Massive Notes 1.12

Key Points : Downlink SDMA; DL SE expressions for MRC

1. Downlink Signal Model

1. So far we have already known that SDMA as a suitable way to improve the SE by an order-to-magnitude or more. Next, we will focus on how SDMA is applied in the Downlink (DL). Figure 1 shows the basic model for the DL.

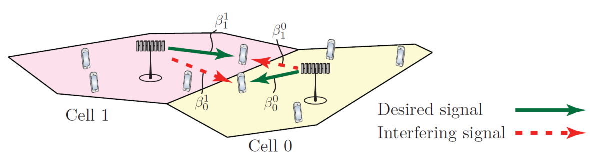

Figure.1 Illustration of the notion of the desired and interfering DL signals in a two-cell network.

2. The main difference from the UL is that the signals are transmitted from BSs instead of from UEs. There are K active UEs in each cell and the serving BS sends a **separate signal** to each of the them using **linear transmit precoding** from an array of M antennas.

3. **Precoding** means that each data signal is sent from all antennas, but **with different amplitude and phase to direct the signal spatially**. This is also called **Beamforming**. However, we don't talk about this technique in note 12.

4. In contrast, precoding means that **each antenna's transmit signal is generated separately in the digital baseband**, which gives full flexibility in the signal generation.

5. $(\mathbf{h}_{0k}^0)^{\mathrm{H}}$ for $k = 1, \ldots, K$ refers to the DL channel response between the BS in cell 0 and its kth **desired UE**. Similarly, $(\mathbf{h}_{0k}^1)^{\mathrm{H}}$ for $k = 1, \ldots, K$ denotes the DL channel response between the BS in cell 1 and the kth UE in cell 0 (i.e., **interfering UE**).

6. The received DL signal $z_{0k} \in \mathbb{C}$ at UE k in cell 0 is modeled as

$$z_{0k} = \underbrace{(\mathbf{h}_{0k}^{0})^{\mathrm{H}} \mathbf{W}_{0k} \zeta_{0k}}_{\text{Desired signal}} + \underbrace{\sum_{i=1, i \neq k}^{K} (\mathbf{h}_{0k}^{0})^{\mathrm{H}} \mathbf{W}_{0i} \zeta_{0i}}_{\text{Intra-cell interference}} + \underbrace{\sum_{i=1}^{K} (\mathbf{h}_{0k}^{1})^{\mathrm{H}} \mathbf{W}_{1i} \zeta_{1i}}_{\text{Inter-cell interference}} + \underbrace{n_0}_{\text{noise}} \tag{1}$$

where $\zeta_{jk} \sim \mathcal{N}_{\mathbb{C}}(0, p)$ is the signal transmitted to the kth UE in cell j and $\mathbf{W}_{jk} \in \mathbb{C}^M$ is the **corresponding unit-norm precoding vector** (i.e., $\|\mathbf{W}_{jk}\| = 1$) that **determines the spatial directivity of the signal**. The receiver noise at this UE is denoted by $n_{0k} \sim \mathcal{N}_{\mathbb{C}}(0, \sigma^2)$.

7. In the LoS case, we have the MISO channel response

$$\mathbf{h}_{jk}^{l} = \sqrt{\beta_j^l} \left[1, e^{2\pi \mathrm{j} d_{\mathrm{H}} \sin(\varphi_{jk}^{l})}, \ldots, e^{2\pi \mathrm{j} d_{\mathrm{H}} (M-1) \sin(\varphi_{jk}^{l})}\right]^{\mathrm{T}} \tag{2}$$

between UE k in cell j and the BS in cell l, where $\varphi_{jk}^{l} \in [0, 2\pi)$.

8. In the NLoS case, we have the MISO channel response

$$\mathbf{h}_{jk}^{l} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_M, \beta_j^l \mathbf{I}_M) \tag{3}$$

9. The precoding vectors $\mathbf{W}_{jk}$, for $k = 1, \ldots, K$ and $j = 0, 1$, can be selected in a variety of ways. As seen from the received signal in (1), each UE is affected by all the precoding vectors.

10. The own precoding vector is multiplied with the channel response from the serving BS, while the other ones cause interference and are multiplied with the channel response from the corresponding transmitting BSs.

11. Hence, the precoding vectors should be selected carefully in the DL, based on knowledge of the channel responses. In order to achieve the maximum array gain, we consider MRC with

$$\mathbf{W}_{jk} = \frac{\mathbf{h}_{jk}^{j}}{\|\mathbf{h}_{jk}^{j}\|} \tag{4}$$

12. Note that $\|\mathbf{W}_{jk}\|^2 = 1$, which implies that the total transmit power of the BS is **constant**, irrespective of the number of antennas. Consequently, the transmit power per BS antenna decreases roughly as $1/M$.

2. DL SE expressions for MRC

13. If the BSs use MRC and the UEs in cell 0 know their **respective effective channels** $(\mathbf{h}_{0k}^{0})^{\mathrm{H}}\mathbf{W}_{0k}$ and the interference variance, then an achievable DL sum SE [bit/s/Hz/cell] in the LoS case is

$$\mathrm{SE}_0^{\mathrm{LoS}} = \sum_{k=1}^{K} \log_2\left(1 + \frac{M}{\sum_{i=1,i\neq k}^{K} g(\varphi_{0i}^{0}, \varphi_{0k}^{0}) + \bar{\beta}\sum_{i=1}^{K} g(\varphi_{1i}^{1}, \varphi_{0k}^{1}) + \frac{1}{\mathrm{SNR}_0}}\right) \tag{5}$$

14. With NLoS channels, a DL sum SE [bit/s/Hz/cell] and a closed-form lower bound are

$$\mathrm{SE}_0^{\mathrm{NLoS}} \geq K \log_2\big(1 + \frac{M-1}{(K-1)\frac{M-1}{M} + K\bar{\beta} + \frac{1}{\mathrm{SNR}_0}}\big) \tag{6}$$

15. Recall the UL sum SE expression

$$\mathrm{SE}_0^{\mathrm{LoS}} = \sum_{k=1}^{K} \log_2\left(1 + \frac{M}{\sum_{i=1,i\neq k}^{K} g(\varphi_{0k}^{0}, \varphi_{0i}^{0}) + \bar{\beta}\sum_{i=1}^{K} g(\varphi_{0k}^{0}, \varphi_{1i}^{0}) + \frac{1}{\mathrm{SNR}_0}}\right) \tag{7}$$

$$\mathrm{SE}_0^{\mathrm{NLoS}} \geq K \log_2\big(1 + \frac{M-1}{(K-1) + K\bar{\beta} + \frac{1}{\mathrm{SNR}_0}}\big) \tag{8}$$

16. So the DL sum SE is very similar to the UL sum SE. The NLoS case only differs in the extra multiplicative term $\frac{M-1}{M}$ in the denominator of (6), which is almost one for large M. The LoS case only differs in the angles that appear in each expression; all angles in the UL are from UEs to the BS in cell 0, while **the DL includes both the angles from the desired UE to all transmitting BSs** and **the angles from the other UEs that these BSs are transmitting to**(representing the directivity of each DL signal).

17. The array gain is M with MRC in both UL and DL, but it is obtained differently. In the UL, the BS makes M observations of the desired signal over its M antennas, each being corrupted by an independent noise term. By **coherently combining** the M signal components, the signal power **grows proportionally to** M while **the noise realizations add incoherently so that the noise variance is unchanged**.

18. In the DL, the M transmit antennas have different channels to the receiving UE (desired UE). Since **the total transmit power is fixed**, the signal power per antenna is reduced as $1/M$ and the signal amplitude as $1/\sqrt{M}$. **With precoding that makes the** M **transmitted signal components add coherently at the UE**, the received signal's amplitude grows as $M/\sqrt{M} = \sqrt{M}$ and the received signal power therefore grows as M. (END)