EX 2 - RESOLVA A EQ. $y' + 2y = e^{2x}$.

$$\frac{dy}{dx} + 2y = e^{2x}; \quad (1)$$

$$\frac{dy}{dx} + P(x)y = f(x), \quad (A)$$

FATOR INTEGRANTE:

$$\mu(x) = e^{\int P(x)\,dx}, \quad (B)$$

$$\therefore P(x) = 2; \quad (2)$$

$$\mu(x) = e^{\int 2\,dx} = e^{2x}; \quad (3)$$

MULTIPLICA A EQ. (1) POR (3) (F.I.):

$$\frac{dy}{dx} \cdot e^{2x} + 2e^{2x} y = e^{2x} \cdot e^{2x}; \quad (4)$$

$$\underbrace{\frac{dy}{dx} \cdot e^{2x} + 2e^{2x}}\ = e^{4x}; \quad (5)$$

$\hookrightarrow$ É A DERIVADA DO PRODUTO $\frac{d}{dx}[y \cdot e^{2x}]$;

$$\frac{d}{dx}[y \cdot e^{2x}] = e^{4x}; \quad (6)$$

- $\int dx = x$
- $\int du = u$

$$d(y \cdot e^{2x}) = e^{4x}\,dx; \quad (7)$$

$$\int d(y \cdot e^{2x}) = \int e^{4x}\,dx; \quad (8)$$

$$y \cdot e^{2x} = \frac{1}{4} e^{4x} + C, \quad (9) \rightarrow (\times) \text{ POR } e^{-2x}$$

$$\boxed{y(x) = \frac{e^{2x}}{4} + C \cdot e^{-2x}} \quad (10)$$

EX 3 - RESOLVA A EQ. $\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$. (1)

- $P(x) = \frac{2x+1}{x}$; (2)

- F.I.: $\mu(x) = e^{\int \left(2 + \frac{1}{x}\right) dx}$; (3)

$$\mu(x) = e^{\int 2 dx + \int \frac{1}{x} dx} = e^{2x + \ln|x|} = e^{2x} \cdot e^{\ln|x|} ; \quad (4)$$

$$\boxed{a^{n+m} = a^n \cdot a^m}$$

$$\mu(x) = e^{2x} \cdot |x| , \quad (5).$$

$$\boxed{\mu(x) = x \cdot e^{2x}} , \quad x > 0 ; \quad (6)$$

$$\boxed{\begin{array}{l} a^{\log_a b} = b \\ e^{\log_e |x|} = |x| = x \\ \qquad x > 0 \end{array}}$$

- MULTIPLICA A EQ. (1) POR $\mu(x)$:

$$\underbrace{\frac{dy}{dx}(x \cdot e^{2x}) + \left(\frac{2x+1}{x}\right) \cdot x \cdot e^{2x} \cdot y}_{} = e^{-2x} \cdot x \cdot e^{2x} ; \quad (7)$$

$$\frac{d}{dx}\left[y \cdot x e^{2x}\right] = x \cdot e^{-2x+2x} ; \quad (8)$$

$$\frac{d}{dx}\left[y \cdot x e^{2x}\right] = x \cdot \cancel{e^{0}} ; \quad (9)$$

$$d\left(y \cdot x \cdot e^{2x}\right) = x\,dx \;; \quad (10)$$

$$\int d\left(y \cdot x \cdot e^{2x}\right) = \int x\,dx \;; \quad (11)$$

$$y \cdot x \cdot e^{2x} = \frac{x^2}{2} + c \;; \quad (12)$$

$$y \cdot (x \cdot e^{2x}) = \frac{x^2}{2} + c \;; \quad (13)$$

$$y(x) = \frac{x^2}{2x e^{2x}} + \frac{c}{x e^{2x}} \;; \quad (14)$$

$$y(x) = \frac{x \cdot e^{-2x}}{2} + \frac{c}{x} e^{-2x} \;; \quad (15) \qquad \boxed{\frac{1}{a} = a^{-1}}$$

ou

$$y(x) = e^{-2x}\left(\frac{x}{2} + \frac{c}{x}\right) \;; \quad (16)$$

EX 4 – RESOLVA O P.V.I.

$$\begin{cases} (x^2+1)\dfrac{dy}{dx} + 4xy = x, & (1) \\ y(2) = 1. \end{cases}$$

$$\boxed{\frac{dy}{dx} + P(x).y = f(x)} \quad (A)$$

REESCREVENDO A EQ. $\boxed{\dfrac{dy}{dx} + \dfrac{4x}{x^2+1}.y = \dfrac{x}{x^2+1}}$ (2)

- $P(x) = \dfrac{4x}{x^2+1}$ (3)

- $\mu(x) = e^{4\int \frac{x}{x^2+1}.dx} = e^{4\int \frac{1}{u}\frac{du}{2}} = e^{\frac{4}{2}\int \frac{1}{u}du}$; (4)

(B)
$u = x^2+1$
$du = 2x\,dx$
$\dfrac{du}{2} = x.dx$

$n.\log b = \log b^n$

$\mu(x) = e^{2\ln|u|} = e^{2\ln|x^2+1|}$; (5)

$\mu(x) = e^{2\ln(x^2+1)} = e^{\ln(x^2+1)^2}$; (6)

$\therefore \boxed{\mu(x) = (x^2+1)^2}$ (7)

- MULTIPLICA A EQ. (2) POR $\mu(x)$:

$$\frac{dy}{dx}\cdot(x^2+1)^2 + \frac{4x}{(x^2+1)}\cdot(x^2+1)^2.y = \frac{x}{(x^2+1)}\cdot(x^2+1)^2 \quad ; (8)$$

SIMPLIFICA SIMPLIFICA

$$\frac{dy}{dx}\cdot(x^2+1)^2 + 4x(x^2+1).y = x(x^2+1) \quad ; (9)$$

TEMOS:

$$\frac{d}{dx}\left[y\cdot(x^2+1)^2\right] = x^3 + x \quad ; \quad (10)$$

$$d\left[y\cdot(x^2+1)^2\right] = (x^3+x)\,dx \quad ; \quad (11)$$

$$\int d\left[y(x^2+1)^2\right] = \int (x^3+x)\,dx \quad ; \quad (12)$$

$$\boxed{y(x^2+1)^2 = \frac{x^4}{4} + \frac{x^2}{2} + c} \quad ; \quad (13)$$

→ SOLUÇÃO GERAL.

PVI ⇒ $y(2) = 1 \Rightarrow \begin{cases} x = 2, \\ y = 1. \end{cases}$ ⇒ EM (13)

$$1\cdot(2^2+1)^2 = \frac{2^4}{4} + \frac{2^2}{2} + c \quad ;$$

$$5^2 = 4 + 2 + c \Rightarrow c = 25 - 6 \; ; \; \therefore \boxed{c = 19}$$

→ EM (13)

$$\boxed{y(x^2+1)^2 = \frac{x^4}{4} + \frac{x^2}{2} + 19}$$

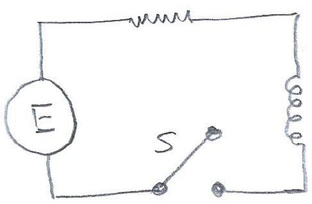
EXEMPLO DE APLICAÇÃO

UM CIRCUITO ELÉTRICO SIMPLES CONSISTE DE UM RESISTOR R E DE UM INDUTOR L LIGADOS EM SÉRIE, ONDE E É UMA FORÇA ELETROMOTRIZ CONSTANTE.

FECHANDO-SE O INTERRUPTOR S EM $t=0$, DECORRE DAS LEIS DA ELETRICIDADE QUE, PARA $t>0$, A CORRENTE i VERIFICA (OBEDECE) A EQ. $\boxed{L\dfrac{di}{dt}+R.i=E}$.

EXPRESSE i COMO FUNÇÃO DO TEMPO.

SOLUÇÃO:



$$\boxed{L.\frac{di}{dt}+R.i=E} \quad (1)$$

QUER: $\boxed{i=f(t)}$ (2)

DADOS:

$t=0 \Rightarrow i=0$

SABEMOS:

EDO, LINEAR, 1ª ORDEM

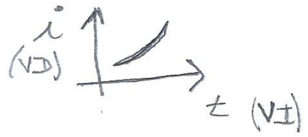
$\boxed{y=f(x)}$

$$\boxed{\frac{dy}{dx}+P(x)\,y=f(x)} \quad (A)$$

$$\boxed{\mu(x)=e^{\int P(x).dx}} \quad (B)$$

NOTAÇÃO - AJUSTE:

$\boxed{i=f(t)}$



$$\boxed{\frac{di}{dt}+P(t).i=f(t)} \quad (C)$$

$$\boxed{\mu(t)=e^{\int P(t).dt}} \quad (D)$$

OU SEJA:

$x \Leftrightarrow t$

$y \Leftrightarrow i$

—— // ——

REESCREVENDO A EQ. (1):

PQ?

$$\boxed{\frac{di}{dt}+\frac{R}{L}\,i=\frac{E}{L}} \quad (3)$$

IMPORTANTE: $\left\{\begin{array}{l} \cdot R \\ \cdot L \\ \cdot E \end{array}\right.$ SÃO CONSTANTES

- $\boxed{P(t) = \frac{R}{L}}$ (4)

- F.I.: $\mu(t) = e^{\int \frac{R}{L} . dt} = e^{\frac{R}{L}\int dt}$; (5)

$\therefore \boxed{\mu(t) = e^{\frac{R}{L} . t}}$; (6)

- (x) A EQ. (3) POR $\mu(t)$:

$$\boxed{\frac{di}{dt} . e^{\frac{R}{L} . t} + \frac{R}{L} . e^{\frac{R}{L} . t} . i = \frac{E}{L} e^{\frac{R}{L} . t}} \quad ; (7)$$

$$\frac{d}{dt}\left[i . e^{\frac{R}{L} . t}\right] = \frac{E}{L} e^{\frac{R}{L} . t} \quad ; (8)$$

$$d\left[i . e^{\frac{R}{L} . t}\right] = \frac{E}{L} . e^{\frac{R}{L} . t} . dt \quad ; (9)$$

$$\int d\left[i . e^{\frac{R}{L} . t}\right] = \frac{E}{L}\int e^{\frac{R}{L} . t} dt \quad ; (10)$$

$$i . e^{\frac{R}{L} . t} = \frac{E}{L}\int e^{u} . \frac{L}{R} du \quad ; (11)$$

$$i . e^{\frac{R}{L} . t} = \frac{E}{\cancel{L}} . \frac{\cancel{L}}{R}\int e^{u} du \quad ; (12)$$

$$\boxed{\begin{array}{l} u = \frac{R}{L} . t \\ \frac{du}{dt} = \frac{R}{L} \\ \frac{L}{R} . du = dt \end{array}}$$

$$i \cdot e^{\frac{R}{L} \cdot t} = \frac{E}{R} e^{u} + C \; ; \quad 13$$

$$\boxed{i \cdot e^{\frac{R}{L} \cdot t} = \frac{E}{R} \cdot e^{\frac{R}{L} \cdot t} + C} \; ; \quad 14$$

FALTA

LEMBRANDO

$$\boxed{\begin{array}{l} \frac{di}{dt} + \frac{R}{L} i = \frac{E}{L} \; ; \; 3 \\ \text{QUER: } i = f(t) \end{array}}$$

USA C.I. (P.V.I.):

$t = 0 \Leftrightarrow i = 0 \Rightarrow$ EM 14

$$0 \cdot e^{0} = \frac{E}{R} \cdot e^{0} + C \Rightarrow \boxed{C = -\frac{E}{R}} \quad 15$$

$\Rightarrow$ EM 14

$$i \cdot e^{\frac{R}{L} \cdot t} = \frac{E}{R} \cdot e^{\frac{R}{L} \cdot t} - \frac{E}{R} \; ; \quad 16$$

"MELHORANDO" 16 : $(\times) \; e^{-\frac{R}{L} \cdot t}$

$$i \cdot \underbrace{e^{\frac{R}{L} \cdot t} \cdot e^{-\frac{R}{L} \cdot t}}_{=1} = \frac{E}{R} \underbrace{e^{\frac{R}{L} t} \cdot e^{-\frac{R}{L} t}}_{=1} - \frac{E}{R} \cdot e^{-\frac{R}{L} \cdot t} \; ; \quad 17$$

$$i(t) = \frac{E}{R} - \frac{E}{R} \cdot e^{-\frac{R}{L} \cdot t} \; ; \quad 18$$

EVIDÊNCIA

$$\boxed{i(t) = \frac{E}{R}\left(1 - e^{-\frac{R}{L} \cdot t}\right)} \quad 19$$

NOTE: $t = 0 \Leftrightarrow i = 0$

$i(0) = \frac{E}{R}\left(1 - e^{0}\right)$;

$i(0) = \frac{E}{R}(1 - 1)$;

$i(0) = 0$; $\Rightarrow$ CI