3ª LISTA

1) VERIFIQUE SE A EQ. É EXATA, SE FOR RESOLVA:

$Mdx + Ndy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = 0$

$M = \frac{\partial f}{\partial x}$

$N = \frac{\partial f}{\partial y}$

a) $\boxed{\underbrace{(2x-1)}_{M} dx + \underbrace{(3y+7)}_{N} dy}$

$M = 2x - 1 \Rightarrow \boxed{\frac{\partial M}{\partial y} = 0}$ // $N = 3y + 7 \Rightarrow \boxed{\frac{\partial N}{\partial x} = 0}$

A EQ. É EXATA

$f(x,y) = \int (2x-1)\,dx + g(y)$

$f(x,y) = 2\frac{x^2}{2} - x + g(y) \Rightarrow \boxed{f(x,y) = x^2 - x + g(y)}$ ①

$\overset{=N}{\frac{\partial f}{\partial y}} = \frac{\partial}{\partial y}\left[x^2 - x + g(y)\right] \Rightarrow 3y + 7 = \frac{dg}{dy} \Rightarrow \int dg = \int (3y+7)\,dy$

$\boxed{g(y) = \frac{3y^2}{2} + 7y}$ ②

② → ①: $f(x,y) = x^2 - x + \frac{3y^2}{2} + 7y \Rightarrow$ Sol: $\boxed{x^2 - x + \frac{3y^2}{2} + 7y = C}$

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b) $\boxed{(5x + 4y)\,dx + (4x - 8y^3)\,dy}$

$M = 5x + 4y \Rightarrow \boxed{\frac{\partial M}{\partial y} = 4}$; $N = 4x - 8y^3 \Rightarrow \boxed{\frac{\partial N}{\partial x} = 4}$ ∴ EQ. EXATA

$f(x,y) = \int (5x + 4y)\,dx + g(y) \Rightarrow \boxed{f(x,y) = \frac{5x^2}{2} + 4yx + g(y)}$ ①

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\frac{5x^2}{2} + 4xy + g(y)\right] \Rightarrow \cancel{4x} - 8y^3 = \cancel{4x} + g'(y)$

$\frac{dg}{dy} = -8y^3 \Rightarrow g(y) = -\frac{8y^4}{4} \Rightarrow \boxed{g(y) = -2y^4}$ ②

② → ①: $\frac{5x^2}{2} + 4xy - 2y^4 = f(x,y)$

Sol: $\boxed{\frac{5x^2}{2} + 4xy - 2y^4 = C}$

(2)

c) $\boxed{\underbrace{(2y^2x-3)}_{M}dx+\underbrace{(2yx^2+4)}_{N}dy=0}$

$M=2y^2.x-3 \Rightarrow \boxed{\frac{\partial M}{\partial y}=4xy}$ // $N=2yx^2+4 \Rightarrow \boxed{\frac{\partial N}{\partial x}=4xy}$

É EXATA

$f(x,y)=\int(2y^2x-3)\,dx+g(y)$

$f(x,y)=2y^2\frac{x^2}{2}-3x+g(y) \Rightarrow \boxed{f(x,y)=x^2y^2-3x+g(y)}$ (1)

$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left[x^2y^2-3x+g(y)\right] \Rightarrow 2yx^2+4=2x^2y+g'(y) \Rightarrow$

$\frac{dg}{dy}=4 \Rightarrow \boxed{g=4y}$ (2) (2) em (1) $\Rightarrow f(x,y)=x^2y^2-3x+4y$

sol. $\boxed{x^2y^2-3x+4y=C}$

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d) $\boxed{\underbrace{(x+y).(x-y)}_{M}dx+\underbrace{x(x-2y)}_{N}dy=0}$

$M=(x+y).(x-y)=x^2-xy+xy-y^2 \Rightarrow M=x^2-y^2 \Rightarrow \boxed{\frac{\partial M}{\partial y}=-2y}$

$N=x^2-2xy \Rightarrow \boxed{\frac{\partial N}{\partial x}=2x-2y}$ $\frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$, ∴ NÃO É EXATA

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e) $\boxed{\left(1+\ln(x)+v.x^{-1}\right)dx=\left(1+\ln(x)\right)dv}$

NESSE CASO: $\frac{\partial M}{\partial v}=\frac{\partial N}{\partial x}$

$M=1+\ln x+v.x^{-1} \Rightarrow \boxed{\frac{\partial M}{\partial v}=\frac{1}{x}}$

$N=1+\ln(x) \Rightarrow \boxed{\frac{\partial N}{\partial x}=\frac{1}{x}}$ COMO $\frac{\partial M}{\partial v}=\frac{\partial N}{\partial x}$, É EQ. EXATA

$M=\frac{\partial f}{\partial x}$ E $N=\frac{\partial f}{\partial v}$

$Mdx+Ndv=0$

$\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial v}dv=0$

$f(x,y)=\int\left(1+\ln(x)+\frac{v}{x}\right)dx+g(v)$

$$f(x,y) = x + \int \underbrace{\ln(x)}_{u}\,\underbrace{dx}_{dV} + y\int \frac{1}{x}\,dx + g(y)$$

$$\int u\,dV = u.V - \int V.du$$

$$u = \ln(x)$$

$$du = \frac{1}{x}\,dx$$

$$dV = dx$$

$$V = x$$

$$x > 0$$

$$f(x,y) = x + \left[x.\ln(x) - \int \cancel{x}\,\frac{1}{\cancel{x}}\,dx\right] + y.\ln|x| + g(y)$$

$$f(x,y) = x + \left[x.\ln x - x\right] + y.\ln|x| + g(y)$$

$$f(x,y) = \cancel{x} + x.\ln x \cancel{- x} + y.\ln|x| + g(y)$$

$$\boxed{f(x,y) = x.\ln(x) + y.\ln(x) + g(y)} \quad 1$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[x.\ln(x) + y.\ln(x) + g(y)\right]$$

$$N = \ln(x) + g'(y) \Rightarrow 1 + \cancel{\ln(x)} = \cancel{\ln(x)} + g'(y)$$

$$\frac{dg}{dy} = 1 \Rightarrow \int dg = \int dy \Rightarrow \boxed{g(y) = y} \quad 2$$

$$2 \to 1 \quad x.\ln(x) + y.\ln(x) + y = f(x,y)$$

Sol. $\boxed{x.\ln(x) + y.\ln(x) + y = C}$

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f) $\boxed{\left(1-\frac{3}{x}+y\right)dx+\left(1-\frac{3}{y}+x\right)dy=0}$

$M=1-3.x^{-1}+y \Rightarrow \boxed{\frac{\partial M}{\partial y}=1}$ // $N=1-3.y^{-1}+x \Rightarrow \boxed{\frac{\partial N}{\partial x}=1}$

É EXATA

$f(x,y)=\int\left(1-\frac{3}{x}+y\right)dx+g(y)=x-3\ln|x|+xy+g(y)$

$\boxed{f(x,y)=x-3\ln x+x.y+g(y)}$ (1)

$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left[x-3.\ln x+x.y+g(y)\right] \rightarrow 1-\frac{3}{y}+\cancel{x}=\cancel{x}+g'(y)$

$\frac{dg}{dy}=1-\frac{3}{y} \Rightarrow \int dg=\int dy-3\int\frac{1}{y}dy \Rightarrow \boxed{g(y)=y-3\ln y}$ (2)

(2) → (1) : $\boxed{x-3\ln(x)+x.y+y-3\ln(y)=C}$

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g) $\boxed{\left[2y-\frac{1}{x}\cos(3x)\right]\frac{dy}{dx}+\frac{y}{x^2}-4x^3+3y\,\text{sen}(3x)=0}$

$$\underbrace{\left[2y-\frac{1}{x}.\cos(3x)\right]}_{N}dy+\underbrace{\left[\frac{y}{x^2}-4x^3+3y.\text{sen}(3x)\right]}_{M}dx=0$$

$M=\frac{y}{x^2}-4x^3+3y.\text{sen}(3x) \Rightarrow \boxed{\frac{\partial M}{\partial y}=\frac{1}{x^2}+3\,\text{sen}(3x)}$

$N=2y-\frac{1}{x}.\cos(3x) \Rightarrow \frac{\partial N}{\partial x}=\frac{\partial}{\partial x}\left[2y-x^{-1}.\cos(3x)\right]$

$\frac{\partial N}{\partial x}=-\left[-x^{-2}.\cos(3x)+x^{-1}.(-1)\,\text{sen}(3x).3\right]$

$\frac{\partial N}{\partial x}=-\left[-\frac{1}{x^2}.\cos(3x)-\frac{3}{x}\,\text{sen}(3x)\right]$ ou

$\boxed{\frac{\partial N}{\partial x}=\frac{1}{x^2}\cos(3x)+\frac{3}{x}\,\text{sen}(3x)}$ $\quad \frac{\partial M}{\partial y}\neq\frac{\partial N}{\partial x}$

∴ NÃO É EQ. EXATA

h) $\boxed{x \cdot \frac{dy}{dx} = 2x \cdot e^{3x} - y + 6x^2}$

$$\underbrace{\left(2x e^{3x} - y + 6x^2\right)}_{M} dx \underbrace{- x}_{N} dy = 0$$

$$M = 2xe^{3x} - y + 6x^2 \Longrightarrow \boxed{\frac{\partial M}{\partial y} = -1} \quad \Bigg\| \quad N = -x \Rightarrow \boxed{\frac{\partial N}{\partial x} = -1}$$

Ε' ΕΧΑΤΗ

$$f(x,y) = \int \left(2x e^{3x} - y + 6x^2\right) dx + g(y)$$

$$f(x,y) = 2\int \underbrace{x}_{u} \cdot \underbrace{e^{3x} dx}_{dv} - y\int dx + 6\int x^2 dx + g(y)$$

$$\int u \cdot dv = uv - \int v \cdot du$$
$$u = x \Rightarrow du = dx$$
$$dv = e^{3x}$$
$$v = \frac{1}{3} e^{3x}$$

$$f(x,y) = 2\left[\frac{x \cdot e^{3x}}{3} - \frac{1}{3}\int e^{3x} dx\right] - y \cdot x + 6\frac{x^3}{3} + g(y)$$

$$f(x,y) = 2\left[\frac{x \cdot e^{3x}}{3} - \frac{e^{3x}}{9}\right] - xy + 2x^3 + g(y)$$

$$\boxed{f(x,y) = \frac{2x e^{3x}}{3} - \frac{2e^{3x}}{9} - x \cdot y + 2x^3 + g(y)} \quad (1)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[\frac{2x e^{3x}}{3} - \frac{2e^{3x}}{9} - x \cdot y + 2x^3 + g(y)\right]$$

$$-x = -x + g'(y) \rightarrow \frac{dg}{dy} = 0 \Rightarrow \boxed{g(y) = c} \quad (2)$$

(2) → (1) :

$$\boxed{\frac{2x e^{3x}}{3} - \frac{2e^{3x}}{9} - xy + 2x^3 = 4}$$

$\boxed{i}$ $\boxed{\underbrace{[tg(x) - sen(x).sen(y)]}_{M} dx + \underbrace{cos(x).\ cos(y)}_{N} dy = 0}$ (6)

$M = tg(x) - sen(x).sen(y) \Rightarrow \boxed{\frac{\partial M}{\partial y} = -sen(x)cos(y)}$

$N = cos(x).cos(y) \rightarrow \boxed{\frac{\partial N}{\partial x} = -sen(x).cos(y)}$ ε' EXATA

$f(x,y) = \int tg(x)\,dx - sen(y).\int sen(x)\,dx + g(y)$

$\boxed{\begin{array}{l} u = cos(x) \\ du = -sen(x)dx \\ -du = sen(x)dx \end{array}}$

$f(x,y) = \int \frac{sen(x)}{cos(x)}dx - sen(y).(-1)cos(x) + g(y)$

$f(x,y) = -\int \frac{1}{u}du + sen(y).cos(x) + g(y) \rightarrow \boxed{f(x,y) = -ln|cos(x)| + cos(x).sen(y) + g(y)}$ (1)

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[-ln|cos(x)| + cos(x).sen(y) + g(y)\right]$

$cos(x).cos(y) = cos(x).cos(y) + g'(y) \rightarrow g'(y) = 0 \Rightarrow \boxed{g = c}$ (2)

Sol.: $\boxed{-ln|cos(x)| + cos(x).sen(y) = c}$

$\boxed{y}$ $\boxed{[2y\,sen(x).cos(x) - y + 2y^2.e^{xy^2}]dx = [x - sen^2(x) - 4xy.e^{xy^2}]dy}$

$M = 2y\,sen(x).cos(x) - y + 2y^2.e^{xy^2}$

$\frac{\partial M}{\partial y} = 2\,sen(x).cos(x) - 1 + 2[2y.e^{xy^2} + y^2e^{xy^2}.2xy]$

$\boxed{\frac{\partial M}{\partial y} = 2\,sen(x).cos(x) - 1 + 4y\,e^{xy^2} + 4xy^3e^{xy^2}}$

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$N = -x + \text{sen}^2(x) + 4xy\, e^{xy^2}$

$\frac{\partial N}{\partial x} = -1 + 2\,\text{sen}(x)\cos(x) + 4y\left[e^{xy^2} + x\, e^{xy^2} . y^2\right]$

$\boxed{\frac{\partial N}{\partial x} = 2\,\text{sen}(x).\cos(x) - 1 + 4y e^{xy^2} + 4xy^3 e^{xy^2}}$ ∴ É EXATA !

$f(x,y) = 2y\int \text{sen}(x).\cos(x)\,dx - y\int dx + 2y^2 \int e^{xy^2} dx + g(y)$

$f(x,y) = 2y\int u\, du - yx + 2y^2 \int e^{v} \frac{dv}{y^2} + g(y)$

$f(x,y) = 2y\frac{u^2}{2} - xy + 2e^{v} + g(y)$

$\boxed{f(x,y) = y.\text{sen}^2(x) - xy + 2.e^{x.y^2} + g(y)}$ 1

$u = \text{sen}(x)$
$du = \cos(x)\,dx$

$v = xy^2$
$dv = y^2 dx$
$\frac{dv}{y^2} = dx$

$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[y.\text{sen}^2(x) - xy + 2e^{xy^2} + g(y)\right]$

$\cancel{-x} + \cancel{\text{sen}^2(x)} + \cancel{4xy\, e^{xy^2}} = \cancel{\text{sen}^2(x)} - \cancel{x} + \underbrace{2e^{xy^2}.\, x2y}_{\cancel{4xye^{xy^2}}} + g'(y)$

$g'(y) = 0 \Rightarrow \boxed{g(y) = c}$ 2

Sol: $\boxed{y.\text{sen}^2(x) - xy + 2e^{xy^2} = c}$