Disturbance Observer

Big picture

Disturbance and uncertainties in mechanical systems:

- system models are important in design: e.g., in ZPET, observer, and preview controls
- inevitable to have uncertainty in actual mechanical systems
- system is also subjected to disturbances

Related control design:

- robust control
- adaptive control

Disturbance observer is one example of robust control.

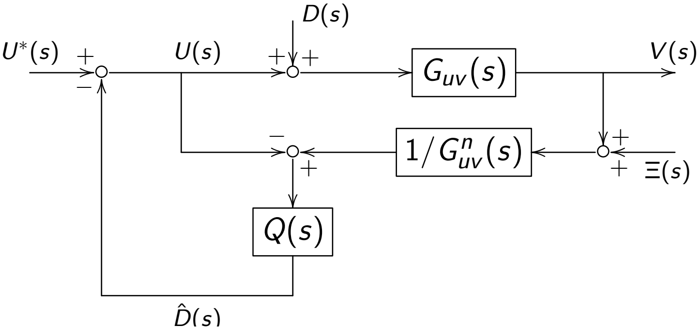
Disturbance observer (DOB)

- introduced by Ohnishi (1987) and refined by Umeno and Hori (1991)

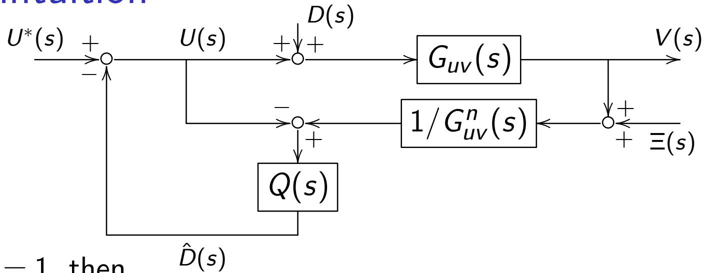
System:

$$V(s) = G_{uv}(s)\left[U(s) + D(s)\right]$$

$u(t)$–input; $d(t)$–disturbance; $v(t)$–output; $G_{uv}(s)$–actual plant dynamics between u and v; $G^n_{nv}(s)$–nominal model



DOB intuition



if $Q(s) = 1$, then

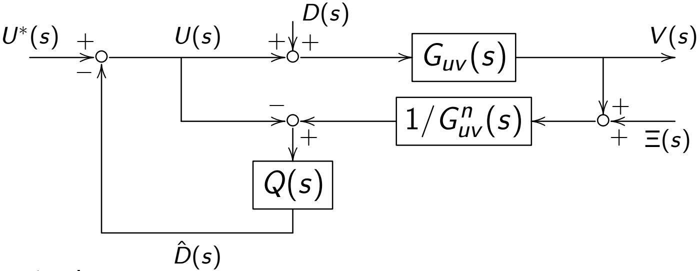
$$U(s) = U^*(s) - \left[\frac{G_{uv}(s)}{G_{uv}^n(s)}(U(s) + D(s)) + \frac{1}{G_{uv}^n(s)}\Xi(s) - U(s)\right]$$

$$\Rightarrow U(s) = \frac{G_{uv}^n(s)}{G_{uv}(s)}U^*(s) - D(s) - \frac{1}{G_{uv}(s)}\Xi(s)$$

$$V(s) = G_{uv}^n(s)U^*(s) - \Xi(s)$$

i.e., dynamics between $U^*(s)$ and $V(s)$ follows the nominal model; and disturbance is rejected

DOB intuition



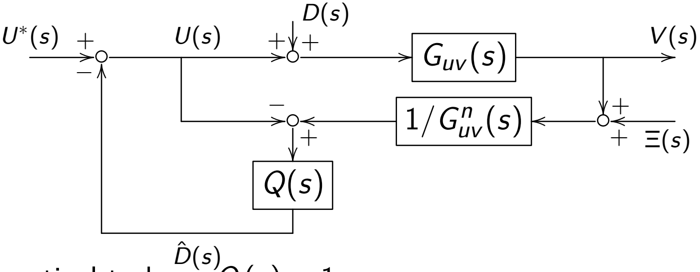
if $Q(s) = 1$, then

$$\hat{D}(s) = \left(\frac{G_{uv}(s)}{G_{uv}^n(s)} - 1\right) U(s) + \frac{1}{G_{uv}^n(s)} \Xi(s) + \frac{G_{uv}(s)}{G_{uv}^n(s)} D(s)$$

$$\approx \frac{1}{G_{uv}(s)} \Xi(s) + D(s) \;\; \text{if } G_{uv}(s) = G_{uv}^n(s)$$

i.e., disturbance $D(s)$ is estimated by $\hat{D}(s)$.

DOB details: causality

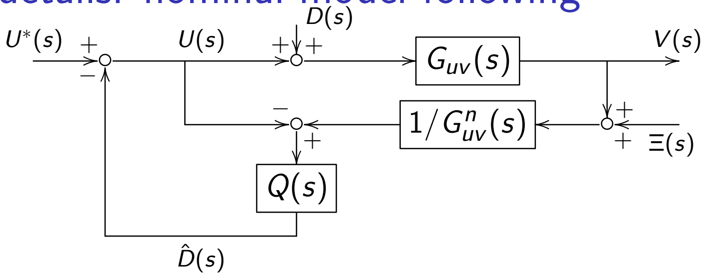


It is impractical to have $Q(s) = 1$.
e.g., if $G_{uv}(s) = 1/s^2$, then $1/G_{uv}^n(s) = s^2$ (not realizable)
$Q(s)$ should be designed such that $Q(s)/G_{nv}^n(s)$ is causal. e.g. (low-pass filter)

$$Q(s) = \frac{1 + \sum_{k=1}^{N-r} a_k (\tau s)^k}{1 + \sum_{k=1}^{N} a_k (\tau s)^k},\ Q(s) = \frac{3\tau s + 1}{(\tau s + 1)^3},\ Q(s) = \frac{6(\tau s)^2 + 4\tau s + 1}{(\tau s + 1)^4}$$

where τ determines the filter bandwidth

DOB details: nominal model following



Block diagram analysis gives

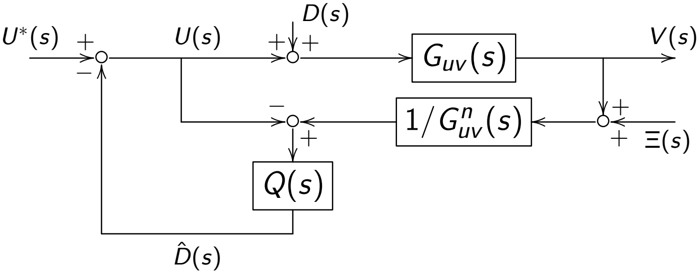
$$V(s) = G^o_{uv}(s)\, U^*(s) + G^o_{dv}(s)\, D(s) + G^o_{\xi v}(s)\, \Xi(s)$$

where

$$G^o_{uv} = \frac{G_{uv} G^n_{uv}}{G^n_{uv} + (G_{uv} - G^n_{uv})\, Q}, \quad G^o_{dv} = \frac{G_{uv} G^n_{uv}(1-Q)}{G^n_{uv} + (G_{uv} - G^n_{uv})\, Q}$$

$$G^o_{\xi v} = -\frac{G_{uv} Q}{G^n_{uv} + (G_{uv} - G^n_{uv})\, Q}$$

DOB details: nominal model following



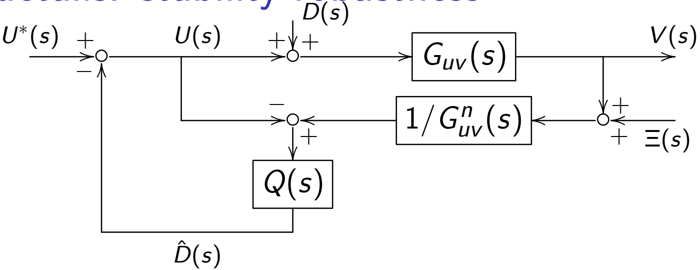
if $Q(s) \approx 1$, we have disturbance rejection and nominal model following:

$$G_{uv}^o \approx G_{uv}^n,\ G_{dv}^o \approx 0,\ G_{\xi v}^o = -1$$

if $Q(s) \approx 0$, DOB is cut off:

$$G_{uv}^o \approx G_{uv},\ G_{dv}^o \approx G_{uv},\ G_{\xi v}^o \approx 0$$

DOB details: stability robustness



$$G_{uv}^o = \frac{G_{uv} G_{uv}^n}{G_{uv}^n + (G_{uv} - G_{uv}^n) Q}, \; G_{dv}^o = \frac{G_{uv} G_{uv}^n (1-Q)}{G_{uv}^n + (G_{uv} - G_{uv}^n) Q}, \; G_{\xi v}^o = -\frac{G_{uv} Q}{G_{uv}^n + (G_{uv} - G_{uv}^n) Q}$$

closed-loop characteristic equation:

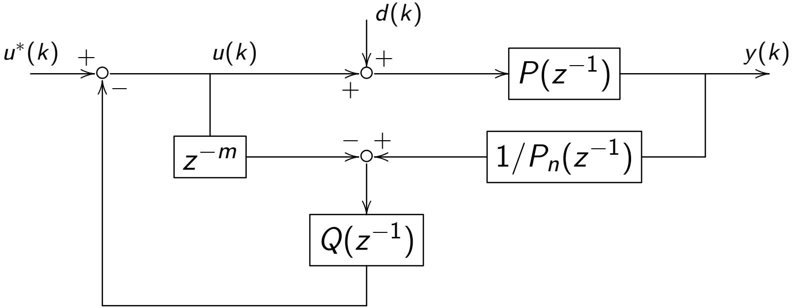
$$G_{uv}^n(s) + (G_{uv}(s) - G_{uv}^n(s)) Q(s) = 0$$
$$\Leftrightarrow G_{uv}^n(s)(1 + \Delta(s) Q(s)) = 0, \text{ if } G_{uv}(s) = G_{uv}^n(s)(1 + \Delta(s))$$

robust stability condition: stable zeros for $G_{nv}^n(s)$, plus

$$||\Delta(j\omega) Q(j\omega)|| < 1, \; \forall \omega$$

Application example

Discrete-time case



where $P\left(z^{-1}\right) \approx z^{-m} P_n\left(z^{-1}\right)$

see more details in, e.g., X. Chen and M. Tomizuka, "Optimal Plant Shaping for High Bandwidth Disturbance Rejection in Discrete Disturbance Observers," in Proceedings of the 2010 American Control Conference, Baltimore, MD, Jun. 30-Jul. 02, 2010, pp. 2641-2646