calcular lim de forma direta ou ver se é contínua

se ñ for, não é diferenciável

Lista 9

1) $f(x) = \begin{cases} x^2, & x > 1 \\ 2x-1, & x \leq 1 \end{cases}$

$$f'_+(1) = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{(1+h)^2 - 1}{h} =$$

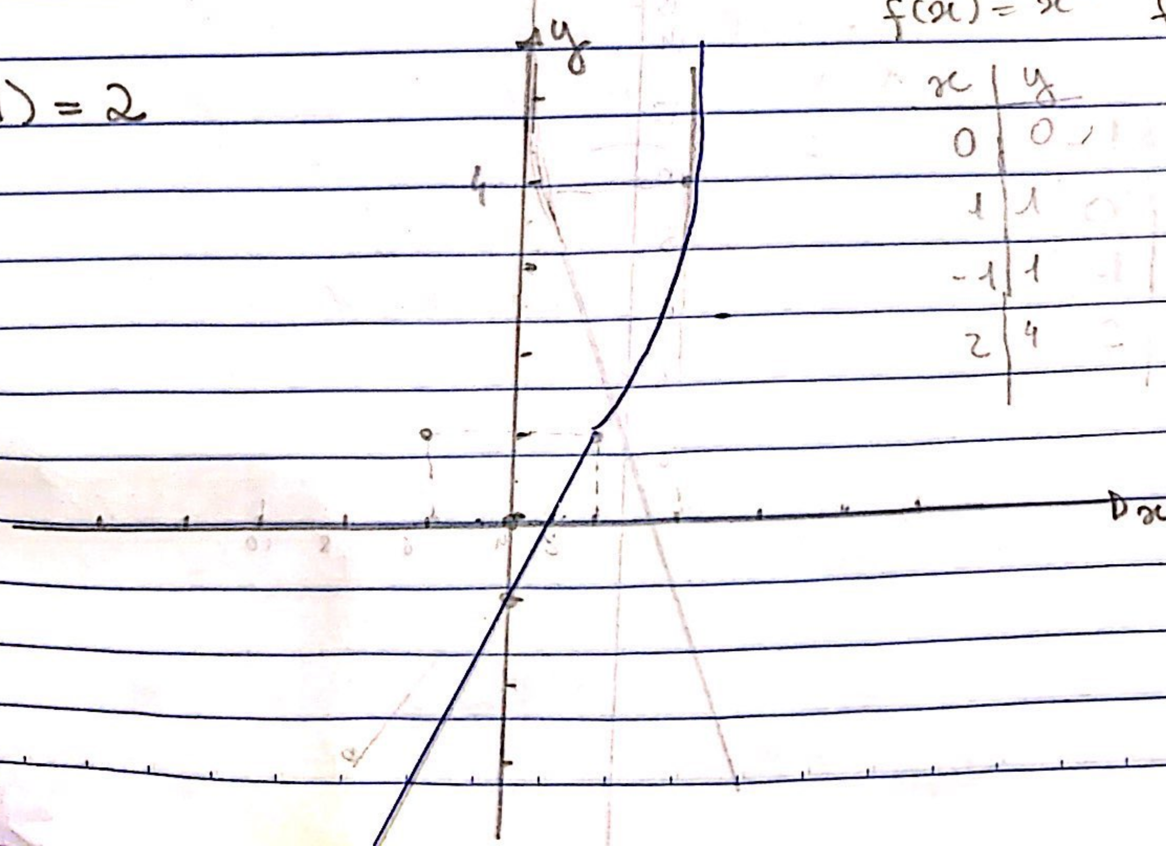
$$= \lim_{h \to 0^+} \frac{1 + 2h + h^2 - 1}{h} = \lim_{h \to 0^+} 2 + h = 2$$

$$f'_-(1) = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{2(1+h) - 1 - 1}{h} =$$

$$= \lim_{h \to 0^-} \frac{2 + 2h - 2}{h} = 2$$

$f'_+(1) = f'_-(1) \Rightarrow$ então $f'(1)$ é diferenciável em $x = 1$

$f'(1) = 2$



$f(x) = x^2$

x	y
0	0
1	1
-1	1
2	4

$f(x) = 2x - 1$

x	y
0	-1
1	1

data / /

b) $g(x) = \begin{cases} x^2 + 2x, & x > 2 \\ 3x + 2, & x \leq 2 \end{cases}$

$g'_+(2) = \lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0^+} \frac{(2+h)^2 + 2(2+h) - 8}{h} =$

$= \lim_{h \to 0^+} \frac{4 + 2h + h^2 + 4 + 2h - 8}{h} = \lim_{h \to 0^+} 4 + h = 4$

$g'_-(2) = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0^-} \frac{3(2+h) + 2 - 8}{h} =$

$= \lim_{h \to 0^-} \frac{6 + 3h - 6}{h} = 3$

Como $g'_+(2) \neq g'_-(2)$, então $g(x)$ não é diferenciável em $x=2$

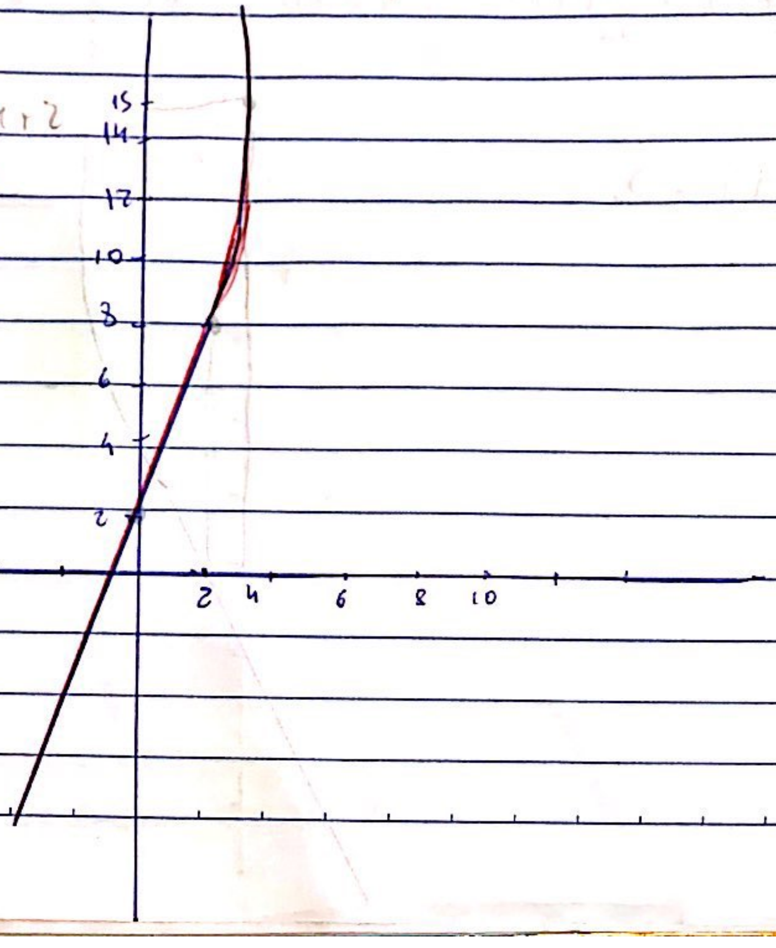
$\nexists\ g'(2)$.

$f(x) = x^2 + 2x$

x	y
2	8
3	15

$f(x) = 3x + 2$

x	y
0	2
2	8



3)

$$f(x) = \begin{cases} 2 - x^2, & x > 0 \\ 2, & x \leq 0 \end{cases} \qquad f'(x)\ ?$$

para $x > 0 \Rightarrow f'(x) = -2x$

para $x < 0 \Rightarrow f'(x) = 0$

$$f'_+(0) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^+} \frac{2 - (0+h)^2 - 2}{h} =$$

$$= \lim_{h \to 0^+} \frac{-h^2}{h} = \lim_{h \to 0^+} -h = 0$$

$$f'_-(0) = \lim_{h \to 0^-} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0^-} \frac{2-2}{h} = 0$$

$f'(0) = 0$

$$f'(x) = \begin{cases} -2x, & x > 0 \\ 0, & x = 0 \\ 0, & x < 0 \end{cases} = \begin{cases} -2x, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

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4) $g(x) = \begin{cases} 3x^2 - 2x, & x > 2 \\ 3x + 2, & x \leq 2 \end{cases}$

para $x > 2 \Rightarrow g'(x) = 6x - 2$

para $x < 2 \Rightarrow g'(x) = 3$

para $x = 2$

$$f'_+(2) = \lim_{h \to 0^+} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0^+} \frac{3(2+h)^2 - 2(2+h) - 8}{h} =$$

$$= \lim_{h \to 0^+} \frac{3(4 + 4h + h^2) - 4 - 2h - 8}{h} = \lim_{h \to 0^+} \frac{\cancel{12} + 12h + 3h^2 - \cancel{12} - 2h}{h} =$$

$$= \lim_{h \to 0^+} 3h + 10 = 10$$

$$f'_-(2) = \lim_{h \to 0^-} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0^-} \frac{3(2+h) + 2 - 8}{h} =$$

$$= \lim_{h \to 0^-} \frac{6 + 3h - 6}{h} = \lim_{h \to 0^-} 3 = 3$$

$f'_+(2) \neq f'_-(2) \Rightarrow \nexists\, f'(2)$

$g'(x) = \begin{cases} 6x - 2, & x > 2 \\ 3, & x < 2 \end{cases}$

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5) $g(x) = \begin{cases} x^2 \operatorname{sen}(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Teorema sanduíche

$g'(0)$?

$-1 \leq \operatorname{sen}(1/x) \leq 1$

$-x^2 \leq x^2 \operatorname{sen}(1/x) \leq x^2$

$f(x) = -x^2$ $\quad$ $h(x) = x^2$

$f'(x) = -2x$ $\quad$ $h'(x) = 2x$

$f'(0) = 0$ $\quad$ $h'(0) = 0$

$g'(0) = 0$

6)

$$g(x) = \begin{cases} |x - [[x]]|, & \text{se } [[x]] = 0 \text{ ou par} \\ |x - [[x+1]]|, & \text{se } [[x]] \text{ é ímpar} \end{cases}$$

$f(x) = \operatorname{sen}\left(\frac{\pi}{2} g(x)\right)$ $\qquad$ $f'(1) = ?$

$\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$ $\qquad$ $\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$

$f(1) = ?$

$[[1]] = 1$ ímpar $\Rightarrow g(1) = |1 - [[1+1]]| = |1-2| = 1$

$f(1) = \operatorname{sen}\left(\frac{\pi}{2} \cdot 1\right) = \operatorname{sen}\frac{\pi}{2} = 1$

0 $\quad$ 1 $\quad$ 2

$0 < x < 1$

$\to [[x]] = 0$

$\to 1 < x < 2 \to [[x]] = 1$

$h \to 0^+ \longrightarrow 1 < 1+h < 2 \Rightarrow [[1+h]] = 1$

pq módulo é positivo.

$g(1+h) = |1+h - [[1+h+1]]| = |1+h-2| = |h-1| = 1-h$

$1 < 1+h < 2$

$2 < 1+h+1 < 3$

$\to [[1+h+1]] = 2$

$1 < 1+h < 2$

$-1 < 1+h-2 < 0$

$-1 < h-1 < 0$

$\to h-1$ é negativo

$f(1+h) = \operatorname{sen}\left(\frac{\pi}{2} g(1+h)\right) = \operatorname{sen}\left(\frac{\pi}{2}(1-h)\right) = \operatorname{sen}\left(\frac{\pi}{2} - \frac{\pi}{2}h\right) =$

$= \underbrace{\operatorname{sen}\frac{\pi}{2}}_{1} \cdot \cos\frac{\pi}{2}h - \underbrace{\cos\frac{\pi}{2}}_{0} \cdot \operatorname{sen}\frac{\pi}{2}h = \cos\frac{\pi}{2}h$

$\hookrightarrow \operatorname{sen}(a-b) = \operatorname{sen}a\cos b - \cos a \operatorname{sen} b$

$\lim\limits_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim\limits_{h \to 0^+} \frac{\cos\frac{\pi}{2}h - 1}{h} = \lim\limits_{h \to 0^+} \frac{(\cos\frac{\pi}{2}h - 1)}{h} \cdot \frac{(\cos\frac{\pi}{2}h + 1)}{(\cos\frac{\pi}{2}h + 1)} =$

p $\operatorname{sen}^2 x + \cos^2 x = 1$

$= \lim\limits_{h \to 0^+} \frac{\cos^2\frac{\pi}{2}h - 1}{h(\cos\frac{\pi}{2}h + 1)} = \lim\limits_{h \to 0^+} \frac{-\operatorname{sen}^2\frac{\pi}{2}h}{h(\cos\frac{\pi}{2}h + 1)} =$

$= \lim\limits_{h \to 0^+} -\underbrace{\frac{\operatorname{sen}\frac{\pi}{2}h}{\frac{\pi}{2}h}}_{1} \cdot \frac{\overset{\operatorname{sen} 0}{\operatorname{sen}\frac{\pi}{2}h}}{\underset{\hookrightarrow \cos 0 + 1}{(\cos\frac{\pi}{2}h + 1)}} \cdot \pi/2 = 0$

$h \to 0^- \Rightarrow 0 < 1 + h < 1 \Rightarrow [[1+h]] = 0$

$g(1+h) = |1+h - [[1+h]]| = |1+h-0| = |1+h| = 1+h$

$\downarrow$

$0 < 1+h < 1$

positivo

$f(1+h) = \operatorname{sen}\left(\frac{\pi}{2}(1+h)\right) = \underbrace{\operatorname{sen}\frac{\pi}{2}}_{1} \cdot \cos\frac{\pi}{2}h + \underbrace{\cos\frac{\pi}{2}}_{0} \cdot \operatorname{sen}\frac{\pi}{2}h =$

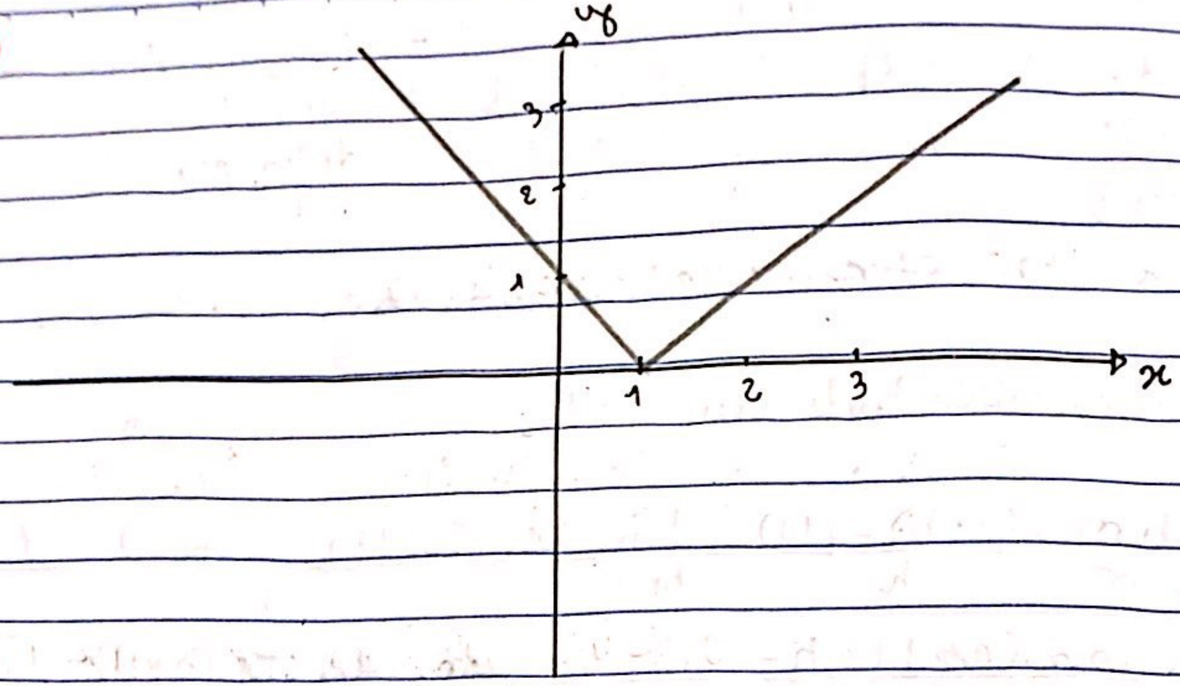
$= \cos\frac{\pi}{2}h$ //

$\lim\limits_{h \to 0^-} \frac{f(1+h) - f(1)}{h} = \lim\limits_{h \to 0^-} \frac{(\cos\frac{\pi}{2}h - 1)}{h} = 0$

$\hookrightarrow$ mesma coisa do de cima

Logo $f'(1) = 0$ //

7)



$f(x) = |x - 1|$

8)

a)

$$y = \sqrt{x(x-1)(x-2)(x-3)}$$

$$\ln(y) = \ln\left[x(x-1)(x-2)(x-3)\right]^{1/2}$$

$$\ln(y) = \frac{1}{2}\left[\ln(x) + \ln(x-1) + \ln(x-2) + \ln(x-3)\right]$$

derivando

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}\right]$$

$$\frac{dy}{dx} = \frac{\sqrt{x(x-1)(x-2)(x-3)}}{2}\left(\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}\right) //$$

b) $y = \dfrac{x^2(x+4)(x-7)}{x^2+1}$

$$\ln(y) = \ln\left(\frac{x^2(x+4)(x-7)}{x^2+1}\right)$$

$$\ln(y) = \ln(x^2) + \ln(x+4) + \ln(x-7) - \ln(x^2+1)$$

derivando

$$\frac{1}{y}\frac{dy}{dx} = \frac{2x}{x^2} + \frac{1}{x+4} + \frac{1}{x-7} - \frac{2x}{x^2+1}$$

$$\frac{dy}{dx} = \frac{x^2(x+4)(x-7)}{x^2+1} \cdot \left(\frac{2x}{x^2} + \frac{1}{x+4} + \frac{1}{x-7} - \frac{2x}{x^2+1}\right) //$$

c) $y = (x^2+4)^x$

$\ln(y) = \ln(x^2+4)^x$

$\ln(y) = x \cdot \ln(x^2+4)$

$\frac{1}{y}\frac{dy}{dx} = \ln(x^2+4) + x \cdot \frac{2x}{x^2+4}$

$\frac{dy}{dx} = y \cdot \left(\ln(x^2+4) + \frac{2x^2}{x^2+4}\right)$

$\frac{dy}{dx} = (x^2+4)^x \cdot \left(\ln(x^2+4) + \frac{2x^2}{x^2+4}\right)$ //

d) $[\cosh x]^{\operatorname{sen} x} = y$

$\ln(y) = \ln(\cosh x)^{\operatorname{sen} x}$

$\ln(y) = \operatorname{sen} x \cdot \ln(\cosh x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \cdot \ln(\cosh x) + \operatorname{sen} x \cdot \frac{\operatorname{senh} x}{\cosh x}$

$\frac{dy}{dx} = y \left[\cos x \ln(\cosh x) + \frac{\operatorname{sen} x \cdot \operatorname{senh} x}{\cosh x}\right]$

$\frac{dy}{dx} = [\cosh x]^{\operatorname{sen} x} \cdot \left[\cos x \ln(\cosh x) + \frac{\operatorname{sen} x \operatorname{senh} x}{\cosh x}\right]$

Funcões hiperbólica

$y = \operatorname{senh} x \rightarrow y' = \cosh x$

$y = \cosh x \rightarrow y' = \operatorname{senh} x$

$y = (x^2+1)^x$

e) $y = (x+1)(x+2)(x+3)$

$\ln(y) = \ln(x+1) + \ln(x+2) + \ln(x+3)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3}$$

$$\frac{dy}{dx} = (x+1)(x+2)(x+3) \cdot \left(\frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3}\right)$$

$$\frac{dy}{dx} = (x+1)(x+2)(x+3) \cdot \frac{(x+2)(x+3) + (x+1)(x+3) + (x+1)(x+2)}{(x+1)(x+2)(x+3)}$$

$$\frac{dy}{dx} = 3x^2 + 12x + 11$$

f) $y = \left(\dfrac{(x-1)(x+2)(x-3)}{x^2+x+1}\right)^{1/3}$

$$\ln(y) = \frac{1}{3}\left[\ln(x-1) + \ln(x-2) + \ln(x-3) - \ln(x^2+x+1)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{3}\left[\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3} - \frac{2x+1}{x^2+x+1}\right]$$

$$\frac{dy}{dx} = \frac{1}{3}\sqrt[3]{\frac{(x-1)(x-2)(x-3)}{x^2+x+1}} \cdot \left(\frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3} - \frac{2x+1}{x^2+x+1}\right) //$$

g) $y = (x^4 + x^2 + 1)^{x^2}$

$$\ln(y) = x^2 \cdot \ln(x^4 + x^2 + 1)$$

$$\frac{1}{y}\frac{dy}{dx} = 2x \cdot \ln(x^4 + x^2 + 1) + x^2 \cdot \frac{4x^3 + 2x}{x^4 + x^2 + 1}$$

$$\frac{dy}{dx} = (x^4 + x^2 + 1)^{x^2} \cdot \left[2x \ln(x^4 + x^2 + 1) + \frac{4x^5 + 2x^3}{x^4 + x^2 + 1}\right]$$

h) $y = [2 + \operatorname{sen} x]^{e^x}$

$$\ln y = e^x \cdot \ln(2 + \operatorname{sen} x)$$

$$\frac{1}{y}\frac{dy}{dx} = e^x \cdot \ln(2 + \operatorname{sen} x) + e^x \cdot \frac{\cos x}{2 + \operatorname{sen} x}$$

$$\frac{dy}{dx} = [2 + \operatorname{sen} x]^{e^x} \cdot e^x \left(\ln(2 + \operatorname{sen} x) + \frac{\cos x}{2 + \operatorname{sen} x}\right) //$$