Massive Notes 2.2

Key Points : Spatial Channel Correlation; A Generative Model for Channel Vectors; Mobility

1. Spatial Channel Correlation

1. The channel response between UE k in cell l and the BS in cell j is denoted by $\mathbf{h}_{lk}^{j} \in \mathbb{C}^{M_j}$, where each of the elements corresponds to the channel response from the UE to one of the BS's M_j antennas.

2. We use $\mathbf{h}_{lk}^{j}$ for UL channel and $(\mathbf{h}_{lk}^{j})^{\mathrm{H}}$ for the DL channel. It is noteworthy that there is only a transpose in practice and the additional conjugation **does not** change the SE.

3. **Spatially Uncorrelated Channel**: A fading channel gain $\|\mathbf{h}\|^2$ and the channel direction $\mathbf{h}/\|\mathbf{h}\|$ are **independent random variables**, and the channel direction is **uniformly distributed** over the unit-sphere in $\mathbb{C}^M$. The channel is **Spatially Correlated** otherwise.

4. An example of a spatially uncorrelated channel model is the **uncorrelated Rayleigh fading** as follows

$$\mathbf{h}_i^0 \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \beta_i^0 \mathbf{I}_M) \quad for \quad i = 0, 1 \tag{1.24}$$

5. But practical channels are generally **spatially correlated**, also known as having **space-selective fading**. Since spatial channel correlation is very important for large arrays (these arrays have a good spatial resolution because of spatial channel correlation), we concentrate on **correlated Rayleigh fading** channels

$$\mathbf{h}_{lk}^{j} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_{M_j}, \mathbf{R}_{lk}^{j}) \tag{2.1}$$

where $\mathbf{R}_{lk}^{j} \in \mathbb{C}^{M_j \times M_j}$ is the **Positive Semi-definite spatial correlation matrix** (it is also called the **Covariance matrix** due to the zero mean), and it is used to describe the **macroscopic propagation effects**. The normalized trace $\beta_{lk}^{j} = \frac{1}{M_j}\mathrm{tr}(\mathbf{R}_{lk}^{j})$ determines

the average channel gain and is also referred to as the **large-scale fading coefficient**. Uncorrelated Rayleigh fading with $\mathbf{R}_{lk}^j = \beta_{lk}^j \mathbf{I}_{M_j}$ is just a special case of this model.
6. We use the **Gaussian distribution** to model the small-scale fading variations. The channel response is assumed to take a new independent realization form this distribution in every coherence block, as a **Stationary Ergodic Random Process**.

7. The eigenstructure of $\mathbf{R}_{lk}^j$ determines the spatial channel correlation of the channel $\mathbf{h}_{lk}^j$. **Strong spatial correlation is characterized by large eigenvalue variations**.

2. A Generative Model for Channel Vectors

8. Let the eigenvalue decomposition of $\mathbf{R} \in \mathbb{C}^{M\times M}$ be given as

$$\mathbf{R} = \underbrace{\mathbf{U}}_{M\times r}\underbrace{\mathbf{D}}_{r\times r}\underbrace{\mathbf{U}^{\mathrm{H}}}_{r\times M}$$

where $\mathbf{D} \in \mathbb{R}^{r\times r}$ is a diagonal matrix containing the $r = \mathrm{rank}(\mathbf{R})$ **Positive Non-zero Eigenvalues** of $\mathbf{R}$ and $\mathbf{U} \in \mathbb{C}^{M\times r}$ consists of the **Associated Eigenvectors**, such that $\mathbf{U}^{\mathrm{H}}\mathbf{U} = \mathbf{I}_r$. Then, $\mathbf{h}$ can be generated as (this is also referred to as the **Karhunen-Loeve expansion** of $\mathbf{h}$)

$$\mathbf{h} = \mathbf{R}^{\frac{1}{2}}\check{\mathbf{e}} = \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{U}^{\mathrm{H}}\check{\mathbf{e}} \sim \mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{e} \tag{2.4}$$

where $\check{\mathbf{e}} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \mathbf{I}_M)$, $\mathbf{e} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_r, \mathbf{I}_r)$

9. The last step implies that the distribution of $\mathbf{h}$ and $\mathbf{U}\mathbf{D}^{\frac{1}{2}}\mathbf{e}$ are **identical**. It is straightforward to verify that $\mathbf{h}$ is a **complex Gaussian vector** with zero mean and spatial correlation matrix $\mathbb{E}\{\mathbf{h}\mathbf{h}^{\mathrm{H}}\} = \mathbf{R}$.

10. What are the limiting assumptions behind this model?

1) the model assumes that the mean value is 0. It is a **pessimistic** assumption.

2) the model assumes that the channel is Gaussian distributed, which is **NOT** completely true in practice.

3. Mobility

11. The **channel fading model** describes **random variations** caused by **microscopic movements** that affect the **multipath propagation**.

12. The **spatial correlation matrix** describes **macroscopic effects** such as **pathloss, shadowing** and **spatial channel correlation**.

13. The capacity analysis assumes **stationary ergodic fading channels** with **fixed statistics**.