MOSTRAR QUE: [1]

$$\int_{-\ell}^{\ell} \operatorname{sen}\left(\frac{m\pi x}{\ell}\right) \cdot \operatorname{sen}\left(\frac{n\pi x}{\ell}\right) dx = \ell \quad \text{se } m = n \qquad [I]$$

$$(I) = \int_{-\ell}^{\ell} \operatorname{sen}\left(\frac{n\pi x}{\ell}\right) \cdot \operatorname{sen}\left(\frac{n\pi x}{\ell}\right) dx ; \qquad [2]$$

$$(I) = \int_{-\ell}^{\ell} \operatorname{sen}^2\left(\frac{n\pi x}{\ell}\right) dx = \int_{-\ell}^{\ell} \left[\frac{1 - \cos\left(\frac{2n\pi x}{\ell}\right)}{2}\right] dx ; \qquad [3]$$

$$(I) = \frac{1}{2}\left[\int_{-\ell}^{\ell} \left[1 - \cos\left(\frac{2n\pi x}{\ell}\right)\right] dx\right] ; \qquad [4]$$

$$2 \cdot (I) = \int_{-\ell}^{\ell} dx - \int_{-\ell}^{\ell} \cos\left(\frac{2n\pi x}{\ell}\right) dx ; \qquad [5]$$

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$$2 \cdot (I) = [x]_{-\ell}^{\ell} - 2\int_{0}^{\ell} \cos\left(\frac{2n\pi x}{\ell}\right) dx ; \qquad [6]$$

$$2 \cdot (I) = [\ell - (-\ell)] - 2\int_{0}^{2n\pi} \cos(u)\left(\frac{\ell}{2n\pi}\right) du ; \qquad [7]$$

$$u = \frac{2n\pi x}{\ell}$$

$$du = \frac{2n\pi}{\ell} \cdot dx$$

$$\frac{\ell}{2n\pi} du = dx$$

$$2.\textcircled{I} = 2l - 2.\frac{l}{2n\pi}\int_0^{2n\pi} \cos(u)\,du \quad ; \quad 8$$

$$2.\textcircled{I} = 2l - \frac{2l}{2n\pi}\left[\operatorname{Sen}(u)\right]_0^{2.n.\pi} \quad ; \quad 9$$

$$2.\textcircled{I} = 2.l - \frac{2l}{2n\pi}\left[\operatorname{Sen}(2.n.\pi) - \operatorname{Sen}(0)\right] ; \quad 10$$

($\operatorname{Sen}(2.n.\pi) \to 0$, $\operatorname{Sen}(0) \to 0$)

$$\textcircled{I} = \frac{2l}{2} \quad ; \quad \text{ENTÃO:} \quad \boxed{\operatorname{Sen}(2n\pi) = 0}$$

$$\boxed{\int_{-l}^{l} \operatorname{Sen}\left(\frac{m.\pi.x}{l}\right).\operatorname{Sen}\left(\frac{n.\pi.x}{l}\right)dx = l} \quad , \quad \boxed{m = n}$$