Ke(KEN)WANG
Massive Notes 1.8
v0.1
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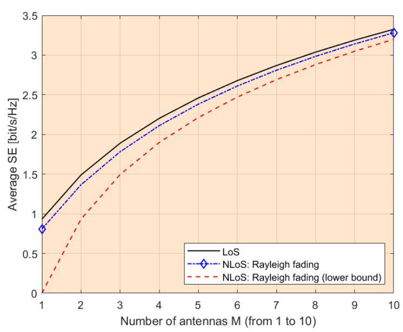
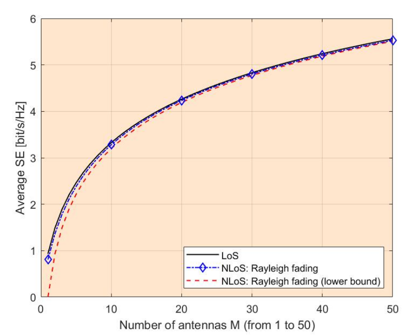
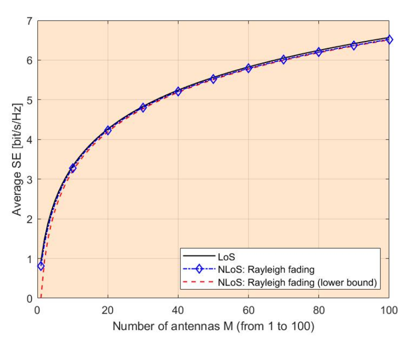
Key Points : Average UL SE and M; Channel Hardening

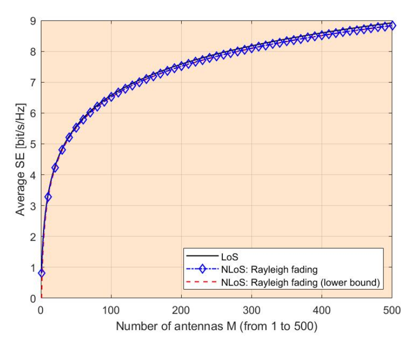
1. Average UL SE and M

1. The figure 1,2,3,4 shows the average SE as a function of the number of BS antennas M when the SNR of the desired UE is fixed at $\text{SNR}_0 = 0$ dB and $\bar{\beta} = -10$ dB. Consider ULA with $d_{\text{H}} = 1/2$ and the UE angles are all uniformly distributed from 0 to 2π. These four figures correspond to $M = 10$, $M = 50$, $M = 100$ and $M = 500$ respectively.









2. Figure 1 shows that, by going from $M = 1$ to $M = 10$ antennas, one can improve the SE from 0.8 to 3.3 bit/s/Hz. This is obtained thanks to the array gain provided bt MRC.

3. The SE a **monotonically increasing function** of M and grows **WITHOUT** limit as $M \to \infty$ (as the figures above shows), in contrast to the power-scaling case that the SE **SATURATED** in the high-SNR regime. This is also because of MRC, which **selectively collects more signal energy from the array**, without collecting more interference energy.

2. Channel Hardening

4. The difference between LoS and NLoS is negligible as figure 2, 3 and 4 show since the channel fading has a gradually smaller impact on the mutual information between the transmitted and received signal as more antennas are added. This is attributed to the **Spatial Diversity** from having multiple receiver antennas that observe independent fading realizations, which are **unlikely to all the nearly zero simultaneously**.

5. The term **Channel Hardening** was used to describe a fading channel that behaves almost **deterministically** due to spatial diversity.

6. In the massive MIMO case, a channel $\mathbf{h}_i^0$ is said to provide **Asymptotic Channel Hardening** if

$$\frac{\|\mathbf{h}_i^0\|^2}{\mathbb{E}\{\|\mathbf{h}_i^0\|^2\}} \to 1 \tag{1}$$

almost surely as $M \to \infty$. The essence of the result is that **the channel variations reduce as more antennas are added**, in the sense that **the normalized instantaneous channel gain converges to the deterministic average channel gain**.

7. In the LoS propagation, (1) is easy to understand. In the NLoS propagation, when $M \to \infty$, the variations becoming increasingly concentrated around its mean value, as more antennas are added. So

$$\frac{\mathbb{E}\{\|\mathbf{h}_i^0\|^2\}}{M} = \beta_i^0 \quad \text{as} \quad M \to \infty \tag{2}$$

then the (1) can be

$$\frac{\|\mathbf{h}_i^0\|^2}{\mathbb{E}\{\|\mathbf{h}_i^0\|^2\}} = \frac{\|\mathbf{h}_i^0\|^2}{M\beta_i^0} \to 1 \quad \text{as} \quad M \to \infty \tag{3}$$

8. But even we have (3), this does **NOT** mean that $\|\mathbf{h}_i^0\|^2$ becomes deterministic. In fact, the standard deviation of $\|\mathbf{h}_i^0\|^2$ grows as $\sqrt{M}$, while the standard deviation of $\|\mathbf{h}_i^0\|^2/M$ goes asymptotically to zero as $\sqrt{M}/M = 1/\sqrt{M}$

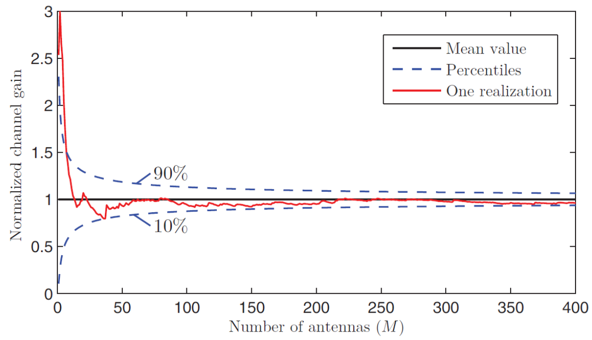


Figure.5 Illustration of the channel hardening phenomenon.

9. Figure.5 shows the channel hardening effect for the M-dimensional channel $\mathbf{h} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \mathbf{I}_M)$. As expected, we have $\|\mathbf{h}_i^0\|^2/\mathbb{E}\{\|\mathbf{h}_i^0\|^2\} \approx 1$ when M is large.

10. In sum, increasing M can improve the SE, even when $M \to \infty$. In contrast, increasing the transmit power Will increase both the signal and interference equally much and give an upper SE limit. However, the SE grows only **logarithmically** with the M, as $\log_2(M)$, which **cannot provide sufficient scalability** to achieve any order-of-magnitude improvement in SE in future cellular networks.

11. We have to know that the limit $M \to \infty$ is **NOT** physically achievable (e.g., if we want to obtain -60 dB channel gain, we have to use one million antennas to collect all the transmitted energy), but asymptotic analysis can still be suitable for investigating the system behavior at practically large antenna number.

3. Simulation to obtain the Figure 1-4

Program 5: averageULSE.m

Listing 1: averageULSE.m

```
%% Define parameters
%Empty workspace and close figures
close all;
clear;

%Define the SNR
SNR = 1;

%Define betabar (strength of inter-cell interference)
betabar = 1e-1;

%Define range of number of BS antennas
M = 1:100; % 10,50,100,500

%Extract the maximal number of BS antennas
Mmax = max(M);
Mmin = min(M);

%Select number of Monte Carlo realizations of the Rayleigh fading
numberOfRealizations = 100000;

%Generate random UE angles from 0 to 2*pi
varphiDesired = 2*pi*rand(1,numberOfRealizations);
varphiInterfering = 2*pi*rand(1,numberOfRealizations);

%Define the antenna spacing (in number of wavelengths)
antennaSpacing = 1/2; %Half wavelength distance

%Preallocate matrices for storing the simulation results
SE_LoS = zeros(Mmax,numberOfRealizations);
SE_NLoS_montecarlo = zeros(Mmax,numberOfRealizations);

%Generate uncorrelated Rayleigh fading realizations
fadingRealizationsDesired = (randn(Mmax,numberOfRealizations)+1i*randn(Mmax,
    numberOfRealizations))/sqrt(2);
fadingRealizationsInterfering = (randn(Mmax,numberOfRealizations)+1i*randn(
    Mmax,numberOfRealizations))/sqrt(2);

%% Go through different channel realizations
```

```matlab
for k = 1:numberOfRealizations

    %Compute the argument, that appears in (1.28), for different UE angles
    argument = (2*pi*antennaSpacing*( sin(varphiDesired(k))  - sin(varphiInterfering(k))) );

    %Compute the SE under line-of-sight (LoS) propagation, for a particular
    %set of UE angles, using (1.27). Note that the g-function is
    %implemented slightly differently from (1.28)
    SE_LoS(:,k) = log2(1 + SNR*M  ./ (  (SNR*betabar/(1-cos(argument)))*(1-cos(argument*M)) ./ M + 1 ) );

    %Compute the SE under NLoS propagation for one fading realization
    SE_NLoS_montecarlo(:,k) = log2(1 + SNR*cumsum(abs(fadingRealizationsDesired(:,k)).^2) ./ ...
        ( betabar*SNR*(abs(cumsum(conj(fadingRealizationsDesired(:,k)).*fadingRealizationsInterfering(:,k))).^2)./...
        cumsum(abs(fadingRealizationsDesired(:,k)).^2) +1));

end

%Compute the lower bound on SE under NLoS propagation in (1.32)
SE_NLoS_lower = log2(1 + SNR*(M-1)  ./ ( SNR*betabar + 1));

%Compute the SE under NLoS propagation using (1.29)
SE_NLoS =  (exp(1/(SNR*betabar))*expint(1/(SNR*betabar))/log(2))*(1./(1-1/betabar).^M -1 );

%First summation in (1.29) by considering all the M values at once
for m = 1:Mmax

    disp([num2str(m) ' antenna numbers out of ' num2str(Mmax)]);

    %First term that depend only on m
    term1 = 1/betabar/(1-1/betabar)^m;

    %Second summation
    for l = 0:Mmax-m

        %Second term that depend only on m and l
        term2 = term1 * (-1).^(M-m-l+1) ./ (SNR.^(M-m-l)  .* factorial(abs(M-m-l)) * log(2));

        %Third and fourth summation
```

```
        term3 = exp(1/SNR)*expint(1/SNR);

        for n = 1:l

            for j = 0:n-1

                term3 = term3 + 1/n/factorial(j)/SNR^j;

            end

        end

        %Determine which of the considered M values that contain the current term
        feasible = (m<=M) & (l<=(M-m));

        %Add the terms to the SE computation
        SE_NLoS(feasible) = SE_NLoS(feasible) + term2(feasible)*term3;

    end

end

%% Plot the simulation results
figure;
hold on; box on;

plot(M,mean(SE_LoS,2),'k-','LineWidth',1);
plot(M(1),mean(SE_NLoS_montecarlo(1,:),2),'bd-.','LineWidth',1);
plot(M,SE_NLoS_lower,'r--','LineWidth',1);
plot(M(10:10:end),mean(SE_NLoS_montecarlo(10:10:end,:),2),'bd','LineWidth',1);
plot(M,SE_NLoS,'b-.','LineWidth',1);

xlabel(['Number of antennas M (from ', num2str(Mmin), ' to ', num2str(Mmax), ')']);
ylabel('Average SE [bit/s/Hz]');

legend('LoS','NLoS: Rayleigh fading','NLoS: Rayleigh fading (lower bound)','Location','SouthEast');
grid on
set(gca, 'color',  [1, 0.9, 0.8]);
```