Massive Notes 1.6

Key Points : MRC; UL SE for the desired UE in the LoS case

1. Maximum Ratio Combining (MRC)

1. The **Cauchy-Schwartz Inequality** means that

$$|a_1b_1 + \cdots + a_nb_n|^2 \leq (|a_1|^2 + \cdots + |a_n|^2) \cdot (|b_1|^2 + \cdots + |b_n|^2) \tag{1}$$

and (1) can be rewritten as

$$|\sum_{i=1}^{n} a_ib_i|^2 \leq \sum_{i=1}^{n} |a_i|^2 \cdot \sum_{i=1}^{n} |b_i|^2 \tag{2}$$

therefore we have

$$\frac{|\sum_{i=1}^{n} a_ib_i|^2}{\sum_{i=1}^{n} |a_i|^2} \leq \cdot \sum_{i=1}^{n} |b_i|^2 \tag{3}$$

2. The benefits of having multiple antennas at the BS appear **when the BS knows the channel response of the desired UE**. This knowledge enables **the BS to coherently combine the received signals from all antennas**. Remember the equation

$$\mathbf{y}_0 = \underbrace{\mathbf{h}_0^0 s_0}_{\text{Desired signal}} + \underbrace{\mathbf{h}_1^0 s_1}_{\text{Interfering signal}} + \underbrace{\mathbf{n}_0}_{\text{Noise}} \tag{4}$$

3. Assume that the channel responses are known at the BS and can be used to select a **Receive Combining Vector** $\mathbf{v}_0 \in \mathbb{C}^M$, and this vector is multiplied with (4) to obtain

$$\mathbf{v}_0^{\mathrm{H}}\mathbf{y}_0 = \underbrace{\mathbf{v}_0^{\mathrm{H}}\mathbf{h}_0^0 s_0}_{\text{Desired signal}} + \underbrace{\mathbf{v}_0^{\mathrm{H}}\mathbf{h}_1^0 s_1}_{\text{Interfering signal}} + \underbrace{\mathbf{v}_0^{\mathrm{H}}\mathbf{n}_0}_{\text{Noise}} \tag{5}$$

where $\mathbf{v}_0^{\mathrm{H}}$ refers to the conjugate transpose of $\mathbf{v}_0$.

4. According to the Cauchy-Schwartz inequality we know that the ratio $|\mathbf{v}_0^{\mathrm{H}}\mathbf{h}_0^0|^2/\|\mathbf{v}_0\|$ can be maximized when $\mathbf{v}_0 = \mathbf{h}_0^0$. It is important to know that $\mathbf{v}_0$ is independent of $\mathbf{h}_1^0$ and $\mathbf{n}_0$.

2. UL SE for the desired UE (1): ULA LoS case

5. Suppose the BS in cell 0 knows the channel responses and applies MRC to the received signal. An achievable UL SE for the desired UE in the LoS case is

$$\mathrm{SE}_0^{\mathrm{LoS}} = \log_2\left(1 + \frac{M}{\bar{\beta} g(\varphi_0^0, \varphi_1^0) + \frac{1}{\mathrm{SNR}_0}}\right) = \log_2\left(1 + \frac{M}{\bar{\beta} g(\varphi_0^0, \varphi_1^0) + \frac{\sigma^2}{p\beta_0^0}}\right) \tag{6}$$

For comparison, remember the SISO LoS case

$$\mathrm{SE}_0^{\mathrm{LoS}} = \log_2\left(1 + \frac{1}{\bar{\beta} + \frac{1}{\mathrm{SNR}_0}}\right) = \log_2\left(1 + \frac{1}{\bar{\beta} + \frac{\sigma^2}{p\beta_0^0}}\right) \tag{7}$$

6. In (6), the function $g(\varphi_0^0, \varphi_1^0)$ is

$$g(\varphi_0^0, \varphi_1^0) = \begin{cases} \frac{\sin^2\left(\pi d_{\mathrm{H}} M (\sin(\varphi_0^0) - \sin(\varphi_1^0))\right)}{M \sin^2\left(\pi d_{\mathrm{H}} (\sin(\varphi_0^0) - \sin(\varphi_1^0))\right)} & \text{if } \sin(\varphi_0^0) \neq \sin(\varphi_1^0) \\ M & \text{if } \sin(\varphi_0^0) = \sin(\varphi_1^0) \end{cases} \tag{8}$$

where φ_0^0 and φ_1^0 are the azimuth angles to the UE0(desired) and UE1(interfering), respectively. Recall the figure below

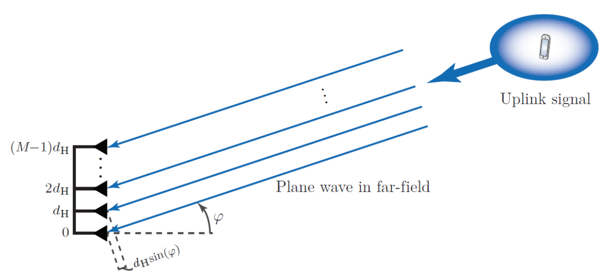

Figure 1: Recall the azimuth angle.

3. The Function $g(\varphi_0^0, \varphi_1^0)$

7. Now we focus on how to obtain (8). We have already known that

$$\mathbf{h}_i^0 = \sqrt{\beta_i^0}\left[1, e^{2\pi j d_{\mathrm{H}} \sin(\varphi_i^0)}, \dots, e^{2\pi j d_{\mathrm{H}}(M-1)\sin(\varphi_i^0)}\right]^{\mathrm{T}} \quad \text{for} \quad i = 0, 1 \tag{9}$$

where $\varphi_i^0 \in [0, 2\pi)$. Therefore

$$g(\varphi_0^0, \varphi_1^0) = \frac{|\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0|^2}{M} \tag{10}$$

assume $\sqrt{\beta_i^0} = 1$, and then $\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0$ is

$$\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0 = 1 \cdot 1 + e^{-2\pi j d_{\mathrm{H}} \sin(\varphi_0^0)} \cdot e^{2\pi j d_{\mathrm{H}} \sin(\varphi_1^0)} + \cdots + e^{-2\pi j d_{\mathrm{H}}(M-1)\sin(\varphi_0^0)} \cdot e^{2\pi j d_{\mathrm{H}}(M-1)\sin(\varphi_1^0)} \tag{11}$$

if $\sin(\varphi_0^0) = \sin(\varphi_1^0)$, then

$$\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0 = M \tag{12}$$

if $\sin(\varphi_0^0) \neq \sin(\varphi_1^0)$, then

$$\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0 = 1 \cdot 1 + e^{-2\pi j d_{\mathrm{H}} \sin(\varphi_0^0)} \cdot e^{2\pi j d_{\mathrm{H}} \sin(\varphi_1^0)} + \cdots + e^{-2\pi j d_{\mathrm{H}}(M-1)\sin(\varphi_0^0)} \cdot e^{2\pi j d_{\mathrm{H}}(M-1)\sin(\varphi_1^0)}$$

$$\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0 = 1 \cdot 1 + e^{2\pi j d_{\mathrm{H}}(\sin(\varphi_1^0) - \sin(\varphi_0^0))} + \cdots + e^{2\pi j d_{\mathrm{H}}(M-1)(\sin(\varphi_1^0) - \sin(\varphi_0^0))} \tag{13}$$

8. According to the **Geometric Series Formula**

$$\sum_{m=0}^{M-1} x^M = \begin{cases} M & \text{for } x = 1 \\ \frac{1-x^M}{1-x} & \text{for } x \neq 1 \end{cases} \tag{14}$$

let $x = e^{2\pi j d_{\mathrm{H}}(\sin(\varphi_1^0) - \sin(\varphi_0^0))}$, so (13) can be

$$\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0 = \frac{1 - e^{2\pi j d_{\mathrm{H}} M(\sin(\varphi_1^0) - \sin(\varphi_0^0))}}{1 - e^{2\pi j d_{\mathrm{H}}(\sin(\varphi_1^0) - \sin(\varphi_0^0))}} \tag{15}$$

9. Finally we have

$$|\mathbf{h}_0^{0\,\mathrm{H}} \mathbf{h}_1^0|^2 = \left|\frac{1 - e^{2\pi j d_{\mathrm{H}} M(\sin(\varphi_1^0) - \sin(\varphi_0^0))}}{1 - e^{2\pi j d_{\mathrm{H}}(\sin(\varphi_1^0) - \sin(\varphi_0^0))}}\right|^2 = \frac{\sin^2\left(\pi d_{\mathrm{H}} M(\sin(\varphi_0^0) - \sin(\varphi_1^0))\right)}{\sin^2\left(\pi d_{\mathrm{H}}(\sin(\varphi_0^0) - \sin(\varphi_1^0))\right)} \tag{16}$$

and (8) obtained.

4. Simulation Results

Program 3: gFunction.m

Listing 1: gFunction.m

```
%% Define parameters
%Empty workspace and close figures
close all;
clear;

%Define the range of BS antennas
Mvalues = [1, 10, 100];

%Angle of the desired UE
varphiDesired = pi/6;

%Range of angles of the interfering UE
varphiInterfererDegrees = [-180 : 1 : 180];
varphiInterfererRadians = varphiInterfererDegrees * (pi/180);

%Define the antenna spacing (in number of wavelengths)
antennaSpacing = 1/2; %Half wavelength distance

%Preallocate matrix for storing the simulation results
gfunction = zeros(length(varphiInterfererDegrees), length(Mvalues));


%% Go through all number of antennas
for m = 1 : length(Mvalues)

    %Generate channel response for the desired UE using (1.23)
    %assume \beta_0^0 = \beta_1^0 = 1
    hdesired = exp(1i * 2 * pi * antennaSpacing * sin(varphiDesired) * (0 : Mvalues(m) - 1)');

    %Go through all angles of interfering UE
    for n = 1 : length(varphiInterfererRadians)

        %Generate channel response for the interfering UE using (1.23)
        hinterfering = exp(1i * 2 * pi * antennaSpacing * sin(varphiInterfererRadians(n)) * (0 : Mvalues(m) - 1)');

        %Compute the g-function in (1.28), using its definition
        gfunction(n, m) = abs(hdesired' * hinterfering) ^ 2 / Mvalues(m);

```

```matlab
    end

end

%% Plot the simulation results
figure;
hold on; box on;

plot(varphiInterfererDegrees, gfunction(:, 1), 'k-', 'LineWidth', 1);
plot(varphiInterfererDegrees, gfunction(:, 2), 'r--', 'LineWidth', 1);
plot(varphiInterfererDegrees, gfunction(:, 3), 'b-.', 'LineWidth', 1);

xlabel('Angle of interfering UE [degree]');
ylabel('$$g(\varphi_0^0, \varphi_1^0)$$', 'Interpreter', 'latex');

set(gca, 'Yscale', 'log');
xlim([-180, 180]);
ylim([1e-5, 1e2]);

legend('M=1', 'M=10', 'M=100', 'Location', 'NorthWest');
grid on
set(gca, 'color',  [1, 0.9, 0.8]);
```

Output 3:

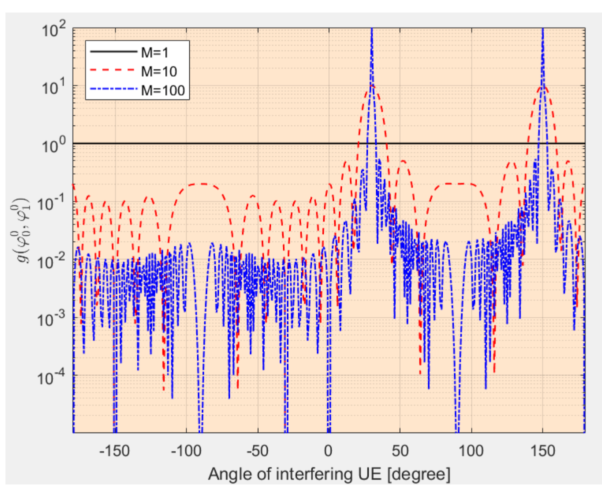

Figure 2: The g function that determines the interference level in an LoS scenario..

5. Results Analysis

10. The figure 2 shows a function $g(\varphi_0^0, \varphi_1^0)$ for a desired UE at the fixed angle $\varphi_0^0 = 30°$, while $\varphi_1^0 \in [-180°, 180°]$ and $d_\mathrm{H} = 1/2$. In the SISO case, we have $g(\varphi_0^0, \varphi_1^0) = 1$ irrespective of the angles. When consider SIMO case, $g(\varphi_0^0, \varphi_1^0)$ depends strongly on the **individual UE angles**. There are **interference peaks** when **the two UEs have the same angle** (i.e., $\varphi_1^0 = 30°$) or **the angles are each others' mirror reflections** (i.e., $\varphi_1^0 = 180° - 30° = 150°$).

11. It is important to note that **the function is equal to M at these peaks** (e.g., when$M = 10$ and $\varphi_1^0 = 30°$, the function $g(\varphi_0^0, \varphi_1^0) = 10$), because the interfering signal is coherently combined by the MRC, just like the desired signal.

12. The interference level oscillates as the interfering UE's angle is varied, but is **approximately $1/M$ times weaker than in the SISO case**. As shown in figure 2, for example, the average levels of the red line and the blue line are 10^{-1} and 10^{-2}, respectively.

13. In sum, the multiple BS antennas help to **suppress interference**, as long as the UE angles are sufficiently different.

6. UL SE for the desired UE (2): Analysis of ULA LoS case

14. From (6), we can know that the SE is characterized by 1) SNR_0, 2) $\bar{\beta}$ and 3) M. Notice that by having M receive antennas, the array collects M times more energy from the desired and interfering signals, and also from the noise. In the LoS case, the gain of the desired signal scales as M, and this **linear scaling** with the number of antennas is called **array gain**.

15. MRC **coherently** combines all the received energy from the desired signal, since the combining vector $\mathbf{v}_0$ is **matched** to the channel response of the desired UE, $\mathbf{h}_0^0$. In contrast, MRC combines the noise $\mathbf{n}_0$ and the interfering signal components $\mathbf{h}_1^0$ **non-coherently** over the array since $\mathbf{v}_0$ is **independent** of $\mathbf{h}_1^0$ and $\mathbf{n}_0$.

16. Recall (6)

$$\mathrm{SE}_0^{\mathrm{LoS}} = \log_2\big(1 + \frac{M}{\bar{\beta} g(\varphi_0^0, \varphi_1^0) + \frac{1}{\mathrm{SNR}_0}}\big) = \log_2\big(1 + \frac{M}{\bar{\beta} g(\varphi_0^0, \varphi_1^0) + \frac{\sigma^2}{p\beta_0^0}}\big) \tag{6}$$

the interference power $\bar{\beta} g(\varphi_0^0, \varphi_1^0)$ can be upper bounded as

$$\bar{\beta} g(\varphi_0^0, \varphi_1^0) \leq \frac{\bar{\beta}}{M} \frac{1}{\sin^2\big(\pi d_\mathrm{H}(\sin(\varphi_0^0) - \sin(\varphi_1^0))\big)} \tag{17}$$

where $\sin(\varphi_0^0) \neq \sin(\varphi_1^0)$, which decreases as $1/M$ when more receive antennas are added.

17. The basic reason that MRC rejects the interfering signal is that **the M antennas provide the BS with M spatial degrees of freedom**, which can be used to **separate the desired signal from the interfering signal**.

18. The **directions** of the LoS channel responses $\mathbf{h}_0^0$ and $\mathbf{h}_1^0$ gradually become **orthogonal** as M increases. This property is called **Asymptotically Favorable Propagation**.

19. Obviously the interference is stronger when the UE's angles are similar to each other. For example, when $d_{\mathrm{H}} M|\sin(\varphi_0^0) - \sin(\varphi_1^0)| < 0.2$, then we have

$$g(\varphi_0^0, \varphi_1^0) \approx M \tag{18}$$

20. The **Angular Interval** becomes smaller as the **aperture** $d_{\mathrm{H}}M$ of the ULA increases. Therefore, the interference is reduced by either **increasing** M and/or **using a larger** $d_{\mathbf{H}}$ as figure 3 and 4 shows.For a given array aperture, it is beneficial to have many antennas rather than widely separated antennas.

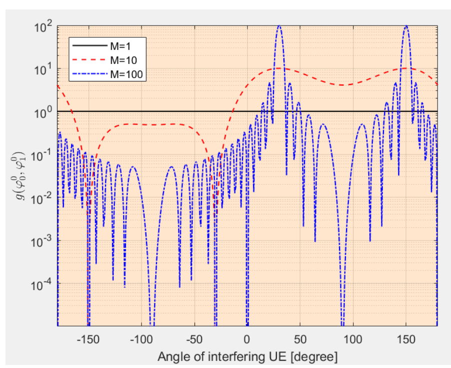

Figure 3: When dH = 0.1

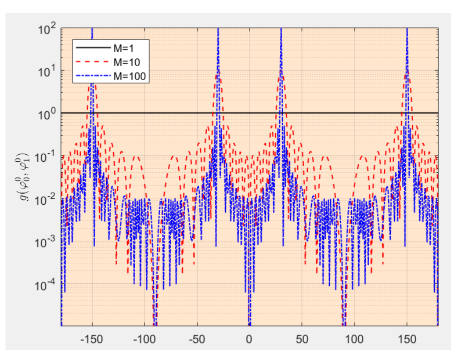

Figure 4: When dH = 1