TRANSFORMADA INVERSA DE LAPLACE

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-s.t}\{f(t)\}dt = F(s)}$$

ESSA EQ. DEFINE A T.L. DA FUNÇÃO $f(t)$; OU SEJA:

TEMOS $f(t) \Rightarrow$ CALCULAMOS $\Rightarrow \boxed{\mathcal{L}\{f(t)\} = F(s)}$

SE:

- TEMOS $F(s)$, COMO ACHAR $f(t)$?
- COMO INVERTER A T.L.?
- PARA UMA DADA $F(s)$ EXISTE APENAS UMA $f(t)$?

PARA RESPONDER ESSA QUESTÕES DEFINE-SE A TRANSFORMADA INVERSA DE LAPLACE:

$$\boxed{f(t) = \mathcal{L}^{-1}\{F(s)\}} \text{ TAL QUE } \boxed{F(s) = \mathcal{L}\{f(t)\}}$$

T. L. DE ALGUMAS FUNÇÕES	TRANSFORMADA INVERSA DE LAPLACE
a) $\mathcal{L}\{1\} = \frac{1}{s}$ $\mathcal{L}\{f(t)\} = F(s)$	a) $1 = \mathcal{L}^{-1}\{\frac{1}{s}\}$ $f(t) = \mathcal{L}^{-1}\{F(s)\}$
b) $\mathcal{L}\{e^{a.t}\} = \frac{1}{s-a}$	b) $e^{a.t} = \mathcal{L}^{-1}\{\frac{1}{s-a}\}$
c) $\mathcal{L}\{\text{SEN}(a.t)\} = \frac{a}{a^2+s^2}$	$\text{SEN}(a.t) = \mathcal{L}^{-1}\{\frac{a}{a^2+s^2}\}$

- TRANSFORMADA INVERSA DE LAPLACE É OUTRA INTEGRAL, CUJO CÁLCULO ENVOLVE O USO DE VARIÁVEIS COMPLEXAS.

- ENCONTRAR TRANSFORMADA INVERSA: CONSULTA TABELA.

CALCULE:

1) $\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s^2+64}\right\}}$

$$\mathcal{L}^{-1}\left\{\frac{1}{64+s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{8^2+s^2}\right\} = \frac{8}{8}\,\mathcal{L}^{-1}\left\{\frac{1}{8^2+s^2}\right\};$$

$$\mathcal{L}^{-1}\left\{\frac{1}{64+s^2}\right\} = \frac{1}{8}\,\mathcal{L}^{-1}\left\{\frac{8}{8^2+s^2}\right\};$$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{1}{64+s^2}\right\} = \frac{1}{8}\,\mathrm{sen}(8.t)}.$$

2) $\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\}}$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} = t^n} \; , \quad n \to \text{INTEIRO}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^{4+1}}\right\} ;$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{4!}{4!}\,\mathcal{L}^{-1}\left\{\frac{1}{s^{4+1}}\right\} ;$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{1}{4!}\,\mathcal{L}^{-1}\left\{\frac{4!}{s^{4+1}}\right\} ;$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{1}{4!}\cdot t^4 = \frac{t^4}{4 \cdot 3 \cdot 2 \cdot 1} ;$$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s^5}\right\} = \frac{t^4}{24}}$$

3) $\boxed{\mathcal{L}^{-1}\left\{\frac{3s+5}{s^2+7}\right\}}$ ① $\boxed{27}$

$$\boxed{\cos(a.t)=\mathcal{L}^{-1}\left\{\frac{s}{a^2+s^2}\right\}}\ ② \quad ; \quad \boxed{\text{SEN}(a.t)=\mathcal{L}^{-1}\left\{\frac{a}{a^2+s^2}\right\}}\ ③$$

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{7+s^2}\right\}=\mathcal{L}^{-1}\left\{\frac{3s}{7+s^2}+\frac{5}{7+s^2}\right\};$$

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{7+s^2}\right\}=\mathcal{L}^{-1}\left\{\frac{3s}{7+s^2}\right\}+\mathcal{L}^{-1}\left\{\frac{5}{7+s^2}\right\};$$

$$'' \quad =\mathcal{L}^{-1}\left\{\frac{3.s}{(\sqrt{7})^2+s^2}\right\}+\mathcal{L}^{-1}\left\{\frac{5}{(\sqrt{7})^2+s^2}\right\};$$

$$'' \quad =3.\mathcal{L}^{-1}\left\{\frac{s}{(\sqrt{7})^2+s^2}\right\}+5.\mathcal{L}^{-1}\left\{\frac{1}{(\sqrt{7})^2+s^2}\right\};$$

$$'' \quad =3.\cos(\sqrt{7}.t)+5.\frac{\sqrt{7}}{\sqrt{7}}\mathcal{L}^{-1}\left\{\frac{1}{(\sqrt{7})^2+s^2}\right\};$$

$$'' \quad =3.\cos(\sqrt{7}.t)+\frac{5}{\sqrt{7}}.\mathcal{L}^{-1}\left\{\frac{\sqrt{7}}{(\sqrt{7})^2+s^2}\right\};$$

$$\boxed{\mathcal{L}^{-1}\left\{\frac{3s+5}{7+s^2}\right\}=3.\cos(\sqrt{7}.t)+\frac{5}{\sqrt{7}}.\text{SEN}(\sqrt{7}.t)}$$