that is computationally scalable for the case study while satisfying all demand within an acceptable optimality gap. However, rather than a supply chain where the products are directly delivered to customers from the production center, a two-echelon logistics network is proposed where a number of intermediate retailers are utilized. Combined with the weekly period deliveries planned by the system, this allows the usage of larger vehicles for bulk deliveries minimize network costs in the long-term approach. The first echelon of the logistic network consists of the larger vehicles departing from the production center to the retailers and the second echelon includes the delivery to the customers from the retailers visited by the production center. By planning a yearly vehicle routing schedule, we hope to identify the best sequence of appropriate decisions to maximize the honey packager company's sales by minimizing total costs and delivering products with greater customer satisfaction. To achieve these goals a mathematical optimization model is introduced. Since the problem combines retailer inventory management with routing decisions, the commercial solvers fails to provide first-rate solutions within acceptable time horizon. Therefore, a three-step clustering based math-heuristic approach to reach high-quality solutions with a minimum loss from optimality is proposed. With the proposed math-heuristic algorithm, the solutions improved on average 16% compared with the optimization model's solution performance in commercial solver with a time limit of four hours. In addition to yielding a better solution than the optimization model in the provided time limit, the proposed algorithm returns drastically shorter computational times when compared to commercial solvers. The defining challenge in our study is the sheer size of the network, consisting of approximately 1500 customers, 45 available retailer locations, and a large fleet of vehicles that need to be given a 52 period (periods are formulated as weeks, resulting in a yearly model) routing plan.

The remainder of the paper is organized as follows: §2 defines the problem and introduces the formulation of the mathematical model. §3 introduces the math-heuristic approach, accordingly; §3.1, §3.2 and §3.3 indicate/declare the steps of the approach as clustering, routing on second echelon and first echelon, respectively. §4 argues the numerical results of the implementation in §4.1, §4.2, §4.3 as details of forecasting, a brief introduction to the known clustering algorithms DBSCAN, K-Means and the comparison between clustering algorithms computational results respectively. Finally, §5 presents the conclusion.

2. Problem Definition and Model Formulation

The problem is inspired by a real life honey packager company that aims to utilize their inventories and vehicles better and work with better routes in order to satisfy all demand of a specific product with a high seasonality effect. The data on retailers, customers and the production center used in this paper are provided by the honey packager company along with the locations and other details. The company-provided problem revolves around a finite horizon with a single production facility and multiple retailers where a fleet of vehicles with finite capacities transport products from the single production facility to multiple customers through the usage of retailers. Production is assumed to be constant, not delayed by any means and unlimited to better manage the supply chain and guarantee demand is met perfectly since back-orders are not allowed. Customer demands vary based on their consumption rates, which indirectly affects when and how much customers receive products. Combined with the difficult-to-predict nature of the demand for the products, direct shipping is required to be implemented to satisfy several customer's demands based on their location and demand. The problem definition, parameters and assumptions are taken from the real life problem in order to ensure the model is easily implementable.

The considered problem is deterministic since the customer demands are forecasted for each period, and no stock-outs are allowed. The vendor is tasked with delivering products to the customers based on their forecasted yearly demand without the customer inventories reaching zero, similar to a vendor managed inventory setting. The fleet of vehicles on both echelons deliver the designated amounts in each period over the yearly planning horizon. In addition, the vendor can practice direct shipping in certain cases like outlier customers, to provide a model that is better suited for real-life cases. The vendors incur costs such as warehouse and vehicle usage costs, inventory cost, distribution and direct shipping costs. The model output consists of a yearly transportation plan for the fleet of vehicles including which routes to take in each period, how much product to carry over these routes, the inventory levels of each warehouse and when to use direct shipping for which customer. It is assumed that the production of the single product is considered to be infinite and constant at all times.

We propose a mathematical formulation solution and a math-heuristic algorithm that includes a clustering method for solving the two-echelon vendor managed inventory routing problem (2E-VMIRP). First, we construct a mathematical model for solving the problem and later show that it is not possible to solve the two-echelon vendor managed inventory routing problem in an accepted time horizon due to the computational challenge of said problem. The complex nature of the problem originates from connecting the inventory management and the routing process which makes getting an exact solution to the problem difficult. Then we propose the math-heuristic algorithm consisting of three steps which are: clustering, second echelon routing and first echelon routing. The idea behind this approach is to divide the problem into three small parts and solve them separately with mathematical formulations that are connected via their inputs and outputs. In clustering we assign retailer(s) to customers to satisfy the demand and cluster the retailer and it's customers. Note that, in order to make the problem more realistic, every customer must be served by only one retailer and/or can be satisfied by direct shipping. In the second step of the algorithm, we focus on the second echelon of the network and take the clusters from the previous step as input. This way, we can develop a feasible and cost-efficient routing structure for each cluster separately. The idea is to use the "good" clusters generated by our own algorithm for routing to save computational time. In the final step of our algorithm, we focus on the first echelon routing taking the routing information from the previous step as an input. This way, the amount of products the retailers must pull from the production center is known and the retailer(s) that will be used for routing is predetermined. The first echelon routing does not include any clusters so, executing the routing for all the retailers and the production center is sufficient. As mentioned before, although our algorithm consists of three steps, the integrated nature of the formulations bound the separate solutions.

The two-echelon vendor managed inventory routing problem in this study consists of a single production center supplying a single type of product to multiple retailers which then satisfies customer demands working in two echelons. The vendors plan inventory replenishment, ordering products from the production center according to the demand based on the forecasted amounts. In the first echelon, we consider that a product is shipped from the production center (i=0) to a set of possible retailers (i=1,...,Re) over a discrete time horizon T via first level vehicles (v=1,...,NV) with routing. In the second echelon, the products delivered to the retailers are distributed to the customers (j=1,...Cu) via second level vehicles (l=1,...,NL).

Before introducing the parameters of the mathematical formulation the assumptions made must be explained. It is assumed that the production of the production center is constant, not delayed by any means and unlimited. It is assumed that loading and unloading times are zero and do not incur any additional costs. Vehicles on the first level exit from the production facility and are routed to retailers, then route back to the production facility so they can be used again in the next period. Customers can only be served by a designated retailer. These customer- retailer pairings are constructed strictly based on transportation cost. It must be signified that back-orders are not allowed by any means. Within the same period, separate vehicles exiting from retailers carry this demand over to the customers on the second level and route back to the retailer where they started to route for further use. It is also possible for a vehicle to route between several customers before returning back to the retailer, the same is also valid for vehicles exiting the production center and routing between retailers before routing back. To ensure demand is met, direct shipping methods are also implemented to the model to neutralize cases where truck capacity is exceeded or where retailers cannot satisfy customer demand from their own inventory. The direct shipping option acts as a new type of transportation where the demand of the customer is satisfied from the production center directly with a noticeably higher cost. Note that, the flow on each arc and routes of every vehicle including their amount of demand on board are recorded for easier interpretation. The mathematical optimization model is formulated as follows:

Table 1: Summary of notation for the original mathematical model

Variables	
$R1^t_{0iv}$	1 if vehicle v is utilized from the production facility to customer j at time t;0 o.w.
$R3^t_{ikv}$	1 if vehicle v is utilized from retailer k to retailer i in t; 0 o.w.
$R5^t_{i0v}$	1 if vehicle v is utilized from retailer k to the production facility at time t; 0 o.w.
$R0^t_{0jl}$	1 if vehicle l is utilized from the production facility to customer j at time t; 0 o.w.
$R2^t_{ijl}$	1 if vehicle l is utilized from retailer i to customer j at time t; 0 o.w.
$R6^t_{mjl}$	1 if vehicle l is utilized from customer m to customer j at time t; 0 o.w.
$R4^t_{jil}$	1 if vehicle l is utilized from customer j to retailer i at time t; 0 o.w.
A_i	1 if retailer i has inventory flow for any given period; 0 o.w.
X^t_{0iv}	product flow on arc the production facility to retailer i by vehicle v at time t.
Z^t_{ikv}	product flow on arc retailer i to retailer k by vehicle v at time t.
Y^t_{ijl}	product flow on arc retailer i to county j by vehicle l at time t.
W^t_{mjl}	product flow on arc customer m to customer j by vehicle l at time t.
Q^t_{0jl}	product flow on arc the production facility to customer j by vehicle l at time t.
DS^t_j	amount of products delivered to customer j by using direct shipping at time t.
I^t_i	inventory level of retailer i at time t.
Parameters	
d^t_j	demand of customer j at time t.
c_{0i}	cost of arc from the production center to retailer i.
c_{ik}	cost of arc from retailer i to retailer k.
c_{ij}	cost of arc from retailer i to customer j.
c_{mj}	cost of arc from customer m to customer j.
c_{0m}	cost of arc from the production center to customer m.
FC	retailer usage cost.
H	inventory holding cost for each product.
CC	cost of carry for each product via routing.
DC	cost of direct shipping per item.
S	inventory capacity for each retailer.
C_v	capacity of vehicles, as number of units for the first level.
C_l	capacity of vehicles, as number of units for the second level.
CR	retailers' periodic distribution capacity
AR	number of different routes that retailers can do in a period.
D_{0i}	distance between production center to retailer i.
D_{ik}	distance between retailer i to retailer k.
D_{ij}	distance between retailer i to customer j.
D_{mj}	distance between customer m to customer j.
$MD1$	the maximum distance a first level vehicle can travel.
$MD2$	the maximum distance a second level vehicle can travel.

Mathematical Model :

$$
\begin{aligned}
\min_{\mathbf{z}} \quad & \sum_l \sum_t \sum_i \sum_j Y^t_{ij} CC + \sum_l \sum_t \sum_m R0^t_{0ml} c_{0m} + \sum_l \sum_t \sum_m R7^t_{m0l} c_{0m} \\
& + \sum_v \sum_t \sum_i R1^t_{0iv} c_{0i} + \sum_t \sum_{i,k} \sum_{i \neq k} \sum_v R3^t_{ikv} c_{ik} + \sum_v \sum_t \sum_i R5^t_{i0v} c_{i0} \\
& + \sum_l \sum_t \sum_i \sum_j R2^t_{ijl} c_{ij} + \sum_l \sum_t \sum_i \sum_j R4^t_{jil} c_{ij} + \sum_l \sum_t \sum_m \sum_j R6^t_{mjl} c_{mj} \\
& + \sum_t \sum_j DS^t_j DC + \sum_t \sum_i I^t_i H + \sum_i A_i FC
\end{aligned} \tag{1}
$$

s.t.

$$I_i^0 = 0 \tag{2}$$

$$\sum_v X_{0iv}^t + I_i^{t-1} + \sum_v \sum_k Z_{kiv}^t = \sum_l \sum_j Y_{ijl}^t + \sum_v \sum_k Z_{ikv}^t + I_i^t \quad \forall i, t \tag{3}$$

$$I_i^t \leq S \quad \forall i, t \tag{4}$$

$$I_i^T = 0 \quad \forall i \tag{5}$$

$$\sum_i \sum_j Y_{ijl}^t \leq C_l \quad \forall l, t \tag{6}$$

$$\sum_j Q_{0jl}^t \leq C_l \quad \forall l, t \tag{7}$$

$$\sum_i X_{0iv}^t \leq C_v \quad \forall i, t \tag{8}$$

$$\sum_i R1_{0iv}^t \leq 1 \quad \forall t, v \tag{9}$$

$$Q_{0jl}^t \leq MR0_{0jl}^t \quad \forall j, t, l \tag{10}$$

$$Y_{ijl}^t \leq MR2_{ijl}^t \quad \forall i, j, l, t \tag{11}$$

$$W_{mjl}^t \leq MR6_{mjl}^t \quad \forall m, j, t, l \tag{12}$$

$$R1_{0iv}^t + \sum_k R3_{kiv}^t = \sum_k R3_{ikv}^t + R5_{i0v}^t \quad \forall i, t, v \tag{13}$$

$$\sum_k \sum_v R3_{kiv}^t + \sum_v R1_{0iv}^t \leq 1 \quad \forall i, t \tag{14}$$

$$R0_{0jl}^t + \sum_i R2_{ijl}^t + \sum_m R6_{mjl}^t = \sum_m R6_{jml}^t + \sum_i R4_{jil}^t + R7_{j0l}^t \quad \forall l, t, j \neq m \tag{15}$$

$$\sum_l Q_{0jl}^t + \sum_i \sum_l Y_{ijl}^t + \sum_m \sum_l W_{mjl}^t - d_j^t + DS_j^t = \sum_m \sum_l W_{jml}^t \quad \forall j, t \tag{16}$$

$$\sum_j \sum_l Y_{ijl}^t \leq CR \quad \forall t, i \tag{17}$$

$$\sum_j \sum_l Q_{0jl}^t \leq CR \quad \forall t \tag{18}$$

$$\sum_j \sum_l R2_{ijl}^t = AR \quad \forall t, i \tag{19}$$

$$\sum_j \sum_l R0_{0jl}^t = AR \quad \forall t \tag{20}$$

$$\sum_j R2_{ijl}^t = \sum_j R4_{jil}^t \quad \forall t, i, l \tag{21}$$

$$\sum_j R0^t_{0jl} = \sum_j R7^t_{j0l} \quad \forall t, l \tag{22}$$

$$\sum_t \sum_v R1^t_{0iv} + \sum_t \sum_k \sum_v R3^t_{kiv} \leq MA_i \quad \forall i \tag{23}$$

$$\sum_i \sum_j R2^t_{ijl} + \sum_j R0^t_{0jl} \leq 1 \quad \forall l, t \tag{24}$$

$$X^t_{0iv} \leq MR1^t_{0iv} \quad \forall i, v, t \tag{25}$$

$$Z^t_{ikv} \leq MR3^t_{ikv} \quad \forall i, k, v, t \tag{26}$$

$$\sum_l R6^t_{mjl} = 0 \quad \forall m = j, t \tag{27}$$

$$\sum_i \sum_l R2^t_{ijl} + \sum_m \sum_l R6^t_{mjl} + \sum_l R0^t_{0jl} \leq 1 \quad \forall t, j \tag{28}$$

$$\sum_v R3^t_{ikv} = 0 \quad \forall i = k, t \tag{29}$$

$$\sum_i R1^t_{0iv} D_{0i} + \sum_i \sum_k R3^t_{ikv} D_{ik} + \sum_i R5^t_{i0v} D_{0i} \leq MD_1 \quad \forall t, v \tag{30}$$

$$\sum_j R0^t_{0jl} D_{0j} + \sum_j R7^t_{j0l} D_{0j} + \sum_i \sum_j R2^t_{ijl} D_{ij} + \sum_m \sum_j R6^t_{mjl} D_{mj}$$
$$+ \sum_i \sum_j R4^t_{jil} D_{ij} \leq MD_2 \quad \forall t, l \tag{31}$$

$$X^t_{0iv}, Y^t_{ijl}, Z^t_{ikv}, W^t_{mjl}, Q^t_{0jl}, I^t_i, DS^t_j \geq 0 \text{ and integer} \tag{32}$$

$$R0^t_{0jl}, R1^t_{0iv}, R2^t_{ijl}, R3^t_{ikv}, R4^t_{jil}, R5^t_{i0v}, R6^t_{mjl}, R7^t_{jil}, A_i \in \{0, 1\} \tag{33}$$

The objective function (1) is the minimization of all the related costs which are cost of carry per product, cost of using the specific arc, cost of direct shipping, holding cost of inventory and retailer usage fixed cost. Constraint (2) sets the initial inventory to zero. Constraint (4) sets the inventory capacity for all the retailers. Constraint (5) ensures that no inventory is left at the end of the last period. Constraint (8) sets the vehicle capacity for the first level while ensuring that a delivery can be smaller or equal to the capacity of the vehicle. Constraints (6) and (7) set the vehicle capacity for the second period while ensuring that a delivery can be smaller or equal to the capacity of the vehicle. Constraint (9) ensures that a vehicle that comes out from the production center can visit one retailer at maximum in a particular period. Constraint (14) enables that a retailer can be visited by only from another retailer or the production center. Constraint (3) sets the inventory flow balance for all the retailers for the first level. Constraint (16) sets the second level product flow, and ensures that all demand is satisfied via available channels. Constraints (13) and (15) ensure that the number of entering and exiting vehicles and their indices are equal for the first and second level. Constraints (10), (11) and (12) force the flow to be present only if that second level arc is used by that second level vehicle. Constraints (25) and (26) force the flow to be present only if that first level arc is used by that first level vehicle. Constraint (17) makes sure that the number of products distributed by a retailer cannot exceed the retailer distribution capacity. Constraint (18) makes sure that number of products distributed to customers by the production center cannot exceed its distribution capacity. Constraints (19) and (20) limit the possible routes a retailer or the productions center can do at any period to the different available routes for every period. Constraint (29) computes that a retailer cannot tour back to itself. Constraint (27) computes that a customer cannot tour back to itself. Constraints (21) and (22) ensure that the vehicles return to their designated starting point whether it is the production center or a retailer. Constraint (24) ensures that only a single retailer can utilize a second level vehicle for any period t. Constraint (28) ensures that visiting is made at most once to satisfy demand of a customer via routes. Constraint (23) limits the retailer to be utilized only if it receives product from the production center or any other retailer. Constraints (30) and (31) ensure that the distances traveled by the first and second level vehicles does not exceed the maximum allowed distance for each of them. Constraints (32) and (33) enforce binary, integer and non-negativity conditions upon the variables.

3. Math-Heuristic Approach

The complex nature of the problem combined with its dimensions forms a computationally challenging problem to be solved in CPLEX. Therefore, a heuristic approach is mandatory for the problem to be solved efficiently. We propose a math-heuristic algorithm consisting of three steps; clustering, second echelon routing and first echelon routing. In the proposed solution we use different mathematical formulations for each of the steps. The first mathematical model aims to create clusters consisting of a single retailer and multiple customers. The customers within the cluster are served by their corresponding retailer. After the clustering is complete, the second mathematical model is solved for each cluster on the second echelon, using the output of the first model. Following after is the third model, which is solving the routing for the first echelon, yielding the routing between the production center and each retailer that is included in a cluster in the previous model. Over the improvement period of the algorithm construction process, we continually generated randomized data sets to observe the wellness of the algorithm compared to the result of the mathematical model.

3.1. Step 1 - Clustering

In this step we aim to effectively cluster the retailers and the customers together. The key point is to select the best possible retailer-customer combinations with a method that does not steer the result too far from the optimal solution. A utility based mathematical model is introduced as the mentioned method. The objective of the model is to pick the retailer-customer pairings with the best demand over distance value, which is called the utility. Also, we incur an addition to the total utility value to consider the effect of the products distributed through used retailers on the objective function. Production center will also be used for distributing products to customers directly, meaning that customers close to the production center can be served via second level vehicles departed from the production center. The maximization of the utility needs an upper-bound to limit the model. Note that the last retailer node which is the dummy retailer corresponds to direct shipping. We introduced direct shipping as a retailer to compare the efficiency and fitness of direct shipping and comparing it to other possible retailers. Keep in mind that the dummy retailer has no capacity and does not involve any parameters while being available for usage every period. The dummy retailer is assumed to be at an equal distance to every customer even though this assumption is not possible in real life.

Table 2: Summary of notation for the clustering mathematical model

Variables	
x_{ij}^t	1 if customer i is assigned to retailer j at time t; 0 o.w.
y_j^t	1 if retailer j is utilized at time t; 0 o.w.
Parameters	
U_{ij}^t	utility of customer i - retailer j pairing (demand/distance) at time t.
MR	possible number of retailers can be used.
MD	tolerated distance for each retailer-customer pair.
DR	minimum demand required to be assigned to retailers for usage.
Abbreviations	
pc	production center.
dm	dummy retailer (direct shipment).

Clustering Mathematical Model :

$$\max_{\mathbf{z}} \quad \frac{1}{\alpha}\sum_{i}\sum_{j\notin dm}\sum_{t} U_{ij}^{t}x_{ij}^{t} + \frac{1}{\beta}\sum_{i}\sum_{j\notin dm}\sum_{t} d_{i}^{t}x_{ij}^{t} \tag{1}$$

s.t.

$$\sum_{j} x_{ij}^{t} = 1 \quad \forall i,t \tag{2}$$

$$\sum_{j\notin\{pc,dm\}} y_{j}^{t} \leq MR \quad \forall t \tag{3}$$

$$x_{ij}^{t} \leq y_{j}^{t} \quad \forall i,j,t \tag{4}$$

$$\sum_{i} d_{i}^{t}x_{ij}^{t} \leq CR \quad \forall j,t \tag{5}$$

$$D_{i}^{t}\ x_{ij}^{t} \leq MD \quad \forall i,j,t \tag{6}$$

$$\sum_{i}\sum_{t} d_{i}^{t}x_{ij}^{t} \geq \sum_{t} DR\ y_{j}^{t} \quad \forall j \notin \{pc,dm\} \tag{7}$$

$$y_{j}^{t} = 1 \quad \forall t, j = \{pc,dm\} \tag{8}$$

$$y_{j}^{t} = y_{j}^{t+1} \quad \forall t, j \notin \{pc,dm\} \tag{9}$$

$$x_{ij}^{t}, y_{j}^{t} \in \{0,1\} \tag{10}$$

The objective function (1) maximizes the utility by also considering the amount of demand distributed via routing, the nature of the objective function is further explained in §*3.1.1*. Constraint (2) allows a customer to only receive products from one retailer. Constraint (3) limits the total number of retailers utilized to not exceed the maximum number of retailers. Constraint (4) allows a retailer can only be assigned to customers if it is utilized. Constraint (5) ensures that product flow from a retailer cannot exceed the retailer's total distribution capacity. Constraint (6) limits the distance traveled from a customer to a retailer to not exceed the maximum tolerated distance. Constraint (7) makes sure that if a retailer is utilized, it meets the minimum demand output requirement. Constraint (8) enables direct shipping/dummy retailer and production center to be used and they are not included in the calculation of maximum retailer capacity. Constraint (9) makes sure that if a retailer is utilized at period t, it can be utilized for all other periods without incurring further costs. Constraint (10) sets the binary condition for the variables.

3.1.1. Determining α and β values

As seen in the objective function of the clustering mathematical model which maximizes utility, also to consider the amount of products delivered via routing it is added to objective function. However, while comparing the average demand and utility parameters' values there is a significant difference between their magnitudes, and this difference affects the objective function critically. To reduce this effect, it is required to normalize both sides of the objective function with designated coefficients. To determine these coefficients, the objective function is separated into two parts and the model is run separately for each part. While the first execution maximizes only the utility function, the second execution aims to only maximize the amount of products delivered via routing. Then the optimal objective values from both models are designated as α and β coefficients and implemented to the original Clustering Mathematical Model. The utility and demand functions are divided by α and β values respectively. With this implementation featuring the optimal values α and β of the two objective functions, the combined objective function can take values in the range of [0,2] and the resulting clustering mathematical model is normalized.

3.2. Step 2 - Second Echelon Routing

The clustering mathematical model, is giving clusters of retailer-customer assignments as an output which we will use as an input in this step. Recall that, the production center will behave like a retailer in this model to ensure customers assigned to the production center are served. The introduced mathematical model is an vehicle routing problem model that minimizes costs and determines the second level routes between the retailer and its assigned customers and between customers. Therefore, all decision variables and parameters related to the first level are removed. The second echelon routing includes the possibility of direct shipping that the model can prefer depending on the condition. Recall that direct shipping is sending the product/s via a different and direct transport with an increased cost to the customer in need without stopping somewhere else. Every cluster taken from the clustering step contains only one retailer and each customer must be served by at most one retailer, which creates the necessity of running the model for each cluster separately.

Second Echelon Routing Mathematical Model :

$$\min_{\mathbf{z}} \quad \sum_{l}\sum_{t}\sum_{i}\sum_{j} Y_{ij}^{t}CC + \sum_{t}\sum_{j} DS_{j}^{t}DC + \sum_{l}\sum_{t}\sum_{i}\sum_{j} R2_{ijl}^{t}c_{ij}$$

$$+ \sum_{l}\sum_{t}\sum_{i}\sum_{j} R4_{jil}^{t}c_{ij} + \sum_{l}\sum_{t}\sum_{m}\sum_{j} R6_{mjl}^{t}c_{mj} \tag{11}$$

s.t.

$$\sum_{l} R6_{mjl}^{t} = 0 \quad \forall t, m = j \tag{12}$$

$$Y_{ijl}^{t} \leq MR2_{ijl}^{t} \quad \forall i, j, l, t \tag{13}$$

$$W_{mjl}^{t} \leq MR6_{mjl}^{t} \quad \forall m, j, t, l \tag{14}$$

$$\sum_{i} R2_{ijl}^{t} + \sum_{m} R6_{mjl}^{t} = \sum_{m} R6_{jml}^{t} + \sum_{i} R4_{jil}^{t} \quad \forall l, t, j \neq m \tag{15}$$

$$\sum_{i} R2_{ijl}^{t} + \sum_{m} R6_{mjl}^{t} \leq 1 \quad \forall t, j \tag{16}$$

$$\sum_{i}\sum_{j} Y_{ijl}^{t} = C_{l} \quad \forall l, t \tag{17}$$

$$\sum_{i}\sum_{l} Y_{ijl}^{t} + \sum_{m}\sum_{l} W_{mjl}^{t} - d_{j}^{t} + DS_{j}^{t} = \sum_{m}\sum_{l} W_{jml}^{t} \quad \forall j, t \tag{18}$$

$$\sum_{j} R2_{ijl}^{t} \leq 1 \quad \forall t, i, l \tag{19}$$

$$\sum_{i}\sum_{j} R2_{ijl}^{t}D_{ij} + \sum_{m}\sum_{j} R6_{mjl}^{t}D_{mj} + \sum_{i}\sum_{j} R4_{jil}^{t}D_{ij} \leq MD_{2} \quad \forall t, l \tag{20}$$

$$\sum_{j} d_{j}^{t} - \sum_{j} DS_{j}^{t} = N_{i}^{t} \quad \forall t, i \tag{21}$$

$$Y_{ijl}^{t}, W_{mjl}^{t}, DS_{j}^{t}, N_{i}^{t} \geq 0 \text{ and integer} \tag{22}$$

$$R2_{ijl}^{t}, R4_{jil}^{t}, R6_{mjl}^{t} \in \{0, 1\} \tag{23}$$

The objective function (11) is the minimization of all the related costs which are cost of carry per product, cost of using the specific arc and cost of direct shipping. Constraint (12) limits the customer to tour back to itself. Constraints (13) and (14) allow the flow to be present only if that arc is used by a second level vehicle. Constraint (15) ensures that number of entering and exiting vehicles and their indices are equal. Constraint (16) allows the routing to be present only in the purpose of satisfying a demand. Constraint (17) is the vehicle capacity constraint for the second level. Constraint (18) is the second level flow constraint, and ensures that all demand is satisfied via available channels. Constraint (19) limits the route to only start from a retailer, arriving to a customer. Constraint (20) ensures that the distance traveled by a second level vehicle does not exceed the maximum allowed distance. Constraint (21) enables the dummy demand of the retailers to be equal to the demand of the customers that has been satisfied by that retailer. The Constraints (22) and (23) are binary, integer and non-negativity conditions for the variables.

$$\sum_j d_j^t - \sum_j DS_j^t = N_i^t \quad \forall t, i \qquad (21)$$

The dummy demand N_i^t is constructed in order to track the amount of products required to the retailer which is equal to the products distributed to the assigned customers from that retailer. This dummy demand is initialized in both echelons but with different agendas. In the second echelon the dummy demand is initialized as a decision variable in order to prevent limitation and allow the formulation to find the optimal amount. However, in the first echelon the dummy demand is initialized as a parameter and takes it's value from the second echelon routing as an input. By being initialized as a parameter the dummy demand forces the production center to send the products that are required to the retailer. This way we also prevent back-orders and ensure that all demand is satisfied.

3.3. Step 3 - First Echelon Routing

In this step we use some of the outputs from the previous mathematical model, the Customer Bubble Model. The retailers that are used in the previous model are all assigned with customer/s, as it is known that they must be opened for the optimal result to be achieved in our next model. It is assumed that those retailers are available and the other ones are not available, calculating the cost of using a retailer accordingly. The introduced mathematical model is an IRP model that minimizes costs and determines the routes between the production center and retailers and between retailers. In the second echelon routing there are several clusters that must be solved and the same model must be run for each of them however, in this model all the retailers and the production center are in the same cluster, so running the model once for the entire cluster will cover all the possible routes.

First Echelon Routing Mathematical Model :

$$\min_{\mathbf{z}} \quad \sum_t \sum_i \sum_v R1^t_{0iv} c_{0i} + \sum_t \sum_{i,k} \sum_{i \neq k} \sum_v R3^t_{ikv} c_{ik}$$

$$+ \sum_t \sum_i \sum_v R5^t_{i0v} c_{0i} + \sum_t \sum_i I^t_i H \tag{24}$$

s.t.

$$I^0_i = 0 \quad \forall i \tag{25}$$

$$I^T_i = 0 \quad \forall i \tag{26}$$

$$I^t_i \leq S_i \quad \forall i, t \tag{27}$$

$$\sum_v X^t_{0iv} + I^{t-1}_i + \sum_v \sum_k Z^t_{kiv} = N^t_i + \sum_v \sum_k Z^t_{ikv} + I^t_i \quad \forall i, t \tag{28}$$

$$\sum_i X^t_{0iv} \leq C_v \quad \forall v, t \tag{29}$$

$$\sum_i R1^t_{0iv} \leq 1 \quad \forall v, t \tag{30}$$

$$R1^t_{0iv} + \sum_k R3^t_{kiv} = \sum_k R3^t_{ikv} + R5^t_{i0v} \quad \forall i, v, t \tag{31}$$

$$\sum_v R1^t_{0iv} + \sum_k \sum_v R3^t_{kiv} \leq 1 \quad \forall i, t \tag{32}$$

$$X^t_{0iv} \leq M R1^t_{0iv} \quad \forall i, v, t \tag{33}$$

$$Z^t_{ikv} \leq M R3^t_{ikv} \quad \forall i, k, v, t \tag{34}$$

$$\sum_v R3^t_{ikv} = 0 \quad \forall i = k, t \tag{35}$$

$$\sum_i R1^t_{0iv} D_{0i} + \sum_i \sum_k R3^t_{ikv} D_{ik} + \sum_i R5^t_{i0v} D_{0i} \leq M D_1 \quad \forall t, v \tag{36}$$

$$X^t_{0iv}, Z^t_{ikv}, I^t_i \geq 0 \text{ and integer} \tag{37}$$

$$R1^t_{0iv}, R3^t_{ikv}, R5^t_{i0v} \in \{0, 1\} \tag{38}$$

The objective function (24) is the minimization of all the related costs which are the cost of using the specific arc and inventory holding cost. Constraint (25) sets the initial inventory of every retailer to zero. Constraint (26) makes sure that at the end of the last period there are no products in the inventory. Constraint (27) sets the inventory capacity of all the retailers. Constraint (28) is the inventory balance constraint for all the retailers, and ensures product needs to be distributed from each used retailer is satisfied by the production center. Constraints (29) and (30) set the vehicle capacity and makes sure that the delivery amounts can be smaller than or equal to the first level vehicle capacity. Constraint (31) ensures that number of entering and exiting vehicles and their indices are equal. Constraint (32) enables a retailer can be visited by another retailer or the production center. Constraints (33) and (34) allow the flow to be present only if that arc is used by a first level vehicle. Constraint (35) makes sure that a retailer cannot tour back to itself. Constraint (36) limits the distance traveled by a first level vehicle to be smaller than or equal to the maximum allowed distance. Constraints (37) and (38) are the binary, integer and non-negativity constraints for the variables.

Table 3: Pseudocode of the proposed math-heuristic algorithm

Clustering	
Inputs:	Utility U_{ij}^t, demand d_i^t, and the clustering model.
Procedure:	**Step 0:** Start with j available retailers (initially j=0), for every visit increase j by one. **Step 1:** Execute the clustering model twice to maximize both side of the clustering model's objective function separately, keep optimal values as α and β. **Step 2:** Using α and β values, execute the initial clustering model to find best retailer-customer pairs. **Step 3:** Keep the quantity of demand carried with direct shipping determined in **Step 2**, together with the number of retailers available. **Step 4:** If $j = 0$, go to **Step 0**, and proceed to **Step 5** without visiting **Step 4**, else go to **Step 5**. **Step 5:** Compare the difference in quantity of demand carried with direct shipping when available retailer equals j and $j-1$. If the difference is bigger than the parameter DR, go to **Step 0**, else break the procedure and keep all customer-retailer pairing results when number of available retailers is equal to $j-1$.
Outputs:	x_{ij}^t values if customer i assigned to retailer j at time t, and y_j^t values if retailer j is used.
Second Echelon	
Inputs:	Customer-retailer assignment list, used retailers list from clustering, and the second echelon routing mathematical model.
Procedure:	**Step 0:** Start with k^{th} index in used retailers list (initially k=0), and keep the value in index as i. **Step 1:** For each retailer, time period t is 1 initially, if $t > t_{max}$, then increase k by one, and go to **Step 0**, else go to **Step 2**. **Step 2:** Indicate which customer is assigned to retailer i at time t by using customer-retailer assignment list, and corresponding parameters. **Step 3:** Execute the second echelon routing mathematical model with indicated customers, and keep amount of product needs to be distributed via routing by retailer i at time t as initialized decision variable N_i^t. Increase t by one, and go to **Step 1**.
Outputs:	Total cost as objective function value and routing output of the second echelon. Amount of product needs to distributed via routing for every used retailer i, and every t as N_i^t.
First Echelon	
Inputs:	N_i^t as parameter; for every used retailer and every t, the first echelon routing mathematical model.
Procedure:	Execute the first echelon routing mathematical model to satisfy product needed to used retailers for distribution.
Outputs:	Total cost as objective function value and routing output of the first echelon. It will be initialized the utilization cost of a retailer for each used retailer.

4. Implementation and Computational Results

4.1. Forecasting Details

The goal of the study is to decrease the delivery cost of a product that has high sales by declaring a new delivery methodology. The current delivery procedure that the honey packager company operates induces high operational costs. Therefore, a new routing system that optimizes the total cost of the delivery/transportation process is developed. Because of the product's increased demand rates, forecasting future sales, and satisfying all demand is important. In consideration of all these, having a good forecast is essential for the planning horizon. Shared past data consisted of the past three years' sales, and was not categorized by any time horizon. To reach better-forecasted results, first the past data was categorized on a weekly basis, and data structure became eligible to get into the forecasting process. With the implementation of the data into well-known time-series analysis tools, next year's forecasted sales are determined. This way the data for the problem becomes deterministic rather than stochastic.

4.2. Selection of the Clustering Algorithm

It is essential to compare and test our proposed clustering algorithm with two known clustering algorithms which are the density-based spatial clustering (DBSCAN) and the K-Means algorithms. The DBSCAN algorithm performs clustering by grouping areas with more objects than a predetermined threshold in each region by calculating the distance of objects from their neighbors. Algorithm takes epsilon distance (eps) and *minpts* values as input parameters, and forms clusters while detecting the outliers set as "Cluster -1", which refers to the possibility of sending via direct shipping option in our case. In our problem k-nearest neighbors of every point is checked to determine the epsilon distance in the dataset, and corresponding distances are sorted. Finally, the knee points of first "k" nearest neighbors are checked, and the mean is taken in order to determine epsilon distance value. Second parameter *minpts*'s value is determined according to the K-NN method that reveals if there is a significant distance increase (knee) between while analyzing a specific number of neighbors. Therefore, the required *minpts* and epsilon distance values for creating a cluster are determined.

Second clustering algorithm in comparison is K-Means. In the K-Means algorithm, many trials with different k values are required to find the most suitable value of k. The elbow method and

Table 4: Clustering - average computational results on randomly generated instances

n	**c**	**r**	**K-Means**	**DBSCAN**	**Proposed Clustering**
3	18	4	172518	205987	**162953**
3	25	5	350759	277897	**244142**

silhouette score are the techniques for determining the ideal cluster size in our case. The elbow method was used to determine the ideal number of clusters by considering the trade-off between cluster size and the amount of error given. On the other hand, the silhouette score metric is used to check the similarity between determined clusters. In our problem, the outputs of the elbow method and the silhouette scores are used to build "k" clusters.

After completing the math-heuristic algorithm and making sure that it works as intended, the clustering section of the math-heuristic algorithm is removed and the K-Means and DBSCAN's outputs are implemented for testing. The rest of the algorithm worked as intended, solving the inventory routing problem. Recall that customers in DBSCAN's outliers set (cluster -1) are served with direct shipping. In the clustering process if any cluster's demand exceeds the retailer distribution capacity, firstly the excessive demand is attempted to be satisfied via direct shipping. If the excess demand is above the tolerance level, the overpopulated cluster gets re-clustered with respect to its nearest neighbor knee value (for DBSCAN) and its corresponding silhouette score (for K-Means). During the implementation of clustering algorithms into the math-heuristic approach; firstly, cluster's centroids are assigned retailers' coordinates accordingly, then points inside the cluster are automatically assigned to the chosen cluster. This way each dataset's results are compared with three different algorithms which are the proposed math-heuristic's clustering section, K-Means, and density-based spatial clustering algorithms.

After many trials made in different dimensions, the proposed clustering algorithm is tested against DBSCAN and K-Means algorithms. Table 4 showcases the average total cost of all algorithms from three different runs from each dimension. In Table 4; n, c, and r stand for number of different datasets tried in the same dimension, number of customers, and number of retailers respectively. Result shows that the proposed clustering algorithm stands out among DBSCAN and K-Means algorithms with its lowest average total cost. From this point on, the proposed clustering algorithm is chosen and implemented to the solution algorithm.

Table 5: Computational result comparisons on randomly generated instances

c	**r**	**t**	**Model Result**	**Gap (%)**	**Algorithm Result**	**CPU (sec)**	**Improv. (%)**
30	4	2	211960	60.7	**197182**	52	7.9%
30	4	2	170139	49.5	**148720**	12	14.8%
40	5	2	198413	48.0	**182240**	72	8.1%
40	5	2	223397	57.4	**204596**	726	10.8%
50	7	3	366869	40.3	**302869**	1404	17.4%
50	7	3	425680	37.6	**410536**	77	6.4%
65	8	3	642517	49.3	**507150**	1135	29.8%
65	8	3	517274	42.6	**428679**	281	30.3%

4.3. Computational Results

The proposed math-heuristic algorithm in §4 was implemented in Python by using Pyomo and CPLEX 12.9.0, and runs on a 10th Generation Intel Core™ i7 Processor and 16 GB RAM personal computer. For the initial solution of the problem which is the original mathematical model, it was implemented CPLEX Studio IDE 20.1.0 on the same personal computer with a maximum running time of four hours.

The Table 5 provides the results of the mathematical model and the proposed algorithm, the optimality gap of the mathematical model after running time of four hours in CPLEX, the amount of time it takes for the algorithm to solve the problem instance as seconds and the improvement on results of the proposed algorithm compared to the mathematical model. Note that, the percentile for improvement being positive means that the algorithm performed better as the smaller result is preferred for the minimization objective for the problem. The n,r and t stand for the number of customers, number of retailers and the time horizon for the specific instance, respectively. The locations of the production center, retailers and the customers on three randomly generated test instances are shown in Figure 2. The amount of time that is required for the proposed algorithm to solve the problem instance is highly dependent on the number of customers that are connected to any retailer. Even though for two different instances sizes are same, computational time to get a solution may differ due to size of clusters imported to the algorithm to be solved. The %Improvement column is included to observe the performance between two different solutions introduced in this paper. Although the computational time of the proposed algorithm may be high in specific instances, it is still much better than the original mathematical model solved in CPLEX. The goal for the algorithm was to reduce the computational time with minimum loss from optimal results. As a result, we decreased the computational time severely and consistently achieved better results than the 4 hours of run time in CPLEX for the original mathematical model. The algorithm's routing results for the test instances in Figure 2 are available in Appendix A.

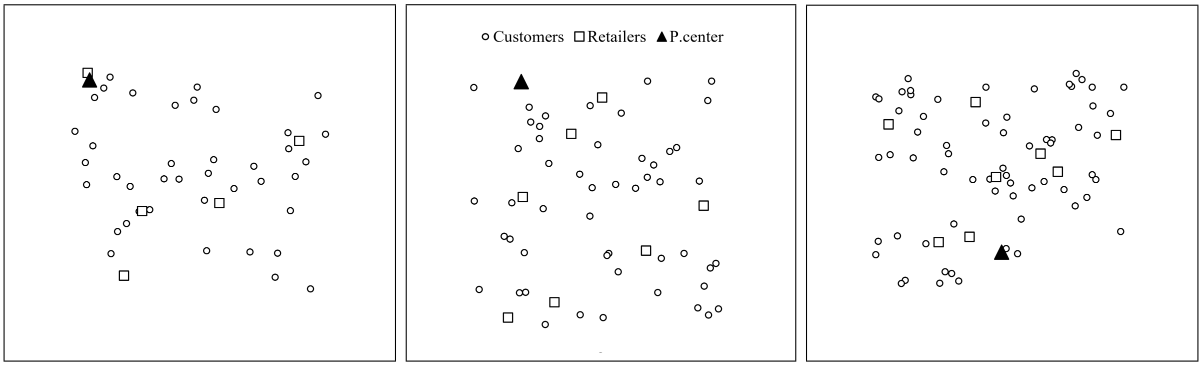

Figure 1: Randomly generated instances with 40, 50, 65 customers respectively

5. Conclusion

The inventory routing problem has been a highly valued topic ever since it was first proposed, with many unique solution methods and different settings. The problem itself is NP-Hard by nature with a fair amount of different objective functions to be decided by the model. The literature review showcases many approaches, with no exact solution in sight due to the problems complexity. The proposed model also fails to be solved in even remotely large settings, once again indicating the complexity of the problem.

The two-echelon vendor managed inventory routing problem is discussed in this paper, and the clustering-based math-heuristic algorithm consists of three parts is proposed. The first phase constructs clusters including a single retailer and multiple customers in each. The customers are paired with their corresponding retailers using an utility based function. The second step solves the routing problem in each cluster independently and the third step solves the routing problem between each cluster's retailer and the production center. The proposed algorithm achieves an on average 16% improvement compared to mathematical model's four hours performance in commercial solver CPLEX on all instances run, while decreasing computational time to minutes. The clustering algorithm is compared to other well-known algorithms in the literature, and the proposed math-heuristic algorithm is compared with the initial model in several instances to yield a proper estimate about the performance of the algorithm.

References

[1] Tamer F Abdelmaguid and Maged M Dessouky. A genetic algorithm approach to the integrated inventory-distribution problem. *International Journal of Production Research*, 44(21):4445–4464, 2006.

[2] Walter J Bell, Louis M Dalberto, Marshall L Fisher, Arnold J Greenfield, Ramchandran Jaikumar, Pradeep Kedia, Robert G Mack, and Paul J Prutzman. Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer. *Interfaces*, 13(6):4–23, 1983.

[3] Teodor Gabriel Crainic, Simona Mancini, Guido Perboli, and Roberto Tadei. *Clustering-based heuristics for the two-echelon vehicle routing problem*, volume 46. CIRRELT Montréal, 2008.

[4] Teodor Gabriel Crainic, Simona Mancini, Guido Perboli, and Roberto Tadei. Multi-start heuristics for the two-echelon vehicle routing problem. In *European Conference on Evolutionary Computation in Combinatorial Optimization*, pages 179–190. Springer, 2011.

[5] Rodolfo Dondo and Jaime Cerdá. A cluster-based optimization approach for the multi-depot heterogeneous fleet vehicle routing problem with time windows. *European journal of operational research*, 176(3):1478–1507, 2007.

[6] Ali Ekici, Okan Örsan Özener, and Gültekin Kuyzu. Cyclic delivery schedules for an inventory routing problem. *Transportation Science*, 49(4):817–829, 2015.

[7] Sevgi Erdoğan and Elise Miller-Hooks. A green vehicle routing problem. *Transportation research part E: logistics and transportation review*, 48(1):100–114, 2012.

[8] Awi Federgruen and Paul Zipkin. A combined vehicle routing and inventory allocation problem. *Operations Research*, 32(5):1019–1037, 1984.

[9] Gianpaolo Ghiani and Gennaro Improta. An efficient transformation of the generalized vehicle routing problem. *European Journal of Operational Research*, 122(1):11–17, 2000.

[10] Chryssi Malandraki and Mark S Daskin. Time dependent vehicle routing problems: Formulations, properties and heuristic algorithms. *Transportation science*, 26(3):185–200, 1992.

[11] Noor Hasnah Moin, Said Salhi, and NAB Aziz. An efficient hybrid genetic algorithm for the multi-product multi-period inventory routing problem. *International Journal of Production Economics*, 133(1):334–343, 2011.

[12] Yang-Byung Park, Jun-Su Yoo, and Hae-Soo Park. A genetic algorithm for the vendor-managed inventory routing problem with lost sales. *Expert Systems with Applications*, 53:149–159, 2016.

[13] Guido Perboli, Roberto Tadei, and Daniele Vigo. The two-echelon capacitated vehicle routing problem: Models and math-based heuristics. *Transportation Science*, 45(3):364–380, 2011.

[14] Rita Ribeiro and Helena Ramalhinho-Lourenço. Multi-period vendor managed inventory systems. 2003.

[15] SUK Rohmer, GDH Claassen, and Gilbert Laporte. A two-echelon inventory routing problem for perishable products. *Computers & Operations Research*, 107:156–172, 2019.

Appendix

Appendix A:

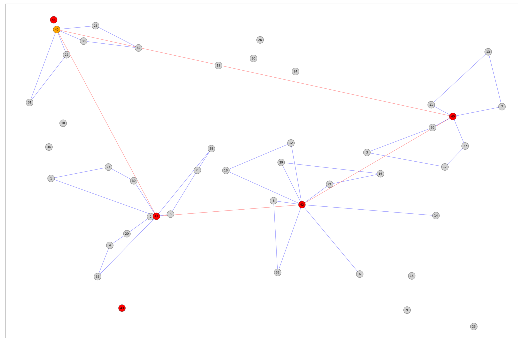

Figure A.1: Algorithm's routing result of 40-5R test instance

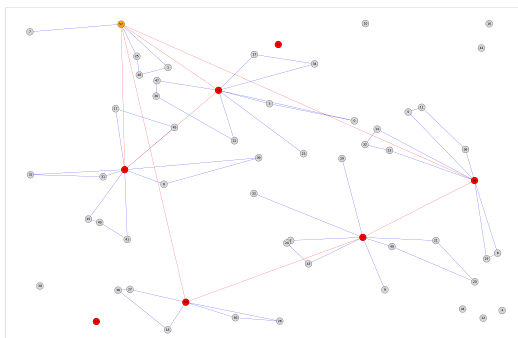

Figure A.2: Algorithm's routing result of 50C-7R test instance

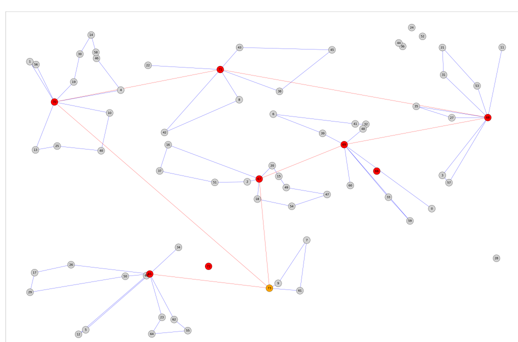

Figure A.3: Algorithm's routing result of 65C-8R test instance