Clase 4 (Youtube)

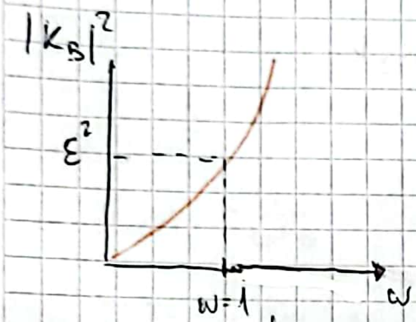
Aproximación de chebyshev y Bessel

chebyshev

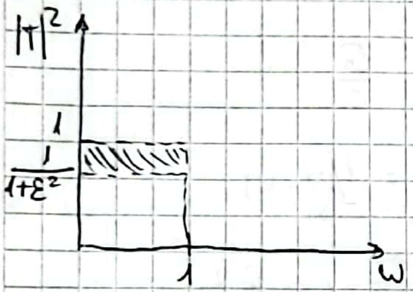
Recordando para Butter

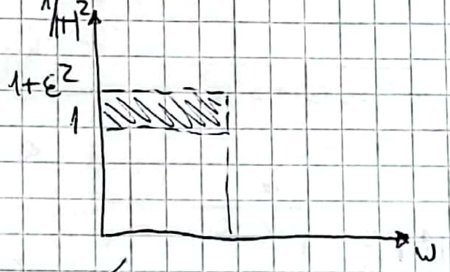
$$|T_B(j\omega)|^2 = \frac{1}{1+\underbrace{\varepsilon^2\omega^{2n}}_{\varepsilon \text{ en butter } = 1}} \rightarrow |K(j\omega)|^2$$

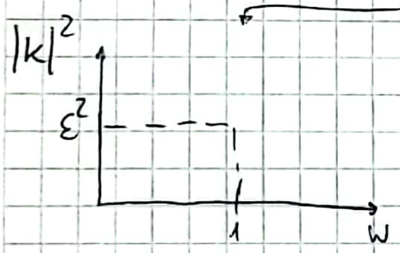
ε en butter = 1, Max planicidad ≠ 1

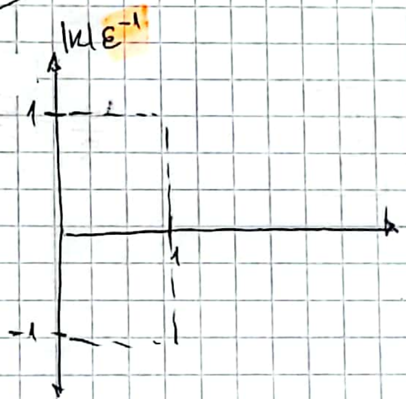


Queremos ver que hace $K(j\omega)$









$$\varepsilon^{-1}|K| = Y$$

$$Y = \varepsilon \cos(nx)$$

$$x = \cos^{-1}(\omega)$$

$$|K| = \varepsilon \cos[n \cos^{-1}(\omega)]$$

$$\cos^{-1}\omega = jz \quad , \quad |\omega| > 1$$

$$\omega = \cos(jz) \quad , \quad \underbrace{\frac{e^{j(jz)} + e^{-j(jz)}}{2}} = \cosh(z)$$

$$\omega = \cosh(z)$$

$$z = \cosh^{-1}(\omega)$$

$$|K| = \varepsilon \cos[j \cosh^{-1}(\omega)] = \cosh[\cosh^{-1}(\omega)]$$

NOTA

HOJA N°

FECHA

$|K| = \varepsilon \cosh[n \cosh^{-1}(\omega)]$

$|K| = C_n(\omega) := \varepsilon \cosh[n \cosh^{-1}(\omega)]$

$$|T_C(j\omega)|^2 = \frac{1}{1 + C_n^2(\omega)}$$

$$\cos n\theta = 2^{n-1} \cos^n \theta - \frac{n}{1!} 2^{n-3} \cos^{n-2} \theta + \frac{n(n-1)}{2!} 2^{n-5} \cos^{n-4} \theta -$$
$$- \frac{n(n-3)(n-5)}{3!} 2^{n-7} \cos^{n-6} \theta \ldots$$

Si $\theta = \cos^{-1}(\omega)$,

$$C_n(\omega) = \cos[n \cos^{-1}(\omega)] = 2^{n-1} \omega^n - \frac{n}{1!} 2^{n-3} \cdot \omega^{n-2} + \ldots$$

$$\boxed{C_n(\omega) = 2\omega\, C_{n-1}(\omega) - C_{n-2}(\omega)}$$

$C_0(\omega) = 1$

$C_1(\omega) = \omega$

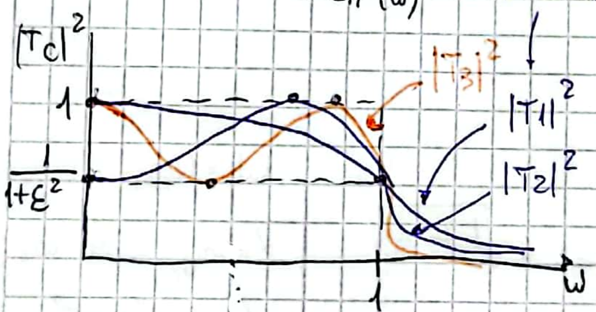
$C_2(\omega) = 2\omega \cdot \omega - 1 = 2\omega^2 - 1$

$C_3(\omega) = 2\omega(2\omega^2 - 1) - \omega = 4\omega^3 - 2\omega - \omega = 4\omega^3 - 3\omega$

$$|T_C(j\omega)|^2 = \frac{1}{1 + C_n^2(\omega)}$$

chebyshev 1° orden

$C_n(\omega) = \varepsilon \cos[n \cos^{-1}(\omega)]$



La cantidad de veces que "toca" entre 1 y $\frac{1}{1+\varepsilon^2}$ es el orden n

$\underline{\omega = \emptyset}$

$\cos^{-1}(\emptyset) = \pi/2$

$\cos(n\,\pi/2)$

n impar $\rightarrow 1$

n par $\rightarrow \frac{1}{1+\varepsilon^2}$

$\omega\ 0 \longrightarrow 1$

$\omega = 1$

$\cos^{-1}(1)$

$\cos(n k 2\pi)$

$1 \ \forall n$ ← "1 para todo n"

NOTA

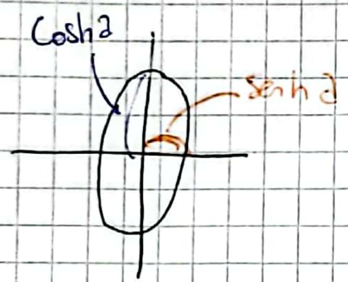
Localización de Polos y Ceros de $|Tc|^2$

$$|Tc(jw)|^2 = \frac{1}{1+C_n^2(w)}\Bigg|_{w=\frac{s}{j}} = \frac{1}{1+C_n^2(s/j)} = T(s)\,T(-s)$$

$$C_n(s/j) = \varepsilon \cosh\left[n \cosh^{-1}(s/j)\right]$$

$$\left.\begin{aligned} \sigma_k &= -\operatorname{Senh} a \operatorname{sen}\left(\frac{2k-1}{2n}\pi\right) \\ w_k &= \cosh a \cos\left(\frac{2k-1}{2n}\pi\right) \end{aligned}\right\}\ k = 1, 2, \dots, n.$$

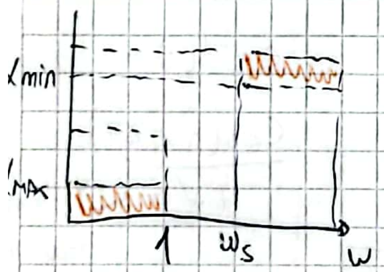
$$\frac{\sigma_k^2}{\operatorname{Senh}(a)} + \frac{w_k^2}{\cosh(a)} = \operatorname{Sen}^2(x) + \cos^2(x) = 1$$



$$a = \frac{1}{n} \operatorname{Senh}^{-1}\frac{1}{\varepsilon}$$

$$|Tc(jw)|^2 = \frac{1}{1+C_n^2(w)}$$

$$C_n(w) = \varepsilon \cdot \cosh\left[n \cdot \cosh^{-1}(w)\right]$$



$\underline{w=1}$

$$\alpha_{MAX} = \frac{1}{|T|} = \sqrt{1+\varepsilon}$$

$$\alpha_{MAX\,dB} = 10 \log(1+\varepsilon^2)$$

$$\varepsilon^2 = 10^{\alpha_{MAX}/10} - 1$$

$w = w_s$

$$\alpha_{min\,dB} = 10 \log\left(1+C_n^2(w_s)\right) =$$

$$= 10 \log\left[1+\varepsilon^2 \cosh^2\left[n \cosh^{-1}(w_s)\right]\right]$$

$\underline{\varepsilon^2 = n}$

NOTA

HOJA N°

FECHA

$$T_4(s) = \frac{\omega_{o1}^2}{s^2 + s\,\frac{\omega_{o1}}{Q_1} + \omega_{o1}^2} =$$

$$\omega_{o1}^2 = \sigma_1^2 + \omega_1^2$$

$$Q_1 = \frac{1}{2\cos\varphi_1}$$

NOTA