Cálculo I
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Lista de exercícios – Integral (parte 2)

Questão 1. Calcule as seguintes integrais por frações parciais:

a) $\int \dfrac{x^4}{x-1}\, dx$

b) $\int \dfrac{3t-2}{t+1}\, dt$

c) $\int \dfrac{y}{(y+4)(2y-1)}\, dy$

d) $\int \dfrac{x^3+4}{x^2+4}\, dx$

e) $\int \dfrac{x^2-5x+16}{(2x+1)(x-2)^2}\, dx$

f) $\int \dfrac{10}{(x-1)(x^2+9)}\, dx$

Questão 2. Calcule as seguintes integrais pela regra da substituição trigonométrica:

a) $\int \dfrac{1}{x^2\sqrt{4-x^2}}\, dx$

b) $\int \dfrac{x^3}{\sqrt{x^2+4}}\, dx$

c) $\int \dfrac{\sqrt{x^2-4}}{x}\, dx$

d) $\int \dfrac{1}{(x^2+1)^2}\, dx$

e) $\int \dfrac{x}{\sqrt{x^2+x+1}}\, dx$

f) $\int \sqrt{x^2+1}\, dx$

Questão 3. **(Difícil)** Calcule as seguintes integrais, deduzindo quais regras usar:

a) $\int \dfrac{e^{\operatorname{arctg} y}}{1+y^2}\, dy$

b) $\int t\cos^2 t\, dt$

c) $\int x^5 e^{-x^3}\, dx$

d) $\int \dfrac{1}{x\sqrt{4x+1}}\, dx$

e) $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$

f) $\int \dfrac{e^{2x}}{1+e^x}\, dx$

g) $\int (2x^2+1)e^{x^2}\, dx$

Gabarito

1a) $\dfrac{x^4}{4} + \dfrac{x^3}{3} + \dfrac{x^2}{2} + x + \ln|x-1| + C$

1b) $3t - 5\ln|t+1| + C$

1c) $\dfrac{\ln|2y-1|}{18} + \dfrac{4\ln|y+4|}{9} + C$

1d) $\dfrac{x^2}{2} - 2\ln(x^2+4) + 2\arctan\left(\dfrac{x}{2}\right) + C$

1e) $-\dfrac{2}{x-2} - \ln|x-2| + \dfrac{3}{2}\ln|2x+1| + C$

1f) $\ln|x-1| - \dfrac{1}{2}\ln(x^2+9) - \dfrac{1}{3}\arctan\left(\dfrac{x}{3}\right) + C$

2a) $-\dfrac{\sqrt{4-x^2}}{4x} + C$

2b) $\dfrac{(x^2-8)\sqrt{x^2+4}}{3} + C$

2c) $\sqrt{x^2-4} + 2\arctan\left(\dfrac{2}{\sqrt{x^2-4}}\right) + C$

2d) $\dfrac{x}{2(x^2+1)} + \dfrac{\operatorname{arctg}(x)}{2} + C$

2e) $\sqrt{x^2+x+1} - \dfrac{1}{2}\operatorname{arcsen}\left(\dfrac{2x+1}{\sqrt{3}}\right) + C$

2f) $\dfrac{x\sqrt{x^2+1}}{2} + \dfrac{\operatorname{arcsen}(x)}{2} + C$

3a) $e^{\operatorname{arctg} y} + C$

3b) $\dfrac{t^2}{4} - \dfrac{\operatorname{sen}^2 t}{8} + \dfrac{\cos^2 t}{8} + \dfrac{t\operatorname{sen} t\cos t}{2} + C$

3c) $-\dfrac{e^{-x^3}(x^3+1)}{3} + C$

3d) $\ln|1 - \sqrt{4x+1}| - \ln|1 + \sqrt{4x+1}| + C$

3e) $2e^{\sqrt{x}}(x - 2\sqrt{x} + 2) + C$

3f) $e^x - \ln(e^x + 1) + C$

3g) $xe^{x^2} + C$