Nuclear Engineering

Units & Conversions

$$\hbar = 1.054\,57 \times 10^{-34}\,\mathrm{J\,s}$$
$$c = 2.997\,925 \times 10^{8}\,\mathrm{m\,s^{-1}}$$
$$hc = 1.240 \times 10^{-6}\,\mathrm{eV\,m}$$
$$k_0 = 8.99 \times 10^{9}\,\mathrm{N\,m^2\,C^{-2}}$$
$$N_0 = 6.0221 \times 10^{23}\,\mathrm{mol^{-1}}$$
$$1\,\mathrm{MeV} = 1.602\,18 \times 10^{-27}\,\mathrm{J}$$
$$1\,\mathrm{MeV/c^2} = 1.782\,66 \times 10^{-30}\,\mathrm{kg}$$
$$1\ \mathrm{AMU} = 1.660\,54 \times 10^{-27}\,\mathrm{kg} = 931.494\,\mathrm{MeV/c^2}$$
$$\mathrm{M_{Proton}} = 928.272\,\mathrm{MeV/c^2}$$
$$\mathrm{M_{Neutron}} = 939.565\,\mathrm{MeV/c^2}$$
$$\mathrm{M_{Electron}} = 0.511\,\mathrm{MeV/c^2}$$
$$1\,\mathrm{b} = 1 \times 10^{-24}\,\mathrm{cm}$$
$$1\,\mathrm{Ci} = 3.7 \times 10^{10}\,\mathrm{Bq}$$
$$1\,\mathrm{C} = 1.602 \times 10^{-19}\,e$$
$$1\,\mathrm{R} = 2.58 \times 10^{-4}\,\mathrm{C\,kg^{-1}}$$

1 Ci is roughly the activity of 1 g of Ra^{226}

Terms & Basic Facts

X_Z^A — X = Chemical Symbol; A = Mass Number (# nucleons); Z = Atomic Number (# protons)

dN particles incident on sphere with σ area dA

Fluence	$\Phi = \dfrac{dN}{dA}$	$[\mathrm{m^{-2}}]$
Fluence Rate, Flux	$\dot{\Phi} = \dfrac{d\Phi}{dt}$	$[\mathrm{m^{-2}\,s^{-1}}]$

For a specified volume,
$R_{\mathrm{in}}, R_{\mathrm{out}}$ are radiant K.E. incident and emerging
Q is a change in rest mass of nuclei or particle

Energy Imparted $\quad \epsilon = R_{\mathrm{in}} - R_{\mathrm{out}} + \sum Q$

Relativistic Kinematics

$$E = T + mc^2 \qquad E = m^2 + p^2$$
$$\beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} \qquad \beta^2 = \frac{\gamma^2 - 1}{\gamma^2}$$
$$\gamma = \frac{T + mc^2}{mc^2} \qquad T = \frac{1}{2}mc^2\beta^2$$

Quantum Mechanics

$\lambda = h/p = h/\gamma mv$	de Broglie wavelength
$E_\gamma = h\nu$	$p_\gamma = E_\gamma/c = h\nu/c$
$\hbar \lesssim \Delta p_x \Delta$	$\hbar \lesssim \Delta E \Delta t$

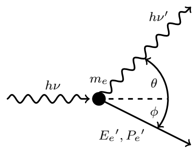
Photoelectric Effect

Electrons eject from a metal, with energy T, from absorption of light with energy $h\nu$

$T_{\max} = h\nu - \phi_0 \quad \phi_0 =$ Work function of metal

Compton Effect

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_e c^2}(1 - \cos\theta)}$$



$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$
$$T_e = h\nu - h\nu'$$
$$T_e = h\nu \frac{1 - \cos\theta}{\frac{m_e c^2}{h\nu} + 1 - \cos\theta}$$
$$T_{\max} = \frac{2h\nu}{2 + m_e c^2/h\nu}$$
$$\tan\phi = \frac{\sin\theta}{(1 + h\nu/m_e c^2)(1 - \cos\theta)}$$
$$\operatorname{atan}\frac{\theta}{2} = \left(1 + \frac{h\nu}{m_e c^2}\right)\tan\phi$$

Radioactive Decay

N – # Atoms	λ – Decay Constant
A – Activity	SA – Specific Activity
$T_{1/2}$ – Half life	M – Molar Mass

$$dN = -\lambda N dt \qquad A = -\frac{dN}{dt} = \lambda N$$
$$\frac{N}{N_0} = \frac{A}{A_0} = e^{-\lambda t} \qquad T_{1/2} = \frac{\ln 2}{\lambda}$$
$$SA = \frac{\mathrm{mol}\ \lambda}{M} = \frac{4.17 \times 10^{23}}{M\ T_{1/2}}$$
$$SA = \frac{1600}{T(\text{years})} \times \frac{226}{A(\text{Mass }\#)}\mathrm{Ci\,g^{-1}}$$

Decay Chains

General Case

$$N_n(t) = \prod_{j=1}^{n-1}\lambda_j \sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{N_i(0)e^{-\lambda_j t}}{\prod_{p=i,p\neq j}^{n}(\lambda_p - \lambda_j)}\right)$$
$$N_n(t) \simeq N\frac{\lambda_j}{\lambda_n}\exp -\lambda_j t \quad \lambda_j \ll \lambda_1 \ldots \lambda_n$$

Decaying Daughter

$$N_2 = \frac{\lambda_1 N_{1_0}}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Secular Equilibrium ($T_1 \gg T_2$)

$$\frac{dN_2}{dt} = A_1 - \lambda_2 N_2$$
$$A_2 = A_1\left(1 - e^{\lambda_2 t}\right) + A_{2_0}e^{-\lambda_2 t}$$

If $A_{2_0} = 0$, $t \gtrsim 7\ T_2$ then $A_1 = A_2$.

Transient Equilibrium ($T_1 \gtrsim T_2$)

$$\lambda_2 N_2 = \frac{\lambda_2\lambda_1 N_{1_0}e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} = A_2\frac{\lambda_2 A_1}{\lambda_2 - \lambda_1}$$

Max A_2 $\qquad t = \dfrac{1}{\lambda_2 - \lambda_1}\log\dfrac{\lambda_2}{\lambda_1}$

Max $A_1 + A_2$ $\qquad t = \dfrac{1}{\lambda_2 - \lambda_1}\log\dfrac{\lambda_2^2}{2\lambda_1\lambda_2 - \lambda_1^2}$

Decay Channels

α Decay: $\mathrm{P}_Z^A \to \mathrm{D}_{Z-2}^{A-4} + \mathrm{He}_2^4$

Usually heavy elements ($Z \geq 83$). Short range, usually radioactive daughters, harmful if inhaled or ingested. Radium, an α emitter seeks bone

$$Q_\alpha = \Delta_P - \Delta_D - \Delta_{\mathrm{He}} = E_\alpha + E_{M_N}$$
$$E_\alpha = \frac{m\ Q_\alpha}{m_\alpha + M_N} \qquad E_{M_N} = \frac{M_N\ Q_\alpha}{m_\alpha + M_N}$$

β^- Decay: $\mathrm{P}_Z^A \to \mathrm{D}_{Z+1}^A + \beta_{-1}^0 + \bar{\nu}_0^0$

Pure β^- emitters: H^3, C^{14}, P^{32}, Sr^{90}, Y^{90}
Mixed β^-/γ emitters: Co^{60}, Cs^{137}
Can penetrate skin, internal hazard. MeV β^- can emit bremsstrahlung in heavy-metal shielding, creating γ hazard

$$Q_{\beta^-} = \Delta_P - \Delta_D = E_{\beta^-} + E_{\bar{\nu}}$$
$$0 \leq E_{\beta^-} \leq Q_{\beta^-} \quad \bar{E}_{\beta^-} \simeq Q_{\beta^-}/3$$

β^+ Decay: $\mathrm{P}_Z^A \to \mathrm{D}_{Z-1}^A + \beta_{+1}^0 + \nu_0^0$

Same effect as electron capture. Accompanied by annihilation γ, $E_\gamma = 0.511\,\mathrm{MeV}$. Competes with EC.

$$Q_{\beta+} = \Delta_P - \Delta_D - 2m_e c^2$$
$$\Delta_P - \Delta_D > 2m_e c^2 = 1.022\,\mathrm{MeV}$$

γ Decay: $\mathrm{P}_Z^A \to \mathrm{D}_Z^A + \gamma_0^0$

γ emissions energies create isotope signatures, basis of γ spectroscopy. Nuclear excited state lifetimes of γ decay typically 10×10^{-10} s, except for much longer lived metastable states. Deeply penetrating, dominates over EC for light nuclei.

$$Q_\gamma = \Delta_P - \Delta_D$$

Internal Conversion (IC)

Alternative to γ emission where nuclear excitation energy is transferred to K or L shell electron, ejecting from atom. Usually followed by fast γ emission. Dominates for heavy nuclei

$$\alpha = \frac{N_e}{N_\gamma} \quad E_e = E^* - E_B$$

α, IC coefficient for nuclear transition, increases with Z^3, decrease with E^*.

Electron Capture (EC):

$\mathrm{P}_Z^A + e_{-1}^0 \to \mathrm{D}_{Z-1}^A + \nu_0^0$

Nucleus captures orbital electron, outgoing neutrino energy $E_\nu = Q_{EC}$. Leaves inner shell vacancy, emitting X-rays and auger electrons of characteristic energies. With electron binding energy E_B,

$$Q_{EC} = \Delta_P - \Delta_D - E_B \quad \Delta_P - \Delta_D > E_B$$

Nuclear Reactions

Mass Excess

$\Delta_{\mathrm{Atom}} = \mathrm{Z}\times\mathrm{M_{Proton}} + (\mathrm{A}-\mathrm{Z})\times\mathrm{M_{Neutron}} - \mathrm{M_{Atom}}$

Energy Released

$a + b \to c + d$ or $a(b,c)d$
Q, energy released, =

$$Q = [(M_a + M_b) - (M_c + M_d)]c^2$$

$Q > 0$ is exothermic, $Q < 0$ is endothermic

Cross Sections

Reaction Rate R, microscopic cross-section σ

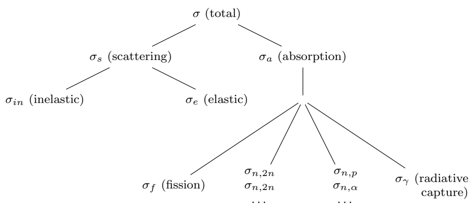
$$R\ [\frac{\#}{\mathrm{cm^2\,s}}] = \sigma\ [\mathrm{cm^2}]\ I\ [\frac{\#}{\mathrm{cm^2\,s}}]\ N_A\ [\frac{\#}{\mathrm{cm^2}}]$$
$$R = \Sigma_T I \qquad \Sigma_T = N\sigma_T$$

Beam I, macroscopic cross-section Σ_T

$$I(x) = I_0 \exp -\Sigma_T x$$
$$\Sigma_T = N_X\sigma_t^X + N_Y\sigma_t^Y + N_Z\sigma_t^Z + \ldots = \frac{1}{\lambda}$$

λ = Mean free path (until 1st interaction)
$\nu\ \Sigma_T = [\mathrm{s^{-1}}]$ Collision Freq., ν = neutron speed

Neutron Cross Section Hierarchy



Radiation In Matter

Single collision, incoming particle (M, E) can transfer at most Q_max to resting particle m

$$Q_{max} = \frac{2\gamma^2 mV^2}{1 + 2\gamma m/M + m^2/M^2} \simeq \frac{4mME}{(M+m)^2}$$

$$Q_{max} \simeq 2\gamma^2 mV^2 = 2\gamma^2 mc^2\beta^2 \quad \gamma m/M \ll 1$$

As E increases, $Q_{max}/E \to 1$

Can define Linear Energy Transfer (LET)

$$L \equiv \frac{dE}{dl}$$

dE is energy lost by **charged** particle due to atomic electron collisions travelling distance dl

Related to Lineal Energy

$$y \equiv \frac{\epsilon}{\bar{l}}$$

ϵ is energy imparted to matter in volume, $\bar{l}$ is mean chord length of volume

Stopping Power, S

Avg total linear energy loss rate in matter

$$S = -\frac{dE}{dx}$$

HCP and β particles in matter eject atomic electrons, creating Delta Rays.

Restricted Stopping Power S_Δ

Linear rate of energy loss due to collisions where energy transfer does not exceed Δ. Related to Linear Energy Transfer (LET).

$$\text{LET}_\Delta = \left(-\frac{dE}{dx}\right)_\Delta \qquad S = \text{LET}_\infty = \left(-\frac{dE}{dx}\right)_\infty$$

Mass Stopping Power

$$\frac{S}{\rho} = -\frac{1}{\rho}\frac{dE}{dx} \left[\text{MeV cm}^2/\text{g}\right] \qquad \rho = \text{density}$$

For compound of multiple elements

$$\frac{S}{\rho} = \sum_i \frac{S_i}{\rho_i}$$

Heavy Charged Particles (HCP)

Stopping Power

Bethe Formula for Stopping Power:

$$-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n_e}{m_e c^2 \beta^2}\left[\ln\frac{2mc^2\beta^2}{I(1-\beta^2)} - \beta^2\right]$$

$$= \frac{5.08\times10^{-31} z^2 n_e}{\beta^2}\left[F(\beta) - \ln I_{\text{eV}}\right] \text{MeV cm}^{-1}$$

$$F(\beta) = \ln\frac{1.02\times10^6\beta^2}{1-\beta^2} - \beta^2$$

z = atomic number of HCP

n_e = electrons per unit volume in medium

I = mean excitation energy of medium

$$I \simeq \begin{cases} 19.0\,\text{eV} & Z = 1 \text{ (hydrogen)} \\ 11.2 + 11.7Z\text{eV} & 2 \le Z \le 13 \\ 52.8 + 8.71Z\text{eV} & Z > 13 \end{cases}$$

$$n_e \ln I = \sum_i N_i Z_i \ln I_i$$

Range $R(\beta)$ [g cm^{-2}]

$$R(\beta) = \frac{M}{z^2}f(\beta) \qquad f(\beta) = \int_0^\beta \frac{g(\beta')}{G(\beta')}d\beta'$$

$$\frac{R_1(\beta)}{R_2(\beta)} = \frac{z_2^2 M_1}{z_1^2 M_2} \qquad R(\beta) = \frac{M}{z^2}R_p(\beta)$$

Where $R_p(\beta)$ is the proton range in the material

Slowing Down Time τ

Time to stop HCP in matter

$$\tau \sim \frac{\text{T}}{V(-dE/dx)} \qquad \text{K.E.} = \text{T, V} = \text{velocity}$$

Electrons / Positrons ($\beta^\pm$)

Can emit through collisional and radiation (bremmstrahlung)

$$\left(-\frac{dE}{dx}\right)^\pm_{\text{tot}} = \left(-\frac{dE}{dx}\right)^\pm_{\text{col}} + \left(-\frac{dE}{dx}\right)^\pm_{\text{rad}}$$

Collisional Stopping Power

$$\left(-\frac{dE}{dx}\right)^\pm_{\text{col}} =$$

$$\frac{5.08\times10^{-31}n_e}{\beta^2}\left[G^\pm(\beta) - \ln I_{\text{eV}}\right] \text{MeV cm}^{-1}$$

$$G^\pm(\beta) = \ln\left(3.61\times10^5\tau\sqrt{\tau+2}\right) + F^\pm(\beta)$$

$$F^-(\beta) = \frac{1-\beta^2}{2}\left[1 + \frac{\tau^2}{8} - (2\tau+1)\ln 2\right]$$

$$F^+(\beta) =$$

$$\ln 2 - \frac{\beta^2}{24}\left[23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2} + \frac{4}{(\tau+2)^3}\right]$$

Radiative Stopping Power

$$\frac{(-dE/dx)^-_{\text{rad}}}{(-dE/dx)^-_{\text{col}}} \simeq \frac{ZE_{e^-}}{800} \qquad Z = \text{Material Z}$$

Radiation Yield Y, amount of energy released as bremmstrahlung while slowing down completely

$$Y \simeq \frac{6\times10^{-4}\,ZT}{1 + 6\times10^{-4}ZT}$$

Range, $R(T)$ [g cm^{-2}]

$$\begin{array}{ll} 0.412\,T^{1.27-0.0954\ln T} & 0.01 \le T \le 2.5 \text{ MeV} \\ 0.530T - 0.106 & T > 2.5 \text{ MeV} \end{array}$$

Slowing Down Time τ

Same as HCP

Photons (γ)

Dose Quantities

Absorbed Dose

For mass dm with mean energy imparted $d\bar{\epsilon}$ by ionizing radiation

$$D \equiv \frac{d\bar{\epsilon}}{dm} \qquad \dot{D} \equiv \frac{dD}{dt}$$

Measured in Gray

$$1\,\text{Gy} = \frac{1\,\text{J}}{1\,\text{kg}} = 100\,\text{rad}$$

For thin layer with constant stopping power,

$$D = \left(\frac{1}{\rho}\frac{dE}{dx}\right)\Phi \quad \Phi = \text{Flux}\left[\#/(\text{cm}^2\,\text{s})\right]$$

Quality Factor

Q weights absorbed dose to the biological effectiveness of **charged** particles producing absorbed dose. Function of LET

Dose Equivalent, H

$$H \equiv QD$$

Used for routine radiation limits, not for acute doses
Measured in Sieverts

$$1\,\text{Sv} = \frac{1\,\text{J}}{1\,\text{kg}} = 100\,\text{rem}$$

Equivalent Dose, H_T

$$H_T \equiv \sum_R w_R D_{T,R}$$

$D_{T,R} \equiv$ Mean tissue dose of radiation type R

$w_R \equiv$ Radiation weighting factor

Radiation weighting factor relates type of particle or radiation for effectiveness of producing an absorbed dose

Effective Dose, E

$$E \equiv \sum_T w_T H_T = \sum_T w_T \sum_R w_R D_{T,R}$$

w_T is the tissue weighting factor, accounting for relative detriment

Radiation Exposure

Annual per capita Eff. Dose (2000 UNSCEAR)

Sources	Annual Effective Dose (mSv)
Background	
Cosmic Rays	0.4
Terrestrial γ Rays	0.5
Inhalation (radon)	1.2
Ingestion	0.3
Total	2.4
Medical	0.4
Atmos. Weapon Tests	0.005
Nuclear Power	0.0002

Nuclear Reactors

Light Water Reactor (LWR)
Reactor which uses ordinary water as coolant
Boiling Water Reactor (BWR)
LWR which allows water to boil in the core. Single loop, steam directly drives generator. Requires powered pumps for circulation and emergency cooling water
Pressurized Water Reactor (PWR)
LWR where high maintained pressure prevents water boiling. Transfers heat to second water loop which is converted to steam for power
Canada Deutrium Uranium (CANDU)
D_2O cooled reactor for better moderation allowing low enriched fuels (nat. uranium). Horizontal fuel bundles, continuous refueling, no outages.
High Temperature Gas Reactor (HTGR)
High-temperature gas-cooled reactor using pressurized helium. Inherently safe, less fuel
Liquid Metal Fast Breeder Reactor (LMFBR)
Fast fission, no moderator, typically sodium cooled. Use natural or enriched uranium or plutonium.

Chain Reactions

For ν neutrons released per fission,

$$\eta = \nu \frac{\sigma_{\text{fission}}}{\sigma_{\text{abs}}}$$

$\frac{\sigma_{\text{fission}}}{\sigma_{\text{abs}}}$ = Prob. of Fission per Absorption

$\eta > 1$ Chain reaction possible

$\eta > 2$ Breeding, creating more fuel, possible

Multiplication factor k,

$$k = \frac{\text{\# Neutrons in Generation i}}{\text{\# Neutrons in Generation i+1}}$$

Can be expressed through the fractional change of neutrons per generation, ρ

$$\rho = (k-1)/k$$

$k < 1$	$\rho < 0$	Subcritical, reactor shutdown
$k = 1$	$\rho = 0$	Critical, maintain reactor power
$k > 1$	$\rho > 0$	Supercritical, reactor startup

With a time θ between neutron generations, the reactor period τ

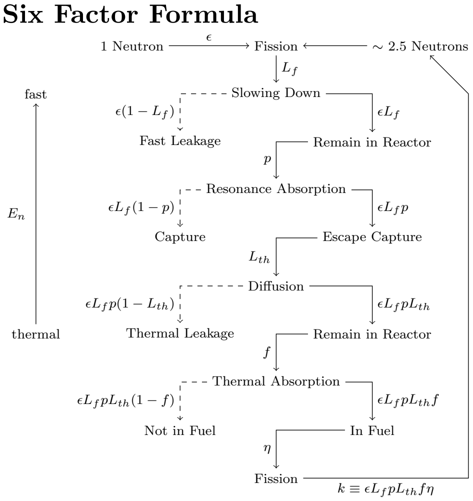
$$\tau = \theta / \ln k$$

Delayed n from fission frag., yield β, increase θ
Reactor Power increases as

$$P/P_0 = e^{t \ln (k)/\theta}$$

$k < 1 + \beta$	Reaction rate controlled by delayed neutrons
$k > 1 + \beta$	Prompt super critical, driven by prompt neutrons, uncontrollable

Six Factor Formula



$$k \equiv \epsilon\ p\ f\ \eta\ L_f\ L_t$$

ϵ	fast fission factor
p	resonance escape prob
f	thermal utilization factor
η	# neutrons per abs. in fuel
L_f	fast non-leak prob
L_t	thermal non-leak prob

$$f = \frac{\Sigma_a^{\text{fuel}}}{\Sigma_a^{\text{all mat}}} \qquad \eta = \frac{\nu \sigma_{\text{fiss}}^{\text{fuel}}}{\sigma_{\text{abs}}^{\text{fuel}}}$$

For infinite reactor with no leakage

$$k_\infty = \epsilon\ p\ f\ \eta$$

Reactor Kinetics

$$k = \frac{P(T)}{L(T)} = \frac{\text{Neutron Production Rate}}{\text{Neutron Loss Rate}}$$

Neutron lifetime l (θ) and reactor period T (τ)

$$l = \frac{N(T)}{L(T)} \qquad T = \frac{l}{k-1}$$

$$N(T) = N_0 e^{t/T}$$

Neutron Diffusion

$$-D\nabla^2 \Phi(\vec{r}) + \Sigma_R \Phi(\vec{r}) = S(\Phi(\vec{r}))$$

$$S(\Phi(\vec{r})) = \nu \Sigma_f \Phi(\vec{r})$$

$$D = 1/3\ \Sigma_{tr}$$

$$\Sigma_{tr} = \Sigma_T - \mu_0 \Sigma_s$$

$$L = \sqrt{D/\Sigma_a} k = \frac{B_m^2}{B_g^2}$$

Misc.

Fission

$\langle E \rangle_{\text{fission}} \simeq 200\,\text{MeV}$

80% = K.E. Fission frag.	3% = K.E. n_{fast}
4% = γ	4% = β decay frag.
5% = ν	4% = (n, γ)