Massive Notes 2.5 pp.231 - 234

Key Points : Channel Hardening; Favorable Propagation

1. Channel Hardening

1. **Channel Hardening** makes a fading channel behave as **deterministic**.

2. This property **alleviates** the need for 1) combating small-scale fading (e.g., by adapting the transmit powers) and 2) improves the DL channel gain estimation.

3. **Channel Hardening** refers to a propagation channel $\mathbf{h}_{jk}^{j}$ provides asymptotic channel hardening if

$$\frac{\|\mathbf{h}_{jk}^{j}\|^2}{\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\}} \to 1 \tag{2.15}$$

almost surely as $M_j \to \infty$.

4. This definition says that the gain $\|\mathbf{h}_{jk}^{j}\|^2$ of an arbitrary fading channel is **close to** its mean value **when there are many antennas**.

5. In other words, the **relative deviation** from the average channel gain $\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\} = \mathrm{tr}(\mathbf{R}_{jk}^{j})$ vanishes asymptotically.

6. However, it does not mean that $\|\mathbf{h}_{jk}^{j}\|^2 \to \mathrm{tr}(\mathbf{R}_{jk}^{j})$, because both these terms generally **diverge** as $M_j \to \infty$, but one can interpret the result as

$$\frac{1}{M_j}\|\mathbf{h}_{jk}^{j}\|^2 - \frac{1}{M_j}\mathrm{tr}(\mathbf{R}_{jk}^{j}) \to 0 \tag{2.16}$$

almost surely as $M_j \to \infty$.

7. With correlated Rayleigh fading, $\mathbf{h}_{jk}^{j} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M_j}, \mathbf{R}_{jk}^{j})$, a sufficient condition for asymptotic channel hardening is that the **spectral norm** $\|\mathbf{R}_{jk}^{j}\|_2$ of the **spatial correlation matrix** is bounded and $\beta_{jk}^{j} = \frac{1}{M_j}\mathrm{tr}(\mathbf{R}_{jk}^{j})$ remains **strictly positive** as $M_j \to \infty$.

8. What is important for practical purposes is not the asymptotic result, but **how close to asymptotic channel hardening we are with a practical number of antennas**. This can be quantified by considering

$$var\left\{\frac{\|\mathbf{h}_{jk}^{j}\|^2}{\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\}}\right\} = \frac{var\{\|\mathbf{h}_{jk}^{j}\|^2\}}{(\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\})^2} \overset{(a)}{=} \frac{\mathrm{tr}((\mathbf{R}_{jk}^{j})^2)}{(\mathrm{tr}(\mathbf{R}_{jk}^{j}))^2} = \frac{\mathrm{tr}((\mathbf{R}_{jk}^{j})^2)}{(M_j\beta_{jk}^{j})^2} \tag{2.17}$$

where (a) follows Lemma B.14 on p.564. This is the variance of the expression in (2.15) and it should be close to **zero** if channel hardening is to be observed.

9. In the special case of **uncorrelated fading**, we have $\mathbf{R}_{jk}^{j} = \beta_{jk}^{j}\mathbf{I}_{M_j}$, and hence (2.17) becomes $1/M_j$, and when $M_j = 100$, $1/M_j$ is small enough to achieve channel hardening.

2. Favorable Propagation

10. **Favorable propagation** makes the directions of two UE channels **asymptotically orthogonal**.

11. The pair of channels $\mathbf{h}_{li}^{j}$ and $\mathbf{h}_{jk}^{j}$ to BS j provide asymptotically favorable propagation if

$$\frac{(\mathbf{h}_{li}^{j})^{\mathrm{H}}\mathbf{h}_{jk}^{j}}{\sqrt{\mathbb{E}\{\|\mathbf{h}_{li}^{j}\|^2\}\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\}}} \to 0 \tag{2.18}$$

almost surely as $M_j \to \infty$.

12. This definition says that **the inner product of the normalized channels**

$$\frac{\mathbf{h}_{li}^{j}}{\sqrt{\mathbb{E}\{\|\mathbf{h}_{li}^{j}\|^2\}}} \overset{\text{(inner product)}}{\times} \frac{\mathbf{h}_{jk}^{j}}{\sqrt{\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\}}} \to 0$$

13. It is noteworthy that favorable propagation **DOES NOT** imply that inner product of $\mathbf{h}_{li}^{j}$ and $\mathbf{h}_{jk}^{j}$ goes to zero. We should clear that it is **the channel directions become orthogonal, but not the channel responses**.

14. For correlated Rayleigh fading channels, the two channels will also exhibit asymptotic channel hardening.

15. One way to quantify

$$var\left\{\frac{(\mathbf{h}_{li}^{j})^{\mathrm{H}}\mathbf{h}_{jk}^{j}}{\sqrt{\mathbb{E}\{\|\mathbf{h}_{li}^{j}\|^2\}\mathbb{E}\{\|\mathbf{h}_{jk}^{j}\|^2\}}}\right\} = \frac{\mathrm{tr}(\mathbf{R}_{li}^{j}\mathbf{R}_{jk}^{j})}{\mathrm{tr}(\mathbf{R}_{li}^{j})\mathrm{tr}(\mathbf{R}_{jk}^{j})} = \frac{\mathrm{tr}(\mathbf{R}_{li}^{j}\mathbf{R}_{jk}^{j})}{M_j^2\beta_{li}^{j}\beta_{jk}^{j}} \tag{2.19}$$

This is a measure of how orthogonal the channel directions are, which determines how much interference the UEs cause to each other. Ideally, the variance in (2.19) should be zero.

16. Note that channel hardening and favorable propagation are two related, but different properties. We described a sufficient condition under which both properties hold, but it is not a necessary condition. Generally speaking, a channel model can have both properties, one of them, or none. For example, the keyhole channel provides favorable propagation, but not channel hardening. In contrast, two LoS channels that have the same azimuth angle provide channel hardening, but not favorable propagation.