

**9ª Lista de Exercícios – Métodos Matemáticos  
(Transformada de Laplace)  
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1) Calcule as transformadas de Laplace:

a)  $L\{e^{at}\}$ ;      R:  $\frac{1}{s-a}$ ;     $s > a$       Resolvido nas notas de aulas

b)  $L\{\text{Sen}^2(t)\}$  ;      R:  $\frac{2}{s(s^2+4)}$       Note que:  $\text{Sen}^2(t) = \frac{1-\text{Cos}(2t)}{2}$

c)  $L\{f(t)\}$ , onde  $f(t) = \begin{cases} 0, & 0 \leq t < 3; \\ 2, & t \geq 3 \end{cases}$ ;      R:  $\frac{2e^{-3s}}{s}$

d)  $L\{t\}$ ;      R:  $\frac{1}{s^2}$ ;       $s > 0$       Resolvido nas notas de aulas

2) Resolva os PVI's usando Transformada de Laplace.

a)  $\frac{dy}{dt} - 5y = 0$ ;     $y(0) = 2$ ;      R:  $f(t) = 2e^{5t}$

b)  $y'' - 6y' + 9y = t^2e^{3t}$ ;     $y(0) = 2$ ;  $y'(0) = 6$ ;

R:  $f(t) = 2e^{3t} + \frac{t^4e^{3t}}{12}$

c)  $X'' + 16X = \text{Cos}(4t)$ ;     $X(0) = 0$ ;  $X'(0) = 1$ ,  $X = f(t)$

R:  $X(t) = \frac{1}{4}\text{Sen}(4t) + \frac{1}{8}t.\text{Sen}(4t)$

d)  $\frac{dN}{dt} = 0,05N$ ;     $N(0) = 20.000$ ;      R:  $N(t) = 20.000e^{0,05t}$

e)  $X'' + 16X = 2 \text{Sen}(4t)$ ;     $X(0) = -\frac{1}{2}$ ;  $X'(0) = 0$ ;

R:  $X(t) = \frac{1}{16}\{\text{Sen}(4t) - 4t\text{Cos}(4t) - \frac{1}{2}\text{Cos}(4t)\}$

3) Calcule as Transformadas Inversas de Laplace de:

a)  $L^{-1}\{\frac{1}{s}\}$  ;      R:  $f(t) = 1$

b)  $L^{-1}\{\frac{1}{s-8}\}$  ;      R:  $f(t) = e^{8t}$

$$\text{c) } L^{-1}\left\{\frac{5s}{(s^2+1)^2}\right\} ; \quad \text{R: } f(t) = \frac{5}{2}x\text{Sen}(x)$$

$$\text{d) } L^{-1}\left\{\frac{1}{s^2-2s+9}\right\} ; \quad \text{R: } f(t) = \frac{1}{\sqrt{8}}e^x\text{Sen}(\sqrt{8}x)$$

$$\text{e) } L^{-1}\left\{\frac{s+4}{s^2+4s+8}\right\} ; \quad \text{R: } f(t) = e^{-2x}\text{Cos}(2x) + e^{-2x}\text{Sen}(2x)$$