2ª LISTA

1) RESOLVA AS EQS. DIF.:

a) $\boxed{2x(y+3) + (x^2-4)\dfrac{dy}{dx} = 0}$

$u = x^2-4$
$du = 2x\,dx$

$V = y+3$
$dV = dy$

$$(x^2-4)\frac{dy}{dx} = -2x(y+3) \Rightarrow \frac{1}{(y+3)}\,dy = \frac{-2x}{(x^2-4)}\,dx \Rightarrow$$

$$\int \frac{1}{(y+3)}\,dy = -\int \frac{2x}{x^2-4}\,dx \Rightarrow \int \frac{1}{V}\,dV = -\int \frac{1}{u}\,du \Rightarrow$$

$$\ln|V| = -\ln|u| + \ln A \Rightarrow \ln|y+3| + \ln|x^2-4| = \ln A$$

$$\ln\Big[|y+3|\cdot|x^2-4|\Big] = \ln A \Rightarrow |y+3|\cdot|x^2-4| = A \Rightarrow$$

$$\frac{|y+3|}{\left(|x^2-4|\right)^{-1}} = A \Rightarrow |y+3| = A\cdot\left(|x^2-4|\right)^{-1} \Rightarrow$$

$$y+3 = \pm A(x^2-4)^{-1} \Rightarrow \boxed{y = \pm A(x^2-4)^{-1} - 3}$$

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b) $\boxed{y\cdot(1+x^3)\dfrac{dy}{dx} + x^2(1+y^2) = 0}$

$$y(1+x^3)\frac{dy}{dx} = -x^2(1+y^2) \Rightarrow \frac{y}{(1+y^2)}\,dy = \frac{-x^2}{(1+x^3)}\,dx \Rightarrow$$

$u = 1+y^2$
$du = 2y\,dy$
$\dfrac{du}{2} = y\cdot dy$

$V = 1+x^3$
$dV = 3x^2dx$
$\dfrac{dV}{3} = x^2dx$

$$\int \frac{y}{1+y^2}\,dy = -\int \frac{x^2}{1+x^3}\,dx \Rightarrow \frac{1}{2}\int \frac{1}{u}\,du = -\frac{1}{3}\int \frac{1}{V}\cdot dV$$

$$\frac{1}{2}\ln|u| = -\frac{1}{3}\ln|V| + \ln C \Rightarrow$$

$$\frac{1}{2}\ln|1+y^2| + \frac{1}{3}\ln|1+x^3| = \ln C \quad (\times 2)$$

②

$$\ln(1+y^2) + \frac{2}{3}\ln|1+x^3| = \ln C^2 \Rightarrow$$

$$\ln(1+y^2) + \ln(|1+x^3|)^{2/3} = \ln C^2 \Rightarrow \ln(1+y^2) + \ln\left[\left(1+x^3\right)^2\right]^{1/3} = \ln \underbrace{C^2}_{=A}$$

$$\ln\left[(1+y^2).(1+x^3)^{2/3}\right] = \ln A \Rightarrow (1+y^2).(1+x^3)^{2/3} = A \Rightarrow$$

$$\frac{1+y^2}{(1+x^3)^{-2/3}} = A \Rightarrow 1+y^2 = A\,(1+x^3)^{-2/3} \Rightarrow \boxed{y^2 = \frac{A}{\sqrt[3]{(1+x^3)^2}} - 1}$$

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c) $\boxed{e^{y}\,\mathrm{sen}(x)\,dx - \cos^2(x)\,dy = 0}$ $\quad\boxed{\begin{array}{c} u = \cos(x) \\ -du = \mathrm{sen}(x)\,dx \end{array}}$

$$\cos^2(x)\,dy = e^{y}.\,\mathrm{sen}(x)\,dx \Rightarrow e^{-y}\,dy = \frac{\mathrm{sen}(x)}{\cos^2(x)}\,dx \Rightarrow$$

$$\int e^{-y}\,dy = \int \frac{\mathrm{sen}(x)}{\cos^2(x)}\,dx \Rightarrow -e^{-y} = -\int \frac{1}{u^2}\,du \Rightarrow$$

$$e^{-y} = \int u^{-2}\,du \Rightarrow e^{-y} = -u^{-1} + C \Rightarrow e^{-y} = -\frac{1}{u} + C$$

$$e^{-y} = -\frac{1}{\cos(x)} + C \Rightarrow \boxed{e^{-y} = -\sec(x) + C}$$

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d) $\boxed{2y\,dx + (xy + 5x)\,dy = 0}$

$$(xy+5x)\,dy = -2y\,dx \Rightarrow x\,(y+5)\,dy = -2y\,dx \Rightarrow$$

$$\frac{y+5}{y}\,dy = -\frac{2}{x}\,dx \Rightarrow \left(1+\frac{5}{y}\right)dy = -\frac{2}{x}\,dx \Rightarrow \int\left(1+\frac{5}{y}\right)dy = -2\int\frac{1}{x}\,dx$$

$$y + 5\ln|y| = -2\ln|x| + C \Rightarrow y + \ln|y|^5 + \ln|x|^2 = C \Rightarrow$$

$$y + \ln\left(|y|^5.\,x^2\right) = C \Rightarrow \ln\left(|y|^5.\,x^2\right) = C - y \Rightarrow$$

$$|y|^5.\,x^2 = e^{C-y} \Rightarrow |y|^5.\,x^2 = \overset{=A}{e^{C}}.\,e^{-y} \Rightarrow \boxed{|y|^5.\,x^2.\,e^{y} = A}$$

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e) $\boxed{(x^2+9)\frac{dy}{dx} + x.y = 0}$

$$\frac{dy}{dx}(x^2+9) = -x.y \Rightarrow \frac{1}{y}\,dy = -\frac{x}{x^2+9}\,dx \Rightarrow \int \frac{1}{y}\,dy = -\int \frac{x}{x^2+9}\,dx \Rightarrow$$

$$\ln|y| = -\frac{1}{2}\int \frac{1}{u}\,du \Rightarrow \ln|y| = -\frac{1}{2}\ln|u| + \ln A$$

$$u = x^2+9 \qquad du = 2x\,dx \qquad \frac{1}{2}du = x\,dx$$

$$\ln|y| + \frac{1}{2}\ln|x^2+9| = \ln A \Rightarrow \ln|y| + \ln(x^2+9)^{\frac{1}{2}} = \ln A$$

$$\ln\left[|y|.(x^2+9)^{\frac{1}{2}}\right] = \ln A \Longrightarrow |y|.(x^2+9)^{\frac{1}{2}} = A \Rightarrow$$

$$\boxed{|y| = \frac{\pm A}{\sqrt{x^2+9}}}$$

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f) $\boxed{x.e^{-y}\,\text{sen}(x)\,dx - y\,dy = 0}$ $\qquad \int u.dv = u.v - \int v.du$

$$y.dy = x.e^{-y}.\text{sen}(x)\,dx \rightarrow y.e^{y}\,dy = x.\text{sen}(x)\,dx \Rightarrow$$

$$\underbrace{\int y.e^{y}.dy}_{\text{II}} = \underbrace{\int x.\text{sen}(x).dx}_{\text{I}} = e^{y}(y-1) = -x.\cos(x) + \text{sen}(x) + C$$

ou

$$\boxed{\text{sen}(x) - x.\cos(x) = e^{y}(y-1) + C}$$

$$\text{I} = \int \underset{u}{x}.\underset{dv}{\text{sen}(x)\,dx} = -x.\cos(x) - \int -\cos(x)\,dx$$

$$\text{I} = \int x.\text{sen}(x)\,dx = -x.\cos(x) + \text{sen}(x)$$

I: $u = x$, $du = dx$, $dv = \text{sen}(x)\,dx$, $v = -\cos(x)$

$$\text{II} = \int \underset{u}{y}.\underset{dv}{e^{y}dy} = y.e^{y} - \int e^{y}\,dy$$

$$\text{II} = \int y.e^{y}dy = y.e^{y} - e^{y} = e^{y}(y-1)$$

II: $u = y$, $du = dy$, $dv = e^{y}\,dy$, $v = e^{y}$

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g) $\boxed{y' = x-1 + x.y - y}$

$\frac{dy}{dx} = (x-1) + y(x-1) \longrightarrow \frac{dy}{dx} = (x-1).[1+y] \Rightarrow \frac{1}{y+1}\, dy = (x-1).dx \Rightarrow$

$\int \frac{1}{y+1}\, dy = \int (x-1)\, dx \Rightarrow \ln|y+1| = \frac{x^2}{2} - x + c \Longrightarrow$

$|y+1| = e^{\frac{x^2}{2} - x + c} \Rightarrow |y+1| = e^{\frac{x^2}{2} - x} . \underbrace{e^{c}}_{=A} \Rightarrow$

$\boxed{y = \pm A\, e^{\frac{x^2}{2} - x} - 1}$

h) $\boxed{e^{x+2y}.dx - e^{2x-y}.dy = 0}$

$e^{x}.e^{2y}.dx = e^{2x}.e^{-y}\, dy \Rightarrow \frac{e^{-y}}{e^{2y}}\, dy = \frac{e^{x}}{e^{2x}}\, dx \Rightarrow$

$e^{-y}.e^{-2y}\, dy = e^{x}.e^{-2x}\, dx \Rightarrow \int e^{-3y}\, dy = \int e^{-x}.dx \Rightarrow$

$-\frac{1}{3} e^{-3y} = -e^{-x} + c \overset{\otimes -3}{\Longrightarrow} e^{-3y} = 3e^{-x} - 3c \Rightarrow$

$\ln e^{-3y} = \ln[3.e^{-x} - 3c] \Rightarrow -3y \cancel{\ln e}^{1} = \ln[3.e^{-x} - 3c] \Rightarrow$

$y = -\frac{1}{3} \ln[3.e^{-x} - 3c]$

i) $\boxed{x^2.y^2.dy = (y+1).dx}$

$u = y+1$
$du = dy$
$y = u-1$

$\frac{y^2}{y+1}\, dy = \frac{1}{x^2}.dx \Rightarrow \int \frac{y^2}{y+1}.dy = \int x^{-2} dx$

$\int \frac{(u-1)^2}{u}\, du = \frac{x^{-1}}{-1} + c \Rightarrow \int \frac{(u^2 - 2u + 1)}{u}\, du = -\frac{1}{x} + c \Rightarrow$

$\int u\, du - 2.\int du + \int \frac{1}{u}\, du = -\frac{1}{x} + c \Longrightarrow$

$$\frac{u^2}{2} - 2u + \ln|u| = -\frac{1}{x} + C \quad \otimes 2 \Rightarrow$$

$$u^2 - 4u + 2\ln|u| = -\frac{2}{x} + 2C \Rightarrow u^2 - 4u + \ln|u|^2 = -\frac{2}{x} + 2C$$

$$(y+1)^2 - 4(y+1) + \ln(y+1)^2 = -\frac{2}{x} + \overset{=A}{2C} \Rightarrow$$

$$\boxed{(y+1)^2 - 4(y+1) + \ln(y+1)^2 = A - \frac{2}{x}}$$

γ) $\boxed{y\cdot\ln(x)\cdot\frac{dx}{dy} = \left(\frac{y+1}{x}\right)^2}$

$$y\cdot\ln(x)\,dx = \frac{(y+1)^2}{x^2}\cdot dy \Rightarrow x^2\cdot\ln(x)\,dx = \frac{(y+1)^2}{y}\cdot dy$$

$$\int x^2\cdot\underbrace{\ln(x)dx}_{u} = \int\frac{(y+1)^2}{y}\,dy \Rightarrow$$

$$\int u\cdot dv = u\cdot v - \int v\cdot du$$

$u = \ln(x)$

$du = \frac{1}{x}\cdot dx$

$dv = x^2\,dv$

$v = \frac{x^3}{3}$

$$\frac{x^3}{3}\cdot\ln(x) - \int\frac{x^3}{3}\,\frac{1}{x}\cdot dx = \int\frac{(y^2+2y+1)}{y}\,dy \Rightarrow$$

$$\frac{x^3}{3}\ln(x) - \frac{1}{3}\int x^2\,dx = \int\left(y+2+\frac{1}{y}\right)dy \Rightarrow$$

$$\frac{x^3}{3}\cdot\ln(x) - \frac{1}{3}\cdot\frac{x^3}{3} = \frac{y^2}{2} + 2y + \ln|y| + C \Rightarrow$$

$$\boxed{\frac{y^2}{2} + 2y + \ln|y| = \frac{x^3}{3}\cdot\ln(x) - \frac{x^3}{9} + C}$$

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e) $\boxed{\operatorname{sen}(3x)\,dx + 2y\cdot\cos^3(3x)\,dy = 0}$

$$2y\cdot\cos^3(3x)\,dy = -\operatorname{sen}(3x)\,dx \Rightarrow 2y\,dy = \frac{-\operatorname{sen}(3x)}{\cos^3(3x)}\,dx \Rightarrow$$

$$2\int y\cdot dy = -\int \frac{\operatorname{sen}(3x)}{\cos^3(3x)}\,dx \Rightarrow$$

$$u = \cos(3x)$$
$$du = -3\operatorname{sen}(3x)dx$$
$$\frac{1}{3}du = -\operatorname{sen}(3x)dx$$

$$\cancel{2}\frac{y^2}{\cancel{2}} = 3\int \frac{1}{u^3}\,du \Rightarrow y^2 = \frac{1}{3}\int u^{-3}\cdot du \Rightarrow$$

$$y^2 = \frac{1}{3}\,\frac{u^{-2}}{-2} + c \Rightarrow y^2 = -\frac{1}{6}\;\frac{1}{u^2} + c$$

$$y^2 = -\frac{1}{6}\;\frac{1}{\cos^2(3x)} + c \Rightarrow y^2 = -\frac{1}{6}\left[\frac{1}{\cos(3x)}\right]^2 + c \Rightarrow$$

$$y^2 = -\frac{1}{6}\left[\sec(3x)\right]^2 + c \Rightarrow \boxed{y^2 = -\frac{1}{6}\sec^2(3x) + c}$$

OBS: OUTRA MANEIRA DE RESOLVER $\int \frac{\operatorname{sen}(3x)}{\cos^3(3x)}dx$:

$$\int \frac{\operatorname{sen}(3x)}{\cos^3(3x)}\,dx = \int \frac{\operatorname{sen}(3x)}{\cos(3x)}\cdot\frac{1}{\cos^2(3x)}\,dx = \int \operatorname{tg}(3x)\cdot\sec^2(3x)\cdot dx$$

$$= \int u\cdot\frac{du}{3} = \frac{1}{3}\,\frac{u^2}{2} + c = \frac{1}{6}\operatorname{tg}^2(3x) + c$$

$$\boxed{\int \frac{\operatorname{sen}(3x)}{\cos^3(3x)}\,dx = \frac{1}{6}\operatorname{tg}^2(3x) + c}$$

$$u = \operatorname{tg}(3x)$$
$$du = \sec^2(3x)\cdot 3\cdot dx$$
$$\frac{1}{3}du = \sec^2(3x)\,dx$$