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Key Points : Basic Impact of Spatial Channel Correlation

1. Basic Impact of Spatial Channel Correlation

1. For **single-user point-to-point MIMO channels** with multiple antennas at both transmitter and receiver, the **spatial channel correlation** is **detrimental**. However, for **multiuser communications with single-antenna UEs**, the picture changes.

2. Why? because it is the **collection of the UE's spatial correlation matrices** that determines the network performance.

3. In fact, 1) The UEs'channels are well modeled as **statistically uncorrelated** because they are generally **physically separated by multiple wavelengths**; 2) although the channel of each UE can exhibit high spatial correlation at the BS, the **spatial correlation matrices can be highly different between UEs**.

4. Consider the UL of a single-cell scenario and assume that the channels are perfectly known. The UEs' channels are distributed as $\mathbf{h}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \mathbf{R}_k)$, for $k = 1, \ldots, K$, and we assume that

$$\mathbf{R}_k = K\mathbf{U}_k\mathbf{U}_k^{\mathrm{H}} \tag{2.9}$$

where $\mathbf{U}_k \in \mathbb{C}^{M\times M/K}$ are tall unitary matrices(i.e., its columns are orthogonal, $\mathbf{U}_k^{\mathrm{H}}\mathbf{U}_k = \mathbf{I}_{M/K}$) and we assume that $\mathbf{U}_k^{\mathrm{H}}\mathbf{U}_j = \mathbf{0}_{M/K\times M/K}$ for all $j \neq k$. The factor K **normalizes** the channel gain such that $\beta_k = \frac{1}{M}\mathrm{tr}(\mathbf{R}_k) = 1$

2. An Example to Understand the Impact of Spatial Channel Correlation

5. Remember (2.4)

$$\mathbf{h} = \mathbf{R}^{\frac{1}{2}}\check{\mathbf{e}} = \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}}\check{\mathbf{e}} \sim \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{e}$$

use K-L expansion, we have

$$\mathbf{h}_k = \sqrt{K}\mathbf{U}_k\mathbf{e}_k \tag{2.10}$$

where $\mathbf{e}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M/K}, \mathbf{I}_{M/K})$

6. Next let's consider the **received UL signal** $\mathbf{y} \in \mathbb{C}^M$ at the BS, which is given by

$$\mathbf{y} = \sum_{i=1}^{K} \mathbf{h}_i s_i + \mathbf{n} \tag{2.11}$$

where $s_i \in \mathbb{C}$, for $i = 1, \ldots, K$, are the UL signals which power is p_i, and $\mathbf{n} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \sigma^2_{\mathrm{UL}}\mathbf{I}_M)$ is receiver noise. Then we have

$$\begin{aligned} \underbrace{\mathbf{U}_k^{\mathrm{H}}}_{\text{Correlation Eigenspace}} \mathbf{y} &= \mathbf{U}_k^{\mathrm{H}}(\sum_{i=1}^{K} \mathbf{h}_i s_i + \mathbf{n}) \\ &= \sum_{i=1}^{K} \sqrt{K}\mathbf{U}_k^{\mathrm{H}}\mathbf{U}_i\mathbf{e}_i s_i + \mathbf{U}_k^{\mathrm{H}}\mathbf{n} = \sqrt{K}\mathbf{e}_k s_k + \check{\mathbf{n}}_k \end{aligned} \tag{2.12}$$

where $\check{\mathbf{n}}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M/K}, \sigma^2_{\mathrm{UL}}\mathbf{I}_{M/k})$, and **the multisuer is divided into K orthogonal single-user channels with M/K effective antennas.**

7. But if all UEs share the same correlation matrix $\mathbf{R} = K\mathbf{U}\mathbf{U}^{\mathrm{H}}$, the detrimental effect of a common correlation matrix is apparent when we multiply the received UL signal $\mathbf{y}$ by the correlation-eigenspace $\mathbf{U}$.