$\boxed{1}$

a) $\boxed{x\dfrac{dy}{dx} + 2y = 3}$

$$\frac{dy}{dx} + \frac{2}{x}\cdot y = \frac{3}{x} \; ; \quad \therefore P(x) = \frac{2}{x} \Rightarrow \mu(x) = e^{2\int \frac{1}{x}\,dx} ,$$

$$\mu(x) = e^{2\ln|x|} = e^{\ln|x|^2} = x^2 ; \quad \therefore \boxed{\mu(x) = x^2}$$

$$\frac{dy}{dx}\cdot x^2 + \frac{2}{x}\cdot x^2 y = \frac{3}{x}x^2 \Rightarrow \frac{dy}{dx}\cdot x^2 + 2x\,y = 3x ,$$

$$\frac{d}{dx}(y\cdot x^2) = 3x \Rightarrow \int d(y\cdot x^2) = 3\int x\,dx \Rightarrow y\cdot x^2 = \frac{3x^2}{2} + C ,$$

$$y = \frac{3}{2}x^2\cdot x^{-2} + C\cdot x^{-2} \Rightarrow \boxed{y = \frac{3}{2} + C\cdot x^{-2}}$$

b) $\boxed{y' = 2y + x^2 + 5}$

$$\frac{dy}{dx} - 2y = x^2 + 5 \Rightarrow \mu(x) = e^{-2\int dx} = e^{-2x}$$

$$\frac{dy}{dx}\cdot e^{-2x} - 2e^{-2x}\cdot y = (x^2+5)\,e^{-2x} \; ;$$

$$\frac{d}{dx}(y\cdot e^{-2x}) = (x^2+5)\,e^{-2x} = x^2\cdot e^{-2x} + 5\cdot e^{-2x} \; ;$$

$$\int d(y\cdot e^{-2x}) = \underbrace{\int x^2 e^{-2x}\,dx}_{\mathrm{I}} + 5\int e^{-2x}\,dx \; ;$$

$$y\cdot e^{-2x} = \left\{-\frac{x^2\cdot e^{-2x}}{2} - \frac{x\,e^{-2x}}{2} - \frac{e^{-2x}}{4}\right\} - \frac{5}{2}e^{-2x} + C \; ;$$

[2]

$$y \cdot e^{-2x} = -\frac{1}{2} e^{-2x}\left[x^2 + x + \frac{1}{2} + 5\right] + c \ ,$$

$$y \cdot e^{-2x} = -\frac{1}{2} e^{-2x}\left[x^2 + x + \frac{1}{2} + \frac{10}{2}\right] + c = -\frac{1}{2} e^{-2x}\left[x^2 + x + \frac{11}{2}\right] + c \ ;$$

$$\boxed{y = -\frac{1}{2}\left[x^2 + x + \frac{11}{2}\right] + c\, e^{-2x}}$$

CALCULO DA INTEGRAL:

$u = x^2 \Rightarrow du = 2x\,dx$

$dv = e^{-2x}\,dx$

$v = -\frac{1}{2} e^{-2x}$

$u = x$

$du = dx$

$$\text{I} = \int x^2 . e^{-2x}\,dx = -\frac{x^2 e^{-2x}}{2} - \int -\frac{1}{2} e^{-2x} . 2x\,dx \ ;$$

$$\text{I} = \frac{-x^2 e^{-2x}}{2} + \int x . e^{-2x}\,dx$$

$$\text{I} = \frac{-x^2 . e^{-2x}}{2} + \left\{\frac{-x e^{-2x}}{2} - \int -\frac{1}{2} e^{-2x}\,dx\right\} ,$$

$$\text{I} = \frac{-x^2 . e^{-2x}}{2} - \frac{x . e^{-2x}}{2} + \frac{1}{2}\int e^{-2x}\,dx \Big\} \ ;$$

$$\boxed{\text{I} = \frac{-x^2 . e^{-2x}}{2} - \frac{x . e^{-2x}}{2} - \frac{1}{4} e^{-2x}}$$

c) $\boxed{x\,dy = [x\,\text{sen}(x) - y]\,dx}$

$$x\frac{dy}{dx} = x\,\text{sen}(x) - y \Rightarrow x\frac{dy}{dx} + y = x.\text{sen}(x) \quad ,$$

$$\boxed{\frac{dy}{dx} + \frac{1}{x}y = \text{sen}(x)} \quad , \quad \mu(x) = e^{\int \frac{1}{x}dx} = e^{\ln|x|} = |x| \; ;$$

$$\boxed{\mu(x) = x} \quad , \quad x > 0$$

$$\frac{dy}{dx}.x + \frac{1}{x}.y.x = x.\text{sen}(x) \Rightarrow \frac{d}{dx}(y.x) = x.\text{sen}(x) \quad ,$$

$$\int d(y.x) = \int \underbrace{x}_{u}.\underbrace{\text{sen}(x)dx}_{dv} \; ;$$

$u = x \Rightarrow du = dx$,
$dv = \text{sen}(x)\,dx$,
$v = -\cos(x)$

$$y.x = -x.\cos(x) - \int -\cos(x)\,dx \; ;$$

$$y.x = -x.\cos(x) + \text{sen}(x) + C \; ; \quad (\times)\; x^{-1}$$

$$\boxed{y = -\cos(x) + x^{-1}.\text{sen}(x) + x^{-1}.C} \quad \text{ou}$$

$$y = \frac{1}{x}\text{sen}(x) - \cos(x) + \frac{C}{x}.$$

d) $\boxed{(1+x^2)\,dy + (xy + x^3 + x)\,dx = 0}$ $\qquad \boxed{\dfrac{dy}{dx} + P(x)y = g(x)}$

$$(1+x^2)\frac{dy}{dx} + xy + x^3 + x = 0 \;;$$

$$\frac{dy}{dx} + \frac{x}{(1+x^2)}\,y = \frac{-(x^3+x)}{1+x^2} = \frac{-x\,\cancel{(1+x^2)}}{\cancel{(1+x^2)}} \;;$$

$$\boxed{\frac{dy}{dx} + \frac{x}{1+x^2}\cdot y = -x} \quad ① \qquad \therefore\; P(x) = \frac{x}{1+x^2}$$

$$\mu(x) = e^{\int \frac{x}{1+x^2}dx} = e^{\frac{1}{2}\int \frac{1}{v}dx} = e^{\frac{1}{2}\ln|1+x^2|} \;; \qquad \boxed{\begin{array}{l} v = 1+x^2 \\ \dfrac{dv}{dx} = 2x \\ \dfrac{1}{2}dv = x.dx \end{array}}$$

$$\therefore\; \boxed{\mu(x) = (1+x^2)^{\frac{1}{2}}} \quad ②$$

TEMOS ENTÃO:

$$\frac{dy}{dx}(1+x^2)^{\frac{1}{2}} + \frac{x}{(1+x^2)}(1+x^2)^{\frac{1}{2}}\cdot y = -x(1+x^2)^{\frac{1}{2}} \;;$$

$$\frac{dy}{dx}(1+x^2)^{\frac{1}{2}} + \frac{x}{(1+x^2)^{\frac{1}{2}}}\cdot y = -x(1+x^2)^{\frac{1}{2}} \;;$$

$$\frac{d}{dx}\left[y\cdot(1+x^2)^{\frac{1}{2}}\right] = -x(1+x^2)^{\frac{1}{2}} \;;$$

$$\int d\left[y(1+x^2)^{\frac{1}{2}}\right] = -\int x\,(\underbrace{1+x^2}_{v})^{\frac{1}{2}}dx \;;$$

$$y\cdot(1+x^2)^{\frac{1}{2}} = -\frac{1}{2}\int v^{\frac{1}{2}}.dv = -\frac{1}{\cancel{2}}\,v^{\frac{3}{2}}\cdot\frac{\cancel{2}}{3} + C \;;$$

$$y\cdot(1+x^2)^{\frac{1}{2}} = -\frac{1}{3}(1+x^2)^{3/2} + C \;; \qquad \{(\times)\text{ POR } (1+x^2)^{-\frac{1}{2}}$$

$$y = -\frac{1}{3}(1+x^2)^{\frac{3}{2}-\frac{1}{2}} + C\,(1+x^2)^{-\frac{1}{2}} \;;$$

$$\boxed{y = -\frac{1}{3}(1+x^2) + \frac{C}{\sqrt{1+x^2}}} \;;$$

$\boxed{e}$ $\boxed{\cos(x)\dfrac{dy}{dx} + \operatorname{sen}(x)\cdot y = 1}$

$$\frac{dy}{dx} + \frac{\operatorname{sen}(x)}{\cos(x)}\cdot y = \frac{1}{\cos(x)} \quad , \quad P(x) = \frac{\operatorname{sen}(x)}{\cos(x)} \;;$$

$$\int P(x)\,dx = \int \frac{\operatorname{sen}(x)}{\cos(x)}\cdot dx = -\int \frac{1}{v}\,dv = -\ln|v| \;;$$

$$\boxed{\begin{aligned} v &= \cos(x) \\ -dv &= \operatorname{sen}(x)\,dx \end{aligned}}$$

$$\int P(x)\,dx = -\ln|\cos(x)| = \ln|\cos(x)|^{-1} \;;$$

$$\mu(x) = e^{\ln|\cos(x)|^{-1}} = |\cos(x)|^{-1} = \frac{1}{|\cos(x)|} = |\sec(x)|$$

$$\therefore \boxed{\mu(x) = \sec(x)}$$

MULTIPLICANDO A EQ. POR $\mu(x)$:

$$\frac{dy}{dx}\cdot\sec(x) + \overset{tg(x)}{\frac{\operatorname{sen}(x)}{\cos(x)}}\cdot\sec(x)\cdot y = \frac{1}{\cos(x)}\cdot\sec(x) \;;$$

$$\frac{d}{dx}\left(y\cdot\sec(x)\right) = \sec^2(x) \;;$$

$$\int d\left(y\cdot\sec(x)\right) = \int \sec^2(x)\,dx \,,$$

$$y\sec(x) = tg(x) + C \,,$$

$$y = \frac{\frac{\text{Sen}(x)}{\cos(x)}}{\frac{1}{\cos(x)}} + \frac{c}{\text{Sec}(x)} = \text{Sen}(x) + c.\cos(x) \ ;$$

$$\therefore \boxed{y = \text{Sen}(x) + c.\cos(x)}$$

f) $\boxed{\cos^2(x).\,\text{Sen}(x)\,dy + \left[y.\cos^3(x) - 1\right] dx = 0}$

$$\cos^2(x).\,\text{Sen}(x)\,\frac{dy}{dx} + y.\cos^3(x) - 1 = 0 \ ; \qquad (\div)\cos^2(x).\,\text{Sen}(x)$$

$$\frac{dy}{dx} + \frac{\cos^3(x)}{\cos^2(x).\,\text{Sen}(x)}.\,y = \frac{1}{\cos^2(x).\text{Sen}(x)} \ ,$$

$$\frac{dy}{dx} + \frac{\cos(x)}{\text{Sen}(x)}.\,y = \text{Sec}^2(x).\,\text{Cossec}(x) \ ; \qquad \therefore \boxed{P(x) = \frac{\cos(x)}{\text{Sen}(x)}}$$

$$\mu(x) = e^{\int \frac{\cos(x)}{\text{Sen}(x)}.dx} = e^{\int \frac{1}{v}\,dv} = e^{\ln|v|} \ ; \qquad \boxed{\begin{array}{l} v = \text{Sen}(x) \\ dv = \cos(x)\,dx \end{array}}$$

$$\mu(x) = e^{\ln|\text{Sen}(x)|} = |\text{Sen}(x)| \ ; \quad \boxed{\mu(x) = \text{Sen}(x)}$$

$$\text{Sen}(x) > 0 .$$

MULTIPLICA A EQ. POR $\mu(x)$

$$\frac{dy}{dx}.\,\text{Sen}(x) + \frac{\cos(x)}{\cancel{\text{Sen}(x)}}.\,\cancel{\text{Sen}(x)}.\,y = \frac{\cancel{\text{Sen}(x)}}{\cos^2(x).\,\cancel{\text{Sen}(x)}} \ ,$$

$$\frac{dy}{dx}.\,\text{Sen}(x) + \cos(x).\,y = \text{Sec}^2(x) \ ,$$

$$\frac{d}{dx}\left(y \cdot \operatorname{sen}(x)\right) = \sec^2(x) \quad ;$$

$$\int d(y \cdot \operatorname{sen}(x)) = \int \sec^2(x)\, dx \Rightarrow y \cdot \operatorname{sen}(x) = tg(x) + C \, ;$$

$$y = \frac{\frac{\cancel{\operatorname{sen}(x)}}{\cos(x)}}{\frac{\cancel{\operatorname{sen}(x)}}{1}} + C \cdot \frac{1}{\operatorname{sen}(x)} = \frac{1}{\cos(x)} + C \cdot \frac{1}{\operatorname{sen}(x)} \quad ;$$

$$\boxed{y = \sec(x) + C \cdot \operatorname{cossec}(x)}$$

$\boxed{9}$ $\boxed{[1 - \cos(x)]\,dy + [2y \operatorname{sen}(x) - tg(x)]\,dx = 0}$

$$[1 - \cos(x)] \frac{dy}{dx} + 2 \operatorname{sen}(x) \cdot y = tg(x) \quad ; \quad (\div)\, [1 - \cos(x)]$$

$$\frac{dy}{dx} + \frac{2 \operatorname{sen}(x)}{1 - \cos(x)} \cdot y = \frac{tg(x)}{1 - \cos(x)} \quad ;$$

$$P(x) = \frac{2 \operatorname{sen}(x)}{1 - \cos(x)} \Rightarrow \mu(x) = e^{2\int \frac{\operatorname{sen}(x)}{1 - \cos(x)}\, dx} \quad ,$$

$$\boxed{\begin{aligned} v &= 1 - \cos(x) \\ \frac{dv}{dx} &= -(-\operatorname{sen}(x)) \\ dv &= \operatorname{sen}(x) \cdot dx \end{aligned}}$$

$$\mu(x) = e^{2\int \frac{1}{v}\, dv} = e^{2 \ln|v|} = e^{\ln [1 - \cos(x)]^2} \quad ,$$

$$\therefore \boxed{\mu(x) = [1 - \cos(x)]^2}$$

MULTIPLICANDO A EQ.

$$\frac{dy}{dx}\cdot[1-\cos(x)]^2+\frac{2\,\mathrm{sen}(x)}{\cancel{[1-\cos(x)]}}[1-\cos(x)]^{\cancel{2}\,1}\cdot y=\frac{tg(x)[1-\cos(x)]^{\cancel{2}}}{\cancel{[1-\cos(x)]}};$$

$$\frac{dy}{dx}\cdot[1-\cos(x)]^2+2\,\mathrm{sen}(x)[1-\cos(x)]\cdot y=tg(x)[1-\cos(x)];$$

$$\frac{d}{dx}\left[y\cdot[1-\cos(x)]^2\right]=tg(x)-tg(x)\cdot\cos(x);$$

$$d\left[y\cdot[1-\cos(x)]^2\right]=\left\{tg(x)-\frac{\mathrm{sen}(x)}{\cancel{\cos(x)}}\cdot\cancel{\cos(x)}\right\}dx;$$

$$\int d\left[y\cdot[1-\cos(x)]^2\right]=-\int\frac{\mathrm{sen}(x)}{\cos(x)}\,dx-\int\mathrm{sen}(x)\,dx;$$

$$l=\cos(x)$$
$$\frac{dl}{dx}=-\mathrm{sen}(x)$$
$$-dl=\mathrm{sen}(x)\,dx$$

$$y\cdot[1-\cos(x)]^2=(-1)\cdot\ln|\cos(x)|+\cos(x)+C;$$

$$y\cdot[1-\cos(x)]^2=\ln|\cos^{-1}(x)|+\cos(x)+C;$$

$$y[1-\cos(x)]^2=\ln\left(\frac{1}{|\cos(x)|}\right)+\cos(x)+C;$$

$$y[1-\cos(x)]^2=\ln|\sec(x)|+\cos(x)+C;$$

$$y=[1-\cos(x)]^{-2}\left\{\ln|\sec(x)|+\cos(x)+C\right\};$$

(h) $\boxed{e^{\frac{x^2}{2}+x}\left[x+x^2(x+1)\right]=\frac{dy}{dx}+(x+1).y}$ $\boxed{9}$

$$\frac{dy}{dx}+(x+1).y=e^{\frac{x^2}{2}+x}\left[x+x^2(x+1)\right] \quad , \quad \therefore P(x)=x+1 .$$

$$\boxed{\mu(x)=e^{\int(x+1)dx}=e^{\frac{x^2}{2}+x}}$$ MOLTIPLICA A EQ. POR $\mu(x)$:

$$\frac{dy}{dx}.e^{\frac{x^2}{2}+x}+(x+1).e^{\frac{x^2}{2}+x}.y=e^{\frac{x^2}{2}+x}\left[x+x^2(x+1)\right].e^{\frac{x^2}{2}+x} \quad ;$$

$$\frac{d}{dx}\left[y.e^{\frac{x^2}{2}+x}\right]=e^{\frac{x^2}{2}+x}\, x\left[1+x.(x+1)\right]e^{\frac{x^2}{2}+x} \quad ;$$

$$\int d\left[y.e^{\frac{x^2}{2}+x}\right]=\int \underbrace{x.e^{\frac{x^2}{2}+x}}_{u}\,\underbrace{\left[1+x(x+1)\right].e^{\frac{x^2}{2}+x}\,dx}_{du} \quad ;$$ VER RODAPÉ

$$y.e^{\frac{x^2}{2}+x}=\int u\,du=\frac{u^2}{2}+C \quad ;$$

$$y.e^{\frac{x^2}{2}+x}=\frac{1}{2}\left[x.e^{\frac{x^2}{2}+x}\right]^2+C; \quad \Longrightarrow (x)\ \boxed{e^{-\left(\frac{x^2}{2}+x\right)}}$$

$$u=x.e^{\frac{x^2}{2}+x} \Rightarrow \frac{du}{dx}=e^{\frac{x^2}{2}+x}+x\,e^{\frac{x^2}{2}+x}(x+1)$$

$$du=\left[1+x(x+1)\right]e^{\frac{x^2}{2}+x}\,dx$$

$$\boxed{y = \frac{1}{2}x^2 e^{\frac{x^2}{2}+x} + c \cdot e^{-\frac{x^2}{2}-x}}$$

i) $\boxed{x\,\frac{dy}{dx} + 2y = e^x + \ln(x)}$ $\div x$

$$\frac{dy}{dx} + \boxed{\frac{2}{x}} \cdot y = \frac{e^x}{x} + \frac{\ln(x)}{x}\ ; \quad P(x) = \frac{2}{x}$$

$$\mu(x) = e^{2\int \frac{1}{x}dx} = e^{2\ln|x|} = e^{\ln|x|^2}\ ; \therefore \boxed{\mu(x) = x^2}$$

MULTIPLICA A EQ. POR $\mu(x)$:

$$\frac{dy}{dx}\cdot x^2 + \frac{2}{\cancel{x}}\cdot x^2 \cdot y = \frac{e^x}{\cancel{x}}\cdot x^2 + \frac{x^2 \ln(x)}{\cancel{x}}\ ;$$

$$\frac{dy}{dx}\cdot x^2 + 2xy = x \cdot e^x + x\ln(x)\ ;$$

$$\frac{d}{dx}[y \cdot x^2] = x \cdot e^x + x \cdot \ln(x)\ ;$$

$$\int d[y\,x^2] = \underbrace{\int x \cdot e^x \cdot dx}_{A} + \underbrace{\int x \cdot \ln(x)\,dx}_{B}\ ; \quad \text{I}$$

$$y \cdot x^2 = \overbrace{[x \cdot e^x - e^x]}^{A} + \overbrace{\left[\frac{x^2 \cdot \ln(x)}{2} - \frac{x^2}{4}\right]}^{B} + c\ ; \quad (x)\ x^{-2}$$

$$y = x^{-1}e^x - x^{-2}e^x + \frac{1}{2}\ln(x) - \frac{1}{4} + c \cdot x^{-2}\ ; \text{ ou}$$

$$\boxed{y = \frac{e^x}{x} - \frac{e^x}{x^2} + \frac{1}{2}\ln(x) - \frac{1}{4} + \frac{c}{x^2}}$$

SOLUÇÃO DAS INTEGRAIS (A) E (B):

$$(A) = \int \underbrace{x}_{u} \cdot \underbrace{e^x \, dx}_{dv} = x \cdot e^x - \int e^x \, dx \; ;$$

$u = x \Rightarrow du = dx$

$dv = e^x \, dx$

$v = e^x$

$$\boxed{(A) = \int x \cdot e^x \cdot dx = x \cdot e^x - e^x}$$

$$(B) = \int \underbrace{x}_{dv} \cdot \underbrace{\ln(x)}_{u} \, dx = \frac{x^2 \ln(x)}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} \cdot dx \; ;$$

$u = \ln(x)$

$du = \frac{1}{x} \, dx$

$dv = x \, dx$

$v = \frac{x^2}{2}$

$$(B) = \frac{x^2 \cdot \ln(x)}{2} - \frac{1}{2} \cdot \frac{x^2}{2} = \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} \; ;$$

7) $\boxed{\dfrac{dy}{dx} + y = \dfrac{1 - e^{-2.x}}{e^{x} + e^{-x}}}$ $\quad P(x) = 1$

$\mu(x) = e^{\int dx} = e^{x}$, MULTIPLICA A EQ. POR e^{x}:

$\dfrac{dy}{dx} \cdot e^{x} + e^{x} \cdot y = \dfrac{(1 - e^{-2x})}{e^{x} + e^{-x}} \cdot e^{x}$;

$\dfrac{d}{dx}[y \cdot e^{x}] = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \Rightarrow \int d[y \cdot e^{x}] = \int \left(\dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right) \cdot dx$,

$y \cdot e^{x} = \int \frac{1}{v} dv = \ln|v| + C$;

$y \cdot e^{x} = \ln|e^{x} + e^{-x}| + C$; $\quad (x)\, e^{-x}$

$\boxed{y = e^{-x} \left\{ \ln|e^{x} + e^{-x}| + C \right\}}$

$$\boxed{\begin{aligned} v &= e^{x} + e^{-x} \\ \frac{dv}{dx} &= e^{x} - e^{-x} \\ dv &= (e^{x} - e^{-x})\, dx \end{aligned}}$$