CÁLCULOS DAS T. L. :

(a) $\boxed{f(t) = 1}$, $t \geq 0$

$$\boxed{\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-s.t}\,[f(t)].dt = F(s)} \quad (1)$$

$$\mathcal{L}\{1\} = \lim_{A \to +\infty} \int_0^{A} e^{-s.t}\,[1].dt \quad ; (2)$$

$$\mathcal{L}\{1\} = \lim_{A \to +\infty} \int_0^{-s.A} e^{u}\left(-\frac{1}{s}\right) du \quad , (3)$$

$$\mathcal{L}\{1\} = \lim_{A \to +\infty} \left(-\frac{1}{s}\right)\left[e^{u}\right]_0^{-s.A} \quad ; (4)$$

SE $s < 0$ DIVERGE

$$\mathcal{L}\{1\} = \left(-\frac{1}{s}\right)\left\{e^{-\infty} - e^{0}\right\} \quad ; (5)$$

$$\mathcal{L}\{1\} = -\frac{1}{s}\left\{\frac{1}{e^{\infty}} - 1\right\} \quad ; (6)$$

$$\therefore \boxed{\mathcal{L}\{1\} = \frac{1}{s}} \quad (7)$$

ou

$$\boxed{F(s) = \frac{1}{s}} \quad , \quad s > 0 \quad (8)$$

$u = -s.t$

$\frac{du}{dt} = -s$

$-\frac{du}{s} = dt$

PARA $t = 0$

$u = 0$

PARA $t = A$

$u = -s.A$

SE $s < 0$ O INTEGRANDO EM (2) DIVERGE

b) $\boxed{f(t) = e^{a.t}}$, $t \geq 0$

$$\mathcal{L}\{e^{a.t}\} = F(s) = \int_0^{\infty} e^{-s.t} \cdot e^{a.t} . dt = \lim_{A \to \infty} \int_0^{A} e^{-(s-a).t} dt ,$$

$$F(s) = \lim_{A \to \infty} \int_0^{(s-a).A} e^{-u} \frac{du}{s-a} ;$$

$u = (s-a).t$

$\frac{du}{dt} = s-a$

$dt = \frac{du}{(s-a)}$

$$F(s) = \frac{1}{s-a} \lim_{A \to \infty} \int_0^{(s-a).A} e^{-u} . du ;$$

$$F(s) = \frac{1}{(s-a)} \lim_{A \to \infty} \Big[- e^{-u} \Big]_0^{(s-a).A} ;$$

$$F(s) = \frac{1}{(s-a)} \left\{ \lim_{A \to \infty} e^{0} - \lim_{A \to \infty} e^{-(s-a).A} \right\} ;$$

$$F(s) = \frac{1}{(s-a)} \left\{ 1 - \lim_{A \to \infty} \underbrace{\frac{1}{e^{(s-a).A}}}_{0} \right\} = \frac{1}{s-a}$$

$$\therefore \boxed{\mathcal{L}\{e^{at}\} = \frac{1}{s-a}}$$

DEVEMOS TER $s-a > 0 \Rightarrow \boxed{s > a}$.

OBS: SE $s \leq a$ A INTEGRAL DIVERGE.

C) $\boxed{f(t) = t}$, $t \geq 0$

$$\mathcal{L}\{t\} = F(s) = \int_0^{\infty} e^{-s.t} . t . dt = \lim_{A \to \infty} \int_0^{A} \underbrace{t}_{u} . \underbrace{e^{-s.t} dt}_{dV} ; \quad (1)$$

$$\int_a^b u . dV = u . V \Big|_a^b - \int_a^b V . du$$

$\boxed{u = t} \Rightarrow du = dt$;

$\boxed{dV = e^{-s.t} dt} \Rightarrow V = -\frac{1}{s} e^{-s.t}$;

SUBSTITUINDO EM (1):

$$\mathcal{L}\{t\} = F(s) = \lim_{A \to \infty} \left\{ \left[-\frac{t . e^{-s.t}}{s} \right]_0^A - \int_0^A -\frac{1}{s} e^{-s.t} dt \right\} ;$$

$$\mathcal{L}\{t\} = \lim_{A \to \infty} \left\{ \left(\frac{0 . e^{0}}{s} - \frac{A . e^{-s.A}}{s} \right) + \frac{1}{s} \left(-\frac{1}{s} \right) \left[e^{-s.t} \right]_0^A \right\} ;$$

$$F(s) = \cancel{\lim_{A \to \infty} \left(-\frac{1}{s} \right) . \frac{A}{e^{s.A}}}^{\,0} + \frac{1}{s^2} \lim_{A \to \infty} \left[e^{0} - e^{-s.A} \right]$$

$$F(s) = \frac{1}{s^2} \left[1 - \lim_{A \to \infty} \cancel{\frac{1}{e^{s.A}}}_{\,0} \right] = \frac{1}{s^2} ,$$

$$\therefore \boxed{\mathcal{L}\{t\} = \frac{1}{s^2}} , \quad s > 0$$

- TRANSFORMADA LINEAR

(LINEARIDADE)

- PROPRIEDADE MUITO → IMPORTANTE
→ USADA

f_1 E f_2 ⇒ DUAS FUNÇÕES, TAL QUE EXISTEM $\mathcal{L}\{f_1\}$ E $\mathcal{L}\{f_2\}$

$$\mathcal{L}\{c_1 f_1 + c_2 f_2\} = \int_0^{\infty} e^{-s.t}[c_1 f_1 + c_2 f_2].dt;$$

$$\mathcal{L}\{c_1.f_1 + c_2 f_2\} = \int_0^{+\infty} e^{-s.t} c_1.f_1.dt + \int_0^{+\infty} e^{-s.t} c_2 f_2.dt;$$

$$\mathcal{L}\{c_1.f_1 + c_2 f_2\} = c_1 \int_0^{+\infty} e^{-s.t}.f_1.dt + c_2 \int_0^{+\infty} e^{-s.t}.f_2.dt;$$

$$\boxed{\mathcal{L}\{c_1.f_1 + c_2 f_2\} = c_1 \mathcal{L}\{f_1\} + c_2\{f_2\}}$$

c_1, c_2 ⇒ CONSTANTES

DOS RESULTADOS DA FOLHA 6:

- $\boxed{\mathcal{L}\{1\} = \frac{1}{s}}$, 1
- $\boxed{\mathcal{L}\{t\} = \frac{1}{s^2}}$ 3
- $\boxed{\mathcal{L}\{e^{at}\} = \frac{1}{s-a}}$ 2
- $\boxed{\mathcal{L}\{\operatorname{sen}(a.t)\} = \frac{a}{a^2+s^2}}$ 4

DA LINEARIDADE:

$$\boxed{\mathcal{L}\{c_1.f_1 + c_2.f_2\} = c_1\,\mathcal{L}\{f_1\} + c_2\,\mathcal{L}\{f_2\}} \quad (5)$$

EX: CALCULE $\mathcal{L}\{3.t - 5.\operatorname{sen}(2.t)\}$

SOLUÇÃO

$$\mathcal{L}\{3t - 5\operatorname{sen}(2t)\} = 3\,\mathcal{L}\{t\} - 5\,\mathcal{L}\{\operatorname{sen}(2.t)\};$$

$$\mathcal{L}\{3.t - 5\operatorname{sen}(2.t)\} = 3\left(\frac{1}{s^2}\right) - 5\left(\frac{2}{2^2+s^2}\right);$$

$$\mathcal{L}\{3.t - 5\operatorname{sen}(2.t)\} = \frac{3}{s^2} - \frac{10}{4+s^2};$$

$$\text{''} \qquad = \frac{3(4+s^2) - 10\,s^2}{s^2(4+s^2)};$$

$$\mathcal{L}\{3.t-5.\,\mathrm{SEN}(2.t)\} = \frac{12+3s^2-10s^2}{s^2(4+s^2)};$$

$$\boxed{\mathcal{L}\{3t-5\,\mathrm{SEN}(2t)\} = \frac{12-7s^2}{s^2(4+s^2)}}$$

$s > 0$