5.

$$r_1: \begin{cases} y = 1 \\ x + 2 = \dfrac{-z+4}{2} \end{cases} \qquad r_2: \begin{cases} x = 3 \\ y = 2t - 1 \\ z = -t + 3 \end{cases}$$

$$r_1: \begin{cases} x = -2 + s \\ y = 1 \\ z = 4 - 2s \end{cases} \qquad \dfrac{-z+4}{2} = 2t \qquad -z = -4 + 2t \; .(-1) \qquad z = 4 - 2t$$

a) São paralelas?

$\vec{v}_{r_1} = (1, 0, -2)$ $\quad \vec{v}_{r_2} = (0, 2, -1)$

$\dfrac{0}{2} = 0$ e $\dfrac{-2}{-1} = 1$ $\quad 0 \neq 1$ logo não são paralelas

São concorrentes?

$$y_{r_1} = y_{r_2} \qquad x_{r_1} = x_{r_2} \qquad z_{r_1} = z_{r_2}$$
$$1 = 2t - 1 \qquad -2 + s = 3 \qquad 4 - 2.5 = -1 + 3$$
$$t = \dfrac{2}{2} = 1 \qquad s = 5 \qquad -6 = 2$$

$-6 = 2$ não é possível, dessa forma as retas não são concorrentes.

R: Como as retas não são paralelas e nem concorrentes, elas só podem ser reversas.

b) distância de r_1 à r_2

$$d = \frac{|\vec{PQ} \cdot \vec{v}_{r_1} \times \vec{v}_{r_2}|}{\|\vec{v}_{r_1} \times \vec{v}_{r_2}\|}$$

$P = (-2, 1, 4)$ $Q = (3, -1, 3)$

$\vec{PQ} = (3-(-2), -1-1, 3-4)$

$\vec{PQ} = (5, -2, -1)$

$$d = \frac{|\langle (5, -2, -1)(4, 1, 2) \rangle|}{\|(4, 1, 2)\|}$$

$$\vec{v}_{r_1} \times \vec{v}_{r_2} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 1 & 0 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 2 \end{array}\right|$$

$$d = \frac{|20 - 2 - 2|}{\sqrt{4^2 + 1^2 + 2^2}}$$

$$d = \frac{|16|}{\sqrt{16 + 1 + 4}}$$

$$d = \frac{16}{\sqrt{21}}$$

$\vec{v}_{r_1} \times \vec{v}_{r_2} = 2k - (-4i - j)$

$= 4i + j + 2k$

$\vec{v}_{r_1} \times \vec{v}_{r_2} = (4, 1, 2)$

c) $r_1 \subset \pi_1$ e $r_2 \subset \pi_2$ equações de π_1 e π_2:?

- como r_1 está contida em π_1, o ponto $(-2, 1, 4) \in \pi_1$
- como r_2 está contida em π_2, o ponto $(3, -1, 3) \in \pi_2$
- $S_1 \parallel r_2$, portanto $\vec{v}_{r_2} = (0, 2, -1) \in M_1$
- $S_2 \parallel r_1$, portanto $\vec{v}_{r_1} = (1, 0, -2) \in M_2$

$$S_1: \begin{cases} x = -2 \\ y = 1 + 2t \\ z = 4 - t \end{cases} \qquad r_1: \begin{cases} x = -2 + s \\ y = 1 \\ z = 4 - 2s \end{cases}$$

$$\pi_1: \begin{cases} x = -2 + s & \to s = x + 2 \\ y = 1 + 2t & \to t = \frac{y-1}{2} \\ z = 4 - t - 2s \end{cases}$$

$$z = 4 - \left(\frac{y-1}{2}\right) - 2 \cdot (x + 2)$$

$$z = \cancel{4} - \left(\frac{y-1}{2}\right) - 2x \cancel{-4}$$

$$z = \frac{\frac{y-1}{2}}{-1} - 2x$$

$$z = \frac{\frac{y-1}{2} + 2x}{-1}$$

$$-z = \frac{y-1}{2} + 2x$$

$$-z - 2x = \frac{y-1}{2}$$

$$y - 1 = -2z - 4x$$

$$-4x - 2z - y + 1 = 0$$

$$\pi_1: -4x - y - 2z + 1 = 0$$

$$S_2: \begin{cases} x = 3 + t \\ y = -1 \\ z = 3 - 2t \end{cases} \qquad r_2: \begin{cases} x = 3 \\ y = -1 + 2s \\ z = 3 - s \end{cases}$$

$$\pi_2: \begin{cases} x = 3 + t & \rightarrow t = x - 3 \\ y = -1 + 2s & \rightarrow s = \frac{y+1}{2} \\ z = 3 - 2t - s \end{cases}$$

$$z = 3 - 2(x-3) - \left(\frac{y+1}{2}\right)$$

$$z = 3 - 2x + 6 - \left(\frac{y+1}{2}\right)$$

$$\pi_2: -4x - y - 2z + 17 = 0$$

$$\frac{y+1}{2} = 9 - 2x - z$$

$$y + 1 = 18 - 4x - 2z$$

d) Distância entre π_1 e π_2?

$\pi_1: -4x - y - 2z + 1 = 0$ $\qquad d = -1$

$\pi_2: -4x - y - 2z + 17 = 0$ $\qquad d = -17$

distância entre dois planos:

$$d(\pi_1, \pi_2) = d(P, \pi_2) = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d(\pi_1, \pi_2) = \frac{|-1-(-17)|}{\sqrt{(-4)^2 + (-1)^2 + (-2)^2}}$$

$$d(\pi_1, \pi_2) = \frac{|16|}{\sqrt{16+1+4}}$$

$$d(\pi_1, \pi_2) = \frac{16}{\sqrt{21}}$$

e) r perpendicular a a r_1 e r_2?

$\vec{v}_{r_1} = (1, 0, -2)$ $\qquad \vec{v}_{r_2} = (0, 2, 1)$ $\qquad \vec{v}_{r_1} \times \vec{v}_{r_2} = (4, 1, 2)$

$r \perp r_1$ $\qquad P = (-2, 1, 4)$

$$\vec{a} = (\vec{v}_{r_1} \times \vec{v}_{r_2}) \times \vec{v}_{r_1} = \left|\begin{array}{ccc|cc} i & j & k & i & j \\ 4 & 1 & 2 & 4 & 1 \\ 1 & 0 & -2 & 1 & 0 \end{array}\right.$$

$$\vec{a} = -2i + 2j - k + 8j$$

$$\vec{a} = -2i + 10j - k$$

$$\vec{a} = (-2, 10, -1)$$

equação geral: $-2(x+2) + 10(y-1) - (z-4) = 0$

$: -2x \cancel{-4} + 10y - 10 - z \cancel{+4} = 0$

$: -2x + 10y - z = 10$

$r_2: x = 3, \quad y = -1 + 2t \qquad z = -t + 3$

$-2.3 + 10(-1 + 2t) - (3 - t) = 10$
$-6 - 10 + 20t - 3 + t = 10$
$21t = 29$
$t = \frac{29}{21}$

Substituindo:
$x = 3$

$y = -1 + 2 . \frac{29}{21}$ $\qquad$ $z = 3 - \frac{29}{21}$

$y = \frac{-21 + 58}{21}$ $\qquad$ $z = \frac{63 - 29}{21}$

$y = \frac{37}{21}$ $\qquad$ $z = \frac{34}{21}$

$$r: \begin{cases} x = 3 + 4t \\ y = \frac{37}{21} + t \\ z = \frac{34}{21} + 2t \end{cases}$$

f) Coordenadas em pontos $A = r_1 \cap r$ $B = r_2 \cap r$

substitui a variável para ficar diferente de r

$$r: \begin{cases} x = 3 + 4t \\ y = \frac{37}{21} + t \\ z = \frac{34}{21} + 2t \end{cases} \qquad r_1: \begin{cases} x = -2 + s \\ y = 1 \\ z = 4 - 2s \end{cases} \qquad r_2: \begin{cases} x = 3 \\ y = -1 + 2u \\ z = 3 - u \end{cases}$$

$$A = \begin{cases} x = 3 + 4t = -2 + s \\ y = \frac{37}{21} + t = 1 \\ z = \frac{34}{21} + 2t = 4 - 2s \end{cases}$$

$$t = 1 - \frac{37}{21}$$

$$t = \frac{21 - 37}{21}$$

$$t = \frac{-16}{21}$$

$$3 + 4\left(-\frac{16}{21}\right) = -2 + s$$

$$3 - \frac{64}{21} + 2 = s$$

$$\frac{63 - 64 + 42}{21} = s$$

$$\frac{41}{21} = s$$

$$\frac{34}{21} + 2 \cdot \left(\frac{-16}{21}\right) = 4 - 2 \cdot \left(\frac{41}{21}\right)$$

$$\frac{34}{21} - \frac{32}{21} = \frac{84}{21} - \frac{82}{21}$$

$$\frac{2}{21} = \frac{2}{21}$$ → são iguais, portanto verdadeira

$x = -2 + \frac{41}{21}$ $\quad y = 1$ $\quad z = 4 - 2 \cdot \frac{41}{21}$

$x = \frac{-1}{21}$ $\quad z = \frac{2}{21}$

$$A = \left(\frac{-1}{21}, 1, \frac{2}{21} \right)$$

$$B: \begin{cases} x = 3 + 4\lambda = 3 \\ y = \frac{37}{21} + \lambda = -1 + 2u \\ z = \frac{34}{21} + 2\lambda = 3 - u \end{cases}$$

$3 + 4\lambda = 3$

$4\lambda = 0$

$\lambda = 0$

$\frac{37}{21} + 0 = -1 \cdot 2u$

$\frac{37 + 21}{21} = 2u$

$\frac{58}{21} = 2u$

$u = \frac{58}{21} \cdot \frac{1}{2}$

$u = \frac{58}{42}$ $\div 2$

$u = \frac{29}{21}$

$\frac{34}{21} + 2 \cdot 0 = 3 - \frac{29}{21}$

$\frac{34}{21} = \frac{63 - 29}{21}$

$\frac{34}{21} = \frac{34}{21}$

↳ iguais, portanto verdadeiro

tilibra

$x = 3 + 4.0$ $\quad y = \frac{37}{21} + 0$ $\quad z = \frac{34}{21} + 2.0$

$x = 3$ $\quad y = \frac{37}{21}$ $\quad z = \frac{34}{21}$

$$B = \left(3, \frac{37}{21}, \frac{34}{21}\right)$$