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1- $f(x) = \operatorname{sen}^2 x$ $\quad\quad f'(x)$

$$f'(x) = \lim_{h\to 0} \frac{\operatorname{sen}^2(x+h) - \operatorname{sen}^2 x}{h}$$

$$= \lim_{h\to 0} \frac{(\operatorname{sen} x \cdot \cos h + \cos x \cdot \operatorname{sen} h)^2 - \operatorname{sen}^2 x}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{sen}^2 x \cdot \cos^2 h + 2 \operatorname{sen} x \cos h \cos x \cdot \operatorname{sen} h + \cos^2 x \operatorname{sen}^2 h - \operatorname{sen}^2 x}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{sen}^2 x (\cos^2 h - 1) + 2 \operatorname{sen} x \cdot \cos h \cdot \cos x \cdot \operatorname{sen} h + \cos^2 x \cdot \operatorname{sen}^2 h}{h}$$

$$= \lim_{h\to 0} \frac{\operatorname{sen}^2 x (\cos h + 1)(\cos h - 1)}{h} + \frac{2 \operatorname{sen} x \cdot \cos h \cdot \cos x \operatorname{sen} h}{h} + \frac{\cos^2 x \cdot \operatorname{sen}^2 h}{h}$$

$f'(x) = 0 + 2 \operatorname{sen} x \cdot 1 \cdot \cos x + \cos^2 x \cdot 0 \cdot 1$

$f'(x) = 2 \operatorname{sen} x \cos x + 0$

$f'(x) = 2 \operatorname{sen} x \cdot \cos x$

2-

$$f(x) = \begin{cases} x^2 & x < 2 \\ \sqrt{ax} + b & x \geq 2 \end{cases}$$

$\lim_{x\to -2} x^2 = (-2)^2 = 4$ então $\sqrt{a2} + b = 4$

$b = 4 - \sqrt{a2}$

$f(x) = \sqrt{ax} + 4\sqrt{2a}$

~~f(x) = (a)~~

3_

$$f(x) = \begin{cases} x^3 & x \leq 2 \\ ax^2 + bx + c & x > 2 \end{cases}$$

$\lim_{x \to 2^-} x^3 = 8$

$\lim_{x \to 2^+} a2^2 + b.2 + c$

$4a + 2b + c = 8$

$f''(2)$ então $f'(x)$ tem que ser contínua em $x = 2$

$$f'(x) = \begin{cases} 3x^2 & x \leq 2 \\ 2ax + b & x > 2 \end{cases}$$

$\lim_{x \to 2^+} 4a + b$ $\qquad 4a + b = 12$

$b = 12 - 4a$

$\lim_{x \to 2^-} 3.4 = 12$

$$f''(x) = \begin{cases} 6x & x \leq 2 \\ 2a & x > 2 \end{cases}$$

$\lim_{x \to 2^-} 6.2 = 12$ $\qquad 2a = 12$

$a = 6$

$\lim_{x \to -\infty}$ $\qquad x - 2x$ $\qquad 1 - \cancel{x} \; \cancel{x^2}$

$b = 12 - 4.6$ $\qquad$ $4.6 + 2.(-12) + c = 8$

$b = -12$ $\qquad$ $\cancel{24} - \cancel{24} + c = 8$

$c = 8$

4- y''' de $\quad b^2x^2 + a^2y^2 = a^2b^2 \quad$ a e b = ctc

$$y' = b^2 2x + a^2 \frac{dy}{dx} y^2 = 0$$

$$y' = 2b^2x + a^2 \frac{dy}{dx} y^2 = 0$$

5. $e^{x} + e^{y} = e^{x+y} \quad y' = ?$

$$e^{x+y} = e^{x+y} \qquad e^{u} \cdot u' = e^{y} \cdot y'$$

$$e^{x} + e^{y} \cdot y' = e^{x'} \cdot e^{y} + e^{x} \cdot e^{y'}$$

$$e^{y} \cdot y' - e^{x+y} \cdot y' = e^{x} \cdot e^{y} - e^{x}$$

$$y' \cdot e^{y} - e^{x+y} = e^{x}(e^{y} - 1)$$

$$y' = \frac{e^{x}(e^{y} - 1)}{e^{y} - e^{x+y}}$$

$$y' = \frac{e^{x-y}(e^{y} - 1)}{-e^{x}}$$

$$\cancel{y'' = \frac{(e^{x-y}(e^{y}-1)) \cdot (1 - e^{x}) - e^{x-y}(e^{y}-1)(1-e^{x})'}{(1 - e^{x})^2}}$$

$$\cancel{y'' = \frac{e^{x} - e^{2x-y} \cdot y' - e^{x-y} + e^{x-y} \cdot y'}{(1 - e^{x})^2}}$$

6. $f(x) = \dfrac{2x^3 - 5x^2 + 4x}{x^2 - 2x + 1}$

i) Domínio: $Df = \mathbb{R} - \{1\}$

$x^2 - 2x + 1 = 0$

$(x - 1)(x - 1)$

$x = 1$

tilibra

ii) interseção com eixo das coordenadas: $(0,0)$

$$\frac{2x^3 - 5x^2 + 4x}{x^2 - 2x + 1} = 0$$

$$2x^3 - 5x^2 + 4x = 0$$
$$x(2x^2 - 5x + 4) = 0$$
$$x = 0 \qquad 2x^2 - 5x + 4 = 0$$

↳ não existe valor real

$$\Delta = (-5)^2 - 4 \cdot 2 \cdot 4$$
$$\Delta = 25 - 32$$
$$\Delta = -7$$

R: O ponto é $(0,0)$

iii) Assíntota vertical: $x = 1$

$$\lim_{x \to 1^+}$$

Assíntota horizontal: Não existe

$$\lim_{x \to +\infty} \frac{2x^3 - 5x^2 + 4x}{x^2 - 2x + 1} \Rightarrow \frac{2x - 5 + \cancel{\frac{4}{x}}^{\to 0}}{1 - \cancel{\frac{2}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}} = \frac{+\infty}{1} = +\infty$$

$$\lim_{x \to -\infty} \frac{2x^3 - 5x^2 + 4x}{x^2 - 2x + 1} \Rightarrow \frac{2x - 5 + \cancel{\frac{4}{x}}^{\to 0}}{1 - \cancel{\frac{2}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}} = \frac{-\infty}{1} = -\infty$$

Assintota oblíqua: $y = 2x - 1$

$$M = \lim_{x \to +\infty} \frac{2x^3 - 5x^2 + 4x}{x^2 - 2x + 1} \cdot \frac{1}{x}$$

$$= \lim_{x \to +\infty} \frac{2x^3 - 5x^2 + 4x}{x^3 - 2x^2 + x}$$

$$= \lim_{x \to +\infty} = \frac{2 - \cancel{\frac{5}{x}}^{\to 0} + \cancel{\frac{4}{x^2}}^{\to 0}}{1 - \cancel{\frac{2}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}} = 2$$

$$N = \lim_{x \to +\infty} \frac{2x^3 - 5x^2 + 4x}{x^2 - 2x + 1} - 2x$$

$$= \lim_{x \to +\infty} \frac{2x^3 - 5x^2 + 4x - 2x(x^2 - 2x + 1)}{x^2 - 2x + 1}$$

$$= \lim_{x \to +\infty} \frac{\cancel{2x^3} - 5x^2 + 4x - \cancel{2x^3} + 4x^2 - 2x}{x^2 - 2x + 1}$$

$$= \lim_{x \to \infty} \frac{-x^2 + 2x}{x^2 - 2x + 1}$$

$$= \lim_{x \to \infty} \frac{-1 + \cancel{\frac{2}{x}}^{\to 0}}{1 - \cancel{\frac{2}{x}}^{\to 0} + \cancel{\frac{1}{x^2}}^{\to 0}} = \frac{-1}{1} = -1$$

$y = 2x - 1$

iv) crescimento e decrescimento e ponto crítico:

$$\frac{dy}{dx} = \frac{(6x^2 - 10x + 4)(x^2 - 2x + 1) - (2x - 2)(2x^3 - 5x^2 + 4x)}{(x-1)^4}$$

$$= \frac{2(x-1)(3x^3 - (6x^2 + 3x - 2x + 4x - 2) - (2x^3 - 5x^2 + 4))}{(x-1)^4}$$

$$= \frac{2\cancel{(x-1)}(x^3 - 3x^2 + 3x - 2)}{(x-1)^4}$$

$$= \frac{2(x^3 - 3x + 3x - 2)}{(x-1)^3}$$

$$= \frac{2(x-2)(x^2 - x + 1)}{(x-1)^3}$$

$2(x-2)(x^2 - x + 1) = 0$

$x = 2$ ↳ não tem raiz

máx $= 2$

mín $= 1$

$\sqrt[3]{(x-1)^3} = \sqrt[3]{0}$

$x - 1 = 0$

$x = 1$

cres | decres | cres
(1) | (2)

$$\frac{2(0-2)(0^2 - 0 + 1)}{(0-1)^3} = 0$$

$$\frac{-4}{-1} = 4 \rightarrow \text{crescente}$$