Massive Notes 1.7

Key Points : UL SE for the desired UE in the NLoS case; How to Quantify the Ability of Separating

1. UL SE for the desired UE in the NLoS case

1. The UL SE for the desired UE in the NLoS case is

$$\mathrm{SE}_0^{\mathrm{NLoS}} = \left(\frac{1}{(1-\frac{1}{\bar{\beta}})^M} - 1\right)\frac{e^{\frac{1}{\mathrm{SNR}_0\bar{\beta}}}E_1(\frac{1}{\mathrm{SNR}_0\bar{\beta}})}{\log_e(2)}$$
$$+\sum_{m=1}^{M}\sum_{l=0}^{M-m}\frac{(-1)^{M-m-l+1}}{(1-\frac{1}{\bar{\beta}})^M}\frac{\left(e^{\frac{1}{\mathrm{SNR}_0}}E_1(\frac{1}{\mathrm{SNR}_0})+\sum_{n=1}^{l}\frac{1}{n}\sum_{j=0}^{n-1}\frac{1}{j!\mathrm{SNR}_0^j}\right)}{(M-m-l)!\mathrm{SNR}_0^{M-m-l}\bar{\beta}\log_e(2)} \tag{1}$$

where $n!$ denotes the factorial function and $E_1 = \int_1^\infty \frac{e^{-xu}}{u}du$ denotes the exponential integral.

2. (1) is harder to interpret since the closed-form expression has a complicated structure with several summations and special functions. However, when $M \gg 1$, we can obtain the lower bound as follows

$$\mathrm{SE}_0^{\mathrm{NLoS}} = \mathbb{E}\left\{\log_2(1+\mathrm{SINR})\right\} = \mathbb{E}\left\{\log_2(1+\frac{p\|\mathbf{h}_0^0\|^2}{p\frac{|(\mathbf{h}_0^0)^{\mathrm{H}}\mathbf{h}_1^0|^2}{\|\mathbf{h}_0^0\|^2}+\sigma^2})\right\} \geq \log_2\left(1+\frac{M-1}{\bar{\beta}+\frac{1}{\mathrm{SNR}_0}}\right) \tag{2}$$

3. From (2), we can figure that in the NLoS case, the lower-bounding technique used made the desired signal scale as $(M-1)$, instead of M which is the natural array gain obtained with MRC. However, the difference is negligible when M is large. The interference power in (2) (i.e., $\bar{\beta}$) is **independent** of M, in contrast to the LoS case (i.e., $\mathrm{SE}_0^{\mathrm{LoS}} = \log_2(1+\frac{M}{\bar{\beta}g(\varphi_0^0,\varphi_1^0)+\frac{1}{\mathrm{SNR}_0}}))$, where it decays as $1/M$.

4. This scaling behavior suggests that **NLoS channels provide less Favorable Propagation than LoS channels**. Favorable Propagation means that the directions of the LoS channel responses $\mathbf{h}_0^0$ and $\mathbf{h}_1^0$ gradually become orthogonal as M increases.

5. According to (2), the **Relative Interference Gain** is

$$\frac{1}{\beta_1^0}\frac{|(\mathbf{h}_0^0)^{\mathrm{H}}\mathbf{h}_1^0|^2}{\|\mathbf{h}_0^0\|^2} \tag{3}$$

which determines **how well interference is suppressed by MRC**.

6. For NLoS channels, (3) can be shown to have an $exp(1)$ **Distribution**, and it is irrespectively of the value of M. However, for LoS channels, (3) equals $g(\varphi_0^0, \varphi_1^0)$, which is a function of M and the UE angles.

7. Recall the CDF of an Exponential Distribution is

$$F(x;\lambda) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \tag{4}$$

8. So the figure below considers the LoS case with $M = 10$ and $M = 100$, and shows the CDF over different uniformly distributed UE angles between 0 and 2π when $d_{\mathrm{H}} = 1/2$. The CDF of the small-scale fading with NLoS channels is also shown.

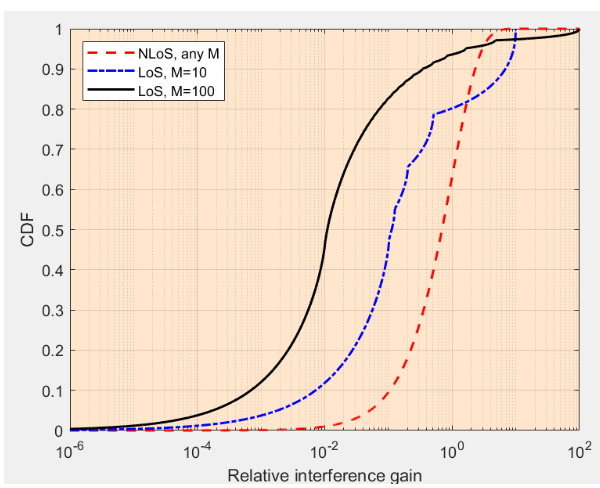

Figure 1: CDF of the relative interference gain.

2. Simulation

Program 4: interferenceCDF.m

Listing 1: interferenceCDF.m

```
%% Define parameters
%Empty workspace and close figures
close all;
clear;

%Define the range of number of BS antennas
M = [10, 100];

%Select the number of Monte Carlo realizations of the Rayleigh fading
numberOfRealizations = 1000000;

%Generate random UE angles from 0 to 2*pi
varphiDesired = 2 * pi * rand(1, numberOfRealizations);
varphiInterfering = 2 * pi * rand(1, numberOfRealizations);

%Define the antenna spacing (in number of wavelengths)
antennaSpacing = 1/2; %Half wavelength distance

%Preallocate matrix for storing the simulation results
interferenceGainLoS = zeros(numberOfRealizations, length(M));

%Compute the argument, that appears in (1.28), for different UE angles
argument = (pi * antennaSpacing * (sin(varphiDesired(:)) - sin(
    varphiInterfering(:))));

%% Go through different number of antennas
for mindex = 1 : length(M)

    %Compute the g-function in (1.28)
    interferenceGainLoS(:, mindex) =  (sin(argument * M(mindex))).^2 ./ (sin
        (argument)).^2 / M(mindex);

end

CDFvalues_LoS = linspace(0, 1, numberOfRealizations);

%Compute the CDF of the relative interference gain for NLoS using the
%Exp(1)-distribution
interferenceGainNLoS = logspace(-6, 2, 10000);
CDFvalues_NLoS = 1 - exp(-interferenceGainNLoS);
```

```
%% Plot the simulation results
figure;
hold on; box on;

plot(interferenceGainNLoS, CDFvalues_NLoS, 'r--', 'LineWidth', 1.5);
plot(sort(interferenceGainLoS(:, 1)), CDFvalues_LoS, 'b-.', 'LineWidth', 1.5);
plot(sort(interferenceGainLoS(:, 2)), CDFvalues_LoS, 'k-', 'LineWidth', 1.5);

set(gca, 'Xscale', 'log');
xlim([1e-6, 1e2]),
xlabel('Relative interference gain');
ylabel('CDF');

legend('NLoS, any M', 'LoS, M=10', 'LoS, M=100', 'Location', 'NorthWest');
grid on
set(gca, 'color',  [1, 0.9, 0.8]);
```

3. Result Analysis

9. The result figure shows that LoS channels often provide several orders-of-magnitude lower interference gains than NLoS channels, but this only applies to the majority of random angle realizations. There is a small probability that the interference gain is larger in LoS than in NLoS; it happens in 18% of the realizations with $M = 10$ and 4% of the realizations with $M = 100$.

10. We have to know that for any finite M, there will be a small angular interval around φ_0^0 where incoming interference will be amplified just as the desired signal. Since the array is unable to separate UEs with such small angle differences, **Time-Frequency** scheduling might be needed to separate them.

4. How to Quantify the Ability of Separating

11. The **favorable propagation** concept provides a way to quantify **the ability to separate UE channels at a BS with many antennas**. The channels $\mathbf{h}_i^0$ and $\mathbf{h}_k^0$ are said to provide **Asymptotically Favorable Propagation** if

$$\frac{(\mathbf{h}_i^0)^{\mathrm{H}}\mathbf{h}_k^0}{\sqrt{\mathbb{E}\{\|\mathbf{h}_i^0\|^2\}\mathbb{E}\{\|\mathbf{h}_k^0\|^2\}}} \to 0 \quad \text{as} \quad M \to \infty \tag{5}$$

12. Note that we **CANNOT** say channel responses become orthogonal just because (END)

$$(\mathbf{h}_i^0)^{\mathrm{H}}\mathbf{h}_k^0 \to 0$$

.