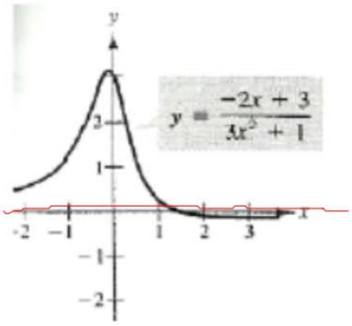
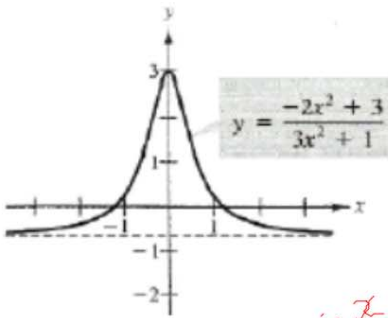
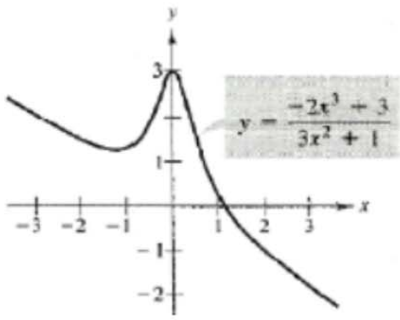
2) Ache as assíntotas horizontais dos gráficos das funções:

a) $y = \dfrac{-2x+3}{3x^2+1}$ b) $y = \dfrac{-2x^2+3}{3x^2+1}$ c) $y = \dfrac{-2x^3+3}{3x^2+1}$







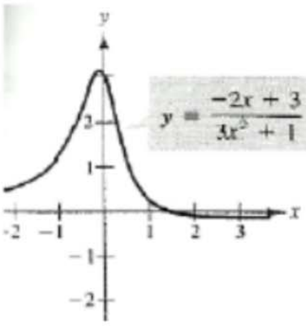
a) $\lim\limits_{x\to+\infty} \dfrac{-2x+3}{3x^2+1} = \lim\limits_{x\to+\infty} \dfrac{-2x \div x}{3x^2 \div x} = \lim\limits_{x\to+\infty} \dfrac{-2}{3x} = 0$

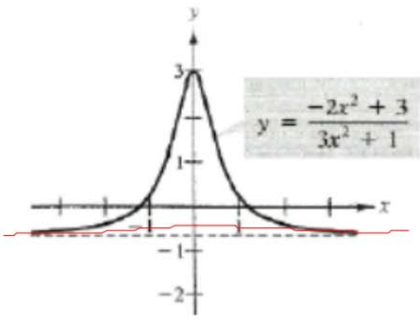
$\lim\limits_{x\to-\infty} \dfrac{-2x+3}{3x^2+1} = \lim\limits_{x\to-\infty} \dfrac{-2x}{3x^2} = \lim\limits_{x\to-\infty} \dfrac{-2}{3x} = 0$

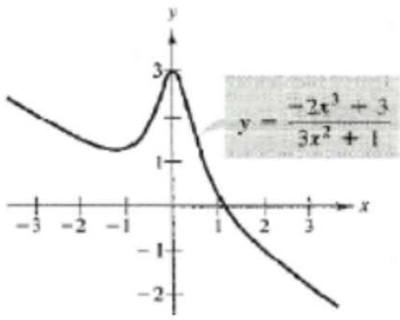
ASSÍNTOTA HORIZONTAL: $y = 0$.

2) Ache as assíntotas horizontais dos gráficos das funções:

a) $y = \dfrac{-2x+3}{3x^2+1}$ b) $y = \dfrac{-2x^2+3}{3x^2+1}$ c) $y = \dfrac{-2x^3+3}{3x^2+1}$







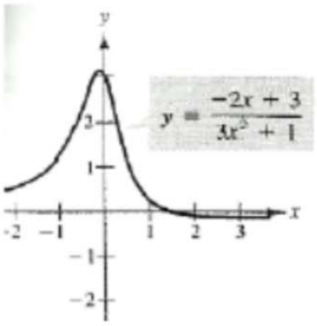
b) $\lim\limits_{x \to +\infty} \dfrac{-2x^2+3}{3x^2+1} = \lim\limits_{x \to +\infty} - \dfrac{2\cancel{x^2}}{3\cancel{x^2}} = \lim\limits_{x \to +\infty} - \dfrac{2}{3} = -\dfrac{2}{3}$

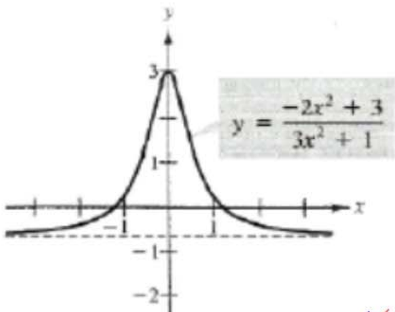
$\lim\limits_{x \to -\infty} \dfrac{-2x^2+3}{3x^2+1} = \lim\limits_{x \to -\infty} \dfrac{-2\cancel{x^2}}{3\cancel{x^2}} = \lim\limits_{x \to -\infty} - \dfrac{2}{3} = -\dfrac{2}{3}$

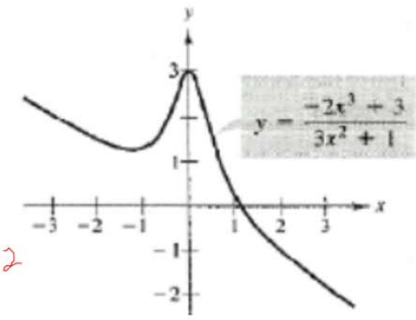
. ASSÍNTOTA HORIZONTAL: $y = -\dfrac{2}{3}$

2) Ache as assíntotas horizontais dos gráficos das funções:

a) $y = \dfrac{-2x+3}{3x^2+1}$ b) $y = \dfrac{-2x^2+3}{3x^2+1}$ c) $y = \dfrac{-2x^3+3}{3x^2+1}$







c) $\lim\limits_{x \to +\infty} \dfrac{-2x^3+3}{3x^2+1} = \lim\limits_{x \to +\infty} \dfrac{-2x^{\cancel{3}} \div x^2}{3\cancel{x^2} \div x^2} = \lim\limits_{x \to +\infty} \dfrac{-2x}{3} = -\infty$

$\lim\limits_{x \to -\infty} \dfrac{-2x^3+3}{3x^2+1} = \lim\limits_{x \to -\infty} \dfrac{-2x}{3} = +\infty$

NÃO EXISTE ASSÍNTOTA HORIZONTAL

3) A população *y* de uma cultura de bactérias segue o modelo da função logística

$$y = \frac{925}{1 + e^{-0,3t}}$$

onde *t* é o tempo em dias. A população tem um limite quando *t* cresce ilimitadamente?

$$\lim_{t \to +\infty} y = \lim_{t \to +\infty} \frac{925}{1 + \frac{e}{1}^{-0,3t}} = \lim_{t \to +\infty} \frac{925}{1 + \left(\frac{1}{e}\right)^{0,3t}} =$$

$$= \lim_{t \to +\infty} \frac{925}{1 + \cancel{\frac{1}{e^{0,3t}}}^{\to 0}} = 923$$

R: SIM, 925 BACTÉRIAS

$$\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}$$

$$\left(\frac{b}{a}\right)^{c} = \frac{b^{c}}{a^{c}}$$

4) A aprendizagem $P(t)$ ao longo de t anos de trabalho de um operário é dada por

$$P(t) = 60 - 20.\, e^{-0,2t}.$$

O que ocorre com a aprendizagem depois de vários anos de trabalho?

$$\lim_{t \to +\infty} P(t) = \lim_{t \to +\infty} \left(60 - 20.\, e^{-0,2t}\right) =$$

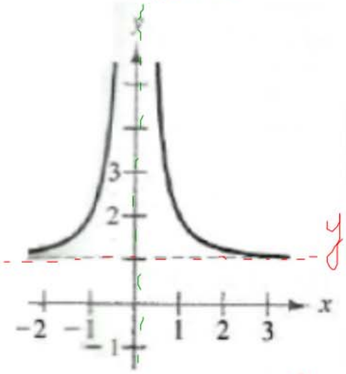
$$= \lim_{t \to +\infty} \left(60 - 20.\, \underbrace{\frac{1}{e^{0,2t}}}_{\to 0}\right) = 60$$

R: A APRENDIZAGEM TENDE A 60.

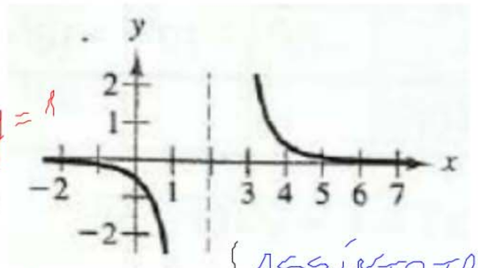
14) Ache as assíntotas horizontais e verticais:

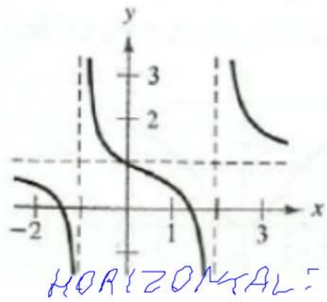
a) $f(x) = \dfrac{x^2+1}{x^2}$ b) $f(x) = \dfrac{4}{(x-2)^3}$ c) $f(x) = \dfrac{x^2-2}{x^2-x-2}$ d) $f(x) = \dfrac{2+x}{1-x}$

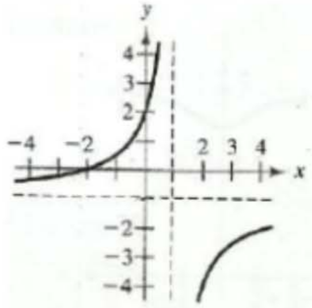
$x = 0$



$y = 1$







a) $f(x) = \dfrac{x^2+1}{x^2}$

ASSÍNTOTA VERTICAL:

$x^2 = 0 \iff x = 0$

$f(0) = \dfrac{1}{0}$

ASSÍNTOTA HORIZONTAL:

$$\lim_{x\to+\infty} \frac{x^2+1}{x^2} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$$

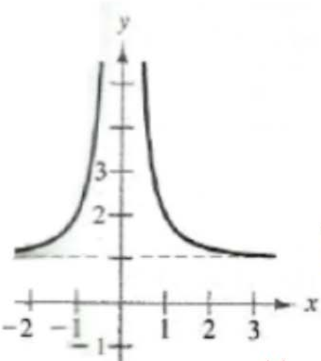
$$\lim_{x\to-\infty} \frac{x^2+1}{x^2} = \lim_{x\to-\infty} 1 = 1$$

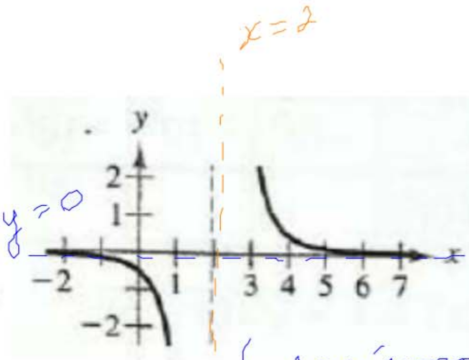
R: ASSÍNTOTA VERTICAL: $x = 0$

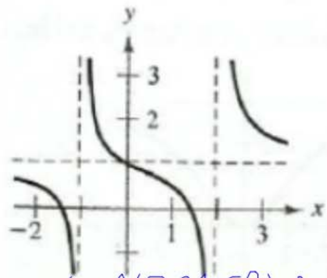
ASSÍNTOTA HORIZONTAL: $y = 1$

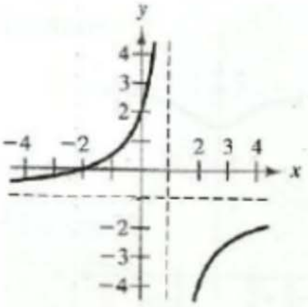
14) Ache as assíntotas horizontais e verticais:

a) $f(x) = \dfrac{x^2+1}{x^2}$ b) $f(x) = \dfrac{4}{(x-2)^3}$ c) $f(x) = \dfrac{x^2-2}{x^2-x-2}$ d) $f(x) = \dfrac{2+x}{1-x}$









b) $f(x) = \dfrac{4}{(x-2)^3}$

ASSÍNTOTA VERTICAL:

$(x-2)^3 = 0 \iff x-2=0 \iff$

$\iff x = 2$

$f(2) = \dfrac{4}{0}$

ASSÍNTOTA HORIZONTAL:

$\lim\limits_{x\to+\infty} \dfrac{4}{(x-2)^3} = 0$

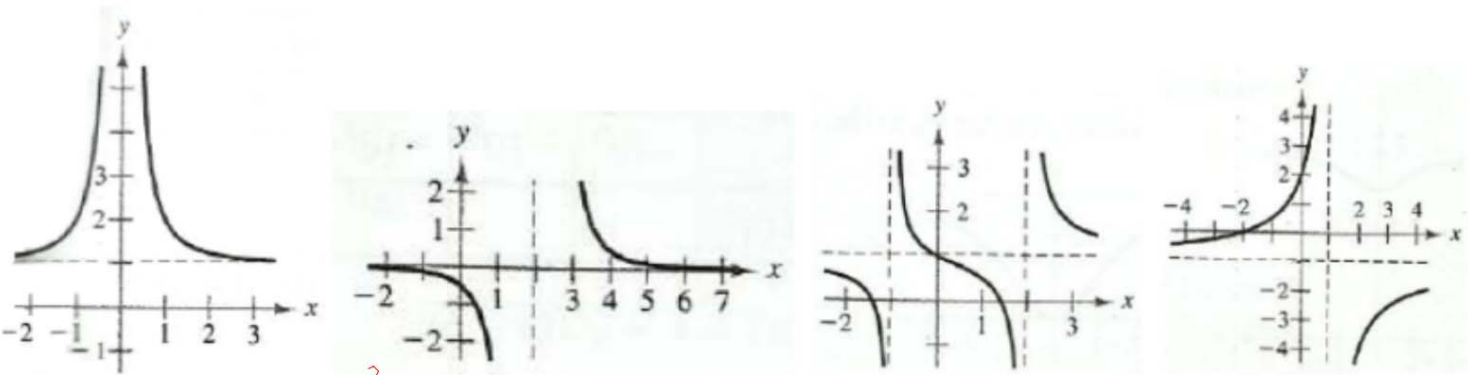
$\lim\limits_{x\to-\infty} \dfrac{4}{(x-2)^3} = 0$

R: ASSÍNTOTA VERTICAL: $x = 2$

ASSÍNTOTA HORIZONTAL: $y = 0$

14) Ache as assíntotas horizontais e verticais:

a) $f(x)=\dfrac{x^2+1}{x^2}$ b) $f(x)=\dfrac{4}{(x-2)^3}$ c) $f(x)=\dfrac{x^2-2}{x^2-x-2}$ d) $f(x)=\dfrac{2+x}{1-x}$



c) $f(x) = \dfrac{x^2-2}{x^2-x-2}$

ASSÍNTOTA VERTICAL

$x^2 - x - 2 = 0$

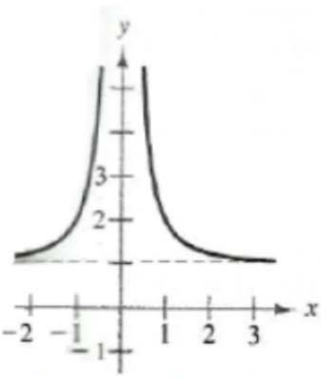
$x = \dfrac{1 \pm \sqrt{1-4\cdot1\cdot(-2)}}{2\cdot1} = \dfrac{1\pm3}{2} \Big\langle \begin{matrix} 2 \\ -1 \end{matrix}$

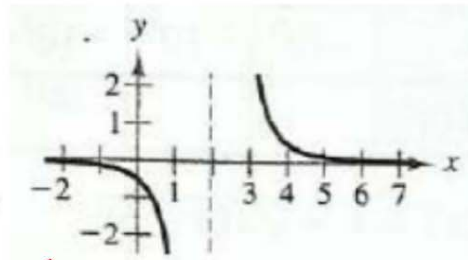
$f(2) = \dfrac{2}{0}$

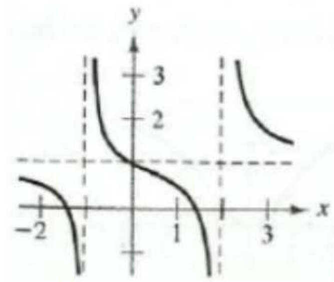
$f(-1) = \dfrac{-1}{0}$

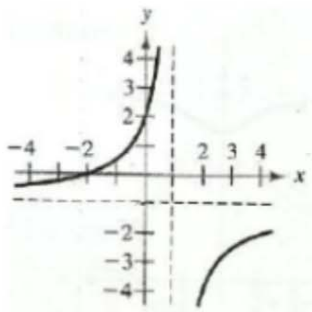
14) Ache as assíntotas horizontais e verticais:

a) $f(x)=\dfrac{x^2+1}{x^2}$ b) $f(x)=\dfrac{4}{(x-2)^3}$ c) $f(x)=\dfrac{x^2-2}{x^2-x-2}$ d) $f(x)=\dfrac{2+x}{1-x}$









c) $f(x) = \dfrac{x^2-2}{x^2-x-2}$

ASSÍNTOTA VERTICAL: $x=-1$ e $x=2$

ASSÍNTOTA HORIZONTAL: $y=1$

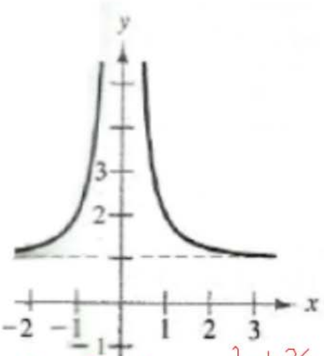
ASSÍNTOTA HORIZONTAL

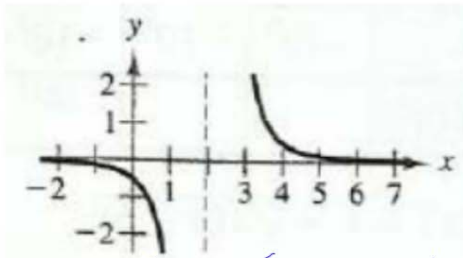
$$\lim_{x\to+\infty} \frac{x^2-2}{x^2-x-2} = \lim_{x\to+\infty} \frac{x^2}{x^2} = \lim_{x\to+\infty} 1 = 1$$

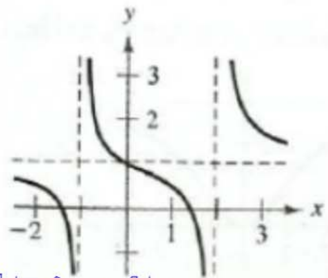
$$\lim_{x\to-\infty} \frac{x^2-2}{x^2-x-2} = \lim_{x\to-\infty} 1 = 1$$

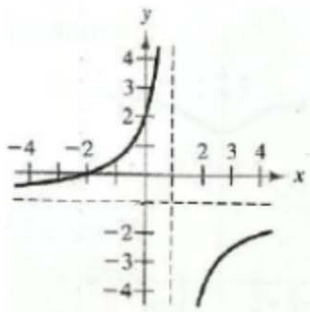
14) Ache as assíntotas horizontais e verticais:

a) $f(x)=\dfrac{x^2+1}{x^2}$ b) $f(x)=\dfrac{4}{(x-2)^3}$ c) $f(x)=\dfrac{x^2-2}{x^2-x-2}$ d) $f(x)=\dfrac{2+x}{1-x}$









d) $f(x)=\dfrac{2+x}{1-x}$

ASSÍNTOTA VERTICAL:

$1-x=0 \iff x=1$

$f(1)=\dfrac{3}{0}$

ASSÍNTOTA HORIZONTAL:

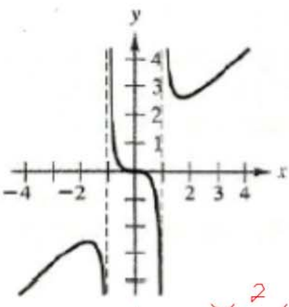
$$\lim_{x\to+\infty}\frac{2+x}{1-x}=\lim_{x\to+\infty}\frac{x}{-x}=\lim_{x\to+\infty}(-1)=-1$$

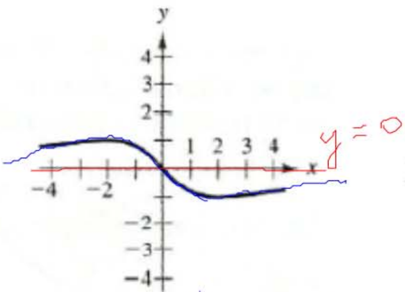
$$\lim_{x\to-\infty}\frac{2+x}{1-x}=\lim_{x\to-\infty}(-1)=-1$$

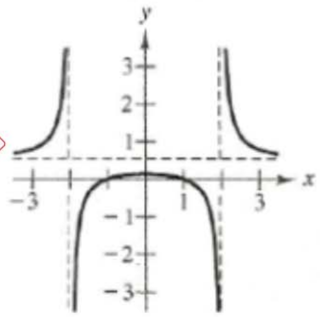
ASSÍNTOTA VERTICAL: $x=1$

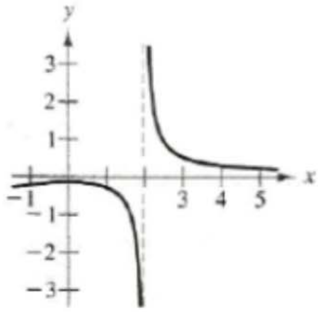
ASSÍNTOTA HORIZONTAL: $y=-1$

e) $f(x) = \dfrac{x^3}{x^2 - 1}$ f) $f(x) = \dfrac{-4x}{x^2 + 4}$ g) $f(x) = \dfrac{x^2 - 1}{2x^2 - 8}$ h) $f(x) = \dfrac{x^2 + 1}{x^3 - 8}$









h) $f(x) = \dfrac{x^2+1}{x^3-8}$

ASSÍNTOTA VERTICAL

$x^3 - 8 = 0 \Leftrightarrow x^3 = 8 \Leftrightarrow$

$\Leftrightarrow x = \sqrt[3]{8} \Leftrightarrow x = 2$

$f(2) = \dfrac{5}{0}$

ASSÍNTOTA HORIZONTAL

$$\lim_{x \to +\infty} \frac{x^2+1}{x^3-8} = \lim_{x \to +\infty} \frac{x^2}{x^3} = \lim_{x \to +\infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{x^2+1}{x^3-8} = \lim_{x \to -\infty} \frac{1}{x} = 0$$

ASSÍNTOTA VERTICAL: $x = 2$

ASSÍNTOTA HORIZONTAL: $y = 0$

15) Associe cada função ao seu gráfico. Recorra às assíntotas horizontais como auxílio.

i) $f(x)=\dfrac{3x^2}{x^2+2}$ ii) $f(x)=\dfrac{2x}{\sqrt{x^2+2}}$ iii) $f(x)=\dfrac{x}{x^2+2}$

iv) $f(x)=2+\dfrac{x^2}{x^4+1}$ v) $f(x)=5-\dfrac{1}{x^2+1}$ vi) $f(x)=\dfrac{2x^2-3x+5}{x^2+1}$

i) $\lim\limits_{x\to+\infty}\dfrac{3x^2}{x^2+2}=\lim\limits_{x\to+\infty}\dfrac{3x^2}{x^2}=\lim\limits_{x\to+\infty}3=3$

$\lim\limits_{x\to-\infty}\dfrac{3x^2}{x^2+2}=\lim\limits_{x\to-\infty}3=3$

ii) $\lim\limits_{x\to+\infty}\dfrac{2x}{\sqrt{x^2+2}}=\lim\limits_{x\to+\infty}\dfrac{2x}{\sqrt{x^2}}=\lim\limits_{x\to+\infty}\dfrac{2x}{x}=\lim\limits_{x\to+\infty}2=2$

$\lim\limits_{x\to-\infty}\dfrac{2x}{\sqrt{x^2+2}}=\lim\limits_{x\to-\infty}\dfrac{2x}{\sqrt{x^2}}=\lim\limits_{x\to-\infty}\dfrac{2x}{-x}=\lim\limits_{x\to-\infty}(-2)=-2$

CÁLCULO DIFERENCIAL E INTEGRAL I
Profª. Me. Mylane dos Santos Barreto

15) Associe cada função ao seu gráfico. Recorra às assíntotas horizontais como auxílio.

i) $f(x) = \dfrac{3x^2}{x^2+2}$ ii) $f(x) = \dfrac{2x}{\sqrt{x^2+2}}$ iii) $f(x) = \dfrac{x}{x^2+2}$

iv) $f(x) = 2 + \dfrac{x^2}{x^4+1}$ v) $f(x) = 5 - \dfrac{1}{x^2+1}$ vi) $f(x) = \dfrac{2x^2-3x+5}{x^2+1}$

iii) $\lim\limits_{x \to +\infty} \dfrac{x}{x^2+2} = \lim\limits_{x \to +\infty} \dfrac{x}{x^2} = \lim\limits_{x \to +\infty} \dfrac{1}{x} = 0$

$\lim\limits_{x \to -\infty} \dfrac{x}{x^2+2} = \lim\limits_{x \to -\infty} \dfrac{1}{x} = 0$

iv) $\lim\limits_{x \to +\infty} \left(2 + \dfrac{x^2}{x^4+1}\right) = \lim\limits_{x \to +\infty} \left(2 + \dfrac{x^2}{x^4}\right) = \lim\limits_{x \to +\infty} \left(2 + \overset{0}{\cancel{\dfrac{1}{x^2}}}\right) = 2$

$\lim\limits_{x \to -\infty} \left(2 + \dfrac{x^2}{x^4+1}\right) = \lim\limits_{x \to -\infty} \left(2 + \dfrac{1}{x^2}\right) = 2$ $f(0) = 2 + \dfrac{0^2}{0^4+1} = 2$

15) Associe cada função ao seu gráfico. Recorra às assíntotas horizontais como auxílio.

i) $f(x)=\dfrac{3x^2}{x^2+2}$ ii) $f(x)=\dfrac{2x}{\sqrt{x^2+2}}$ iii) $f(x)=\dfrac{x}{x^2+2}$

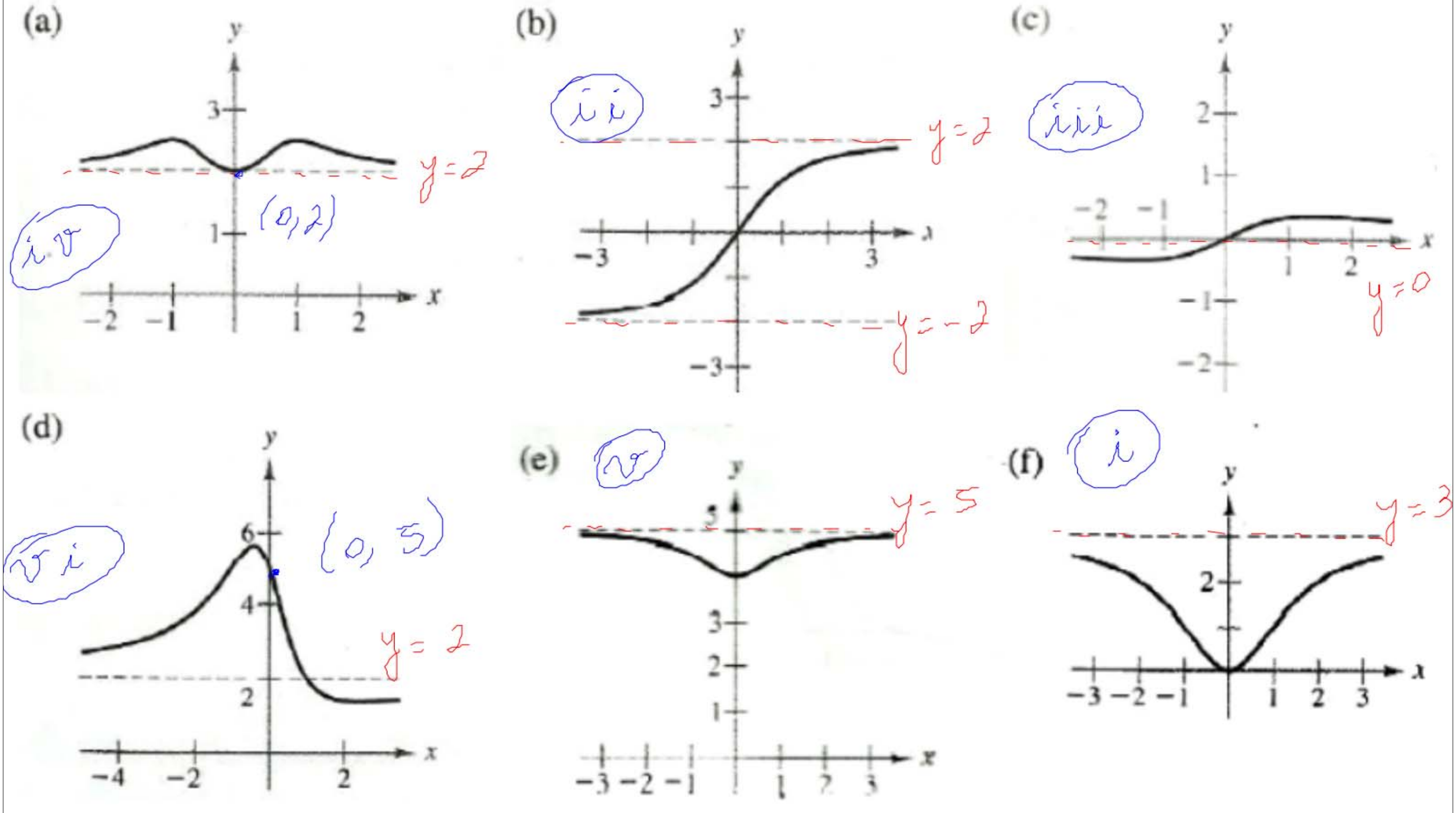
iv) $f(x)=2+\dfrac{x^2}{x^4+1}$ v) $f(x)=5-\dfrac{1}{x^2+1}$ vi) $f(x)=\dfrac{2x^2-3x+5}{x^2+1}$

v) $\lim\limits_{x\to+\infty}\left(5-\cancel{\dfrac{1}{x^2+1}}^{\,0}\right)=5$

$\lim\limits_{x\to-\infty}\left(5-\dfrac{1}{x^2+1}\right)=5$

vi) $\lim\limits_{x\to+\infty}\dfrac{2x^2-3x+5}{x^2+1}=\lim\limits_{x\to+\infty}\dfrac{2x^2}{x^2}=\lim\limits_{x\to+\infty}2=2$

$\lim\limits_{x\to-\infty}\dfrac{2x^2-3x+5}{x^2+1}=\lim\limits_{x\to-\infty}2=2$ $f(0)=5$


(a)
iv
y = 2
(0,2)
(b)
ii
y = 2
y = -2
(c)
iii
y = 0
(d)
vi
(0, 5)
y = 2
(e)
v
y = 5
(f)
i
y = 3

17) Determine os limites, se existir:

a) $\lim\limits_{x \to +\infty} \dfrac{4x - 3}{2x + 5}$

Solução:

$$\lim_{x \to +\infty} \frac{4x - 3}{2x + 5} = \frac{\lim\limits_{x \to +\infty} (4x - 3) \div x}{\lim\limits_{x \to +\infty} (2x + 5) \div x} = \frac{\lim\limits_{x \to +\infty} 4 - \lim\limits_{x \to +\infty} \dfrac{3}{x}}{\lim\limits_{x \to +\infty} 2 + \lim\limits_{x \to +\infty} \dfrac{5}{x}} = \frac{4 - 3 \cdot \lim\limits_{x \to +\infty} \dfrac{1}{x}}{2 + 5 \cdot \lim\limits_{x \to +\infty} \dfrac{1}{x}} = \frac{4}{2} = 2$$

b) $\lim_{x \to +\infty} \dfrac{4x + 2x^2}{-7 + 3x^3} = \lim_{x \to +\infty} \dfrac{2x^2}{3x^3} = \lim_{x \to +\infty} \dfrac{2}{3x} = 0$

c) $\lim_{x \to -\infty} \dfrac{2x^2 - x + 5}{4x^3 - 1} = \lim_{x \to -\infty} \dfrac{\cancel{2}x^2}{\overset{2}{\cancel{4}}x^3} = \lim_{x \to -\infty} \dfrac{1}{2x} = 0$

d) $\lim_{x \to -\infty} \left(x^2 + 5\right) = +\infty$

e) $\lim_{x \to +\infty} \left(x^3 + 1365\right) = +\infty$

f) $\lim\limits_{x \to +\infty} (3 - x) = -\infty$

g) $\lim\limits_{x \to +\infty} (x^2 - x) = +\infty$

h) $\lim_{x \to +\infty} \left(x^3 - x^2 - x + 1\right)$ $= +\infty$

i) $\lim_{x \to +\infty} \left(- x^4 + 7x^3 - x^2 + x + 1\right)$ $= -\infty$

j) $\lim_{x \to +\infty} \dfrac{x + 1}{x^2 + 1} = \lim_{x \to +\infty} \dfrac{x}{x^2} = \lim_{x \to +\infty} \dfrac{1}{x} = 0$

k) $\lim_{x \to -\infty} \dfrac{x^2 - 5x + 8}{x + 3} = \lim_{x \to -\infty} \dfrac{x^2}{x} = \lim_{x \to -\infty} x = -\infty$

l) $\lim_{x \to +\infty} \dfrac{1 - x + x^2 + 5x^3}{4 + x^3} = 5$

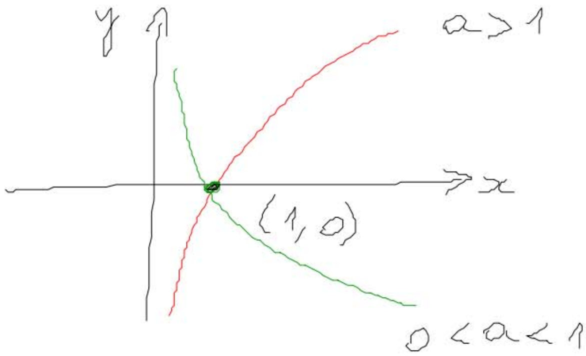
m) $\lim_{x \to -\infty} \dfrac{1 - 4x^3}{5x^3 - 8} = -\dfrac{4}{5}$

n) $\lim_{x \to +\infty} \left(10 + e^{-x}\right) = \lim_{x \to +\infty} \left(10 + \cancelto{0}{\frac{1}{e^{x}}}\right) = 10$

$f(x) = \log_a x$

o) $\lim_{x \to +\infty} \left[\ln\left(x + 1\right)\right] = +\infty$

x	$\ln(x+1)$
20	3,04
50	3,93
100	4,62



p) $\lim\limits_{x \to +\infty} \dfrac{2x^3 - 5x^2 + 3}{x^2 + 4x - 1}$

q) $\lim\limits_{x \to 2} f(x)$ onde $f(x) = \begin{cases} x^2 - 1, & x \leq 2 \\ x + 1, & x > 2 \end{cases}$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (x^2 - 1) = 3$

$\lim\limits_{x \to 2} f(x) = 3$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (x + 1) = 3$