Massive Notes 1.10

Key Points : SE expressions (lower bound) for the case of MRC;

1. SE expressions (lower bound) for the case of MRC

1. Consider we use MRC, then the closed-form SE expressions (lower bound)[bit/s/Hz/cell] are

$$\mathrm{SE}_0^{\mathrm{LoS}} = \sum_{k=1}^{K} \log_2 \left(1 + \frac{M}{\sum_{i=1, i\neq k}^{K} g(\varphi_{0k}^0, \varphi_{0i}^0) + \bar{\beta} \sum_{i=1}^{K} g(\varphi_{0k}^0, \varphi_{1i}^0) + \frac{1}{\mathrm{SNR}_0}} \right) \tag{1}$$

$$\mathrm{SE}_0^{\mathrm{NLoS}} \geq K \log_2 \big(1 + \frac{M-1}{(K-1) + K\bar{\beta} + \frac{1}{\mathrm{SNR}_0}} \big) \tag{2}$$

2. (1) and (2) are more complicated than the previous SE expressions due to the **addition of intra-cell interference** and **the greater amount of inter-cell interference**.

3. In the LoS case, SDMA results in the **summation** of K SE expressions, one per desired UE. The desired signal gains **inside** the logarithms increase **linearly** with M and thus **every UE experiences the full array gain when using MRC**.

4. The drawback of SDMA is seen from the **denominator**, where the interference terms contain contributions from $K-1$ intra-cell UEs (i.e., $\sum_{i=1, i\neq k}^{K} g(\varphi_{0k}^0, \varphi_{0i}^0)$) and K inter-cell UEs (i.e., $\bar{\beta} \sum_{i=1}^{K} g(\varphi_{0k}^0, \varphi_{1i}^0)$).

5. Recall the function $g(\varphi, \psi)$ in Notes 1.6 we have

$$\bar{\beta} g(\varphi, \psi) \leq \frac{\bar{\beta}}{M} \frac{1}{\sin^2\big(\pi d_{\mathrm{H}}(\sin(\varphi) - \sin(\psi))\big)} \tag{3}$$

therefore, for any $\sin(\varphi) \neq \sin(\psi)$, the function $g(\varphi, \psi)$ **decreases** as $1/M$. Moreover, we notice that the desired signal power **grows as** M. So the interference power is **proportional to** K/M, then the signal-to-interference ratio (SIR) becomes $M/(K/M) = M^2/K$. If we want a constant SIR as K grows, then $M^2/K = K \Rightarrow M = \sqrt{K}$. In other words, we can thus serve multiple UEs and still maintain roughly the same SINR per UE if M is increased proportionally to $\sqrt{K}$ to counteract the increased interference.

6. The gain from SDMA is easily see from (2); there is a factor K **in front of the logarithm** that shows that the sum SE **increases proportionally** to the number of UEs. This multiplicative factor is known as the **Multiplexing Gain** and achieving this gain is the main point with SDMA.

7. Inside the logarithm, the desired signal power **increases linearly** with M, while the intra-cell interference power $K - 1$ and the inter-cell interference power $K\bar{\beta}$ **increase linearly** with K.

8. This means that, as we add more UEs, we can **counteract** the increasing interference by **adding a proportional amount of additional BS antennas**; more precisely, we can maintain roughly the same SINR per UE by **increasing M jointly with K to keep the antenna-UE ratio M/K fixed**.

9. It is worthy pointing out that in the NLoS case, we need more antennas to suppress interference with MRC than in the LoS case, where M only needs to increase as $\sqrt{K}$. The reason is that **all interfering UEs cause substantial interference in the NLoS case**, while **only the ones with sufficiently similar angles to the desired UE does that in the LoS case**.

2. Simulation Analysis

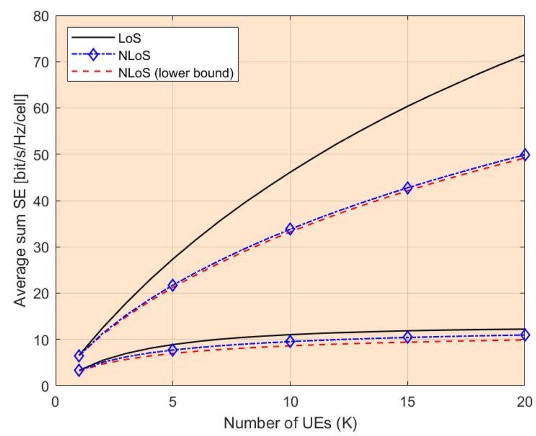

Figure 1. MRC

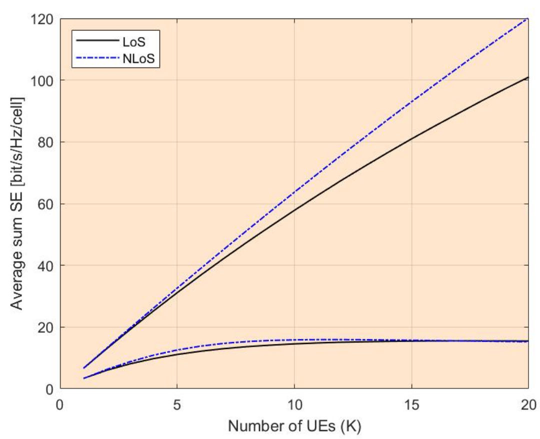

Figure 2. M-MMSE

10. Figure 1 and 2 shows the average sum SE as a function of the number of UEs per cell, for either $M = 10$ or $M = 100$ antennas. The sum SE with MRC is shown in figure 1 based on the analytic formulas from (1), while Monte-Carlo simulations are used for M-MMSE combining in figure 2. In both cases, the SNR is fixed at $\mathrm{SNR}_0 = 0\mathrm{dB}$ and the strength of the inter-cell interference is $\bar{\beta} = -10\mathrm{dB}$. The antenna spacing is $d_{\mathrm{H}} = 1/2$ in the LoS case and **the results are averaged over different independent UE angles**, all being **uniformly distributed** from 0 to 2π.

11. These two figures show that the sum SE is **slowly increasing function** of K in the case of $M = 10$, because **the BS does not have enough spatial degrees of freedom to separate the UEs** — neither by MRC or by M-MMSE combining.

12. However, this behavior is completely different when $M = 100$ antennas are used since **the channel response of each UE is then a 100-dimensional vector but there are only up to 20 UEs per cell** so the UE channels only span a small portion of the spatial dimensions that the BS can **resolve**.

13. Consequently, the sum SE increases almost **linearly with the number of UEs** and we can achieve a roughly K-fold improvement in sum SE over a single-user scenario.

14. When we use MRC, the sum SE is **considerably lower** with NLoS than with LoS; but in the M-MMSE case, the result is different. The reason for this is that each UE is affected by interference from many UEs in the NLoS case, while **only a few UEs with similar angles** cause strong interference in the LoS case,

and if the interference is ignored, as with MRC, the SE is lower in the NLoS case due to **the larger sum interference power**.

15. However, **it is easier for M-MMSE combining to reject interference in NLoS than in LoS**, where there might be a few UEs with channels that are nearly parallel to the desired UE's channel. That is why in the figure 2, the SE is higher in the NLoS when using M-MMSE.

16. We call M/K as **Antenna-UE ratio**. Figure 3 shows that the sum SE obtained by M-MMSE combining when $M/K \in \{1, 2, 4, 8\}$. The SE grows almost **linearly** with K in all four cases, as expected from (1) and (2). The **steepness** of the curves increases as M/K increases, since it becomes easier to suppress the interference when $M \gg K$. (END)

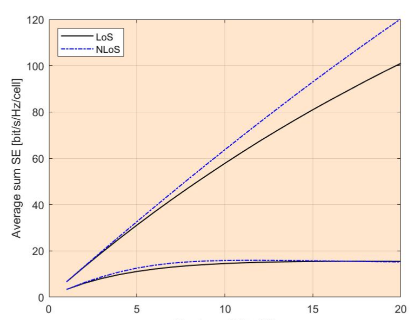

Figure 3. When the number of antennas increases with K with different fixed antenna-UE ratios M/K. We say that $M/K \geq 4$ is the preferred operating regime for multiuser MIMO.