

# IMEP 2014 Lectures 11 and 12

## The Balassa-Samuelson model, UIP and the flex-price model of exchange rates

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# Outline of today's lectures

## Lecture 11

- The Balassa-Samuelson model
- Implications for PPP and the real exchange rate
- Empirical evidence on the B-S model
- Theoretical issues

## Lecture 12

- Uncovered Interest Parity (UIP)
- The flex-price monetary model of the exchange rate
- Solving the flex-price monetary model
- Policy implications

# Lecture 11 – The Balassa-Samuelson model

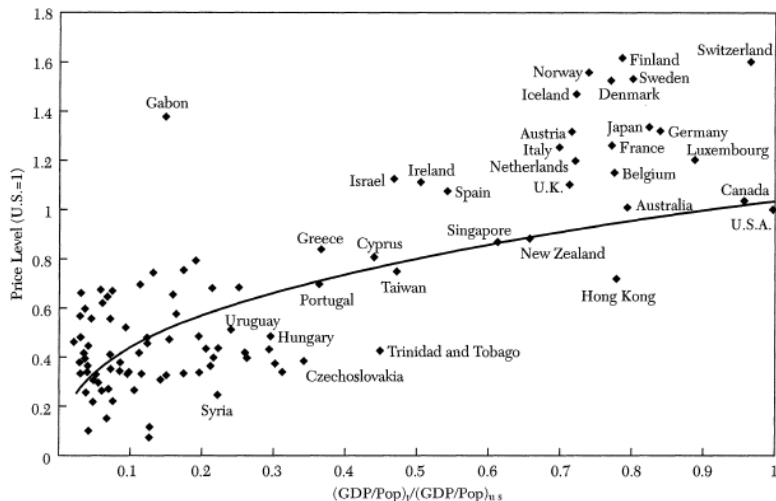
Key reading:

- ① Obstfeld and Rogoff, Chapter 4.1 and 4.2.3

# The Balassa-Samuelson model

- This model is motivated by a striking empirical observation: rich countries have much higher price levels than poor ones!
- See, for example, the data in O&R Fig 4.1, Rogoff (1996) Table 3, and the next slide
- The B-S model rationalises PPP deviations based on the interaction between traded and non-traded goods sectors
- In particular, it explains higher price levels in rich countries by appealing to higher productivity in the traded goods sector
- The key point is that the consumer price index will increase if non-tradable prices increase, since  $P = (P^T)^\alpha (P^{NT})^{1-\alpha}$

# International price levels vs GDP per head



Source: Rogoff (1996), Fig 3

# A simple model of the B-S hypothesis

- The model here is a simplified version of Sarno and Taylor, pp. 89-90
- Consider a small open economy that uses capital and labour to produce tradable and non-tradable goods:

$$Y^T = A^T (L^T)^\beta (K^T)^{1-\beta} \quad \text{and} \quad Y^{NT} = A^{NT} (L^{NT})^\beta (K^{NT})^{1-\beta}$$

- Here  $A^T$  is productivity in the traded goods sector,  $A^{NT}$  is productivity in the non-traded goods sector, and  $0 < \beta < 1$
- The world interest rate is given by  $r$ . Capital is mobile internationally but labour is not.
- Capital and labour can move freely between home economy sectors

# A simple model of the B-S hypothesis

- Because capital is mobile internationally, the rental rate paid to capital in both traded and non-traded sectors is equal to  $r$
- The real wage paid to workers is equal to  $w$  in both sectors, which may differ from the world wage rate
- PPP will hold in the tradable goods sector, so that

$$P^T = SP^{T*}$$

- The tradable goods price is therefore determined outside the model. Let's suppose it's equal to 1 to make things simple.

# A simple model of the B-S hypothesis

- Each sector maximises profits:

$$\max_{L^T, K^T} Y^T - wL^T - rK^T; \quad \max_{L^{NT}, K^{NT}} P^{NT} Y^{NT} - wL^{NT} - rK^{NT}$$

- The first-order conditions are

$$w = \beta A^T (K^T / L^T)^{1-\beta}, \quad r = (1 - \beta) A^T (L^T / K^T)^\beta$$

$$w = \beta P^{NT} A^{NT} (K^{NT} / L^{NT})^{1-\beta}, \quad r = (1 - \beta) P^{NT} A^{NT} (L^{NT} / K^{NT})^\beta$$

- Set the equations for  $r$  equal to each other and rearrange for  $P^{NT}$ :

$$P^{NT} = \frac{A^T}{A^{NT}} \left( \frac{L^T / K^T}{L^{NT} / K^{NT}} \right)^\beta$$

- Since the labour-capital ratio will be the same in both sectors, this equation simplifies to

$$P^{NT} = \frac{A^T}{A^{NT}}$$



# Implications of the B-S model

- The equation  $P^{NT} = \frac{A^T}{A^{NT}}$  is a statement of the B-S hypothesis
- It says the price of non-tradables will be higher in countries which are relatively more productive in the traded goods sector
- Since the price of non-tradables rises with relative productivity, so will the price level  $P$
- Consequently, differentials in relative productivity in the traded goods sector will lead to different national price levels

# Implications of the B-S model

- This model can potentially explain why price levels are higher in rich countries, as technological progress has been faster in manufacturing than in services – see Rogoff (1996, p. 658)
- The B-S effect leads to an increase in  $P$ , so the real exchange rate  $Q = \frac{SP^*}{P}$  will fall
- In other words, the model predicts an appreciation of the real exchange rate in rich countries
- The size of the B-S effect will determine how far  $Q$  falls below 1

# The economics behind the B-S hypothesis

- There are 4 key assumptions behind the B-S effect:
  - 1 The price of traded goods cannot vary from the level required by PPP, so only the relative price of non-traded goods changes
  - 2 Wages must be equal across home sectors because labour can move freely from one sector to another
  - 3 Capital is mobile internationally, so the return on capital in both sectors is equal to the world interest rate
  - 4 Because labour is immobile internationally, home workers cannot escape low wages by moving abroad

# The economics behind the B-S hypothesis

- The economic intuition for the B-S effect is as follows
- Suppose productivity is rising at a faster rate in the traded sector. In the absence of any other changes, wages in this sector would also rise at this faster rate.
- The only way the non-traded goods sector can prevent an exodus of workers to the traded goods sector is by raising the price of its goods so as to keep wage growth at the same level as in the traded sector
- The larger the difference between productivity growth in the traded and non-traded sectors, the larger the rise in non-traded prices, and the larger the appreciation in the real exchange rate

# Empirical evidence on the Balassa-Samuelson model

- Empirical evidence on the B-S effect is quite mixed
- Postwar labour productivity growth in manufacturing has been higher in Japan than the US, while the US remains more productive in services (see O&R, pp. 212-14)
- As predicted by the B-S model, Japan's real exchange rate against the US has appreciated.
- However, the evidence is weaker for other industrialised economies like those in the OECD and the Asia-Pacific region
- In some cases, the B-S effect is not evident in real exchange rates even at long horizons (Rogoff 1996, p. 662; Sarno and Taylor 2002, p. 92)

# The Balassa-Samuelson model: theoretical issues

- One can also question the B-S model on theoretical grounds
- **Example:** the model assumes that labour is completely immobile internationally. If labour were partially mobile internationally, the predicted Balassa-Samuelson effect would be smaller
- The model also assumes improvements in productivity DO NOT spill-over from rich to poor countries. This is probably unreasonable in the long run because technology diffuses across borders.
- Rogoff (1996, p. 662) argues that diffusion of technology across borders could explain why PPP holds in the long run

# Balassa-Samuelson model summary

- The B-S model is an important theory because it provides an equilibrium explanation for the long run deviations from PPP
- It predicts that countries with relatively strong productivity growth in their traded sectors should experience higher inflation and an appreciation in their real exchange rate
- This effect does seem to have some support in the data, but in many countries it is not evident even at longer horizons. So, while we do not have enough evidence to reject the B-S hypothesis outright, we cannot accept it either.
- The PPP puzzle therefore remains and is unlikely to be solved anytime soon!

# Lecture 12

## UIP and the flex-price model of the exchange rate

Key reading:

- ① Copeland, Chapter 3.1: Uncovered interest rate parity
- ② Obstfeld and Rogoff, Chapter 8.2 to 8.2.2 and 8.2.7



# Uncovered interest parity (UIP)

- UIP: the expected rate of depreciation of the exchange rate will be related to the difference between home and foreign interest rates
- To see why, recall our two-period model of the current account with investment and perfect capital mobility

- This model led us to conclude that

$$1 + MPK = 1 + \text{world interest rate}$$

- Due to arbitrage in the home country, the domestic interest rate will equal the marginal product of capital (MPK), so that

$$1 + i = 1 + i^*$$

- **What, if anything, is wrong with this conclusion?**

# Uncovered interest parity (UIP)

- In the two-period model, the world interest rate is denominated in units of *home output*, but things are not so simple in practice...
- **Example:** if we lend money to the US government by buying their debt (ie bonds), we will receive a return in \$ next period
- To use this return to buy consumption at home, we will have to convert it back into Pounds at the prevailing exchange rate  $S_{+1}$
- And to make the original purchase of bonds one period earlier, we would need to exchange Pounds for \$s at rate  $1/S$
- Therefore, the return on investing in foreign bonds is

$$(1 + i^*) \frac{S_{+1}}{S}$$

# UIP under perfect foresight

- Let  $1 + i$  be the return on home bonds and assume that there is perfect capital mobility
- Applying a simple arbitrage argument leads us to the conclusion that

$$1 + i = (1 + i^*) \frac{S_{+1}}{S} \quad (\text{UIP under perfect foresight})$$

- Again  $MB = MC$ : the LHS is the return from investing £1 more in home bonds, while the RHS is the return that could have been earned on foreign bonds if that £1 had been invested abroad
- **Example:** Suppose that  $i^* = 0.05$  and you know that the Pound will depreciate by 10% over the next year. What is  $i$  equal to?

# UIP under uncertainty

- In practice, we do not know the future exchange rate with certainty
- Consequently, domestic residents must compare the return on home bonds with the *expected return* on foreign bonds
- This leads us to the general UIP condition:

$$1 + i = (1 + i^*) \frac{S^e}{S} \quad (\text{UIP, general case})$$

where  $S^e$  is the expected future exchange rate

- **Exercise:** if  $i = 0.1$  and  $i^* = 0.05$ , what expected rate of depreciation in the home currency is consistent with UIP?

- **UIP and the policy trilemma**

- 1 If we have a credible fixed exchange rate regime then  $S^e = S$ , so that  $i = i^*$ . An economy with no restrictions on capital flows and a fixed exchange rate cannot operate an independent monetary policy.
- 2 If the exchange rate is floating, then in general  $S^e \neq S$ . As a result,  $i \neq i^*$  so that the domestic economy can operate an independent monetary policy while also benefiting from full capital mobility.

# Case study I: UK exit from the ERM

- The UK joined the ERM in Oct 1990 but was forced to exit on 16 Sep 1992 after widespread speculation that the Pound was overvalued
- **Can we understand this chain of events using UIP?**
- Yes, it seems so...
- The UK was in recession, partly because it had to match the high level of interest rates in Germany – ie  $i = i^*$  since the exchange rate was expected to remain fixed.
- Recession and high inflation led to the expectation that the Pound would depreciate – ie  $S^e > S$

# Case study I: UK exit from the ERM

- In an attempt to maintain the exchange rate, the UK used up large amounts of reserves, but the expectation of depreciation remained
- Consequently, it became difficult – and costly – to remain in the ERM, leading to UK exit and a 15% depreciation in the Pound
- Consistent with these events, the UIP condition predicts that an expected depreciation will lead to an actual depreciation
- We can see this by rearranging the UIP condition:

$$S = S^e \frac{(1+i^*)}{(1+i)}$$

## Case study II: Exchange rates and interest rates

- On the face of it, UIP appears to be at odds with our expectation that a cut in interest rates will be followed by a depreciation in the home currency
- However, closer inspection shows that this is not the case – if  $i$  falls and  $i^*$  is constant,  $\frac{S^e}{S}$  must fall to ensure that UIP holds
- In other words, there must be an expected future appreciation of the home currency. The only way this is possible is if the currency depreciates immediately after a cut in interest rates.
- For further discussion, see Bank of England (1999) – last para, p. 4



# UIP an the flex-price monetary model

- UIP is an important ingredient in monetary models of exchange rates
- For the remainder of this lecture we will focus on the flex-price monetary model of the exchange rate
- UIP enters the flex-price monetary model as one of the equations.  
The other equations relate to money supply, money demand and PPP.

# The flex-price monetary model of the exchange rate

- As a starting point for thinking about how monetary policy affects exchange rates, let's add the exchange rate into a model with perfectly flexible prices
- The flex-price monetary model relates exchange rates to monetary policy via money market equilibrium and the PPP assumption
- This model is appropriate for studying the implications of monetary policy for exchange rates when
  - 1 We are interested in the long run effects of monetary policy
  - 2 There is high inflation or hyperinflation

# The flex-price monetary model of the exchange rate

- Consider a small open economy with open capital markets and exogenous output  $Y$
- The money supply  $M$  is set by the central bank. Money demand and money supply must be equal in equilibrium, so  $M^d = M$ .
- The demand for money in the domestic economy is given by

$$m_t^d = p_t - \eta i_{t+1} + \phi y_t \quad \eta, \phi > 0$$

where  $m^d = \ln M^d$ ,  $p = \ln P$ , and  $y = \ln Y$

- Here,  $i$  is the domestic nominal interest rate, which represents the opportunity cost of holding money

# The flex-price monetary model of the exchange rate

- Since PPP holds it follows that

$$P_t = S_t P_t^* \implies p_t = e_t + p_t^*$$

where  $p^* = \ln P^*$  and  $e = \ln S$  is the log nominal exchange rate

- We also assume UIP holds and investors have *rational expectations*:

$$1 + i_{t+1} = (1 + i_{t+1}^*) E_t \left( \frac{S_{t+1}}{S_t} \right)$$

- In logs, the UIP condition can be approximated by

$$i_{t+1} = i_{t+1}^* + \overbrace{E_t e_{t+1} - e_t}^{\text{Expected rate of depreciation}}$$

where the fact that  $E_t e_t = e_t$  has been used

# Rational expectations: I

- Under rational expectations, future expectations come from the predictions of the model
- If we have a term such as  $E_t(\cdot)$ , then we must calculate the rational expectation of whatever is inside the brackets, conditional on information up to and including period  $t$
- Rational expectations will be correct *on average* if the economy is hit by random shocks which average out to zero
- The rational expectations approach rules out 'bubbles' or expectations which are not linked to economic fundamentals

# Rational expectations: II

- To solve models with rational expectations, we use the law of iterated expectations (LIE)
- LIE states that today's expectation of an expected value formed *subsequently* is equal to today's expectation

- **LIE examples**

①  $E_t(E_{t+1}z_{t+2}) = E_t z_{t+2}$

②  $E_t(E_{t+2}z_{t+3}) = E_t z_{t+3}$

③  $E_t(z_t) = z_t$

④  $E_t(\bar{z}) = \bar{z}$  when  $\bar{z}$  is a constant

# Solving the flex-price monetary model

- If we substitute for  $i_{t+1}$  using UIP and for  $p_t$  using PPP, the money demand function reads as follows:

$$m_t^d = e_t + p_t^* - \eta i_{t+1}^* - \eta(E_t e_{t+1} - e_t) + \phi y_t$$

- Set  $m_t^d = m_t$  and rearrange to get

$$x_t - e_t = -\eta(E_t e_{t+1} - e_t)$$

where  $x_t = m_t - \phi y_t + \eta i_{t+1}^* - p_t^*$  consists of *exogenous variables*

- Rearranging for  $e_t$ ,

$$e_t = \frac{1}{(1+\eta)} x_t + \frac{\eta}{(1+\eta)} E_t e_{t+1} \quad (1)$$

- It is fairly straightforward to solve this linear equation

# Solving the flex-price monetary model

- Since  $e_t = \frac{1}{(1+\eta)}x_t + \frac{\eta}{(1+\eta)}E_t e_{t+1}$ , the next period's exchange rate is given by the same equation, updated by one period:

$$e_{t+1} = \frac{1}{1+\eta}x_{t+1} + \frac{\eta}{1+\eta}E_{t+1}e_{t+2}$$

- Substituting for  $e_{t+1}$  in (1) and using LIE,

$$\begin{aligned} e_t &= \frac{1}{1+\eta}x_t + \frac{\eta}{1+\eta}E_t \left( \overbrace{\frac{1}{1+\eta}x_{t+1} + \frac{\eta}{1+\eta}E_{t+1}e_{t+2}}^{e_{t+1}} \right) \\ &= \frac{1}{1+\eta}x_t + \frac{\eta}{(1+\eta)^2}E_t x_{t+1} + \frac{\eta^2}{(1+\eta)^2}E_t E_{t+1}e_{t+2} \end{aligned} \quad (2)$$



# Solving the flex-price monetary model

- Substitute for  $e_{t+2} = \frac{1}{1+\eta}x_{t+2} + \frac{\eta}{1+\eta}E_{t+2}e_{t+3}$  in (2) and use LIE:

$$\begin{aligned} e_t &= \frac{1}{1+\eta}x_t + \frac{\eta}{(1+\eta)^2}E_t x_{t+1} + \frac{\eta^2}{(1+\eta)^2}E_t \left( \overbrace{\frac{1}{1+\eta}x_{t+2} + \frac{\eta}{1+\eta}E_{t+2}e_{t+3}}^{e_{t+2}} \right) \\ &= \frac{1}{1+\eta}x_t + \frac{\eta}{(1+\eta)^2}E_t x_{t+1} + \frac{\eta^2}{(1+\eta)^3}E_t x_{t+2} + \frac{\eta^3}{(1+\eta)^3}E_t e_{t+3} \quad (3) \end{aligned}$$

- The emerging patterns in (2) and (3) tells us that the exchange rate can be written as a function of its expected value  $T$  periods ahead:

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{t+T-1} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t x_s + \left( \frac{\eta}{1+\eta} \right)^T E_t e_{t+T} \quad (4)$$

where  $T$  is a positive integer

# Solving the flex-price monetary model

- To reach a full solution, we let  $T \rightarrow \infty$  in (4):

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t x_s + \lim_{T \rightarrow \infty} \left( \frac{\eta}{1+\eta} \right)^T E_t e_{t+T}$$

- If there are no speculative bubbles in the exchange rate, then:

$$\lim_{T \rightarrow \infty} \left( \frac{\eta}{1+\eta} \right)^T E_t e_{t+T} = 0$$

- In other words, the no-bubbles assumption rules out solutions where the exchange rate is explosive

# Solving the flex-price monetary model

- Our full solution for the exchange rate is therefore

$$\begin{aligned} e_t &= \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t x_s \\ &= \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t \{ m_s - \phi y_s + \eta i_{s+1}^* - p_s^* \} \quad (5) \end{aligned}$$

where I have substituted for  $x_s = m_s - \phi y_s + \eta i_{s+1}^* - p_s^*$

- This is a full solution because we have only exogenous variables and known constants on the RHS of (5)

# Policy implications of the flex-price monetary model

- Our exchange rate solution gives us important insights:
  - ① The nominal exchange rate must be viewed as an *asset price*, because its value depends on expectations about the future
  - ② Due to the role of future expectations, the nominal exchange rate can be quite sensitive to monetary shocks
  - ③ Looser monetary policy implies a depreciation in the nominal exchange rate. This applies both to monetary policy today (ie higher  $m_t$ ) and expected future policy (ie higher  $E_t m_{t+1}$ ,  $E_t m_{t+2}$ , etc.)
- **Policy implication:** countries with high inflation will see their currencies depreciate unless tight monetary policy today is matched with a credible commitment to tight future policy

# Monetary Policy Example 1

- Suppose the money supply is increased from  $\bar{m}$  to  $2\bar{m}$  today and in all future periods. What is the impact on the exchange rate?
- Using (5), we have:

$$\begin{aligned} e_t^{before} &= \frac{1}{1+\eta} \left[ \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} \bar{m} + \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t \{ \eta i_{s+1}^* - \phi y_s - p_s^* \} \right] \\ &= \bar{m} + \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t \{ \eta i_{s+1}^* - \phi y_s - p_s^* \} \end{aligned}$$

$$e_t^{after} = 2\bar{m} + \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t \{ \eta i_{s+1}^* - \phi y_s - p_s^* \}$$

- Therefore,  $e_t^{after} - e_t^{before} = \bar{m}$ . So the nominal exchange rate depreciates, with the amount of depreciation equal to the increase in the money supply.

## Monetary Policy Example 2

- Suppose that  $i^* = y = p^* = 0$ , so that

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t m_s$$

- Consider an unanticipated announcement on date  $t = 0$  that the money supply will rise permanently to  $\bar{m}'$  on a future date  $T$ :

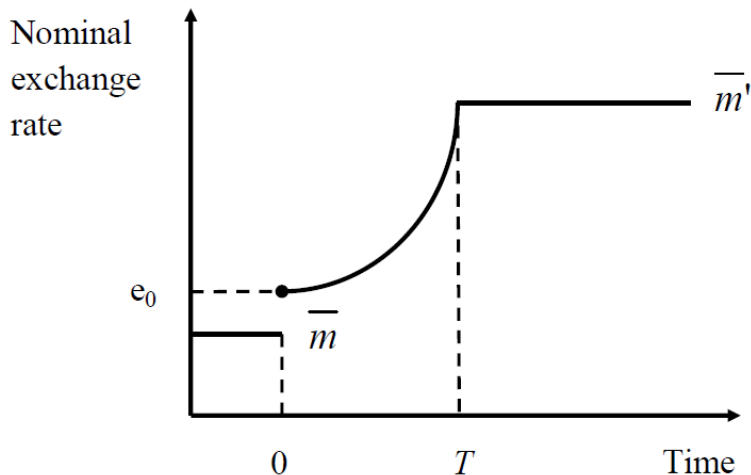
$$m_t = \bar{m} \quad \text{for } t < T \quad \text{and} \quad m_t = \bar{m}' \quad \text{for } t \geq T$$

- The exchange rate is given by

$$e_t = \bar{m} + \left( \frac{\eta}{1+\eta} \right)^{T-t} (\bar{m}' - \bar{m}) \quad \text{for } t < T \quad \text{and} \quad e_t = \bar{m}' \quad \text{for } t \geq T$$

- Therefore, the exchange rate depreciates prior to date  $T$ , in anticipation of the the future rise in the money supply

# A rise in the money supply at $T$ anticipated at date 0



- For the mathematical solution, see the appendix to these lecture slides

# Next time...

- We will relax the assumption that prices are perfectly flexible
- This gives us the Dornbusch overshooting model for exchange rates
- We will consider the predictions of this model and how they compare with those of the flex-price model we have just studied
- We will solve the Dornbusch model graphically and using equations



## Appendix: Example 2 Solution

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t m_s \quad \Longrightarrow \quad \text{Split this into two sums:}$$

$$= \frac{1}{1+\eta} \left[ \sum_{s=t}^{T-1} \left( \frac{\eta}{1+\eta} \right)^{s-t} E_t m_s + \sum_{s=T}^{\infty} \left( \frac{\eta}{1+\eta} \right)^{s-T} E_t m_s \left( \frac{\eta}{1+\eta} \right)^{T-t} \right]$$

Multiply out the brackets and simplify each term:

$$\begin{aligned} &= \frac{1}{1+\eta} \frac{\left( 1 - \left( \frac{\eta}{1+\eta} \right)^{T-t} \right)}{\left( 1 - \frac{\eta}{1+\eta} \right)} \bar{m} + \left( \frac{\eta}{1+\eta} \right)^{T-t} \bar{m}' \\ &= \bar{m} + \left( \frac{\eta}{1+\eta} \right)^{T-t} (\bar{m}' - \bar{m}) \end{aligned}$$

**Note:**  $m_t = \bar{m}$  for  $t < T$  and  $m_t = \bar{m}'$  for  $t \geq T$  is used in line 3

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