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Key Points : Uplink Pilot Transmission

1. Uplink Pilot Transmission

1. It is important for BS j to have estimates of the channels from the UEs in cell j. Channel estimates from interfering UEs in order cells can also be useful to perform **Interference Suppression** during data transmission. In each coherence block, there are τ_p samples are reserved for UL pilot signaling. The pilot sequence of UE k in cell j is denoted by $\phi_{jk} \in \mathbb{C}^{\tau_p \times 1}$.

2. ϕ_{jk} is assumed to have **unit-magnitude elements** to obtain a **constant** power level, and this implies that

$$\|\phi_{jk}\|^2 = \phi_{jk}^{\mathrm{H}} \phi_{jk} = \tau_p$$

where the elements of ϕ_{jk} are **scaled** by the UL transmit power as $\sqrt{p_{jk}}$ and then transmitted as the signal s_{jk} in (2.5)

$$\begin{aligned} \mathbf{y}_j &= \sum_{l=1}^{L} \sum_{k=1}^{K_l} \mathbf{h}_{lk}^{j} s_{lk} + \mathbf{n}_j \\ &= \underbrace{\sum_{k=1}^{K_j} \mathbf{h}_{jk}^{j} s_{jk}}_{\text{Desired Signals}} + \underbrace{\sum_{l=1, l\neq j}^{L} \sum_{i=1}^{K_l} \mathbf{h}_{li}^{j} s_{li}}_{\text{Inter-cell Interference}} + \underbrace{\mathbf{n}_j}_{\text{Noise}} \end{aligned} \tag{2.5}$$

over τ_p UL samples, leading to the received UL signal $\mathbf{Y}_j^p \in \mathbb{C}^{M_j \times \tau_p}$ at BS j.

3. The the $\mathbf{Y}_j^p$ is given by

$$\mathbf{Y}_j^p = \underbrace{\sum_{k=1}^{K_j} \sqrt{p_{jk}} \mathbf{h}_{jk}^{j} \phi_{jk}^{\mathrm{T}}}_{\text{Desired Pilots}} + \underbrace{\sum_{l=1, l\neq j}^{L} \sum_{i=1}^{K_l} \sqrt{p_{li}} \mathbf{h}_{li}^{j} \phi_{li}^{\mathrm{T}}}_{\text{Inter-cell Pilots}} + \underbrace{\mathbf{N}_j^p}_{\text{Noise}} \tag{3.1}$$

where $\mathbf{N}_j^p \in \mathbb{C}^{M_j \times \tau_p}$ is **the Independent Additive Receiver Noise** with i.i.d. The elements of $\mathbf{N}_j^p$ distributed as $\mathcal{N}_{\mathbb{C}}(0, \sigma_{\mathrm{UL}}^2)$.

4. Suppose that BS j wants to estimate the channel $\mathbf{h}_{li}^j$ from an arbitrary UE i in cell l. The BS can then **Multiply/Correlate** $\mathbf{Y}_j^p$ with the pilot sequence ϕ_{li}^* of this UE, given as

$$\mathbf{y}_{jli}^p = \mathbf{Y}_j^p \phi_{li}^* = \sum_{l'=1}^{L} \sum_{i'=1}^{K_{l'}} \sqrt{p_{l'i'}} \mathbf{h}_{l'i'}^j \phi_{l'i'}^{\mathrm{T}} \phi_{li}^* + \mathbf{N}_j^p \phi_{li}^* \tag{3.2}$$

5. For the kth UE in the BS's own cell, (3.2) can be expressed as

$$\mathbf{y}_{jjk}^p = \mathbf{Y}_j^p \phi_{jk}^* =$$

$$\underbrace{\sqrt{p_{jk}} \mathbf{h}_{jk}^j \phi_{jk}^{\mathrm{T}} \phi_{jk}^*}_{\text{Desired Pilot}} + \underbrace{\sum_{i=1, i \neq k}^{K_j} \sqrt{p_{ji}} \mathbf{h}_{ji}^j \phi_{ji}^{\mathrm{T}} \phi_{jk}^*}_{\text{Intra-cell Pilots}} + \underbrace{\sum_{l=1, l \neq j}^{L} \sum_{i=1}^{K_l} \sqrt{p_{li}} \mathbf{h}_{li}^j \phi_{li}^{\mathrm{T}} \phi_{jk}^*}_{\text{Inter-cell Pilots}} + \underbrace{\mathbf{N}_j^p \phi_{jk}^*}_{\text{Noise}} \tag{3.3}$$

if the pilot sequences of two UEs are **Orthogonal**(i.e., $\phi_{li}^{\mathrm{T}} \phi_{jk}^* = 0$), then the corresponding interference term in (3.3) (i.e., intra-cell pilots and inter-cell pilots) **vanishes** and **DOES NOT** affect the estimation. Therefore **ideally** we have

$$\mathbf{y}_{jjk}^p = \mathbf{Y}_j^p \phi_{jk}^* = \underbrace{\sqrt{p_{jk}} \mathbf{h}_{jk}^j \phi_{jk}^{\mathrm{T}} \phi_{jk}^*}_{\text{Desired Pilot}} + \underbrace{\mathbf{N}_j^p \phi_{jk}^*}_{\text{Noise}} = \underbrace{\sqrt{p_{jk}} \tau_p \mathbf{h}_{jk}^j}_{\text{Desired Pilot}} + \underbrace{\mathbf{N}_j^p \phi_{jk}^*}_{\text{Noise}}$$

6. However, it is **impossible** to assign mutually orthogonal pilots to all UEs in practice. If we define the set

$$\mathcal{P}_{jk} = \left\{ (l, i) : \phi_{li} = \phi_{jk}, \quad l = 1, \ldots, L; i = 1, \ldots, K_j \right\} \tag{3.4}$$

that means UE i in cell l uses the same pilot as UE k in cell j. Then (3.3) simplifies to

$$\mathbf{y}_{jjk}^p = \mathbf{Y}_j^p \phi_{jk}^* =$$

$$\underbrace{\sqrt{p_{jk}} \tau_p \mathbf{h}_{jk}^j}_{\text{Desired Pilot}} + \underbrace{\sum_{(l(\neq j), i(\neq k)) \in \mathcal{P}_{jk}} \sqrt{p_{jk}} \tau_p \mathbf{h}_{jk}^j}_{\text{Interfering Pilots}} + \underbrace{\mathbf{N}_j^p \phi_{jk}^*}_{\text{Noise}} \tag{3.5}$$

interfering pilots term means the inferences come from different cells which share the same pilot as desired cell. We also note that $\mathbf{N}_j^p \phi_{jk}^* \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M_j}, \sigma_{\mathrm{UL}}^2 \tau_p \mathbf{I}_{M_j})$, since the pilot sequence are deterministic and $\|\phi_{jk}\|^2 = \tau_p$.

7. The signal $\mathbf{y}_{jjk}^{p}$ is a **sufficient statistic** for estimating $\mathbf{h}_{jk}^{j}$ since there is **NO loss** in useful information as compared to using the originally received signal $\mathbf{Y}_{j}^{p}$.