IMEP 2014 Lectures 13 and 14:

The sticky-price model of the exchange rate

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Outline of today's lectures

Lecture 13

- Introducing sticky prices
- Sticky versus fixed prices
- Introducing the sticky-price Dornbusch model

Lecture 14

- Dornbusch overshooting graphically
- Dornbusch overshooting analytically
- Other predictions of the Dornbusch model

Lecture 13

The sticky-price monetary model of the exchange rate

Key reading:

1. Obstfeld and Rogoff, Chapter 9.1 to 9.2.1

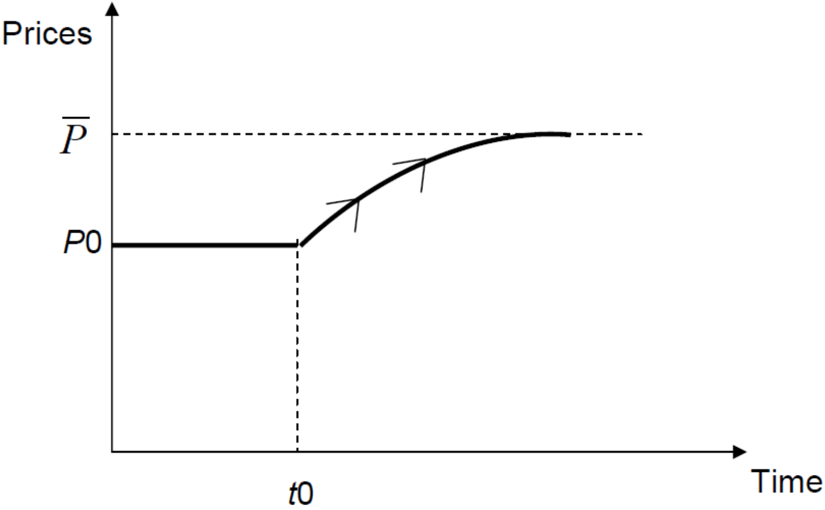
Introducing sticky prices

- In the flex-price monetary model of the exchange rate, prices are perfectly flexible. As a result, PPP holds continuously – ie in the short and long run.
- This is at odds with the PPP puzzle – there are large and persistent deviations from PPP in the short run, though PPP does appear to be a long run relationship to which real exchange rates revert
- Dornbusch proposed a monetary model that could produce short term PPP deviations and has the feature that PPP holds in the long run
- To do so, he introduced 'sticky prices' into the monetary model

Sticky versus fixed prices

- The terms 'sticky prices' and 'fixed prices' are sometimes used interchangeably, but there is an important difference between the two
- Fixed prices literally remain fixed, whereas sticky prices are fixed for only a short period of time and then gradually adjust as time goes on
- Therefore, sticky prices are a compromise between between flexible prices and fixed prices
- In the long run, sticky prices have the same implications as flexible prices, but the short run implications can be very different
- See Copeland Ch 4.2 for a more detailed discussion of the difference between fixed and sticky prices

Price adjustment under sticky prices



Motivating sticky prices

- Average frequency of price changes in the US economy

Duration	Study	Notes
$\approx$ 1 year	BCLR (1998)	Survey evidence
4 months	BK (2004)	Includes sales prices
8 months	NS (2008)	Excludes sales prices
7 months	KK (2008)	Excludes sales prices

Sources: Bils and Klenow (2004), Blinder et al. (1998), Nakamura and Steinsson (2008), Klenow and Kryvtsov (2008)

The Dornbusch model

- The Dornbusch model basically builds upon the flex-price model of the exchange rate by adding in sticky prices
- When prices are sticky, aggregate demand determines output. Therefore, the model also introduces an aggregate demand equation.
- Aggregate demand is modelled as depending on the deviation of the real exchange rate from its PPP value
- The domestic economy is a small open economy with perfect capital mobility. As a result, UIP holds at all dates.
- This difference in adjustment speeds of goods prices and financial prices is crucial for understanding how the model works

The Dornbusch model: equations

- We consider a perfect foresight version as in O&R Chapter 9.2.1
- This means we do not need to include the expectations operator E_t
- We assume that the foreign nominal interest rate is fixed at i^*
- Hence UIP is given by

$$i_{t+1} = i^* + e_{t+1} - e_t$$

where

$$i = \text{domestic nominal interest rate}$$

$$e = \text{log nominal exchange rate } (= \ln S)$$

The Dornbusch model: equations

- The money demand equation is given by

$$m_t^d = p_t - \eta i_{t+1} + \phi y_t \qquad \eta, \phi > 0$$

where $m^d = \ln M^d$, $p = \ln P$, and $y = \ln Y$

- There is equilibrium in the money market, so

$$m^d = m$$

where $m =$ log nominal money supply

- The money supply is determined outside the model by monetary policy

The Dornbusch model: equations

- Since PPP does not hold, the real exchange rate will vary over time
- The log real exchange rate q is:

$$q_t = e_t + p^* - p_t$$

where $p = \log P$ and $p^* = \log P^*$ is constant

- Aggregate demand depends on the deviation of the real exchange rate from its PPP value $\overline{q}$:

$$y_t^d = \overline{y} + \delta(\underbrace{e_t + p^* - p_t}_{q_t} - \overline{q}), \qquad \delta > 0$$

where $\overline{y}$ is the full employment level of output

- Aggregate supply is demand determined, so $y_t = y_t^d$

The Dornbusch model: equations

- The home price level p_t is *predetermined* and so cannot respond to date t shocks

- But it does adjust slowly over time in response to excess demand

- The adjustment process is described by a Phillips curve:

$$p_{t+1} - p_t = \psi(y_t^d - \overline{y}) + \widetilde{p}_{t+1} - \widetilde{p}_t, \qquad \psi > 0$$

where $\widetilde{p}_t = e_t + p^* - \overline{q}$

- Here, $\widetilde{p}_t$ is the price level consistent with consistent with zero excess demand – ie $y_t^d = \overline{y}$

The Dornbusch model: equations

- The RHS of the Phillips curve consists of two terms:

  1. $\psi(y_t^d - \overline{y})$ = price adjustment due to excess demand in the current period
  2. $\widetilde{p}_{t+1} - \widetilde{p}_t$ = price level adjustment needed to keep up with expected inflation

- Since $\widetilde{p}_{t+1} - \widetilde{p}_t = e_{t+1} - e_t$, we can write the Phillips curve as

$$p_{t+1} - p_t = \psi(y_t^d - \overline{y}) + e_{t+1} - e_t$$

The Dornbusch model: a summary

- The Dornbusch model envisages a world where prices in goods markets adjust slowly but financial prices adjust rapidly
- This appears to be consistent with what we see when we compare retail prices against those determined in financial markets
- The aggregate demand equation states that a depreciation in the home currency will lead to an increase in domestic demand
- After a shock, the economy will gradually return to its long run equilibrium, with prices adjusted according to the Phillips curve

- Now that our description of the Dornbusch model is complete, we are in a position to solve it!

- We will solve the Dornbusch model both graphically and analytically in Lecture 14

- We will also conduct some policy experiments in order to understand how a change in the money supply affects the real and nominal exchange rate

- This will lead us to Dornbusch's celebrated *exchange rate overshooting* result

Lecture 14

Exchange rate overshooting in the Dornbusch model

Key reading:

1. Obstfeld and Rogoff, Chapter 9.2.2. to 9.2.5

Solving the Dornbusch model

- In this lecture, we will solve the Dornbusch model in two ways:

  1. A graphical approach using a 'phase diagram'
  2. An algebraic solution similar to Lecture 12

- These two methods are complementary – understanding one will help you to understand the other even better!

- In an exam, you can use whichever method you are most comfortable with, but I will expect to see the steps outlined here

Solving the Dornbusch model graphically

- **Step 1:** reduce model to two equations and two endogenous variables:
  1. Real exchange rate q
  2. Nominal exchange rate e
- We have already shown that

$$p_{t+1} - p_t = \psi(y_t^d - \overline{y}) + e_{t+1} - e_t$$

- Since $e_{t+1} - e_t = q_{t+1} - q_t + p_{t+1} - p_t$,

$$q_{t+1} - q_t = -\psi(y_t^d - \overline{y})$$

$$\Longrightarrow q_{t+1} - q_t = -\psi\delta(q_t - \overline{q}) \qquad (1)$$

  where we assume $\psi\delta < 1$

- In a steady-state equilibrium, q must be constant (ie $\Delta q = 0$). By (1), this happens when $q = \overline{q}$.

Solving the Dornbusch model graphically

- We are now looking for an expression for e in terms of q and exogenous variables

- Substitute for p_t, i_{t+1}, and y_t on RHS of the money demand equation:

$$m_t = \underbrace{p^* + e_t - q_t}_{p_t} \overbrace{-\eta i^* - \eta(e_{t+1} - e_t)}^{-\eta i_{t+1}} + \overbrace{\phi \overline{y} + \phi\delta(q_t - \overline{q})}^{\phi y_t}$$

- Setting $i^* = p^* = \overline{y} = 0$,

$$m_t = -\eta(e_{t+1} - e_t) + e_t - q_t + \phi\delta(q_t - \overline{q})$$

- Rearranging for $e_{t+1} - e_t$, we find that

$$e_{t+1} - e_t = \frac{e_t - m_t}{\eta} - \frac{(1-\phi\delta)q_t}{\eta} - \frac{\phi\delta\overline{q}}{\eta} \qquad (2)$$

Solving the Dornbusch model graphically

- The *steady state* of (2) occurs when the money supply is constant at $\overline{m}$ and $q = \overline{q}$
- Let's call the steady-state nominal exchange rate $\overline{e}$
- At steady-state, (2) reads as follows:

$$\underbrace{0}_{\overline{e}-\overline{e}} = \frac{\overline{e}-\overline{m}}{\eta} - \frac{(1-\phi\delta)\overline{q}}{\eta} - \frac{\phi\delta\overline{q}}{\eta} \Longrightarrow \overline{e} = \overline{m} + \overline{q}$$

- Since $p^* = 0$, it follows that

$$\overline{p} = \overline{e} - \overline{q} = \overline{m}$$

Solving the Dornbusch model graphically

- We now have the two equations we were looking for:

$$q_{t+1} - q_t = -\psi\delta(q_t - \overline{q}) \qquad (1)$$

$$e_{t+1} - e_t = \frac{e_t - m_t}{\eta} - \frac{(1-\phi\delta)q_t}{\eta} - \frac{\phi\delta\overline{q}}{\eta} \qquad (2)$$

- **Step 2:** Consider plotting these equations with e on the vertical axis and q on the horizontal axis

- (1) does not depend on e, so it is a vertical line at $q = \overline{q}$

- (2) has a steady-state $\overline{e} = \overline{m} + \overline{q}$, so it must intersect (1) at $(\overline{q}, \overline{e})$

Solving the Dornbusch model graphically

- From (2), the nominal exchange rate is constant when

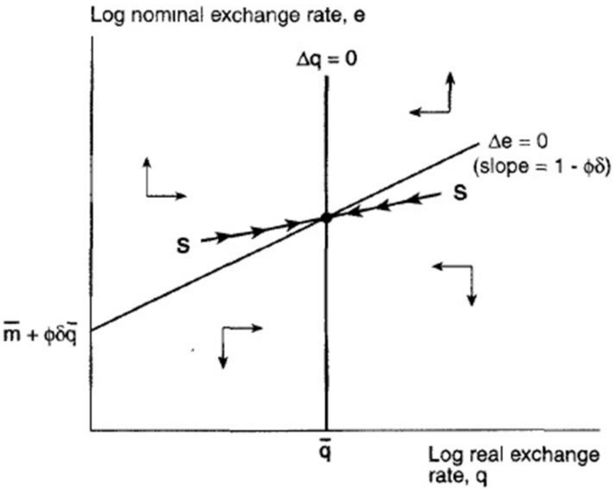
$$e_t = m_t + (1 - \phi\delta)q_t + \phi\delta\overline{q}$$

- The reason is that this implies $e_{t+1} - e_t = 0$ on the LHS

- The equation for e_t is a line with slope $1 - \phi\delta$ and intercept $m_t + \phi\delta\overline{q}$

- If we fix m_t at $\overline{m}$ we can solve graphically

Graphical solution of the Dornbusch model (O&R, Fig 9.4)



- **Key concept:** the SS curve is the model's *saddlepath*

Diagram analysis

- The saddlepath SS shows the unique path along which there is convergence to the steady-state
- The arrows on this path indicate the direction of convergence
- At the steady-state, the saddlepath and the $\Delta q = 0$ and $\Delta e = 0$ schedules intersect
- The steady-state nominal exchange rate is not labelled, but we know it equals $\overline{m} + \overline{q}$

Diagram analysis...cont'd

- The position of the $\Delta e = 0$ schedule depends on the money supply $\overline{m}$

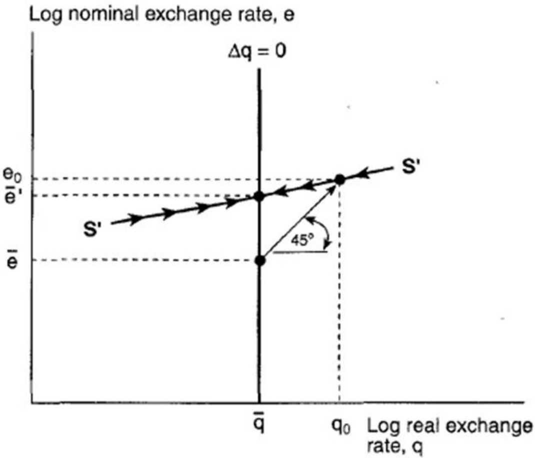
- **Example:** a rise in m will shift up the $\Delta e = 0$ schedule, leading to a new steady-state with a different saddlepath

- This property means that we can easily assess the effects of an increase in the money supply...

An unanticipated increase in the money supply

- Consider an unanticipated permanent increase in the money supply from $\overline{m}$ to $\overline{m}'$
- Since prices can adjust fully in the long run, both the nominal exchange rate and the price level will rise by the amount of the increase in the money supply, while the real exchange rate will remain unchanged at $\overline{q}$
- We will therefore end up at a new long run equilibrium at point $(\overline{q}, \overline{e}')$, where $\overline{e}' = \overline{m}' + \overline{q}$
- **What about the transition to the new long run equilbrium?**
- This question leads to the Dornbusch *overshooting* result...

Exchange rate overshooting (O&R, Fig 9.5)



- The unanticipated increase in money supply takes place at date 0

Diagram analysis

- Due to the unanticipated increase in the money supply, the saddlepath shifts up to S'S'
- As a result, the economy initially jumps to (q_0, e_0), where both the real exchange rate and the nominal exchange rate exceed their new long run equilibrium values
- The term *exchange rate overshooting* describes this phenomenon
- The nominal exchange rate overshoots its new equilibrium by $e_0 - \overline{e}'$
- The real exchange rate overshoots its new equilibrium by $q_0 - \overline{q}$

Diagram analysis cont'd

- After the overshoot, the exchange rate is expected to appreciate until it reaches $\overline{e}'$
- The assumption that prices are initially fixed is crucial, because it means that the initial rise in e is not matched by an equal rise in prices
- As a result, the real exchange rate rises to q_0

The economics of ER overshooting

- As a starting point, consider money market equilibrium. A rise in the money supply will lower the nominal interest rate i.

- In other words, i falls to ensure that money demand rises to meet the higher money supply $\overline{m}'$

- Because the shock was not forecast, UIP must hold immediately *after* the initial shock has hit

- Since i has fallen, UIP predicts a future appreciation:

$$e_1 = i_1 - i^* + e_0$$

where $i_1 - i^*$ is negative

The economics of ER overshooting...cont'd

- Likewise, UIP predicts an appreciation in the next period:

$$e_2 = i_2 - i^* + e_1$$

where $i_2 - i^*$ is negative

- The same goes for every future period until $i = i^*$
- That point will only be reached when prices have fully adjusted and we have reached the new long run equilbrium $(\overline{q}, \overline{e}')$
- In other words, we must have an initial depreciation of the exchange rate, followed by a continuous appreciation
- This is only possible if the exchange rate initially overshoots!

ER overshooting and the money market

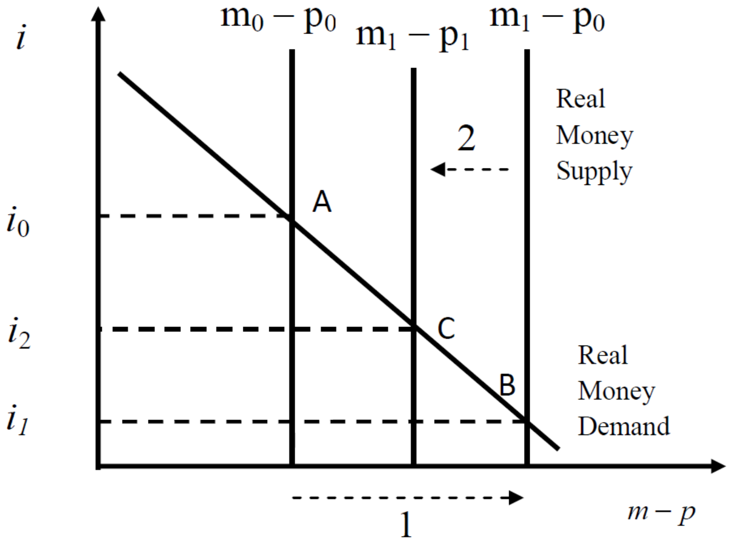
- It should be clear that the UIP condition is crucial for the exchange rate overshooting result

- Sticky prices are also crucial – it is the slow adjustment of prices which ensures that the domestic interest rate remains below i^* for many periods

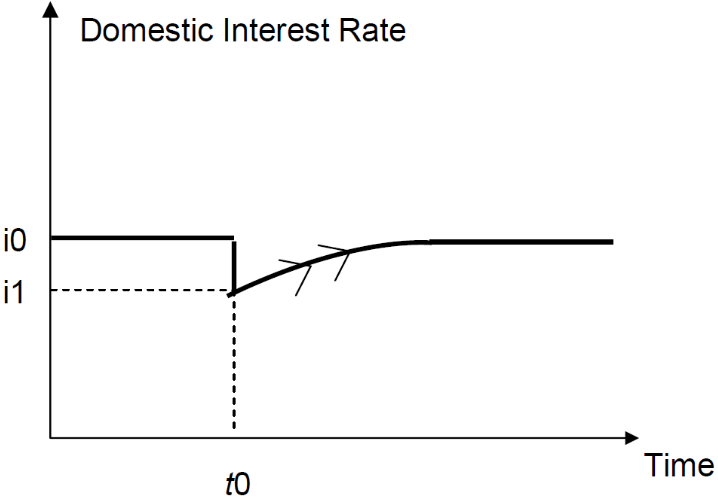
- In particular, prices start to rise gradually after the shock has hit

- This starts to reduce the real money supply until it eventually returns to its starting value, which is consistent with $i = i^*$

The money market after a positive money supply shock



Interest rates after a positive money supply shock



Solving the Dornbusch model analytically

- Our starting point is Eq (1):

$$q_{t+1} - q_t = -\psi\delta(q_t - \overline{q})$$

- Adding $q_t - \overline{q}$ to both sides,

$$q_{t+1} - \overline{q} = (1 - \psi\delta)(q_t - \overline{q}) \qquad (3)$$

- The same relationship holds between periods t and $t-1$:

$$q_t - \overline{q} = (1 - \psi\delta)(q_{t-1} - \overline{q}) \qquad (4)$$

- Substituting (4) into (3) gives

$$q_{t+1} - \overline{q} = (1 - \psi\delta)^2(q_{t-1} - \overline{q}) \qquad (5)$$

Solving the Dornbusch model analytically

- We now require an expression for $q_{t-1} - \overline{q}$. Lagging (4) by one period on both sides,

$$q_{t-1} - \overline{q} = (1 - \psi\delta)(q_{t-2} - \overline{q}) \qquad (6)$$

- Substituting (6) into (5), we find that

$$q_{t+1} - \overline{q} = (1 - \psi\delta)^3 (q_{t-2} - \overline{q}) \qquad (7)$$

- A clear pattern is emerging: $q - \overline{q}$ is related to its value j periods ago by the coefficient $(1 - \psi\delta)^j$

- Consequently, the solution in any period $s \geq t$ is given by

$$q_s - \overline{q} = (1 - \psi\delta)^{s-t}(q_t - \overline{q})$$

Solving the Dornbusch model analytically

- To make further progress, we need an equation to solve for e_t
- From our graphical solution we know that

$$e_{t+1} - e_t = \frac{e_t - m_t}{\eta} - \frac{(1-\phi\delta)q_t}{\eta} - \frac{\phi\delta\overline{q}}{\eta}$$

- Rearranging for e_t and subtracting $\overline{q}$ from both sides,

$$e_t - \overline{q} = \frac{\eta}{1+\eta}(e_{t+1} - \overline{q}) + x_t \qquad (8)$$

where $x_t = \frac{1-\phi\delta}{1+\eta}(q_t - \overline{q}) + \frac{1}{1+\eta}m_t$

- Since x_t consists of one exogenous variable (m_t) and one we have solved for $(q_t - \overline{q})$, we can look for a solution with x on the RHS

Solving the Dornbusch model analytically

- Updating our equation for $e_t - \overline{q}$ in (8) by one period,

$$e_{t+1} - \overline{q} = \tfrac{\eta}{1+\eta}(e_{t+2} - \overline{q}) + x_{t+1} \qquad (9)$$

- Substituting (9) into (8), we find that

$$e_t - \overline{q} = \tfrac{\eta^2}{(1+\eta)^2}(e_{t+2} - \overline{q}) + x_t + \tfrac{\eta}{1+\eta}x_{t+1}$$

- Doing the same for $e_{t+2} - \overline{q}$ gives

$$e_t - \overline{q} = \tfrac{\eta^3}{(1+\eta)^3}(e_{t+3} - \overline{q}) + x_t + \tfrac{\eta}{1+\eta}x_{t+1} + \tfrac{\eta^2}{(1+\eta)^2}x_{t+2}$$

- If we carry on substituting, we find

$$e_t - \overline{q} = \sum_{s=t}^{t+T-1} \left(\frac{\eta}{1+\eta}\right)^{s-t} x_s + \left[\frac{\eta}{(1+\eta)}\right]^T (e_{t+T} - \overline{q}) \qquad (10)$$

where T is a positive integer

Solving the Dornbusch model analytically

- Taking the limit of (10) as $T \rightarrow \infty$ gives

$$e_t - \overline{q} = \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} x_s + \lim_{T \longrightarrow \infty} \left[\frac{\eta}{(1+\eta)}\right]^T (e_{t+T} - \overline{q}) \qquad (11)$$

- If we assume there are no exchange rate 'bubbles, then

$$\lim_{T \rightarrow \infty} \left[\frac{\eta}{(1+\eta)}\right]^T (e_{t+T} - \overline{q}) = 0$$

- Therefore, our exchange rate solution is

$$\begin{aligned} e_t &= \overline{q} + \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} x_s \\ &= \overline{q} + \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta}\right)^{s-t} [m_s + (1 - \phi\delta)(q_s - \overline{q})] \qquad (12) \end{aligned}$$

Solving the Dornbusch model analytically

- If m is constant at $\overline{m}$ we can simplify this to

$$\begin{aligned} e_t &= \overline{q} + \overline{m} + \frac{1-\phi\delta}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} (q_s - \overline{q}) \\ &= \overline{q} + \overline{m} + \frac{1-\phi\delta}{1+\eta} (q_t - \overline{q}) \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} (1-\psi\delta)^{s-t} \end{aligned} \tag{13}$$

  where our solution $q_s - \overline{q} = (1-\psi\delta)^{s-t}(q_t - \overline{q})$ has been used

- Simplifying (13) and rearranging,

$$e_t = \overline{m} + \overline{q} + \tfrac{1-\phi\delta}{1+\psi\delta\eta}(q_t - \overline{q}) \tag{14}$$

- This is the saddlepath SS. Its slope is positive if $\phi\delta < 1$.

ER overshooting analytically

- Consider an unanticipated permanent increase in the money supply from $\overline{m}$ to $\overline{m}'$ at date 0
- From our assumption that $p^* = 0$, the date 0 real exchange rate is $q_0 = e_0 - \overline{m}$, where $\overline{m}$ is the date 0 price level
- From (14), the date 0 nominal exchange rate is

$$e_0 = \overline{m}' + \overline{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}(q_0 - \overline{q})$$

- Therefore,

$$e_0 = \overline{m}' + \overline{q} + \frac{1-\phi\delta}{1+\psi\delta\eta}(e_0 - \overline{m} - \overline{q})$$

- This implies that

$$e_0 = \overline{m} + \overline{q} + \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}(\overline{m}' - \overline{m}) \qquad \text{and} \qquad q_0 = \overline{q} + \frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}(\overline{m}' - \overline{m})$$

ER overshooting analytically: interpretation

- The previous slide confirms our graphical results: the nominal and real exchange rate rise above their long run equilibrium levels at date 0
- Both the nominal and real exchange rate overshoot by the same amount $\frac{1+\psi\delta\eta}{\phi\delta+\psi\delta\eta}(\overline{m}' - \overline{m})$ – hence the 45$^{\text{o}}$ line on Fig 9.5
- The solutions show that larger money supply shocks (i.e. larger $\overline{m}'$) will lead to larger 'overshoots' in exchange rates

Real interest rates and the Dornbusch model

- The model predicts that the real interest rate differential is inversely related to the real exchange rate:

$$r_t - r_t^* = -\psi\delta(q_t - \overline{q})$$

- Here, r = home real interest rate and r^* = foreign real interest rate
- See O&R Section 9.2.5 for a more detailed discussion
- In order to test whether the Dornbusch model is a valid description of the world, we can check if this relationship holds true using data
- In next week's lectures, we will consider this and other empirical tests of monetary exchange rate models

THE END

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