

S. 145 Nr. 7

Hauptbedingung: $A = a \times b$

Nebenbedingung: $a = x$; $b = f(x)$

Zielfunktion: $Z(x) = x \times f(x)$

$$\Leftrightarrow Z(x) = x \times e^{-x}$$

$$u = x; v = e^{-x}; u' = 1; v' = -e^{-x}$$

$$Z'(x) = 1 \times e^{-x} + (-e^{-x} \times x)$$

$$\Leftrightarrow Z'(x) = e^{-x} \times (1 - x)$$

$$u_2 = e^{-x}; u'_2 = -e^{-x}; v_2 = 1 - x; v'_2 = -1$$

$$Z''(x) = -1 \times e^{-x} + (-e^{-x} \times (1 - x))$$

$$\Leftrightarrow -e^{-x} \times (1 - x)$$

notwendige Bedingung für EST: $Z'(x) = 0$

$$Z'(x) = 0$$

$$\Leftrightarrow e^{-x} \times (1 - x) = 0 \quad e^{-x} \neq 0 \quad \forall x \in \mathbb{R}$$

$$\Leftrightarrow 1 - x = 0 \quad | -1$$

$$\Leftrightarrow -x = -1 \quad | \times (-1)$$

$$\Leftrightarrow x = 1$$

hinreichende Bedingung für EST: $Z'(x) = 0 \wedge Z''(x) \neq 0$

$$Z''(1) = -e^{-1} \times (1 - 1)$$

$$\Leftrightarrow -e^{-1} \times 0$$

$$= 0$$

VZW:

$$Z'(0) = e^{-0} \times (1 - 0)$$

$$\Leftrightarrow Z'(0) = 1 \times 1 = 1$$

$$Z'(2) = e^{-2} \times (1 - 2)$$

$$\Leftrightarrow Z'(2) = e^{-2} \times (-1)$$

Y-Wert: $f(1) = e^{-1}$

Punkt P bei $P(1|e^{-1})$

Flächeninhalt: $A = 1 \times e^{-1} = e^{-1}$

S. 145 Nr. 8

$$f(x) = 10x \times e^{-x^2}$$

$$\text{Hauptbedingung: } A = \frac{a \times b}{2}$$

$$\text{Nebenbedingung: } a = x; b = f(x)$$

$$\text{Zielfunktion: } A(x) = \frac{x \times f(x)}{2}$$

$$\Leftrightarrow x \times 10x \times e^{-x^2} \times \frac{1}{2}$$

$$\Leftrightarrow 5x^2 \times e^{-x^2}$$

Ableitung:

$$u = 5x^2; v = e^{-x^2}; u' = 10x;$$

$$s = e^x; t = -x^2; s' = e^x; t' = -2x;$$

$$v' = s'(t(x)) \times t'(x)$$

$$v' = e^{-x^2} \times (-2x)$$

$$A'(x) = 5x^2 \times (e^{-x^2} \times (-2x)) + 10x \times e^{-x^2}$$

$$\Leftrightarrow A'(x) = e^{-x^2} \times (10x + 5x^2 \times (-2x))$$

$$\Leftrightarrow A'(x) = e^{-x^2} \times (10x - 10x^3)$$

$$u_2 = e^{-x^2}; v_2 = 10x - 10x^3;$$

$$s = e^x; t = -x^2; s' = e^x; t' = -2x;$$

$$u'_2 = s'(t(x)) \times t'(x) = e^{-x^2} \times (-2x);$$

$$v'_2 = 10 - 30x^2$$

$$A''(x) = e^{-x^2} \times (-2x) \times (10x - 10x^3) + (10 - 30x^2) \times e^{-x^2}$$

$$\Leftrightarrow A''(x) = e^{-x^2} \times (-20x^2 - 30x^2 - 10x^3 + 10)$$

$$\Leftrightarrow A''(x) = e^{-x^2} \times (-50x^2 - 10x^3 + 10)$$

notwendige Bedingung für EST: $A'(x) = 0$

$$A'(x) = 0$$

$$e^{-x^2} \times (10x - 10x^3) = 0 \quad e^{-x^2} \neq 0 \quad \forall x \in \mathbb{R}$$

$$\Leftrightarrow 10x - 10x^3 = 0$$

$$\Leftrightarrow x \times (10 - 10x^2) = 0$$

Satz des Nullproduktes: $x_1 = 0$

$$10 - 10x^2 = 0 \quad | -10$$

$$\Leftrightarrow -10x^2 = -10 \quad | \div (-10)$$

$$\Leftrightarrow x^2 = 1 \quad | \sqrt{}$$

$$\Leftrightarrow x = \sqrt{1}$$

$$x_2 = 1$$

hinreichende Bedingung für EST: $A'(x) = 0 \wedge A''(x) \neq 0$

$$A''(0) = e^{-0^2} \times (-50 \times 0^2 - 10 \times 0^3 + 10)$$

$$\Leftrightarrow 1 \times (0 - 0 + 10)$$

$$\Leftrightarrow 10 > 0 \rightarrow TP$$

$$A''(1) = e^{-1^2} \times (-50 \times 1^2 - 10 \times 1^3 + 10)$$

$$\Leftrightarrow e^1 \times (-50 \times 1 - 10 \times 1 + 10)$$

$$\Leftrightarrow e^1 \times (-50 - 10 + 10)$$

$$\Leftrightarrow -50e < 0 \rightarrow HP$$

TP entfällt im Sachzusammenhang.

Flächeninhalt:

$$A(-50e) = 5 \times (-50e)^2 \times e^{50e^2}$$

$$A(-50e) = 5 \times 2500e^2 \times e^{50e^2}$$

$$A(-50e) = 12500e^2 \times e^{50e^2} FE$$