# **Project Work**

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```
library(ISLR)
library(tidyverse)
## - Attaching packages -
                                                                – tidyverse 1.3.0 —
## ✓ ggplot2 3.3.3
                       √ purrr
                                 0.3.4
## ✓ tibble 3.1.0

✓ dplyr 1.0.5
## / tidyr 1.1.3
                       ✓ stringr 1.4.0
## / readr 1.4.0
                       ✓ forcats 0.5.1
## -- Conflicts ---
                                                         — tidyverse_conflicts() —
## x dplyr::filter() masks stats::filter()
## x dplyr::lag() masks stats::lag()
library(tidyr)
library(readr)
library(ggplot2)
library(dplyr)
library(forcats)
library(carData)
library(class)
library(devtools)
## Loading required package: usethis
library(gam)
## Loading required package: splines
## Loading required package: foreach
##
## Attaching package: 'foreach'
```

```
## The following objects are masked from 'package:purrr':
##
##
       accumulate, when
## Loaded gam 1.20
library(glmnet)
## Loading required package: Matrix
##
## Attaching package: 'Matrix'
## The following objects are masked from 'package:tidyr':
##
##
       expand, pack, unpack
## Loaded glmnet 4.1-1
library(leaps)
library(methods)
library(openxlsx)
library(scales)
##
## Attaching package: 'scales'
  The following object is masked from 'package:purrr':
##
##
       discard
##
  The following object is masked from 'package:readr':
##
##
##
       col_factor
library(splines)
library(stats)
library(caret)
```

```
## Loading required package: lattice

##
## Attaching package: 'caret'

## The following object is masked from 'package:purrr':
##
## lift
```

```
#library(readr)
#data2019 <- read_csv("STATS_project_data2019.csv")
#View(data2019)
```

```
library(readxl)
data2019 <- read_excel("STATS_project_data2019.xlsx")</pre>
```

```
# Remove the variables we don't want
data2019 <- data2019 %>%
    select(-ANO4, -TRIMESTRE,-COMPONENTE,-CODUSU,-NRO_HOGAR,-Birth_Loc_Specific, -Loc
ation_5y_Specific,-Read_write_recode,-Schooling_recode,-NIVEL_ED_recoded,-ESTADO,-CAT
_INAC,-Income_job,-Income_scholarship, -Income_no_work, -IPCF,-NIVEL_ED,-Schooling,-R
ead_write)
```

library(broom)

```
#Recode Vars taken as integer as categorical
data2019$CAT OCUP <- as.factor(data2019$CAT OCUP)</pre>
data2019$Sex <- as.factor(data2019$Sex)</pre>
data2019$REGION <- as.factor(data2019$REGION)</pre>
data2019$Relative Rel <- as.factor(data2019$Relative Rel)
data2019$Civil State <- as.factor(data2019$Civil State)</pre>
#data2019$Schooling <- as.factor(data2019$Schooling)
#data2019$NIVEL ED <- as.factor(data2019$NIVEL ED)
data2019$Highest level <- as.factor(data2019$Highest level)
data2019$Type of school <- as.factor(data2019$Type of school)
data2019$`finished?` <- as.factor(data2019$`finished?`)</pre>
data2019$Birth Location <- as.factor(data2019$Birth Location)
data2019$Ownership <- as.factor(data2019$Ownership)</pre>
data2019$House_Type <- as.factor(data2019$House_Type)</pre>
data2019$Studio <- as.factor(data2019$Studio)</pre>
data2019$IH II 01 <- as.factor(data2019$IH II 01)</pre>
data2019$IH II 02 <- as.factor(data2019$IH II 02)
data2019$IP III 04 <- as.factor(data2019$IP III 04)</pre>
data2019$IP III 05 <- as.factor(data2019$IP III 05)</pre>
data2019$IP III 06 <- as.factor(data2019$IP III 06)</pre>
```

```
# Rename column where names is code
names(data2019)[names(data2019) == "IH_II_01"] <- "Computer_house"
names(data2019)[names(data2019) == "IH_II_02"] <- "Internet_house"
names(data2019)[names(data2019) == "IP_III_04"] <- "Internet_use"
names(data2019)[names(data2019) == "IP_III_05"] <- "Computer_use"
names(data2019)[names(data2019) == "IP_III_06"] <- "Cellphone_use"</pre>
```

# Initial investigation 1: ignoring nonlinearity (for now)

Use ordinary least squares (OLS) regression, forward and/or backward selection, and LASSO to build initial models for your quantitative outcome as a function of the predictors of interest. (As part of data cleaning, exclude any variables that you don't want to consider as predictors.)

- These models should not include any transformations to deal with nonlinearity. You'll explore this in the next investigation.
- Note: If you have highly collinear/redundant variables, you might see the message "Reordering variables and trying again" and associated warning() s about linear dependencies being found. Sometimes stepwise selection is able to handle the collinearity/redundancy by modifying the order of the variables tried. If collinearity/redundancy cannot be handled and causes an error, try reducing nvmax.

##Note It is noteworthy the fact that we started fitting the models with individual's income as outcome variable in the natural scale. Only when we got to the residuals plot we realized we needed to log transform the variable to account for non-continuous distribution of residuals or heteroskedasticity. We log transformed the variable and re-traced the process but for the sake of interpretability, we are stiking with the natural scale for now.

That's completely

#### **OLS**

\*Note: We kept the original OLS models to compare residual plots bellow

```
mod1 <- lm(Income_individual ~ Literacy_Index+Sex+Birth_Location+CAT_OCUP+Civil_State
+Highest_level+Cellphone_use+Internet_use+ Computer_use+ Age, data = data2019)
summary(mod1)</pre>
```

```
##
## Call:
##
   lm(formula = Income_individual ~ Literacy_Index + Sex + Birth_Location +
##
       CAT_OCUP + Civil_State + Highest_level + Cellphone_use +
       Internet_use + Computer_use + Age, data = data2019)
##
##
## Residuals:
##
       Min
                 1Q
                    Median
                                 3Q
                                        Max
##
    -73464
             -8292
                      -1112
                               5111 1642734
##
## Coefficients:
##
                     Estimate Std. Error t value Pr(>|t|)
                                           -2.640 0.008294 **
## (Intercept)
                     -3560.759
                                 1348.796
                                             9.013 < 2e-16 ***
## Literacy_Index
                      2047.797
                                  227.197
## Sex2
                     -4561.713
                                  200.031 -22.805
                                                   < 2e-16 ***
                                             3.771 0.000163 ***
## Birth Location2
                      1268.776
                                  336.442
## Birth Location3
                      2341.698
                                  288.152
                                            8.127 4.52e-16 ***
                                  595.578 -0.782 0.434429
## Birth Location4
                      -465.529
                                           -2.203 0.027605 *
## Birth Location5
                     -2075.497
                                  942.160
## Birth_Location9
                    -5622.637
                                 6125.785
                                           -0.918 0.358695
## CAT OCUP1
                                  702.696
                                          24.667 < 2e-16 ***
                     17333.102
## CAT OCUP2
                      5822.716
                                  331.584
                                           17.560
                                                   < 2e-16 ***
## CAT OCUP3
                     13199.429
                                  229.878
                                           57.419
                                                   < 2e-16 ***
## CAT_OCUP4
                                 1685.685
                                          -2.013 0.044071 *
                     -3394.074
## CAT OCUP9
                     -2725.913
                                21367.652
                                           -0.128 0.898488
## Civil State2
                      1176.670
                                  306.090
                                            3.844 0.000121 ***
## Civil State3
                      2322.998
                                  437.821
                                            5.306 1.13e-07 ***
## Civil State4
                      3871.796
                                  524.968
                                            7.375 1.67e-13 ***
## Civil State5
                     -3608.130
                                  291.161 -12.392 < 2e-16 ***
## Civil State9
                     -2558.198
                                12777.654
                                          -0.200 0.841318
## Highest level1
                      2286.799
                                 3439.338
                                            0.665 0.506121
                                 1495.991 -3.090 0.002005 **
## Highest level2
                     -4622.205
```

```
## Highest level3
                   -3559.696
                               1764.270 -2.018 0.043632 *
## Highest level4
                               1763.005 -3.550 0.000386 ***
                   -6258.179
## Highest level5
                   -4557.700
                               1963.588 -2.321 0.020285 *
## Highest level6
                   -6501.692
                               2134.393 -3.046 0.002319 **
## Highest level7
                   -3082.126
                               2115.504 -1.457 0.145144
                               2446.080 9.170 < 2e-16 ***
## Highest level8
                   22430.338
## Highest level9
                               1898.742 1.654 0.098112 .
                    3140.726
## Highest level99 -16965.338
                               9336.962 -1.817 0.069222 .
                                          0.985 0.324445
## Cellphone use2
                     369.912
                                375.402
## Cellphone use9
                    1985.503 12296.952 0.161 0.871729
## Internet use2
                               332.085 -6.649 2.97e-11 ***
                   -2208.172
## Internet use9
                   -5744.849
                               7277.502 -0.789 0.429883
                               226.905 -14.198 < 2e-16 ***
## Computer use2
                   -3221.570
                               4459.193 -0.750 0.453330
## Computer use9
                   -3343.862
## Age
                     277.369
                                  7.586 36.561 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 21370 on 49944 degrees of freedom
     (23 observations deleted due to missingness)
##
## Multiple R-squared: 0.2363, Adjusted R-squared: 0.2358
## F-statistic: 454.5 on 34 and 49944 DF, p-value: < 2.2e-16
```

```
modl_output <- broom::augment(mod1, newdata = data2019)
head(mod1_output)</pre>
```

```
## # A tibble: 6 x 34
##
     Ind Interview REGION MAS 500 AGLOMERADO Relative Rel Sex
                                                                     Age Civil State
##
             <dbl> <fct>
                           <chr>
                                        <dbl> <fct>
                                                            <fct> <dbl> <fct>
## 1
                 1 43
                                             2 1
                                                            1
                                                                      44 3
                           S
                 1 43
                                             2 1
                                                            1
                                                                      59 2
## 2
                           S
## 3
                 1 43
                           S
                                             2 2
                                                                      62 2
                 1 43
                           S
                                             2 3
                                                            1
                                                                      26 5
## 4
## 5
                 1 43
                           S
                                             2 3
                                                            1
                                                                      23 5
## 6
                 1 43
                           S
                                             2 1
                                                                      26 5
##
  # ... with 26 more variables: Type_of_school <fct>, Highest_level <fct>,
       finished? <fct>, last yr approved <chr>, Birth Location <fct>,
## #
       Location_5y <dbl>, CAT_OCUP <fct>, JOB_N <dbl>, Income_individual <dbl>,
## #
## #
       ITF <dbl>, person id <dbl>, Internet use <fct>, Computer use <fct>,
##
       Cellphone use <fct>, House Type <fct>, Room N <dbl>, Ownership <fct>,
## #
       Self Room <dbl>, Self Room Sleep <dbl>, Studio <fct>, Studio N <dbl>,
       Computer house <fct>, Internet house <fct>, Literacy Index <dbl>,
## #
## #
       .fitted <dbl>, .resid <dbl>
```

#### Subset Selection: Backward stepwise selection

```
set.seed(25)
back_step_mod <- train(
    Income_individual~ Literacy_Index+Sex+Birth_Location+CAT_OCUP+Civil_State+Highest
    _level+Cellphone_use+Internet_use+ Computer_use+Age,
    data = data2019,
    method = "leapBackward",
    tuneGrid = data.frame(nvmax = 1:10),
    trControl = trainControl(method = "cv", number = 10, selectionFunction = "oneSE")
,
    metric = "MAE",
    na.action = na.omit
)</pre>
```

```
## Warning in leaps.setup(x, y, wt = weights, nbest = nbest, nvmax = nvmax, : 1
## linear dependencies found
```

## Reordering variables and trying again:

```
summary(back step mod)
```

```
## Subset selection object
## 34 Variables (and intercept)
##
                    Forced in Forced out
## Literacy Index
                        FALSE
                                    FALSE
## Sex2
                        FALSE
                                    FALSE
## Birth Location2
                        FALSE
                                    FALSE
## Birth Location3
                        FALSE
                                    FALSE
## Birth Location4
                        FALSE
                                    FALSE
## Birth Location5
                                    FALSE
                        FALSE
## Birth Location9
                        FALSE
                                    FALSE
## CAT_OCUP1
                        FALSE
                                    FALSE
## CAT OCUP2
                        FALSE
                                    FALSE
## CAT_OCUP3
                        FALSE
                                    FALSE
## CAT OCUP4
                        FALSE
                                    FALSE
## CAT_OCUP9
                        FALSE
                                    FALSE
## Civil_State2
                        FALSE
                                    FALSE
## Civil State3
                                    FALSE
                        FALSE
## Civil State4
                        FALSE
                                    FALSE
## Civil State5
                        FALSE
                                    FALSE
## Civil_State9
                        FALSE
                                    FALSE
## Highest level1
                        FALSE
                                    FALSE
## Highest level2
                        FALSE
                                    FALSE
## Highest level3
                        FALSE
                                    FALSE
```

```
Highest level4
                         FALSE
                                     FALSE
## Highest_level5
                                     FALSE
                         FALSE
## Highest level6
                         FALSE
                                     FALSE
## Highest level7
                         FALSE
                                     FALSE
## Highest_level8
                         FALSE
                                     FALSE
## Highest level9
                         FALSE
                                     FALSE
## Highest level99
                         FALSE
                                     FALSE
## Cellphone_use2
                         FALSE
                                     FALSE
## Cellphone use9
                         FALSE
                                     FALSE
## Internet use2
                         FALSE
                                     FALSE
## Internet use9
                         FALSE
                                     FALSE
   Computer_use2
                         FALSE
                                     FALSE
## Computer use9
                         FALSE
                                     FALSE
## Age
                         FALSE
                                     FALSE
## 1 subsets of each size up to 7
   Selection Algorithm: backward
##
             Literacy Index Sex2 Birth Location2 Birth Location3 Birth Location4
##
      (1)
       (1)
##
   2
##
      (1)
             " * "
   3
             " * "
##
                              . .
##
      (1)
   5
             "*"
##
       (
        1)
##
##
             Birth Location5 Birth Location9 CAT OCUP1 CAT OCUP2 CAT OCUP3
##
   1
        1)
        1)
##
##
   3
        1)
                                                                      " * "
                                                                      " * "
        1
##
   4
##
        1)
                               " * "
                                                           " "
##
       (1)
                                                " * "
                                                           " * "
                                                                      11 🕹 11
##
        1)
   7
##
             CAT_OCUP4 CAT_OCUP9 Civil_State2 Civil_State3 Civil_State4
##
   1
       (1)
##
   2
        1)
        1)
##
   3
##
        1
          )
##
        1
          )
      (1)
##
   6
                                   11 11
                                                 .. ..
##
        1)
##
             Civil State5 Civil State9 Highest level1 Highest level2 Highest level3
                           . .
##
   1
       (1)
      (1)
##
   2
##
   3
        1)
##
        1
           )
##
   5
        1
##
        1)
```

```
##
      (1)
##
            Highest_level4 Highest_level5 Highest_level6 Highest_level7
##
##
   2
      (1)
##
##
##
##
      (1)
##
   7
##
            Highest level8 Highest level9 Highest level99 Cellphone use2
## 1
      (1)
##
        1
          )
##
   3
##
##
   5
        1
##
##
   7
      (1)
##
            Cellphone use9 Internet use2 Internet use9 Computer use2 Computer use9
##
      (1)
##
      (1)
   2
##
##
        1)
##
##
      (1)
##
          )
##
            Age
##
        1)
##
          )
##
##
##
##
        1
  6
## 7
      (1)
```

```
coef(back_step_mod$finalModel, id = back_step_mod$bestTune$nvmax)
```

```
##
                                                         CAT OCUP1
                                                                         CAT OCUP2
      (Intercept) Literacy Index
                                              Sex2
##
      -17390.0283
                        2466.8234
                                       -4155.8548
                                                        19185.0613
                                                                         6416.4241
##
        CAT OCUP3 Highest level8
                                               Age
##
       13965.8905
                       28848.0022
                                          351.3907
```

## **LASSO**

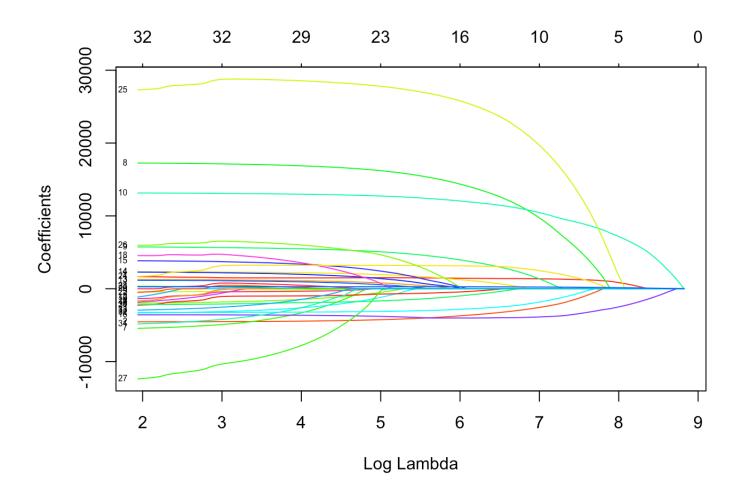
Note to self: You can also look at the two forms of penalized models with this tuneGrid: ridge regression and lasso regression. alpha = 0 is pure ridge regression, and alpha = 1 is pure lasso regression. You can fit a mixture of the two models (i.e. an elastic net) using an alpha between 0 and 1. For example, alpha = 0.05

would be 95% ridge regression and 5% lasso regression. Note to self: According to the internet, the sequence is the amount of lambda values we explore, starting from a particular number, finishing in a different one. The jumps/steps (amount of models we fit) is defined by length.out and it usually jumps with multiples of 10.

```
set.seed(253)
lasso_mod<- train(
    Income_individual ~ Literacy_Index+Sex+Birth_Location+CAT_OCUP+Civil_State+Highes
t_level+Cellphone_use+Internet_use+ Computer_use+Age,
    data = data2019,
    method = "glmnet",
    trControl = trainControl(method = "cv", number = 10, selectionFunction = "oneSE")
,
    tuneGrid = data.frame(alpha = 1, lambda = seq(0,1000, length.out = 100)),
    metric = "MAE",
    na.action = na.omit
)</pre>
```

#### ##Examine

```
# Plot coefficient paths as a function of lambda
plot(lasso_mod$finalModel, xvar = "lambda", label = TRUE, col = rainbow(20))
```



# Codebook for which variables the numbers correspond to
rownames(lasso\_mod\$finalModel\$beta)

```
##
    [1] "Literacy_Index"
                            "Sex2"
                                               "Birth Location2"
                                                                  "Birth_Location3"
    [5] "Birth Location4"
                                              "Birth Location9" "CAT OCUP1"
##
                           "Birth Location5"
    [9] "CAT_OCUP2"
                            "CAT_OCUP3"
                                               "CAT_OCUP4"
                                                                  "CAT_OCUP9"
##
   [13] "Civil_State2"
                            "Civil_State3"
                                               "Civil_State4"
                                                                  "Civil_State5"
##
   [17] "Civil State9"
                            "Highest level1"
                                               "Highest level2"
                                                                  "Highest level3"
   [21] "Highest level4"
                           "Highest level5"
                                               "Highest level6"
                                                                  "Highest level7"
   [25] "Highest level8"
                                               "Highest level99"
                            "Highest level9"
                                                                  "Cellphone use2"
## [29] "Cellphone_use9"
                            "Internet_use2"
                                               "Internet_use9"
                                                                  "Computer_use2"
                            "Age"
## [33] "Computer_use9"
```

PUT ANY RELEVANT TEXT/RESPONSES/INTERPRETATIONS HERE Interpretation OLS: -The average value of Individual Income when all the predictors are set to 0 corresponds to the intercept ARG\$8056.2. - Then, we can say that for one unit increase in the the Literacy index we associate an increase of ARG\$1514.6 in individual's income, holding all other predictors constant. Interpretation for Stepwise Selection: -The best model choosen with oneSE is the one with 7 predictors. In the one stage, the predictor remaining is

CAT\_OCUP3, which corresponds to the category employee/worker. Whether a person belongs or not to that category seems to impact their income greatly, which makes sense. The second most important predictor seems to be Age. The third is Literacy, as I suspected, and the fourth is CAT\_OCUP1 which signals whether a person has a leadership role in their job or not. Additionally, the model picked up Highest\_level8 as the fifth predictor, which corresponds to having attended to Graduate School (Masters, MBA, PhD). In general for backward step-wise selection, variables removed first could be viewed as the least important predictors, and variable that remain until the end could be viewed as the most important predictors. Interpretation for Lasso: From just examining the plot above we can see that some of the \

Estimate test performance of the models from these different methods. Report and interpret (with units) these estimates along with a measure of uncertainty in the estimate (SD is most readily available from <code>caret</code>).

Compare estimated test performance across methods. Which method(s) might you prefer?

```
Be very careful! These are *training* estimates of
Test Metrics For OLS
                                                           performance! You should use caret mod$results to obtain
                                                           the cross-validated estimates of test performance. We use
  # Your code
                                                           the following code to **graphically** explore training set
                                                           residuals (because we don't have a test set):
 mean(abs(mod1 output$.resid),na.rm=TRUE)
                                                           data %>%
                                                             mutate(pred = predict(...), residuals = Y - pred)
 ## [1] 10812.28
                                                           You should still use caret to fit the OLS model (method =
                                                           "Im") and look at $results to get estimates of test
                                                           performance.
 mean(mod1 output$.resid^2,na.rm=TRUE)
 ## [1] 456170460
```

#### Test Metrics for Stepwise Selection

```
# Look at accuracy/error metrics for the different subset sizes
# If you want to sort the table of results, use arrange() from dplyr
back_step_mod$results %>% arrange(MAE)
```

```
##
      nvmax
                RMSE
                       Rsquared
                                      MAE
                                            RMSESD RsquaredSD
                                                                  MAESD
## 1
         10 21234.72 0.23416517 11042.38 4116.832 0.06153874 694.4541
          9 21261.29 0.23218020 11089.10 4118.770 0.06253801 712.8859
## 2
          8 21310.35 0.22856637 11130.06 4116.526 0.06126135 701.9552
##
  3
          7 21373.63 0.22381625 11192.71 4098.222 0.05895647 679.9858
##
##
  5
          6 21478.07 0.21599135 11305.27 4094.588 0.05893365 704.3734
          5 21656.98 0.20287251 11400.23 4117.168 0.05790163 684.0452
##
   7
          4 21774.16 0.19398937 11483.82 4094.982 0.05395613 654.8115
##
          2 22275.11 0.15537183 11509.24 4056.190 0.04510042 683.1004
##
  8
   9
          3 21879.39 0.18593402 11607.97 4096.273 0.05188570 653.6835
##
## 10
          1 23175.36 0.08371103 13192.98 3954.987 0.02081807 169.9558
```

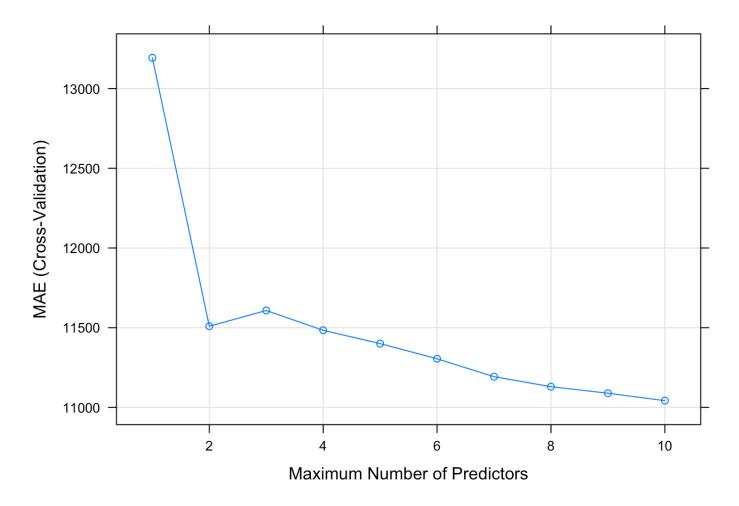
```
# What tuning parameter gave the best performance?
# i.e. What subset size gave the best model?
back_step_mod$bestTune
```

```
## nvmax
## 7 7
```

# Obtain the coefficients for the best model
coef(back\_step\_mod\$finalModel, id = back\_step\_mod\$bestTune\$nvmax)

```
CAT_OCUP2
##
      (Intercept) Literacy_Index
                                                       CAT_OCUP1
                                             Sex2
##
      -17390.0283
                        2466.8234
                                      -4155.8548
                                                      19185.0613
                                                                       6416.4241
##
        CAT OCUP3 Highest level8
                                              Age
##
       13965.8905
                       28848.0022
                                         351.3907
```

```
# Plot metrics for each model in the sequence
plot(back_step_mod)
```



#### Test Metrics for LASSO

lasso mod\$results %>% arrange(MAE)

```
##
       alpha
                 lambda
                            RMSE
                                   Rsquared
                                                 MAE
                                                       RMSESD RsquaredSD
                                                                             MAESD
##
              353.53535 21136.12 0.2442148 10734.64 3748.399 0.04413983 140.0113
##
           1
              363.63636 21138.36 0.2441160 10734.68 3749.031 0.04414769 140.2292
              343.43434 21133.91 0.2443123 10734.75 3747.767 0.04413147 139.8606
##
           1
              373.73737 21140.68 0.2440137 10734.86 3749.654 0.04415511 140.4286
              333.3333 21131.69 0.2444124 10734.98 3747.134 0.04412345 139.6634
##
              383.83838 21143.00 0.2439128 10735.12 3750.263 0.04416282 140.6718
##
           1
              393.93939 21145.28 0.2438163 10735.38 3750.803 0.04416733 140.9472
##
##
           1
              323.23232 21129.54 0.2445090 10735.49 3746.491 0.04411487 139.3761
              404.04040 21147.62 0.2437167 10735.78 3751.333 0.04417125 141.2440
##
##
  10
           1
              313.13131 21127.46 0.2446018 10736.14 3745.838 0.04410584 139.0979
              414.14141 21150.03 0.2436140 10736.40 3751.849 0.04417440 141.4737
##
  11
              424.24242 21152.06 0.2435438 10736.60 3752.435 0.04418775 141.6936
##
  12
              434.34343 21153.96 0.2434852 10736.68 3753.003 0.04420012 141.9160
           1
##
  13
##
  14
                        21155.91 0.2434246 10736.86 3753.564 0.04421229 142.1507
```

L.L.	<u>,</u>	1 5	1	202 02020	21125 46	0 2446005	10726 02	2745 170	0 04400630	120 0200
		15	1						0.04409638	
		16	1						0.04422410	
##	<del>/</del>	17	1	464.64646	21159.85	0.2433065	10737.25	3754.693	0.04424061	142.6311
##	ŧ	18	1	474.74747	21161.82	0.2432506	10737.43	3755.276	0.04425826	142.9081
##	ŧ	19	1	484.84848	21163.83	0.2431930	10737.78	3755.852	0.04427573	143.1185
##	¥	20	1	292.92929	21123.53	0.2447756	10737.89	3744.510	0.04408648	138.6221
##	¥	21	1	494.94949	21165.90	0.2431336	10738.33	3756.421	0.04429304	143.2672
		22	1						0.04407632	
		23	1						0.04431001	
##	<del>/</del>	24	1	515.15152	21170.16	0.2430093	10739.73	3757.540	0.04432670	143.6462
##	ŧ	25	1	272.72727	21119.79	0.2449420	10740.06	3743.156	0.04406618	138.1469
##	<del>/</del>	26	1	525.25253	21172.36	0.2429443	10740.52	3758.089	0.04434332	143.8561
##	¥	27	1	262.62626	21118.00	0.2450216	10741.38	3742.466	0.04405516	137.8853
##	¥	28	1	535,35354	21174.60	0.2428775	10741.40	3758.632	0.04435975	144.1153
		29	1						0.04437601	
		_	_							
		30	1						0.04404210	
##	<b>#</b>	31	1						0.04439219	
##	<del>/</del>	32	1	242.42424	21114.42	0.2451873	10744.22	3741.041	0.04402866	137.1019
##	<del>/</del>	33	1	565.65657	21181.60	0.2426666	10744.76	3760.225	0.04440819	144.7521
##	<del>/</del>	34	1	232.32323	21112.71	0.2452670	10745.75	3740.322	0.04401642	136.7810
##	¥	35	1	575.75758	21184.02	0.2425928	10746.14	3760.742	0.04442401	144.9776
##	¥	36	1	222.22222	21111.06	0.2453438	10747.38	3739.608	0.04400567	136.5231
##	¥	37	1	585.85859	21186.48	0.2425171	10747.62	3761.254	0.04443963	145.2159
		38	1						0.04399544	
		39	1						0.04445506	
		40	1						0.04447014	
##	<del>/</del>	41	1	202.02020	21107.99	0.2454872	10750.98	3738.180	0.04398523	135.9497
##	ŧ	42	1	616.16162	21194.14	0.2422786	10752.46	3762.750	0.04448493	145.9111
##	ŧ	43	1	191.91919	21106.57	0.2455530	10753.03	3737.440	0.04397329	135.6391
##	¥	44	1	626.26263	21196.78	0.2421955	10754.22	3763.239	0.04449942	146.1270
##	¥	45	1	181.81818	21105.25	0.2456138	10755.34	3736.681	0.04395985	135.3046
##	¥	46	1	636,36364	21199.47	0.2421104	10756.07	3763.721	0.04451370	146.3418
		47	1						0.04394591	
		48	_						0.04452778	
			1							
		49	1						0.04454164	
		50	1						0.04392965	
		51	1						0.04455559	
##	ŧ	52	1	151.51515	21101.82	0.2457692	10763.41	3734.231	0.04390884	134.4986
##	ŧ	53	1	676.76768	21210.56	0.2417576	10763.70	3765.586	0.04456910	147.1666
##	¥	54	1	686.86869	21213.40	0.2416686	10765.57	3766.036	0.04458203	147.3651
##	¥	55	1	141.41414	21100.85	0.2458119	10766.48	3733.353	0.04389021	134.2319
		56	1						0.04459473	
		57	1						0.04460720	
			_							
		58	1						0.04387021	
		59	1						0.04461943	
		60	1						0.04384837	
##	<del>/</del>	61	1	727.27273	21225.17	0.2412927	10773.61	3767.777	0.04463162	148.1576
1										

```
##
   62
              737.37374 21228.18 0.2411970 10775.55 3768.217 0.04464550 148.4159
##
              111.11111 21098.35 0.2459254 10776.49 3730.526 0.04382536 133.2271
   63
           1
##
   64
           1
              747.47475 21231.22 0.2410998 10777.50 3768.649 0.04465889 148.6806
##
   65
              757.57576 21234.31 0.2410005 10779.51 3769.074 0.04467207 148.9345
           1
##
   66
              101.01010 21097.65 0.2459580 10780.18 3729.551 0.04380269 132.8216
              767.67677 21237.44 0.2408991 10781.57 3769.494 0.04468503 149.1798
##
   67
           1
              777.7778 21240.61 0.2407957 10783.69 3769.908 0.04469776 149.4110
##
   68
           1
               90.90909 21097.03 0.2459875 10784.08 3728.550 0.04377945 132.5066
##
   69
           1
              787.87879 21243.83 0.2406901 10785.86 3770.316 0.04471026 149.6487
           1
##
   70
##
   71
           1
              797.97980 21247.08 0.2405828 10788.10 3770.719 0.04472267 149.8873
               80.80808 21096.50 0.2460122 10788.23 3727.536 0.04375539 132.2185
##
   72
           1
              808.08081 21250.31 0.2404797 10790.33 3771.144 0.04473791 150.1191
##
   73
           1
               70.70707 21096.09 0.2460316 10792.53 3726.474 0.04372922 131.9100
##
   74
           1
              818.18182 21253.55 0.2403768 10792.59 3771.565 0.04475265 150.3218
   75
           1
##
              828.28283 21256.84 0.2402719 10794.90 3771.980 0.04476718 150.5345
##
   76
           1
               60.60606 21095.76 0.2460476 10796.97 3725.414 0.04370419 131.6378
##
   77
           1
##
   78
           1
              838.38384 21260.16 0.2401649 10797.30 3772.390 0.04478151 150.7451
##
   79
           1
              848.48485 21263.53 0.2400559 10799.77 3772.794 0.04479564 150.9567
               50.50505 21095.57 0.2460561 10801.62 3724.332 0.04368035 131.4656
   80
##
           1
              858.58586 21266.94 0.2399447 10802.31 3773.193 0.04480956 151.1812
##
           1
   81
              868.68687 21270.39 0.2398315 10804.92 3773.586 0.04482328 151.4501
##
   82
           1
               40.40404 21095.50 0.2460598 10806.24 3723.251 0.04365903 131.3424
##
           1
   83
##
   84
           1
              878.78788 21273.81 0.2397225 10807.54 3773.955 0.04483603 151.6915
              888.88889 21277.20 0.2396173 10810.21 3774.331 0.04485028 151.9017
##
   85
           1
               30.30303 21095.55 0.2460568 10810.89 3722.146 0.04363796 131.1841
##
   86
           1
              898.98990 21280.61 0.2395117 10812.95 3774.700 0.04486331 152.1072
##
   87
           1
               20.20202 21095.75 0.2460455 10815.20 3720.997 0.04361708 131.0584
##
   88
           1
##
   89
              909.09091 21284.06 0.2394042 10815.81 3775.064 0.04487612 152.2947
           1
              919.19192 21287.55 0.2392946 10818.75 3775.422 0.04488872 152.5258
##
   90
           1
               10.10101 21095.59 0.2460656 10819.79 3719.913 0.04360682 131.3089
##
   91
           1
                0.00000 21095.60 0.2460789 10821.63 3719.620 0.04361118 131.4670
##
   92
           1
              929.29293 21291.07 0.2391830 10821.74 3775.775 0.04490108 152.7773
##
   93
           1
              939.39394 21294.64 0.2390693 10824.85 3776.122 0.04491322 153.0055
##
   94
           1
              949.49495 21298.25 0.2389535 10828.10 3776.464 0.04492513 153.2316
##
   95
           1
##
              959.59596 21301.89 0.2388363 10831.49 3776.799 0.04493677 153.4453
   96
           1
##
   97
           1
              969.69697 21305.48 0.2387245 10834.88 3777.147 0.04495154 153.7328
              979.79798 21309.08 0.2386141 10838.29 3777.457 0.04496357 154.0283
##
   98
           1
              989.89899 21312.70 0.2385027 10841.79 3777.764 0.04497493 154.3151
##
  99
## 100
           1 1000.00000 21316.36 0.2383892 10845.43 3778.066 0.04498608 154.5771
```

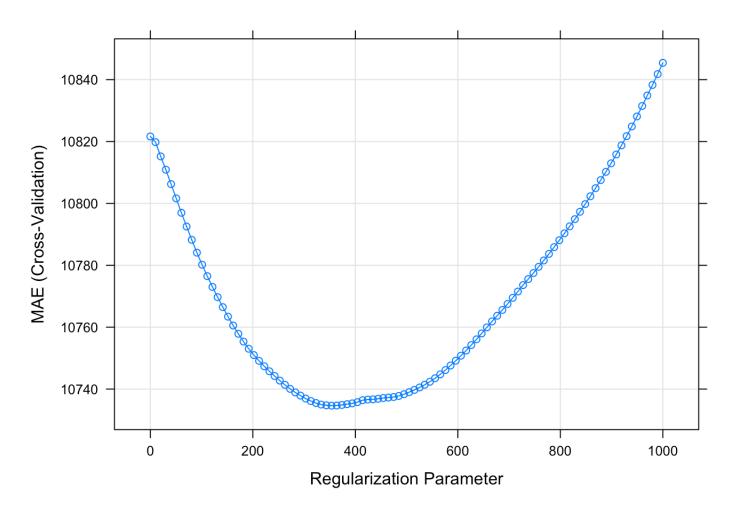
```
# Identify which tuning parameter (lambda) is "best"
lasso mod$bestTune
```

```
## alpha lambda
## 75 1 747.4747
```

#Same thing coded differently, we look at the coefficients for the best lambda model
#coef(lasso\_mod\_log\$finalModel, 747)
coef(lasso\_mod\$finalModel, lasso\_mod\$bestTune\$lambda)

```
## 35 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept)
                   -3936.9274
## Literacy_Index 1396.6896
## Sex2
                   -3123.5300
## Birth Location2
## Birth Location3
                     500.1259
## Birth Location4
## Birth Location5
## Birth_Location9
## CAT OCUP1
                  12093.0945
                   2685.7548
## CAT_OCUP2
                  11279.9739
## CAT OCUP3
## CAT OCUP4
## CAT OCUP9
## Civil State2
## Civil State3
## Civil State4
## Civil State5
                 -3970.3985
## Civil_State9
## Highest_level1
## Highest level2
## Highest_level3
## Highest level4
## Highest_level5
## Highest_level6
## Highest level7
                    2973.7376
## Highest level8
                  22923.2886
## Highest level9
## Highest_level99
## Cellphone use2
## Cellphone use9
## Internet use2
                    -195.3898
## Internet use9
## Computer use2
                   -2392.9450
## Computer_use9
## Age
                     259.7562
```

```
# Plot a summary of the performance of the different models
plot(lasso_mod)
```



#### PUT ANY RELEVANT TEXT/RESPONSES/INTERPRETATIONS HERE

How do the error metrics compare? OLS: The MAE for the 10-variable OLS is 10812.28. This means that on average this OLS model is off in its percentage predictions about a person's income by about 10812 pesos.

Step-wise: The MAE for the 10-variable step-wise model is 11042.38. This means that on average this OLS model is off in its percentage predictions about a person's income by about 11042.38 pesos.

Lasso: Based on the information in <code>lasso\_mod\$results</code>, we can see that the lowest estimated test MAE (10734.64) shows up for lambda = 353.5. However, the algorithm decided to set the "best"  $\lambda = 747$  model with a MAE of 10777.50 pesos. This means that on average this LASSO model is off in its percentage predictions about a person's income by about 10778 pesos. \

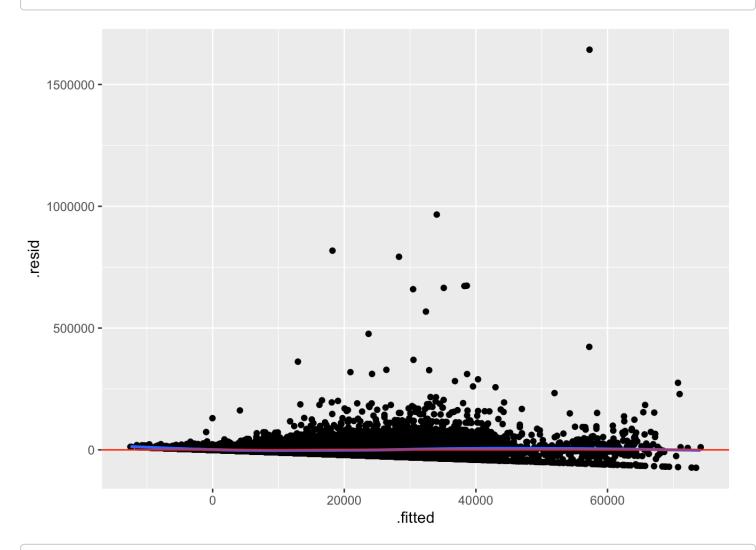
Use residual plots to evaluate whether some quantitative predictors might be better modeled with nonlinear relationships.

```
ggplot(mod1_output, aes(x = .fitted, y = .resid)) +
    geom_point() +
    geom_smooth() +
    geom_hline(yintercept = 0, color = "red")
```

```
## geom_smooth() using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```

## Warning: Removed 23 rows containing non-finite values (stat\_smooth).

## Warning: Removed 23 rows containing missing values (geom\_point).

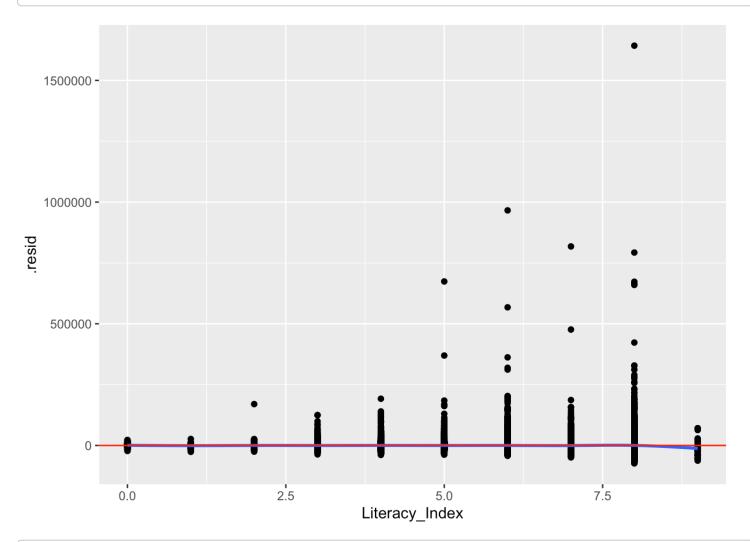


```
# Residuals vs. Literacy index
ggplot(mod1_output, aes(x = Literacy_Index, y = .resid)) +
   geom_point() +
   geom_smooth() +
   geom_hline(yintercept = 0, color = "red")
```

```
## `geom_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```

```
## Warning: Removed 23 rows containing non-finite values (stat_smooth).
```

## Warning: Removed 23 rows containing missing values (geom\_point).

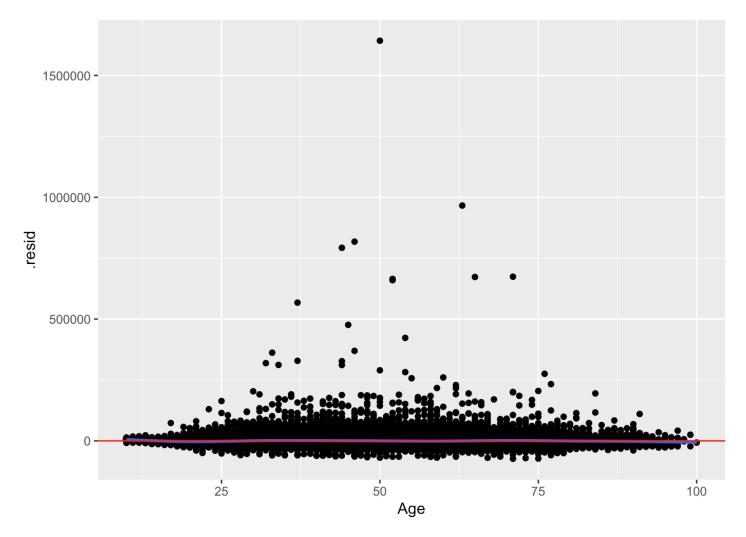


```
# Residuals vs. Age
ggplot(mod1_output, aes(x = Age, y = .resid)) +
    geom_point() +
    geom_smooth() +
    geom_hline(yintercept = 0, color = "red")
```

```
## geom_smooth() using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```

```
## Warning: Removed 23 rows containing non-finite values (stat_smooth).
```

## Warning: Removed 23 rows containing missing values (geom\_point).



PUT ANY RELEVANT TEXT/RESPONSES/INTERPRETATIONS HERE Considering the data used only has 2 quantitative predictors, our plots show the residuals vs. the Literacy\_Index and Age predictors. The Age distributions look okay, but the Literacy index may indicate the need for a log transform because the distribution of the residuals towards the right seems to hit at heteroskedasticity. We should log transform the outcome Individual Income to see how that affects all our investigation, not only the plot It is worth noticing that even when Literacy is numerical, it does have a delimited set of values, which is depicted in the plot as the vertical columns of residuals. Even when no transformation seems necessary, the warning sign mentions the use of the gam method to plot it which is a good sign that we might want to try non-linear investigations. \

```
Data2019 <- data2019 %>% mutate(log_income = log(Income_individual+1))

## Warning in log(Income_individual + 1): NaNs produced

## Veccommend taking a look at the log_income variable to see if anything is amiss.

### Duplicate data

## Duplicate data

## Duplicate data

### data_new[is.na(data_new) | data_new == "Inf"] <- NA ## Replace NaN & Inf with NA data_new[is.na(data_new) | data_new == "NaN"] <- NA
```

```
mod1_log <- lm(log_income ~ Literacy_Index+Sex+Birth_Location+CAT_OCUP+Civil_State+Hi
ghest_level+Cellphone_use+Internet_use+ Computer_use+Age,data = data_new, na.action =
na.omit)
summary(mod1_log)</pre>
```

```
##
## Call:
## lm(formula = log income ~ Literacy Index + Sex + Birth Location +
##
       CAT_OCUP + Civil_State + Highest_level + Cellphone_use +
##
       Internet use + Computer use + Age, data = data new, na.action = na.omit)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
                                            Max
## -13.2478 -1.1983 -0.0168
                                1.6433
                                        10.1126
##
## Coefficients:
##
                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                   -0.102346
                               0.180035 - 0.568 \ 0.569713
## Literacy Index -0.066566
                               0.030651 -2.172 0.029881 *
## Sex2
                   -0.123641
                               0.026935
                                         -4.590 4.44e-06 ***
## Birth Location2 0.505625
                               0.045410 11.135 < 2e-16 ***
                                         6.085 1.17e-09 ***
## Birth Location3 0.237649
                               0.039055
## Birth Location4 -0.176912
                               0.081143 -2.180 0.029243 *
## Birth Location5 -0.125105
                               0.131129 - 0.954 0.340060
## Birth Location9
                   0.636839
                               1.232836
                                         0.517 0.605463
                               0.103331 45.077 < 2e-16 ***
## CAT OCUP1
                    4.657846
## CAT OCUP2
                    4.270733
                               0.045943 92.958 < 2e-16 ***
## CAT OCUP3
                    5.091827
                               0.030913 164.717 < 2e-16 ***
## CAT OCUP4
                   -0.582917
                               0.218782 -2.664 0.007716 **
## CAT OCUP9
                   -4.022862
                               2.755771 -1.460 0.144353
                               0.041561 -15.133 < 2e-16 ***
## Civil State2
                   -0.628923
## Civil State3
                   -0.241102
                               0.059882 -4.026 5.68e-05 ***
                                          4.501 6.77e-06 ***
## Civil State4
                    0.320384
                               0.071176
## Civil_State5
                   -0.798903
                               0.039320 -20.318 < 2e-16 ***
## Civil State9
                    2.095173
                               4.088108
                                          0.513 0.608301
## Highest level1
                    0.124616
                               0.459121
                                          0.271 0.786067
## Highest level2
                    0.316609
                               0.200285
                                          1.581 0.113932
## Highest level3
                    0.943204
                               0.235046
                                          4.013 6.01e-05 ***
## Highest_level4
                                          2.444 0.014510 *
                    0.578385
                               0.236608
## Highest_level5
                    1.217729
                               0.262021
                                          4.647 3.37e-06 ***
                                          4.422 9.79e-06 ***
## Highest level6
                    1.269573
                               0.287083
## Highest level7
                    1.507984
                               0.284451
                                          5.301 1.15e-07 ***
## Highest level8
                    1.949667
                               0.335503
                                          5.811 6.24e-09 ***
## Highest level9
                    2.941281
                               0.251068
                                         11.715 < 2e-16 ***
## Highest_level99
                    0.139335
                               2.766755
                                          0.050 0.959835
                               0.049750 -3.707 0.000210 ***
## Cellphone use2
                   -0.184407
```

```
## Cellphone use9
                    2.465357
                               1.721898
                                        1.432 0.152217
## Internet use2
                               0.044558
                                          3.797 0.000147 ***
                    0.169168
                               1.144391 -1.977 0.048094 *
## Internet use9
                   -2.261993
## Computer use2
                   0.123892
                               0.030536 4.057 4.97e-05 ***
## Computer use9
                   -0.206686
                               0.627158 - 0.330 \ 0.741734
                               0.001027 \ 109.967 < 2e-16 ***
## Age
                    0.112973
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.755 on 45852 degrees of freedom
##
     (4115 observations deleted due to missingness)
## Multiple R-squared: 0.6396, Adjusted R-squared: 0.6393
## F-statistic: 2393 on 34 and 45852 DF, p-value: < 2.2e-16
```

```
mod1_log_output <- broom::augment(mod1_log, newdata = data_new)
head(mod1_log_output)</pre>
```

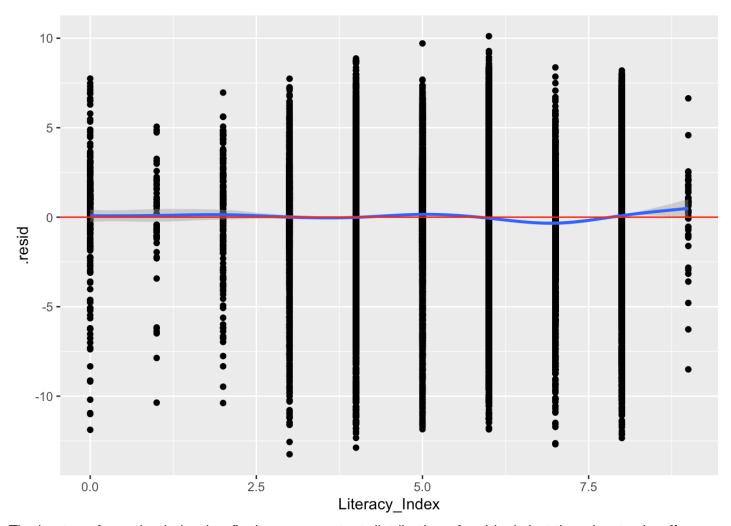
```
## # A tibble: 6 x 35
     Ind Interview REGION MAS 500 AGLOMERADO Relative Rel Sex
##
                                                                     Age Civil State
##
             <dbl> <fct> <chr>
                                        <dbl> <fct>
                                                            <fct> <dbl> <fct>
## 1
                 1 43
                           S
                                             2 1
                                                            1
                                                                      44 3
                 1 43
                                             2 1
                                                                      59 2
## 2
                           S
                                                            1
## 3
                 1 43
                           S
                                             2 2
                                                            2
                                                                      62 2
## 4
                 1 43
                           S
                                             2 3
                                                            1
                                                                      26 5
## 5
                 1 43
                                             2 3
                           S
                                                            1
                                                                      23 5
                                             2 1
                                                                      26 5
## 6
                 1 43
                           S
                                                            2
  # ... with 27 more variables: Type of school <fct>, Highest level <fct>,
##
## #
       finished? <fct>, last_yr_approved <chr>, Birth_Location <fct>,
       Location 5y <dbl>, CAT OCUP <fct>, JOB N <dbl>, Income individual <dbl>,
## #
       ITF <dbl>, person id <dbl>, Internet use <fct>, Computer use <fct>,
## #
       Cellphone use <fct>, House_Type <fct>, Room_N <dbl>, Ownership <fct>,
## #
## #
       Self Room <dbl>, Self Room Sleep <dbl>, Studio <fct>, Studio N <dbl>,
       Computer house <fct>, Internet house <fct>, Literacy Index <dbl>,
## #
## #
       log income <dbl>, .fitted <dbl>, .resid <dbl>
```

```
# Residuals vs. Literacy index
ggplot(mod1_log_output, aes(x = Literacy_Index, y = .resid)) +
    geom_point() +
    geom_smooth() +
    geom_hline(yintercept = 0, color = "red")
```

```
## geom_smooth() using method = 'gam' and formula 'y ~ s(x, bs = "cs")'
```

## Warning: Removed 4115 rows containing non-finite values (stat\_smooth).

## Warning: Removed 4115 rows containing missing values (geom\_point).



The log transformation helped to fix the non-constant distribution of residuals but there is a trade-off on interpretability. We are staying with natural scales in investigation #1.

Compare insights from variable importance analyses from the different methods (stepwise and LASSO, but not OLS). Are there variables for which the methods reach consensus? What insights are expected? Surprising?

• Note that if some (but not all) of the indicator terms for a categorical predictor are selected in the final models, the whole predictor should be treated as selected.

## **LASSO**

```
# Create a boolean matrix (predictors x lambdas) of variable exclusion
bool_predictor_exclude <- lasso_mod$finalModel$beta==0

# Loop over each variable
var_imp <- sapply(seq_len(nrow(bool_predictor_exclude)), function(row) {
    # Extract coefficient path (sorted from highest to lowest lambda)
    this_coeff_path <- bool_predictor_exclude[row,]
    # Compute and return the # of lambdas until this variable is out forever
    ncol(bool_predictor_exclude)-which.min(this_coeff_path)+1
})

# Create a dataset of this information and sort
var_imp_data <- tibble(
    var_name = rownames(bool_predictor_exclude),
    var_imp = var_imp
)
var_imp_data %>% arrange(desc(var_imp))
```

```
## # A tibble: 34 x 2
##
      var name
                      var imp
      <chr>
                        <dbl>
##
                           75
##
   1 Cellphone use9
##
    2 CAT OCUP3
                           74
##
    3 Age
                           74
    4 Civil State5
                           73
##
## 5 Literacy Index
                           70
##
    6 Highest level8
                           66
   7 CAT OCUP1
                           65
##
##
    8 Sex2
                           64
## 9 Highest level7
                           64
## 10 Computer use2
                           62
## # ... with 24 more rows
```

#### PUT ANY RELEVANT TEXT/RESPONSES/INTERPRETATIONS HERE

In this case, the more persistent variables for LASSO are the same selected in backward stepwise selection except for the most persistent predictor for this model, which seems to be the Cellphone\_use9 variable, which is not very relevant considering that it is NA's for "Use of cellphone in the last three months". It is a bit surprising considering we always talk about how backward and forward subset selection may not arrive to the same "best" model because they don't compute all the possibilities but generally they do their job. Leaving aside the Cellphone\_use9, from the analysis we can see that Lasso method does coincide that the most important predictors are CAT\_OCUP3, which corresponds to the category employee/worker, Civil\_State5, which corresponds to being single and the Literacy index. Additionally we see the lasso model picked up Highest\_level8, which corresponds to having attended to Graduate School (Masters, MBA, PhD), in the same way subset selection did.

# **Investigation 2: Accounting for nonlinearity**

Update your stepwise selection model(s) and LASSO model to use natural splines for the quantitative predictors.

- You'll need to update the model formula from y ~ . to something like
   y ~ cat\_var1 + ns(quant\_var1, df) + ....
- It's recommended to use few knots (e.g., 2 knots = 3 degrees of freedom).
- Note that ns(x,3) replaces x with 3 transformations of x. Keep this in mind when setting nvmax in stepwise selection.

## **Stepwise**

```
set.seed(253)

spline_back_step_mod2 <- train(
    Income_individual~ ns(Literacy_Index,3)+Sex+Birth_Location+CAT_OCUP+Civil_State+H
ighest_level+Cellphone_use+Internet_use+ Computer_use+ns(Age,3),
    data = data2019,
    method = "leapBackward",
    tuneGrid = data.frame(nvmax = 1:14), #nvmax=number or vars.
    trControl = trainControl(method = "cv", number = 10),
    metric = "MAE",
    na.action = na.omit
)</pre>
```

```
## Warning in leaps.setup(x, y, wt = weights, nbest = nbest, nvmax = nvmax, : 1
## linear dependencies found
```

```
## Reordering variables and trying again:
```

```
summary(spline_back_step_mod2)
```

```
## Subset selection object
## 38 Variables (and intercept)
##
                           Forced in Forced out
## ns(Literacy Index, 3)1
                               FALSE
                                          FALSE
## ns(Literacy Index, 3)2
                                          FALSE
                               FALSE
## ns(Literacy Index, 3)3
                               FALSE
                                          FALSE
## Sex2
                               FALSE
                                          FALSE
## Birth Location2
                                          FALSE
                               FALSE
## Birth Location3
                               FALSE
                                          FALSE
## Birth Location4
                               FALSE
                                          FALSE
## Birth Location5
                               FALSE
                                          FALSE
```

```
## Birth Location9
                                FALSE
                                            FALSE
## CAT OCUP1
                                FALSE
                                            FALSE
## CAT_OCUP2
                                FALSE
                                            FALSE
## CAT OCUP3
                                FALSE
                                            FALSE
## CAT OCUP4
                                FALSE
                                            FALSE
## CAT OCUP9
                                FALSE
                                            FALSE
## Civil_State2
                                FALSE
                                            FALSE
## Civil_State3
                                FALSE
                                            FALSE
## Civil State4
                                FALSE
                                            FALSE
## Civil State5
                                FALSE
                                            FALSE
## Civil State9
                                FALSE
                                            FALSE
## Highest_level1
                                FALSE
                                            FALSE
## Highest_level2
                                FALSE
                                            FALSE
## Highest level3
                                FALSE
                                            FALSE
## Highest level4
                                FALSE
                                            FALSE
## Highest_level5
                                FALSE
                                            FALSE
## Highest_level6
                                FALSE
                                            FALSE
## Highest_level7
                                FALSE
                                            FALSE
## Highest level8
                                FALSE
                                            FALSE
## Highest level9
                                FALSE
                                            FALSE
## Highest level99
                                FALSE
                                            FALSE
## Cellphone use2
                                FALSE
                                            FALSE
## Cellphone_use9
                                FALSE
                                            FALSE
## Internet use2
                                FALSE
                                            FALSE
## Internet use9
                                FALSE
                                            FALSE
## Computer use2
                                FALSE
                                            FALSE
## Computer use9
                                FALSE
                                            FALSE
## ns(Age, 3)1
                                FALSE
                                            FALSE
## ns(Age, 3)2
                                FALSE
                                            FALSE
## ns(Age, 3)3
                                FALSE
                                            FALSE
## 1 subsets of each size up to 14
## Selection Algorithm: backward
##
              ns(Literacy_Index, 3)1 ns(Literacy_Index, 3)2 ns(Literacy_Index, 3)3
## 1
      (1)
                                       .. ..
                                                                .. ..
              . .
## 2
      (1)
                                                                " * "
## 3
      (1)
              .. ..
##
   4
      (1)
##
      (1)
      (1)
##
   6
   7
        1)
##
                                                                " * "
## 8
      (1)
                                       .. ..
##
   9
      (1)
                                                                " * "
##
       (1)
   10
## 11
       (1)
       ( 1
##
   12
##
       (1)
                                                                " + "
   13
                                                                " * "
## 14
         1)
```

```
##
               Sex2 Birth Location2 Birth Location3 Birth Location4 Birth Location5
##
         1)
   1
##
         1
            )
##
   3
         1
##
         1
##
   5
         1
##
   6
         1
##
   7
         1
           )
##
   8
         1
##
   9
         1
##
   10
          1
                " * "
##
   11
          1
##
   12
           1
##
   13
          1
##
   14
          1
##
               Birth Location9 CAT OCUP1 CAT OCUP2 CAT OCUP3 CAT OCUP4 CAT OCUP9
                                   "
                                                                         "
##
   1
       (1)
                                                                       "
                                                                         - 11
##
   2
         1
##
   3
         1
           )
##
         1
                                                           " * "
##
   5
         1
##
   6
         1)
                                                           " * "
                                   " * "
##
         1
            )
##
   8
         1
                                   " * "
                                                           " * "
         1
##
   9
##
   10
          1
##
   11
                                                           " * "
##
   12
          1
                                                           " * "
                                               " * "
##
          1
   13
                                                           " * "
          1)
##
   14
##
               Civil State2 Civil State3 Civil State4 Civil State5 Civil State9
##
         1)
   1
##
   2
         1)
##
   3
         1
       (
            )
##
         1
##
   5
         1
##
         1
   6
##
   7
         1
##
   8
         1
##
         1
   9
##
   10
          1
##
   11
           1
##
   12
          1
##
   13
          1
##
   14
          1
##
               Highest_level1 Highest_level2 Highest_level3 Highest_level4
       (1)
##
   1
```

```
##
       (1)
##
   3
         1
         1)
##
##
   5
         1
##
##
   7
         1)
##
   8
         1
   9
        1)
##
##
   10
          1
##
   11
          1
                                ##
   12
          1
##
   13
        ( 1
            )
##
          1)
   14
##
               Highest_level5 Highest_level6 Highest_level7 Highest_level8
##
       (1)
   1
##
        1)
       (
##
   3
        1)
##
   4
         1
         1)
##
   5
##
   6
         1
           )
         1)
                                                                   " * "
##
   7
                                . .
##
   8
         1)
                                                                   " * "
##
   9
       (
         1)
##
   10
          1)
                                                                   ....
##
   11
          1
##
   12
          1
                                " * "
                                                                   " * "
        (1)
##
   13
##
   14
          1)
                                                                   " * "
##
               Highest_level9 Highest_level99 Cellphone_use2 Cellphone_use9
                                                                    11 11
##
   1
       (1)
                                                                      "
##
         1)
   2
       (
##
   3
         1)
                                11 11
##
   4
         1)
         1)
##
   5
       (
##
   6
         1)
##
         1)
                                11 11
##
   8
         1
##
   9
       ( 1
           )
##
   10
          1)
##
   11
          1
##
   12
        (1)
##
   13
          1
##
          1)
   14
##
               Internet_use2 Internet_use9 Computer_use2 Computer_use9 ns(Age, 3)1
       (1)
##
   1
                                                                                " * "
##
   2
         1)
       (
                                                                                " * "
##
   3
       (1)
```

```
" * "
##
       (1)
                                                                               " * "
## 5
       (1)
                                                                               " * "
        1)
## 6
                                               .. ..
                                                                               " * "
##
   7
         1)
##
   8
##
   9
       (1)
##
   10
          1
                                               " * "
##
   11
        (1)
##
   12
          1
                                               " * "
        (1)
## 13
                                               " * "
## 14
        (1)
##
               ns(Age, 3)2 ns(Age, 3)3
## 1
       (1)
       (1)
##
   2
                              "
       (1)
##
   3
##
## 5
       (1)
##
         1)
##
   7
       (1)
##
   8
        1)
                            " * "
       (1)
##
   9
## 10
        (1)
               " * "
##
   11
         1
        ( 1
## 12
                             " * "
## 13
        ( 1
        (1)
## 14
```

```
# Look at accuracy/error metrics for the different subset sizes
# If you want to sort the table of results, use arrange() from dplyr
spline_back_step_mod2$results %>% arrange(MAE)
```

```
##
      nvmax
                RMSE
                       Rsquared
                                     MAE
                                            RMSESD RsquaredSD
                                                                 MAESD
## 1
         14 21165.22 0.24107381 10809.47 3724.964 0.04384898 131.0383
##
  2
         13 21196.66 0.23884205 10821.67 3733.309 0.04507788 155.8003
##
         12 21205.57 0.23817622 10824.09 3732.406 0.04490156 153.6167
  3
         11 21222.37 0.23692563 10834.52 3724.088 0.04425554 151.7327
##
## 5
         10 21300.01 0.23118036 10894.72 3713.738 0.04292123 148.9044
##
  6
          9 21354.86 0.22710708 10903.06 3710.179 0.04174570 137.6669
          8 21438.07 0.22100174 11075.57 3718.096 0.04197517 178.0557
## 7
##
  8
          7 21577.65 0.21086429 11190.32 3779.432 0.04513555 200.0228
          6 21670.44 0.20389066 11294.99 3762.111 0.04313215 171.6772
## 9
          4 21910.07 0.18577803 11434.75 3783.104 0.04575279 495.5613
## 10
## 11
          5 21822.13 0.19241715 11450.29 3779.249 0.04741749 479.5312
## 12
          3 22031.99 0.17647248 11639.67 3772.488 0.04423440 450.7492
## 13
          2 22461.53 0.14344219 11996.36 3770.294 0.04006222 379.2329
          1 23219.11 0.08284268 13193.21 3657.685 0.02120588 180.4436
## 14
```

spline\_back\_step\_mod2\$bestTune

```
## nvmax
## 14 14
```

# Obtain the coefficients for the best model
coef(spline back step mod2\$finalModel, id = spline back step mod2\$bestTune\$nvmax)

```
##
               (Intercept) ns(Literacy Index, 3)3
                                                                        Sex2
##
                 10286.720
                                          14013.732
                                                                   -4308.312
##
          Birth Location3
                                          CAT OCUP1
                                                                   CAT OCUP2
                  2030.464
                                                                    6182.729
##
                                          17596.431
##
                 CAT OCUP3
                                       Civil State5
                                                             Highest level6
                 13801.351
                                          -4526.323
                                                                   -3234.324
##
           Highest level8
##
                                      Internet use2
                                                               Computer use2
##
                 24420.379
                                          -2376.977
                                                                   -3280.910
##
               ns(Age, 3)1
                                        ns(Age, 3)2
                                                                 ns(Age, 3)3
##
                 13810.487
                                          19185.984
                                                                   21795.493
```

#### **LASSO**

```
set.seed(253)
spline_lasso_mod2 <- train(
    Income_individual ~ ns(Literacy_Index,3)+Sex+Birth_Location+CAT_OCUP+Civil_State+
Highest_level+Cellphone_use+Internet_use+ Computer_use+ns(Age,3),
    data = data2019,
    method = "glmnet",
    trControl = trainControl(method = "cv", number = 10, selectionFunction = "oneSE")
,
    tuneGrid = data.frame(alpha = 1, lambda = seq(0,1000, length.out = 100)),#Ask Les
lie about length.out
    metric = "MAE",
    na.action = na.omit
)</pre>
```

```
# Identify which tuning parameter (lambda) is "best"
spline_lasso_mod2$bestTune
```

```
## alpha lambda
## 54 1 535.3535
```

```
coef(spline_lasso_mod2$finalModel, lasso_mod$bestTune$lambda)
```

```
## 39 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept)
                          11775.7161
## ns(Literacy Index, 3)1
## ns(Literacy_Index, 3)2
## ns(Literacy Index, 3)3 9487.5276
## Sex2
                          -3319.9355
## Birth_Location2
## Birth_Location3
                            674.0065
## Birth Location4
## Birth Location5
## Birth Location9
## CAT OCUP1
                         11278.0200
## CAT_OCUP2
                          2017.8451
## CAT OCUP3
                          10699.1916
## CAT_OCUP4
## CAT OCUP9
## Civil State2
## Civil_State3
## Civil State4
                          1951.8274
## Civil State5
                          -5057.9354
## Civil State9
## Highest level1
## Highest_level2
                          -136.9092
## Highest_level3
## Highest level4
## Highest_level5
## Highest level6
## Highest_level7
                           2166.9123
                          21113.3501
## Highest_level8
## Highest level9
## Highest level99
## Cellphone use2
## Cellphone use9
## Internet_use2
## Internet use9
## Computer use2
                          -1847.6912
## Computer_use9
## ns(Age, 3)1
                          16152.1577
## ns(Age, 3)2
                           5921.7745
## ns(Age, 3)3
                           2078.5224
```

```
# Create a boolean matrix (predictors x lambdas) of variable exclusion
bool_predictor_exclude <- spline_lasso_mod2$finalModel$beta==0

# Loop over each variable
var_imp <- sapply(seq_len(nrow(bool_predictor_exclude)), function(row) {
    # Extract coefficient path (sorted from highest to lowest lambda)
    this_coeff_path <- bool_predictor_exclude[row,]
    # Compute and return the # of lambdas until this variable is out forever
    ncol(bool_predictor_exclude)-which.min(this_coeff_path)+1
})

# Create a dataset of this information and sort
var_imp_data <- tibble(
    var_name = rownames(bool_predictor_exclude),
    var_imp = var_imp
)
var_imp_data %>% arrange(desc(var_imp))
```

```
## # A tibble: 38 x 2
##
      var name
                             var imp
      <chr>
                                <dbl>
##
## 1 Cellphone use9
                                   74
##
    2 CAT OCUP3
                                   73
## 3 Civil State5
                                   73
## 4 ns(Age, 3)1
                                   71
                                   70
## 5 ns(Literacy Index, 3)3
## 6 Highest level8
                                   65
                                   64
## 7 ns(Age, 3)2
## 8 Sex2
                                   63
## 9 CAT OCUP1
                                   63
## 10 Highest level7
                                   60
## # ... with 28 more rows
```

Compare insights from variable importance analyses here and the corresponding results from Investigation 1. Now after having accounted for nonlinearity, have the most relevant predictors changed?

 Note that if some (but not all) of the spline terms are selected in the final models, the whole predictor should be treated as selected.

## PUT ANY RELEVANT TEXT/RESPONSES/INTERPRETATIONS HERE

Using splines transformations for quantitative variables showed us that they remain among the most important predictors when we account for non-linearity in both methods. When one transformation is selected, it accounts for the importance of the whole variable. We can further the exploration using the GAMs \

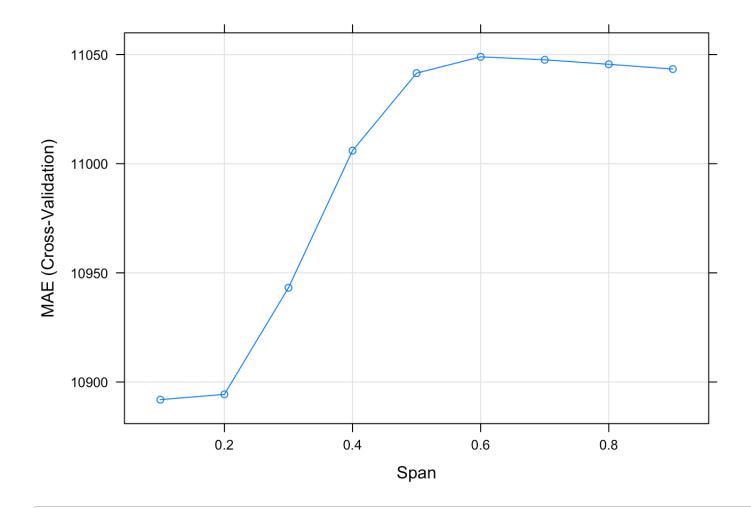
Fit a GAM using LOESS terms using the set of variables deemed to be most relevant based on your investigations so far.

```
# Your code
set.seed(253)
gam mod <- train(</pre>
    Income individual ~ Literacy Index+Sex+Birth Location+CAT OCUP+Civil State+Highes
t level+Cellphone use
    +Internet use+Computer use+Age,
    data = data2019,
    method = "gamLoess",
    tuneGrid = data.frame(degree = 1, span = seq(0.1, 0.9, by = 0.1)),
    trControl = trainControl(method = "cv", number = 8, selectionFunction = "best"),
    metric = "MAE",
    na.action = na.omit
)
## Warning in gam.lo(data[["lo(Age, span = 0.1, degree = 1)"]], z, w, span = 0.1, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.1, degree = 1)"]], z, w, span = 0.1, :
## upperlimit 99.445
## Warning in gam.lo(data[["lo(Age, span = 0.1, degree = 1)"]], z, w, span = 0.1, :
## extrapolation not allowed with blending
## Warning in gam.lo(data[["lo(Age, span = 0.2, degree = 1)"]], z, w, span = 0.2, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.2, degree = 1)"]], z, w, span = 0.2, :
## upperlimit 99.445
## Warning in gam.lo(data[["lo(Age, span = 0.2, degree = 1)"]], z, w, span = 0.2, :
## extrapolation not allowed with blending
## Warning in gam.lo(data[["lo(Age, span = 0.3, degree = 1)"]], z, w, span = 0.3, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.3, degree = 1)"]], z, w, span = 0.3, :
## upperlimit 99.445
```

```
## Warning in gam.lo(data[["lo(Age, span = 0.3, degree = 1)"]], z, w, span = 0.3, :
## extrapolation not allowed with blending
## Warning in gam.lo(data[["lo(Age, span = 0.4, degree = 1)"]], z, w, span = 0.4, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.4, degree = 1)"]], z, w, span = 0.4, :
## upperlimit 99.445
## Warning in gam.lo(data[["lo(Age, span = 0.4, degree = 1)"]], z, w, span = 0.4, :
## extrapolation not allowed with blending
## Warning in gam.lo(data[["lo(Age, span = 0.5, degree = 1)"]], z, w, span = 0.5, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.5, degree = 1)"]], z, w, span = 0.5, :
## upperlimit 99.445
## Warning in gam.lo(data[["lo(Age, span = 0.5, degree = 1)"]], z, w, span = 0.5, :
## extrapolation not allowed with blending
## Warning in gam.lo(data[["lo(Age, span = 0.6, degree = 1)"]], z, w, span = 0.6, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.6, degree = 1)"]], z, w, span = 0.6, :
## upperlimit 99.445
## Warning in gam.lo(data[["lo(Age, span = 0.6, degree = 1)"]], z, w, span = 0.6, :
## extrapolation not allowed with blending
## Warning in gam.lo(data[["lo(Age, span = 0.7, degree = 1)"]], z, w, span = 0.7, :
## eval 100
## Warning in gam.lo(data[["lo(Age, span = 0.7, degree = 1)"]], z, w, span = 0.7, :
## upperlimit 99.445
```

```
## Warning in gam.lo(data[["lo(Age, span = 0.7, degree = 1)"]], z, w, span = 0.7, :
 ## extrapolation not allowed with blending
 ## Warning in gam.lo(data[["lo(Age, span = 0.8, degree = 1)"]], z, w, span = 0.8, :
 ## eval 100
 ## Warning in gam.lo(data[["lo(Age, span = 0.8, degree = 1)"]], z, w, span = 0.8, :
 ## upperlimit 99.445
 ## Warning in gam.lo(data[["lo(Age, span = 0.8, degree = 1)"]], z, w, span = 0.8, :
 ## extrapolation not allowed with blending
 ## Warning in gam.lo(data[["lo(Age, span = 0.9, degree = 1)"]], z, w, span = 0.9, :
 ## eval 100
 ## Warning in gam.lo(data[["lo(Age, span = 0.9, degree = 1)"]], z, w, span = 0.9, :
 ## upperlimit 99.445
 ## Warning in gam.lo(data[["lo(Age, span = 0.9, degree = 1)"]], z, w, span = 0.9, :
 ## extrapolation not allowed with blending
Cellphone_use9 75
CAT OCUP3 74
Age 74
Civil State 573
Literacy Index 70
Highest_level8 66
CAT_OCUP1 65
Sex2
```

plot(gam\_mod)

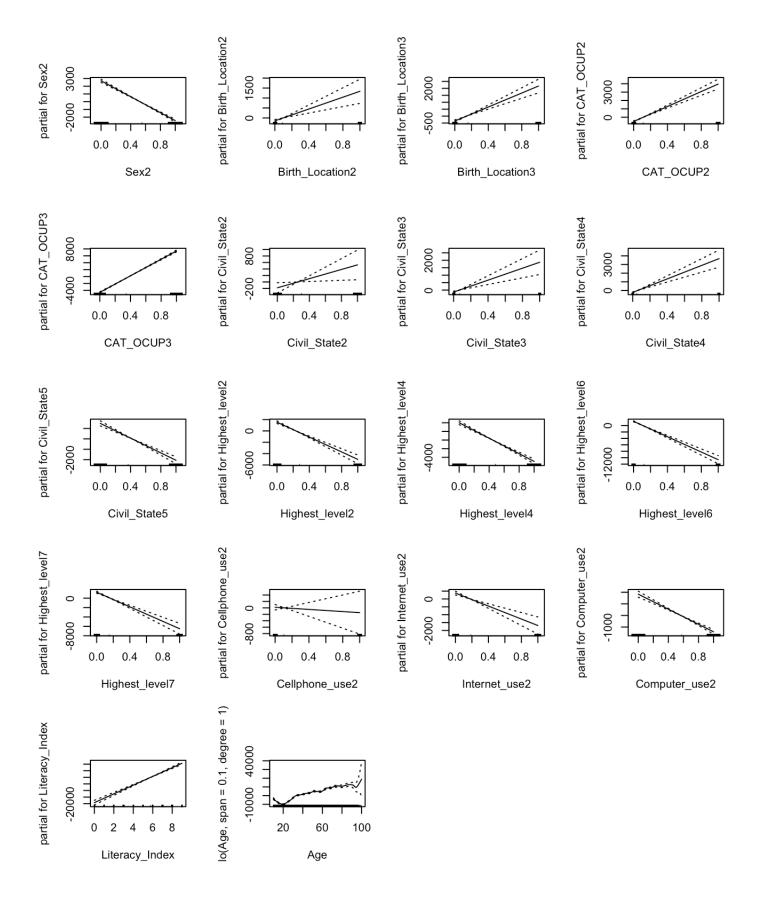


gam\_mod\$bestTune

## span degree
## 1 0.1 1

gam\_mod\$results %>%
 filter(span==gam\_mod\$bestTune\$span)

par(mfrow = c(3,4)) # Sets up a grid of plots
plot(gam\_mod\$finalModel, se = TRUE) # Dashed lines are +/- 2 SEs



#### PUT ANY RELEVANT TEXT/RESPONSES/INTERPRETATIONS HERE

• How does test performance of the GAM compare to other models you explored?

The MAE of the GAM is 10891.92. This means that on average this model is off in its percentage predictions about a person's income by about 10892 pesos.

Do you gain any insights from the GAM output plots for each predictor?

Since most of the variables are categorical (indicator) variables, we observe linear plots. We can see that among individuals that are same in all other characteristics, those of Sex2 and Civil\_State5 have lower average income than others. On the other hand, among individuals that are same in all other characteristics, those of CAT\_OCUP 2&3 and Birth\_Location 2&3 have higher average income than others. \

# Summarize investigations

Decide on an overall best model based on your investigations so far. To do this, make clear your analysis goals. Predictive accuracy? Interpretability? A combination of both?

Our analysis goal is to accurately predict the income of individuals based on different factors, which we believe will ultimately help us predict their digital access and consequent level of education disruption during COVID. Of course, we are hoping that these predicts will be interpretable as well. The overall best model so far for this is the Lasso, which had a MAE of 10777, the lowest so far.

Societal impact

Ok, but perhaps in considering the MAESD there aren't huge differences between the models. Simplicity and variable importance investigations might help distinguish preferred models in this case.

Are there any harms that may come from your analyses and/or how the data were collected? What cautions do you want to keep in mind when communicating your work?

It is important that we communicate our analyses in a nuanced and non-exaggerated way, based on the relatively significant level of error we have faced so far. If not, this could misguide development programs working in education and digital access. This particular set of analyses, which focuses on income as an outcome of interest, has to effectively address intersectionality and the connection between different social dynamics, in order to ensure holistic development efforts, which do not discriminate against a certain group of people based on pre-existing conditions. If educational and digital access related programmes are not carried out effectively - based on solid evidence - then they could impact the learning, growth and consequently life of many people, causing more harm than good. Thus, it is important to exercise the utmost caution to ensure accuracy and mindfulness of analyses.