

MCME

Junior Set Answer Pamphlet

2021 December

This pamphlet contains the official solutions to all 10 problems on the Junior Problems Set of the 1st 2021 Mani-Cretin Mathematics Examination. These solutions are for reference only and are by no means the only solutions to these problems nor the most effective solutions. More effective and highly creative solutions can be submitted to the MCMA at:

MCMA-keystone@outlook.com

The MCMA will NOT accept any proposed solutions more than a week after the competition has been held. The MCMA will only feature some of the proposed solutions. In order to become a temporary problem creator/solver for the next MCME, you must pass the MCIME.

Created by Tata Li (李奥), founder of the Mani-Cretin Mathematics Association.
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MCMA

1. If $(123456 - 100000 - 20000 - 3000 - 400 - 52)^{50} = (5432 - 5000 - 400 - 30)^x$, What is x ?

$$\begin{aligned} 4^{50} &= 2^n \\ n &= \log_2(4^{50}) \\ n &= 100 \end{aligned}$$

(By Tata Li)

[1 point]

2. There is a total of 4936 people in Mani Country and Cretin Country combined. Cretin Country's population is $\frac{1}{3}$ of Mani Country's population, and half of Mani's population is female. What is the male population of Mani Country?

$$\begin{aligned} \text{Let } x &\text{ be the population of Cretin Country} \\ 4x &= 4936 \\ x &= 1234 \\ 1234(3/2) &= 1851 \end{aligned}$$

(By Tata Li)

[1 point]

3. Tim's age 10 years ago is 6 more than half of Max's age 10 years ago. Additionally, Tim's age 3 years later is 3 less than $\frac{3}{4}$ of Max's age. What is Tim's current age?

$$\begin{aligned} \text{Let } t &\text{ be Tim's current age, let } m &\text{ be Max's current age} \\ \textcircled{1} \quad 2t - m &= 4 &\text{ Write the first information to an equation} \\ \textcircled{2} \quad 4/3t - m &= -1 &\text{ Write the second information to an equation} \\ 2/3t &= 5 &\text{ Eliminate } m \text{ by } \textcircled{1} - \textcircled{2} \\ t &= 7.5 \end{aligned}$$

(By Tata Li)

[1 point]

4. turtle and a rabbit were racing to a target 2022 meters away and started at the same time. The rabbit started at a speed of 30km/hr. By half way, the rabbit slept; meanwhile, the turtle kept moving forward at a constant speed, 0.3km/hr. When the turtle is 100 meters away from the target, and the rabbit awakes and began running at speed of 30km/hr.

A) Who reaches the target first?

B) How many minutes did the rabbit sleep?

$$\begin{aligned} \text{Rabbit Speed: } &30000\text{m}/60\text{min}=500\text{m}/\text{min} \\ \text{Turtle Speed: } &300\text{m}/60\text{m}=5\text{m}/\text{min} \\ \text{Rabbit reaches the target first} \\ 5/1 &= 1922/x, \text{ so } x=384.4 \\ 500/1 &= 1011/y, \text{ so } x=2.022 \\ 384.4 - 2.022 &= 382.378 \approx 382 \text{ min} \end{aligned}$$

(By Tata Li)

[2 points]

5. Simplify $x(x^{-2})(x^3)(x^{-4})(x^5)(x^{-6})\dots\dots (x^{99})(x^{-100})$.

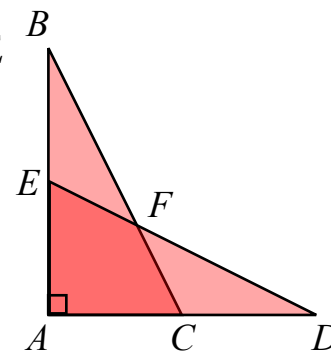
$$\begin{aligned} \text{Method 1: Find the pattern} \\ \text{Method 2: Add all of the exponents together} \\ \text{Answer: } &x^{(-50)} \text{ or } 1/(x^{50}) \end{aligned}$$

(By Tata Li)

[2 points]

6. $\triangle ABC$ and $\triangle AED$ are two congruent right triangles, and $BA = 2BE$.
What is the area of the darker region ($AEFC$)? $BA = 10$.

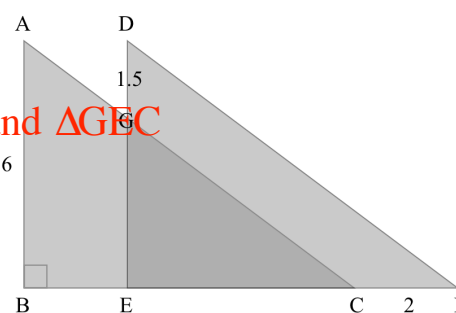
Since triangles with same base and height have same areas
 $\triangle AEF = \triangle AFC = \triangle CFD$
 $\triangle AEF + \triangle AFC + \triangle CFD = \triangle ADE = 25$
 Quadrilateral $AEFC = (25/3) * 2 = 50/3$



(By Tata Li)
[2 points]

7. Reference the diagram below. Right-Triangle DEF is a congruent replica of $\triangle ABC$ that moved several units to the right. AB is 6 units, DG is 1.5 units, BF is 10 units, and CF is 2 units. What is the area of quadrilateral DGCF?

Use any method to find areas of $\triangle DEF$ and $\triangle GEC$
 Area of $\triangle GEC = 13.5$
 Area of $\triangle DEF = 24$
 $24 - 13.5 = 10.5$



(By Tata Li)
[2 points]

8. The equations $x^2 + y^2 = 16$ and $(x+4)^2 + (y+4)^2 = 16$ depict two circles. What is the area of the overlapping part of two circles?

Graph the circles
 $1/4$ of a circle $= 16\pi/4 = 4\pi$
 Overlapping Part $= (4\pi - 8) * 2 = 8\pi - 16$

(By Tata Li)
[3 points]

9. $f(x) = 3x + 4$, $g(x) = ax + b$, $g(3) = 9$, and $f(g(4)) = 37$. What is the expression for $g(x)$?

Solve a and b, which $a = 2$ and $b = 3$
 $g(x) = 2x + 3$

(By Tata Li)
[3 points]

10. If $3x^2 + 1968x = 297\sqrt{2}$, what is the product between the sum of the roots and the product and the roots?

Use Vieta's formula to find the product and sum of the two roots
 $x_1 + x_2 = -1968/3 = -656$
 $x_1 \cdot x_2 = (-297\sqrt{2})/3 = -99\sqrt{2}$
 $(x_1 + x_2)(x_1 \cdot x_2) = 64944\sqrt{2}$

(By Tata Li and Kevin Shi)
[3 points]