# Massive Notes 1.2

**Key Words : Spectral Efficiency; Channel Capacity**

## 1. Spectral Efficiency

1. The **Nyquist-Shannon Sampling Theorem** implies that the band-limited communication signal that is over this channel (suppose the bandwidth of this channel is $B$ Hz) is completely determined by $2B$ real-valued equal-spaced sampled per second.

2. Accordingly, when we consider the **Complex-Baseband** representation of the signal, $B$ complex-valued samples per second is the more natural quantity.

3. These $B$ samples are **the degrees of freedom** available for designing the communication signal.

4. The **SE** is the amount of information that can be transferred reliably per complex-valued sample. More precisely,The **SE** of an encoding/decoding scheme is **the advantage number of bits of information, per complex-valued sample, that it can reliably transmit over the channel under consideration**.

5. Since the bandwidth of a channel is $B$ Hz, there are $B$ samples per second, so an equivalent unit of the SE is bit per second per Hertz, i.e., $bit/s/Hz$.

6. For fading channels, the SE can be viewed as **the average number of** $bit/s/Hz$ over the fading realizations.

7. Another metric is the **Information Rate** $[bit/s] = \mathbf{SE} \times \mathbf{B}$

8. we commonly consider the sum SE of the channels from all UEs in a cell to the respective BS, which is measured in $bit/s/Hz/cell$

# 2. Channel Capacity

9. The largest achievable SE is of key importance when designing communication systems. The maximum SE is determined by the **Channel Capacity**.

10. Consider a **discrete memoryless channel** with two random variables, input $x$ and output $y$. Any SE smaller or equal to the channel capacity

$$C = \sup_{f(x)} \big(\mathcal{H}(y) - \mathcal{H}(y|x)\big) \tag{1}$$

11. The supremum is taken with respect to **all feasible input distributions** $f(x)$, while $\mathcal{H}(f)$ is the **differential entropy** of the output and $\mathcal{H}(y|x)$ is the **conditional differential entropy** of the output given the input.

12. In wireless communications, we are particularly interested in channels where the received signal is the superposition of a scaled version of the desired signal and **Additive Gaussian Noise**. These channels are often referred to as **Additive White Gaussian Noise** (**AWGN**) channels.

13. Consider a discrete memoryless channel with input $x \in \mathbb{C}$ and output $y \in \mathbb{C}$ given by

$$y = h \cdot x + n \tag{2}$$

where $n \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ is **independent noise**. The input distribution is **power-limited** as $\mathbb{E}\{|x|^2\} \leq p$ and the **channel response** $h \in \mathbb{C}$ is known at the output.

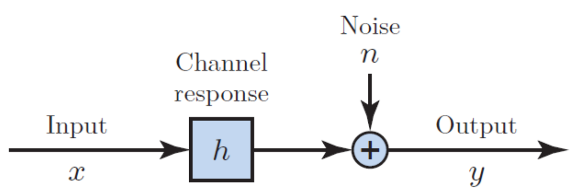


Figure 1: A discrete memoryless channel.

14. If $h$ is deterministic, then the channel capacity is

$$C = \log_2\left(1 + \frac{p|h|^2}{\sigma^2}\right) \tag{3}$$

and is achieved by the input distribution $x \sim \mathcal{N}_{\mathbb{C}}(0, p)$ (i.e., $\mathbb{E}\{|x|^2\} = p$).

15. If $h$ is a realization of a random variable $\mathbb{H}$ that is independent of the signal and noise, then the **ergodic** channel capacity is

$$C = \mathbb{E}\left\{ \log_2 \left(1 + \frac{p|h|^2}{\sigma^2}\right)\right\} \tag{4}$$

this is called a **fading channel** and is achieved by the input distribution $x \sim \mathcal{N}_{\mathbb{C}}(0, p)$ (i.e., $\mathbb{E}\{|x|^2\} = p$).

16. The SE is generally a good performance metric whenever data blocks of thousands of bits are transmitted.

17. In (3) and (4), we can see a signal-to-noise ratio (SNR)-like expression

$$\frac{\textbf{Received Signal Power: } p|h|^2}{\textbf{Noise Power: } \sigma^2} \tag{5}$$

18. It is more convenient to consider the average SNR when describing the quality of a communication channel. We define the average SNR as

$$\mathrm{SNR} = \frac{p\mathbb{E}\{|h|^2\}}{\sigma^2} \tag{6}$$

19. We call $\mathbb{E}\{|h|^2\}$ the **average channel gain**.

20. Transmissions in cellular networks are in general corrupted by **interference** from simultaneous transmissions in the same and other cells.

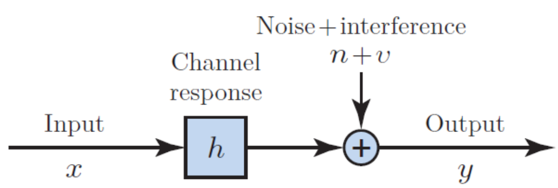


Figure 2: A discrete memoryless interference channel.

21. The interference is not necessarily independent of the input $x$ and the channel $h$. Consider a discrete memoryless interference channel with input $x \in \mathbb{C}$ and output $y \in \mathbb{C}$ given by

$$y = h \cdot x + v + c \tag{7}$$

where $n \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$ is independent noise, the channel response $h \in \mathbb{C}$ is known at the output, and $v \in \mathbb{C}$ is random interference. The input is power-limited as $\mathbb{E}\{|x|^2\} \le p$.

22. It is noteworthy that the exact channel capacity of interference channels is **generally unknown**, but convenient lower bounds can be obtained. Suppose $h$ is deterministic and the interference $v$ has zero mean, a known variance $p_v \in \mathbb{R}_+$ , and is uncorrelated with the input (i.e., $\mathbb{E}\{x * v\} = 0$), then we have

$$C \geq \log_2 \left(1 + \frac{p|h|^2}{p_v + \sigma^2}\right) \tag{8}$$

where the bound is achieved using the input distribution $x \sim \mathcal{N}_{\mathbb{C}}(0, p)$ (i.e., $\mathbb{E}\{|x|^2\} = p$).

23. Now suppose $h \in \mathbb{C}$ is instead a realization of the random variable $\mathbb{H}$ and that $\mathbb{U}$ is a random variable with realization $u$ that affects the interference variance, and the output **know** the realizations of these random variables.

24. If the noise $n$ is conditionally independent of $v$ given $h$ and $u$, then 1) the interference $v$ has conditional zero mean (i.e., $\mathbb{E}\{v|h, u\} = 0$) and 2) conditional variance denoted by $p_v(h, u) = \mathbb{E}\{|v|^2|h, u\}$, and 3) the interference is conditionally uncorrelated with the input (i.e., $\mathbb{E}\{x * v|h, u\} = 0$). Then the **ergodic channel capacity** $C$ is lower bounded as

$$C \geq \mathbb{E}\left\{ \log_2 \left(1 + \frac{p|h|^2}{p_v(h, u) + \sigma^2}\right)\right\} \tag{9}$$

where the bound is achieved using the input distribution $x \sim \mathcal{N}_{\mathbb{C}}(0, p)$ (i.e., $\mathbb{E}\{|x|^2\} = p$).

24. The lower bounds are obtained by treating the interference as an additional source of noise in the decoder. If an interfering signal is very strong, then we can decode it and subtract the interference from the received signal, before decoding the desired signal. But it is not realistic since **there should not be any strongly interfering signal in a well-designed cellular network**. Therefore, **we treat interference as an additional noise in the low-interference regime**.

25. The SE expressions in (8) and (9) have a form typical for wireless communications: the base-two logarithm of one plus the expression(remember (5))

$$\frac{\textbf{Received Signal Power: } p|h|^2}{\textbf{Interference Power: } p_v + \textbf{Noise Power: } \sigma^2} \tag{10}$$

but it is important to note that (10) is an **SINR** only when $h$ and $p_v$ are deterministic. Otherwise this expression is random. We can also call (10) as **instantaneous SINR**.

25. Unfortunately, the capacity results we discussed before are discrete memoryless channels, which are different from practical continuous wireless channels. But we can use some techniques (e.g., OFDM) to solve this problem. (END)