Optimal pensions with endogenous labour supply

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Abstract

We show that a two-part pension system consisting of an income tax and a consumption tax provides optimal capital accumulation without distorting labour supply, thereby achieving the *first-best* for arbitrary initial capital. An economy with too little retirement saving should combine a *negative* income tax with a consumption tax to achieve the first-best without using any lump-sum taxes. Our results are shown in a classic Diamond overlapping generations model that is augmented with labour supply on the intensive margin and stochastic shocks to productivity and population growth.

1 Introduction

In many countries, there is a trend toward greater funding of public pensions. Current projections show that ageing populations will increase the future costs of pension provision significantly given the falling share of young workers, the limited scope for increases in labour supply of existing workers, and other demographic pressures. Despite these known risks, progress in reforming pension systems has been slow, raising several questions. Is it better to gradually phase out existing paygo systems or should policymakers rapidly implement funded pension systems? How should labour be taxed in such systems, given the importance of labour supply in making current pension promises manageable? Is there an optimal pension policy for a world where saving is too low, and if so, what does it look like?

In this paper, we consider these questions using a deliberately simplified model. We set out an overlapping generations model in the spirit of Diamond (1965) that is augmented with *endogenous* labour supply on the intensive margin and stochastic shocks to productivity and population growth. Our main result is that a two-part pension system consisting of an income tax and a consumption tax provides optimal capital accumulation while avoiding a distortion to labour supply, such that the first-best allocation is replicated for any initial capital inherited from past policies. For an economy with *too little* retirement saving, this policy achieves the first-best without using lump-sum taxes. The first-best is the allocation

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chosen by a planner who maximizes a social welfare function with positive weights on the lifetime utilities of all current and future generations and geometric discounting.

The optimal policy depends on the savings rate in the *laissez-faire* economy. If the savings rate is too high relative to the social optimum, combining a positive income tax with a consumption subsidy curbs the capital overaccumulation, while the consumption subsidy prevents a distortion to labour supply by making the *effective* marginal tax on labour *zero*. By contrast, if the savings rate *too low*, the optimal policy combines a *negative* income tax and a positive consumption tax to encourage workers to make extra retirement saving, while avoiding both a labour supply distortion and the use of lump-sum taxes. Such a pension system is in the spirit of policy recommendations that aim to increase private retirement saving (see OECD, 2012), but bolder.[1] While the suggestion of a negative income tax is not new, its role here is to increase saving, not the labour supply of poor households.[2]

Several past papers add endogenous labour supply and pensions in the classic Diamond (1965) model, but most focus on Pareto-improving transitions (see Breyer and Straub, 1993; Andersen and Bhattacharya, 2013) rather than policies which maximize a social welfare function. The closest papers are Abio et al. (2004) and Beetsma et al. (2013). Abio et al. (2004) extend the *steady-state* result of Samuelson (1975) in the classic Diamond model to an economy with endogenous female labour and fertility: they show that *steady-state* utility is still maximized by a paygo pension in this setting, provided the benefit is linked to the number of children. Beetsma et al. (2013) present a two-tier pension system that achieves the first-best allocation in a *two-period* overlapping generations model with intergenerational risk; however, their result does *not* extend to infinitely many periods.[3]

We contribute relative to these works by presenting a policy that yields a first-best *transition* in an infinite-horizon Diamond model. Crucially, the key margin in our analysis – labour supply changes in response to potentially distorting income taxes – is absent in the two papers above. Importantly, our results suggest a way to tackle the dearth of private pension saving using widely available policy instruments: income and consumption taxes.

2 Model

Consider an overlapping generations model with discrete time $t \in \mathbb{N}$ and households with two-period lives. The number of young N_t grows according to $N_t = (1 + n_t)N_{t-1}$, where $n_t > -1$ is the stochastic population growth rate at date t and $N_{-1} > 0$ is given. All members of a generation are identical and lifetime utility of the young born at date t is

$$U_t = \ln(c_{t,y}) + \beta E_t[\ln(c_{t+1,o})] - \theta \frac{l_t^{1+\chi}}{1+\chi}, \qquad \beta, \theta, \chi > 0 \tag{1}$$

where l_t is labour hours, β is a private discount factor, and χ^{-1} is the Frisch elasticity.

[1] For example, OECD (2012, 11,17) notes that pension reforms since the mid-1980s have led "to a reduction in public pension promises in many countries" and calls for "an expanded role for funded, private pensions."

[2] In the economic literature, a negative income tax was proposed by Friedman (1962); the basic idea is that government support should be withdrawn at a low marginal rate to increase incentives to work.

[3] See Section 5 of Beetsma et al. (2013) (p. 153) and their Appendix C for further details.

The economy is closed and output is used for consumption or investment:

$$Y_t = N_t c_{t,y} + N_{t-1} c_{t,o} + K_{t+1} \tag{2}$$

where we assume full depreciation of capital in a generation.

Production is Cobb-Douglas and depends on stochastic total factor productivity A_t:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{3}$$

where $L_t = N_t l_t$ is aggregate labour hours, K_t is aggregate capital and $K_0 > 0$ is given.

Both shocks follow stochastic process which are AR(1) in logs for all $t > 0$:

$$1 + n_t = (1 + n_{t-1})^{\rho_n} (1 + \overline{n})^{(1-\rho_n)} \exp(\varepsilon_{n,t}), \qquad A_t = A_{t-1}^{\rho_A} \exp(\varepsilon_{A,t}) \tag{4}$$

where $\rho_j \in [0, 1)$, $\overline{n}, n_0 > -1, A_0 > 0$ are initial values and $\varepsilon_{j,t} \sim \mathcal{N}(0, \sigma_j^2)$, for $j = n, A$.

2.1 First-best allocation

We consider a planner who wants to maximize a social welfare function which assigns positive weights $\omega_t := \omega^t$ to the lifetime utility of the representative member of each generation:

$$W_0 = E_0 \sum_{t=-1}^{\infty} \omega_t U_t = E_0 \sum_{t=0}^{\infty} \omega^t \left(\ln(c_{t,y}) + \frac{\beta}{\omega} \ln(c_{t,o}) - \frac{\theta}{1+\chi} l_t^{1+\chi} \right) + t.i.p. \tag{5}$$

where $0 < \omega < 1$ is a social discount factor and $t.i.p. := \omega^{-1} \ln(c_{-1,y})$ is a given constant.

The resource constraint (2) in per-worker terms is

$$y_t = c_{t,y} + \frac{c_{t,o}}{1+n_t} + k_{t+1}, \quad \forall t \geq 0 \tag{6}$$

where $y_t := Y_t/N_t = A_t \tilde{k}_t^{\alpha} l_t^{1-\alpha}$ is output per worker, $\tilde{k}_t := k_t/(1+n_t)$ is capital available to each worker in production, and $k_{t+1} := K_{t+1}/N_t$ is saving per young at date t.

The planner's maximization problem can be written recursively as

$$V(k_t, n_t, A_t) = \max_{c_{t,y}, l_t, k_{t+1}} \{ \ln(c_{t,y}) + \frac{\beta}{\omega} \ln(c_{t,o}) - \frac{\theta}{1+\chi} l_t^{1+\chi} + \omega E_t V(k_{t+1}, n_{t+1}, A_{t+1}) \} \tag{7}$$

subject to $c_{t,o} = (1+n_t)(A_t \tilde{k}_t^{\alpha} l_t^{1-\alpha} - c_{t,y} - k_{t+1})$, (4) and $k_0 = K_0/N_{-1} > 0$.

The solution to the problem in (7) must be Pareto efficient since each generation receives a positive welfare weight ω^t. Thus, the planner's chosen allocation is one of many Pareto efficient allocations, but is the only one that maximizes (5) given the discount factor ω.

The first-order optimality conditions are

$$c_{t,o} = \frac{\beta}{\omega}(1+n_t) c_{t,y}, \qquad \theta l_t^{\chi} = \frac{mpl_t}{c_{t,y}}, \qquad \frac{1}{c_{t,y}} = \beta E_t \left[\frac{mpk_{t+1}}{c_{t+1,o}} \right] \tag{8}$$

where $mpl_t := (1-\alpha)y_t/l_t$ and $mpk_t := \alpha y_t/\tilde{k}_t$.

The first equation in (8) shows the optimal balance between generational consumptions, and the second is optimal labour supply on the intensive margin. The third equation in (8) is the Euler equation for optimal capital accumulation. At a riskless steady state, the *modified* Golden Rule holds: $mpk = (1+\overline{n})/\omega$; for a discussion, see De La Croix and Michel (2002).

Using (6) and (8), we can find the first-best allocation chosen by the planner:[4]

$$k_{t+1}^* = \alpha\omega y_t^*, \qquad l_t^* = \left(\frac{(1+\frac{\beta}{\omega})(1-\alpha)}{\theta(1-\alpha\omega)}\right)^{1/(1+\chi)} \tag{9}$$

$$c_{t,y}^* = \frac{\omega(1-\alpha\omega)}{\beta+\omega}y_t^*, \qquad c_{t,o}^* = \frac{\beta(1-\alpha\omega)(1+n_t)}{\beta+\omega}y_t^* \tag{10}$$

where $y_t^* = A_t(\tilde{k}_t^*)^\alpha(l_t^*)^{1-\alpha}$.

The optimal savings rate $\alpha\omega$ depends on the capital share and the social discount factor. Optimal labour supply depends upon this savings rate, the ratio of the private and social discount factors, β/ω, and the labour preference parameters θ, χ. Uninvested output is consumed, with the optimal split between young and old depending on β and ω; see (10).

2.2 Decentralized economy

Households maximize lifetime utility by choosing saving s_t and labour hours l_t. They face an income tax (or subsidy) at rate $\tau \in \mathbb{R}$ and a consumption tax (or subsidy) at rate $\tau_c \in \mathbb{R}$. The proceeds from the two taxes finance a paygo-type pension $P_t \in \mathbb{R}$ to the current old.

The problem solved by a representative young born at date t is

$$\begin{aligned} \max_{s_t,l_t} U_t = \ln(c_{t,y}) + \beta E_t[\ln(c_{t+1,o})] - \theta\frac{l_t^{1+\chi}}{1+\chi} \qquad \text{s.t.} \\ (1+\tau_c)c_{t,y} = (1-\tau)w_t l_t - s_t, \qquad (1+\tau_c)c_{t+1,o} = r_{t+1}s_t + P_{t+1} \end{aligned} \tag{11}$$

where the factor prices w_t, r_t and the pension P_{t+1} are taken as given.

The first-order conditions are

$$\frac{1}{c_{t,y}} = \beta E_t\left[\frac{r_{t+1}}{c_{t+1,o}}\right], \qquad \theta l_t^\chi = \frac{(1-\tau)w_t}{(1+\tau_c)c_{t,y}}. \tag{12}$$

The Euler equation in (12) has the usual form. Intuitively, there is no marginal intertemporal distortion because consumption at both ages is taxed at the same rate τ_c. By comparison, there is a 'tax wedge' $\frac{1-\tau}{1+\tau_c}$ in the intratemporal labour supply condition in (12).

Total contributions to the pension system at date t are

$$C_t = \tau N_t w_t l_t + \tau_c(N_t c_{t,y} + N_{t-1}c_{t,o}). \tag{13}$$

[4]Further details of the planner problem are in the Supplementary Appendix (github.com/MCHatcher).

The pension P_t to each old is given by a transfer rule:

$$P_t = (1 + n_t)[\tau w_t l_t + \tau_c(1 - \Phi)y_t] \tag{14}$$

where $\Phi \in (0, 1)$ is a coefficient (see below for more details).

A representative firm hires capital and labour from households in competitive factor markets at prices r_t and w_t, respectively. The firm maximizes profit each period:

$$\max_{K_t, L_t} \; A_t K_t^{\alpha} L_t^{1-\alpha} - r_t K_t - w_t L_t$$

which yields the factor prices

$$r_t = \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} = mpk_t, \qquad w_t = (1-\alpha) A_t \left(\frac{K_t}{L_t}\right)^{\alpha} = mpl_t. \tag{15}$$

Competitive equilibrium is a set of allocations and prices such that for all $t \geq 0$:

(i) s_t, l_t solve the maximization problem of the date t young given the shocks (4), taxes τ, τ_c, the pension transfer rule (14), and the profit-maximizing factor prices (15).

(ii) The pension system has a balanced budget: $P_t - C_t/N_{t-1} = 0$.

(iii) Aggregate capital equals aggregate saving, $K_{t+1} = N_t s_t$, and both the aggregate resource constraint (2), and the per-person resource constraint (6), hold.

Given (12)–(14) and conditions (i)–(iii), the allocations at the competitive equilibrium are:

$$k_{t+1} = \frac{\tilde{\beta}(1-\alpha)(1-\tau)}{1+\tilde{\beta}} y_t, \qquad l_t = \left(\frac{1+\tilde{\beta}}{\theta}\right)^{1/(1+\chi)} \tag{16}$$

$$c_{t,y} = \frac{k_{t+1}}{\tilde{\beta}(1+\tau_c)}, \qquad c_{t,o} = \left(\frac{\alpha + (1-\alpha)\tau + (1-\Phi)\tau_c}{1+\tau_c}\right)(1+n_t)y_t \tag{17}$$

where $\tilde{\beta} := \frac{\beta\alpha}{\alpha+(1-\alpha)\tau+(1-\Phi)\tau_c}$ and $y_t = A_t \tilde{k}_t^{\alpha} l_t^{1-\alpha}$ as above.

Coefficient Φ is from the pension transfer rule, (14), and must be consistent with the balanced-budget condition in (ii), implying $\Phi = k_{t+1}/y_t$. In the *Supplementary Appendix* we show that Φ must solve a quadratic equation and τ, τ_c, Φ can be set to achieve the first-best.[5]

3 Optimal policy

We now turn to optimal policy. We first show that an income tax or consumption tax *alone* cannot achieve the first-best allocation. We then characterize the optimal two-part pension system (where 'optimal' means that the social welfare function (5) is maximized). To guarantee the existence of an optimal policy, we make the following assumption.

[5]The quadratic is $\tau_c\Phi^2 - [\alpha(1+\beta) + (1-\alpha)\tau + \tau_c]\Phi + \alpha\beta(1-\alpha)(1-\tau) = 0$ by (16) and $\Phi = k_{t+1}/y_t$.

Assumption 1 *We assume that* $\beta(1-\alpha)/\alpha > \omega(1-\omega)$.

Intuitively, if β is too low (or α is too high), then the low savings rate in capital cannot be corrected using τ, τ_c as instruments, because the required transfers are 'too large'. Although this means our policy will not apply to economies with very low discount factors β or very large capital shares α, Assumption 1 holds for a wide range of plausible parameter values.[6]

3.1 Analytical results

We first clarify the conditions for which the *laissez-faire* competitive equilibrium is sub-optimal and then show that a one-part pension system cannot correct this. We then turn to a two-part pension system which is first-best. Proofs of all results appear in the Appendix.

Proposition 1 *Let* $\beta^* := \frac{\alpha\omega}{1-\alpha(1+\omega)}$. *If* $\beta = \beta^*$, *then setting* $\tau = \tau_c = 0$ *(i.e. no intervention) achieves the first-best allocation. If* $\beta \neq \beta^*$, *the laissez-faire competitive equilibrium allocation deviates from the first-best allocation: there is over-saving if* $\beta > \beta^*$ *(under-saving if* $\beta < \beta^*$*). When* $\beta \neq \beta^*$, *no one-part pension system can achieve the first-best allocation.*

Proposition 1 shows that the *laissez-faire* competitive equilibrium (no intervention) yields the first-best *only* if a specific relationship holds between the private discount factor β, the capital income share α, and social discount factor ω. Intuitively, if $\beta = \beta^*$, the saving rate of the young happens to be socially optimal and marginal distortions are avoided absent taxes.

Otherwise, government intervention may be able to raise social welfare given by (5). What Proposition 1 clarifies, however, is that no *one-part* pension system – using τ or τ_c alone – can achieve the first-best. The intuition is quite simple. If $\beta \neq \beta^*$, then saving in equilibrium is suboptimal, being too high if $\beta > \beta^*$ and too low if $\beta < \beta^*$. An income tax τ can restore the optimal saving rate, but will introduce a marginal distortion to labour supply – see (12) – that is absent in (8). A similar argument applies to the consumption tax τ_c.

The above intuition suggests that if the distortionary income tax τ were replaced with a *lump-sum* tax T_t, the first-best could be achieved as there would be no marginal distortion to labour supply. This is indeed the case, as summarized in the following remark.

Remark. *Suppose* $\tau = \tau_c = 0$. *If the young make a lump-sum transfer* $T_t = \tau' w_t l_t$ *to the old in each period, where* $\tau' := \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{(1-\alpha)(\beta+\omega)}$, *then the first-best allocation is achieved for all* t. *(The same conclusion holds with an income tax* τ' *if labour supply is made exogenous.)*

This remark makes clear that the 'friction' which prevents a one-part pension system achieving the first-best allocation (see Proposition 1) is a marginal tax on labour supply. Intuitively, the transfer T_t from young to old is positive in the case of over-saving ($\beta > \beta^*$) and negative in the case of under-saving ($\beta < \beta^*$). We now show that a well-designed two part pension system achieves the first-best despite the friction of a marginal income tax.

[6] Note that $\omega(1-\omega) \leq 1/4$, so Assumption 1 holds for all $\alpha \in (0, 1/2]$, $\beta > 1/4$. Clearly, if $\omega \neq 1/2$, then these (sufficient) conditions can be weakened.

Proposition 2 *The first-best allocation is achieved by a pension system with $\Phi = \alpha\omega$ and*

$$\tau = \frac{\beta(1-\alpha) - \alpha\omega(1+\beta)}{\beta(1-\alpha) - \alpha\omega(1-\omega)}, \qquad \tau_c = -\tau, \qquad P_t = -\tau\alpha(1+n_t)(1-\omega)y_t^* \tag{18}$$

where $P_t > 0$ if and only if $\beta < \beta^*$.

Proposition 2 gives the optimal *two-part* pension system. The consumption tax has equal magnitude to the income tax but *opposite sign* because this makes the *effective* marginal tax on labour *zero* – see Prescott (2004, p. 8) and (12) – thus preventing a marginal distortion to labour supply. When this condition is met, the first-order condition for labour supply in the competitive equilibrium matches the planner's in (8), given that $mpl_t = w_t$. Finally, note that the old receive a positive share of output when τ, τ_c follow the optimal policy.[7]

In the case of over-saving, a positive income tax $\tau > 0$ is combined with a consumption subsidy $\tau_c < 0$. Intuitively, a positive income tax corrects the over-saving by curbing capital accumulation, while setting $\tau_c = -\tau$ eliminates the 'wedge' in the first-order condition for labour supply, (12). Interestingly, the consumption subsidy exceeds the amount raised by the income tax; see (18), so this case results in a *negative* pension (lump-sum tax).

For *under*-saving – i.e. 'strained' pension systems – a *negative* income tax $\tau < 0$ is used alongside a consumption tax $\tau_c > 0$. This raises the savings rate to the socially optimal level while keeping labour supply undistorted (given that $\tau_c = -\tau$). In this case, the pension to the old is *positive*: consumption tax revenue more than pays for the income subsidy; see (18). Thus, while typical policy recommendations call for governments to reduce reliance on paygo, our results suggest that a bold policy of *negative* income taxes alongside a standard consumption tax will raise retirement saving sufficiently, while avoiding lump-sum taxes.

3.2 Discussion

We have presented an optimal pension policy where the two parts – the income tax and consumption tax – are set to be equal and opposite in sign to avoid distorting labour supply. The sign of the income tax is determined by whether the economy's 'natural' savings rate at the *laissez-faire* equilibrium is too high or too low, relative to the social optimum.

In the case of *under*-saving, the optimal policy combines a *negative* income tax and a positive consumption tax, with the old receiving an unfunded pension that is strictly positive. In this important case – which seems of most relevance given demographic trends – there are *no* lump-sum taxes and thus our proposed policy looks highly attractive. In the case of *over*-saving, the pension paid to the old is *negative* (Proposition 2), implying a lump-sum tax. In this case, the policy looks less attractive, but this case also seems of less relevance: retirement saving appears to be too low rather than too high, as argued in the Introduction.

Our policy implications are quite different to existing works. In Abio et al. (2004) there is no consumption tax in the optimal policy because pensions are financed by an income taxes on males, whose labour supply is taken as *inelastic*. As a result, tax distortions to labour supply are absent, and a consumption tax has no role. In Beetsma et al. (2013), the

[7] The share (see (17)) is: $\alpha(1-(1-\omega)\tau) = \frac{\alpha\beta\omega(1-\alpha\omega)}{\beta(1-\alpha)-\alpha\omega(1-\omega)} > 0$ if and only if Assumption 1 holds.

optimal policy in their two-period economy requires a *lump-sum* tax and a distortionary tax to labour income is absent; thus, again, there is no need for a consumption tax. Importantly, while intergenerational risk is a major focus in Beetsma et al. (2013), we instead emphasise a marginal income tax on workers, and recall that their optimality result does *not* hold in an *infinite horizon* overlapping generations economy, while our result does.

In short, our two main policy prescriptions – (i) *equal and opposite* consumption and income taxes to prevent labour supply distortion; and (ii) a *negative* income tax plus a positive pension – seem to be novel, and our optimal policy fills a gap in the existing literature.

3.3 Numerical example

We set the capital share $\alpha = 0.30$ and the social discount factor at $\omega = 0.995$. The labour supply parameters are set at $\theta = 4$ and $\chi = 2$; the latter implies a Frisch elasticity of 0.5. We fix productivity at $A_t = 1$ and population growth at $n_t = 0.05$ in all periods. We set the private discount factor at either $\beta = 0.85$ (over-saving) or $\beta = 0.65$ (under-saving), and initial capital per worker k_0 is set at the steady-state value in the *laissez-faire* economy.

The optimal policy is implemented from date 0 onwards and we simulate the resulting transition dynamics; our exercise models an unanticipated pension reform that immediately implements the new policy.[8] Generational welfare effects are consumption-equivalent gains (or losses) relative to the no-reform counterfactual. The results are shown in Figure 1.

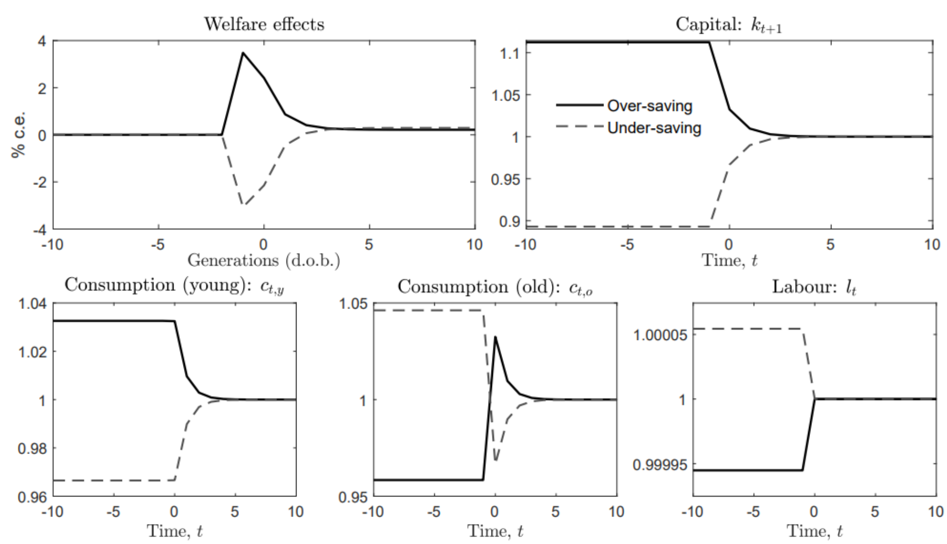

Figure 1: Transition dynamics under the optimal policy for $\beta^* \approx 0.74$. Optimal policy is implemented at date 0. Solid line: case of over-saving: $\beta = 0.85$ $(> \beta^*)$, initial capital $k_0 \approx 0.150$, optimal taxes are $\tau = 0.072$, $\tau_c = -\tau$. Dashed line: case of under-saving: $\beta = 0.65$ $(< \beta^*)$, initial capital $k_0 \approx 0.116$, optimal taxes are $\tau = -0.083$, $\tau_c = -\tau$. Note: % c.e. is the consumption-equivalent increase in lifetime utility, and 'd.o.b.' is date of birth.

[8] If the pension reform takes place at some date $T > 0$, then it generally matters for the transition whether the reform is announced or unannounced; see e.g. Fedotenkov (2016) and the reply by Hatcher (2019).

For the case of over-saving (solid line, $\beta = 0.85$), the optimal policy has capital falling and converging to the optimal long run value (top right). This is achieved by an income tax, and a consumption subsidy, of 7.2%. Consumption when young falls only marginally at date 0 because the wage is unchanged and the effective labour tax is *zero* (only $\tilde{\beta}$ changes); by contrast, consumption when old increases sharply due to the consumption subsidy, with the subsequent reduction in private saving (capital) reducing the consumption increase for later generations (bottom panel).[9] Labour supply 'jumps' to the its new value, which is slightly higher. Looking at the generational welfare effects (top left), we see that there is a Pareto improvement – all generations are better off – given initial capital overaccumulation.[10]

Now consider the case of *under*-saving (dashed lines, $\beta = 0.65$) and 'low' initial capital. An increase in capital is accomplished through a *negative* income tax and a positive consumption tax of about 8.3%. The transition paths are essentially 'mirror images' of the over-saving case; one important difference, however, is that there is *no* Pareto improvement. This is intuitive given that there is capital *under*-accumulation in this case, and we see that both the initial old and the first young generation are hit hard by this policy. It takes several generations until capital has increased sufficiently that subsequent generations are better-off. In short, given the relatively high social discount factor $\omega = 0.995$, the optimal policy trades off the welfare losses of the initial generations against the gains of the subsequent generations in order to maximize the social welfare function.

4 Conclusion

We have presented a simple two-part pension system that achieves the first-best allocation with *endogenous* labour supply for any initial capital. In the case of under-saving, an unfunded pension is paid to the old which is financed by consumption tax revenue net of a labour income subsidy. This policy raises the private savings rate to a socially optimal level, avoids the use of lump-sum taxes, and leaves labour supply undistorted because the consumption tax and the labour income tax are *equal in magnitude but opposite in sign.* Such a policy seems worthy of further investigation given the urgency and apparent difficulty of solving the dearth of private pension saving using widely available policy instruments.

[9] Since $\omega \approx 1$, the pension is near zero (see Proposition 2) and $c_{t=0,o}$ increases around 8% ($\frac{1}{1-0.072} = 1.078$).

[10] Recall that any optimal (first-best) allocation is Pareto efficient as generational weights are positive.

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Appendix

Proof of Proposition 1

Setting $\tau = \tau_c = 0$ in (16)–(17) yields $k_{t+1} = \frac{\beta(1-\alpha)}{1+\beta} y_t$, $l_t = \left(\frac{1+\beta}{\theta}\right)^{\frac{1}{1+\chi}}$, $c_{t,y} = (\frac{1-\alpha}{1+\beta})y_t$, and $c_{t,o} = \alpha(1+n_t)y_t$. Comparing with (9)–(10), the first-best allocation is achieved if and only if $\frac{\beta(1-\alpha)}{1+\beta} = \alpha\omega$, which requires $\beta = \beta^* := \frac{\alpha\omega}{1-\alpha(1+\omega)}$. If $\beta > \beta^*$, then $\frac{\beta(1-\alpha)}{1+\beta} > \frac{\beta^*(1-\alpha)}{1+\beta^*} = \alpha\omega$, so there is over-saving relative to the socially optimal rate; if $\beta < \beta^*$ there is under-saving.

We now show that if $\beta \neq \beta^*$ and one tax is zero, the first-best cannot be replicated. If $\tau_c = 0$, $\tau \in \mathbb{R}$, then by (16)–(17), the ratios k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ equal the planner's in (9)–(10) iff $\tau = \tau' := \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{(1-\alpha)(\beta+\omega)} \neq 0$. But if $\tau = \tau'$, first-order condition in (12) gives (at equilibrium) $l_t^{1+\chi} = \frac{(1-\alpha)(1-\tau)}{\theta c_{t,y}/y_t} \neq (l_t^*)^{1+\chi}$ since $\tau \neq 0$. If $\tau = 0$, $\tau_c \in \mathbb{R}$, then by (16)–(17), the ratios k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ equal the planner's in (9)–(10) iff $\tau_c = \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{\omega(1-\alpha\omega)} \neq 0$, but then the first-order condition in (12) gives $l_t^{1+\chi} = \frac{(1-\alpha)}{\theta(1+\tau_c)c_{t,y}/y_t} \neq (l_t^*)^{1+\chi}$ since $\tau_c \neq 0$. ∎

Proof of Proposition 2

Achieving the first-best requires $\frac{1-\tau}{1+\tau_c} = 1$ (see above) and $k_{t+1}/y_t = \alpha\omega$, so we set $\tau_c = -\tau$, $\Phi = \alpha\omega$. Given this equality, the ratios k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ in (16)–(17) equal the planner's (9)–(10) iff $\tau = \frac{\beta(1-\alpha)-\alpha\omega(1+\beta)}{\beta(1-\alpha)-\alpha\omega(1-\omega)} = \Psi\tau'$, where $\Psi := 1/\left(1 - \frac{\omega(1-\alpha\omega)}{(1-\alpha)(\beta+\omega)}\right) > 1$ by Assumption 1, so $sgn(\tau) = sgn(\tau')$. Labour supply matches the planner's, since $c_{t,y}/y_t = c_{t,y}^*/y_t^*$, so the first-order condition in (12) gives $l_t^{1+\chi} = \frac{(1-\alpha)}{\theta c_{t,y}/y_t} = (l_t^*)^{1+\chi}$. Given equal ratios and $l_t = l_t^*$ for all t, $y_t = y_t^*$, $k_{t+1} = k_{t+1}^*$, $c_{t,y} = c_{t,y}^*$, $c_{t,o} = c_{t,o}^*$ for all t, so the first-best allocation is replicated. By (6), (ii),(13)–(15), $P_t = (1+n_t)[\tau(1-\alpha)y_t^* + \tau_c(y_t^* - k_{t+1}^*)] = (1+n_t)[\tau(1-\alpha) + \tau_c(1-\alpha\omega)]y_t^*$, where $k_{t+1}^* = \alpha\omega y_t^*$ is used. Setting $\tau_c = -\tau$ gives $P_t = -\tau\alpha(1+n_t)(1-\omega)y_t^*$. Since $\omega \in (0,1)$ and $n_t > -1$, $P_t > 0$ if and only if $\tau < 0$. But $sgn(\tau) = sgn(\tau')$, so $\tau < 0 \iff \beta < \beta^*$. ∎

Proof of Remark

The only difference to the case $\tau = \tau'$, $\tau_c = 0$ in the Proposition 1 Proof is that the tax τ' no longer enters the first-order condition for labour supply. Hence, the ratios k_{t+1}/y_t, $c_{t,y}/y_t$, $c_{t,o}/y_t$ equal the planner's in (9)–(10), and the new first-order condition for labour supply gives $l_t^{1+\chi} = \frac{(1-\alpha)}{\theta c_{t,y}/y_t} = (l_t^*)^{1+\chi}$. Given equal ratios and $l_t = l_t^*$ for all t, $y_t = y_t^*$, $k_{t+1} = k_{t+1}^*$, $c_{t,y} = c_{t,y}^*$, $c_{t,o} = c_{t,o}^*$ for all t, and so the first-best allocation is replicated as stated. ∎