Bessel Beam Generation and Propagation Characterization

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A Bessel beam is a wave whose amplitude is described using a Bessel function of the first kind, and it represents a fundamental form of structured light with a wide range of uses, in many industrial and scientific platforms, mainly because of their large Rayleigh range (focal length). Bessel beams are also self-healing, meaning that the beam can be partially obstructed at one point but will regain its structure at a point along the beam axis, hence such beams can provide some resistance to diffraction, and are seen as an alternate form of Gaussian beams in most cases. This research aims to understand more about Bessel beams through simulations, and how to enhance their properties for efficient use.

1.0 Introduction

Typical Bessel beams are usually non-diffractive, and this property helps to keep their shapes over very large distances [1][2]. As a result of the aforesaid property, these beams could be detected several meters away from their source, provided the right parameters are used during simulations[4].

2.0 Motivation

Bessel beam applications are very economical, and take advantage of the very large size of the focus [1][2], without the use of lenses or mirrors [1][2]. This project aims to understand more about Bessel beams by simulating beam propagation through changes in the wave parameters, and seeing how changes in these parameters influence the totality of the beam propagation. A concrete knowledge of how these Bessel beams work, under varying conditions, will help enhance industrial practices that involve the use of the aforesaid beams.

3.0 Theory of Bessel beam Generation

Using a wave with complex amplitude

$$U(r) = A(x,y)e^{-j\beta z},$$

For the above expression to satisfy the Helmholtz equation, $\nabla^2 U + K^2 U = 0$, the term A(x,y) should satisfy

$$\nabla_T^2 A + K_T^2 A = 0 \text{ [4][5] (Two-dimensional Helmholtz equation).}$$

Here, $K_T^2 + \beta^2 = K^2$, and $\nabla_T^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is known as the transverse Laplacian operator.

By method of separation of variables, and using polar coordinates ($x = \rho\cos\varphi, y = \rho\sin\varphi$), the solution to the wave equation becomes : $A(x,y) = A_m J_m(K_T\rho)e^{jm\varphi}, m = 0, \pm 1, \pm 2, \ldots$ [5],
where $J_m(K_T\rho)$ = Bessel function of the first kind, corresponding to its m-th order, $A_m = \text{constant}$.
For clarity, we have excluded solutions of of the wave equation that exist at $\rho = 0$, since they are singular in nature[5].

Considering the case where m = 0, $U(r) = A_0 J_0(K_T\rho)e^{-j\beta z}$, and the intensity $I(\rho,\varphi,z) = |A_0|^2 J_0^2(K_T\rho)$.
We should also note that the intensity appears to be circularly symmetric and varies with ρ, and it does not depend on the z-coordinate, which also helps to avoid the spread of its optical power [4][5], hence the concept of Bessel beams.

4.0 Simulation of Bessel Beam

To enable us simulate a typical Bessel beam, we illuminated a circular disk using a plane monochromatic beam. The disk was placed at the primary focus of a positive lens, so that the waves originating from the edge of the disk was collimated [1]. By eliminating the rest of the incident beam, only the waves at the edge were allowed to propagate resulting in a non-diffracting Bessel beam[1][4][5].

The waves originating from the edges of the disc are seen as spherical waves, propagating with the same phase because of the disc illumination using a monochromatic plane wave. The spherical wave have the same amplitude, which leads to constructive interference along a spot near the axis, Poisson-spot [1][3][4][5].

For this case, we define the intensity as: $I(r,z) \approx I_0 J_0^2(\frac{2\pi\alpha r}{\lambda})$

Here, $\alpha = \frac{a}{r}$, this is the angle of the wave-front near the axis. 2a = diameter of the disc. Let $w(z) = \frac{2.44\lambda z}{\pi a}$. This is known as the width of the beam, and it is directly proportional to the distance, z [1][4]. Below, are samples of the aforesaid beam produced using varying beam parameters.

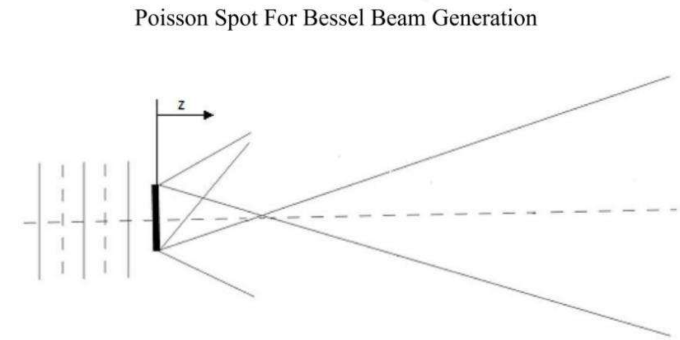
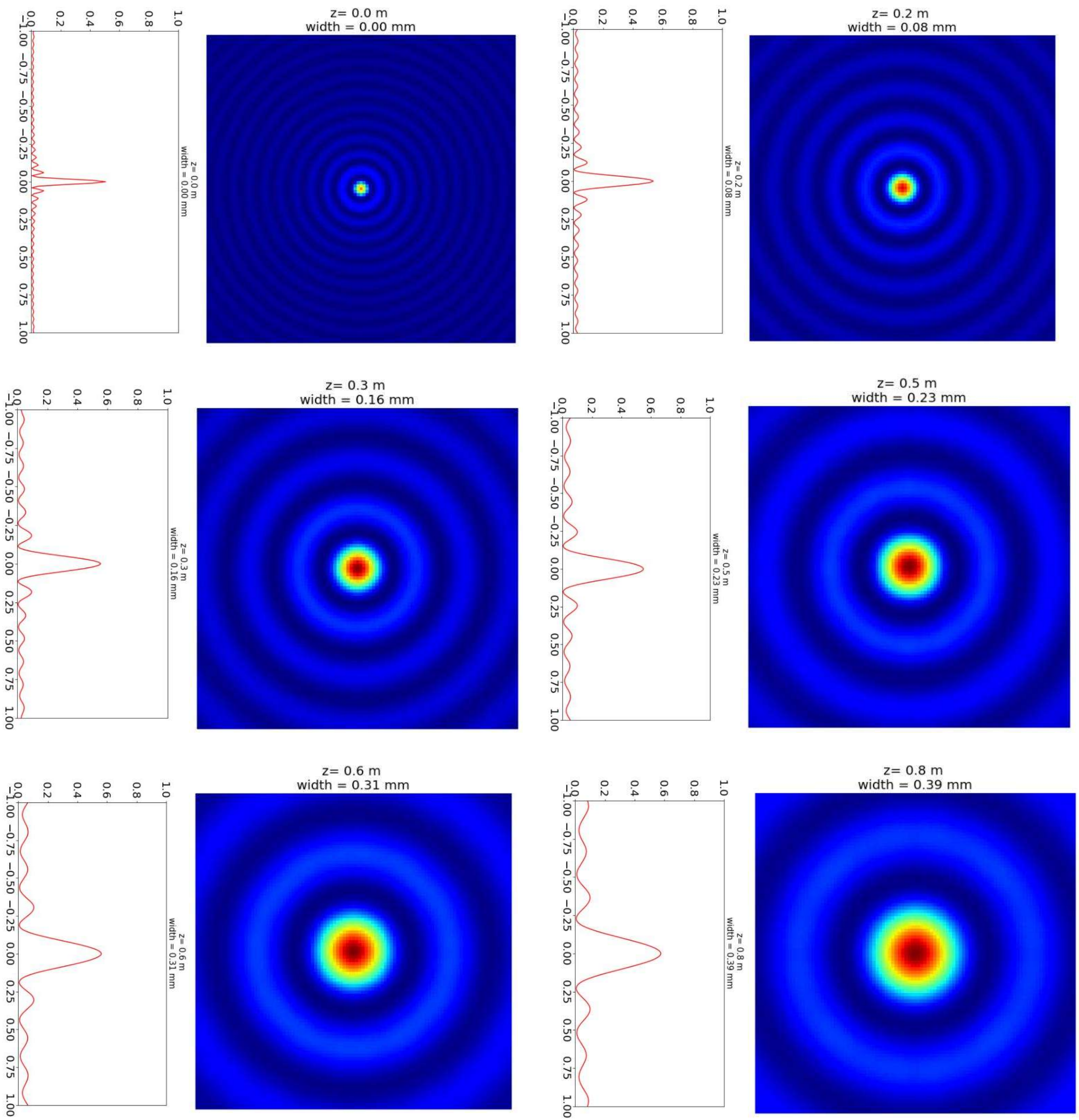


Figure 1 : An Example of the Poisson Spot for the Bessel beam [1]



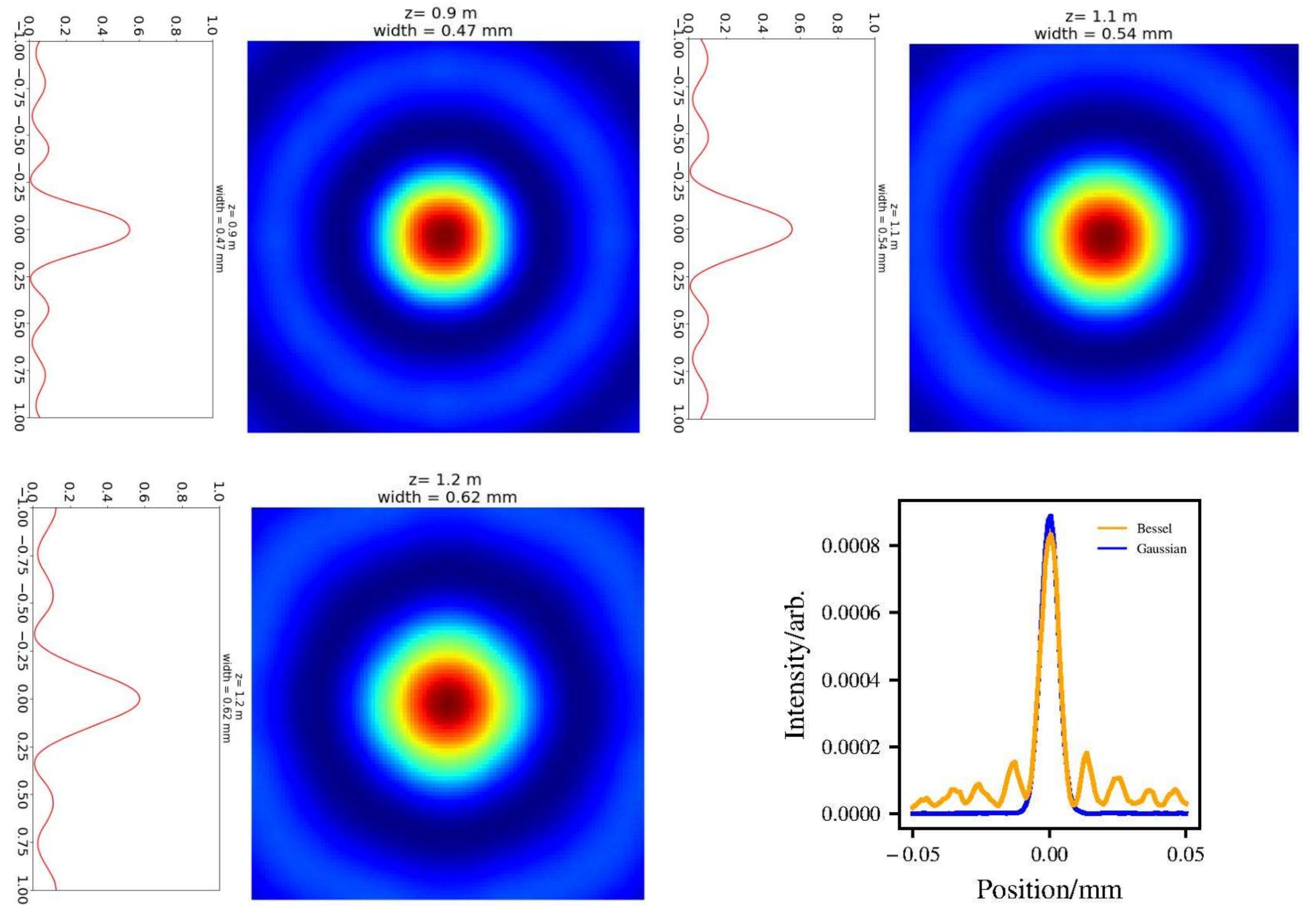


Figure 2 : Set of images showing corresponding propagation of Bessel beam and their intensity distributions; The last image (2b) compares the intensity distribution between Bessel beam and Gaussian beam, as a function of beam position (in millimeter) [3].

5.0 Discussion

Based on the simulations in 4.0, the following are found worthy of note:

1. The intensity distribution of the Bessel beam, decreases oscillatorily on the basis of a slow inverse-power-law decay with ρ, and represented mathematically as:
   $J_0^2(K_T\rho) \approx (2/\pi K_T\rho)\cos^2(K_T\rho - \pi/4)$ [4][5].

2. Owing to its mode of propagation, the transverse RMS width of the Bessel beam is infinite for all value of z, making its divergence to be zero (similar to that of a plane wave)[4][5].

6.0 Conclusion

With these tools in mind, producing customized Bessel beam for industrial purposes can be easily attainable and the process being efficient.

7.0 References

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