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1-a) $\int sen^4(2x)\,dx = \int \left(\frac{u}{2}\right)^5 \cdot sen(u) \cdot du$

$= \frac{1}{32}\int u^5 \cdot sen(u)\,du$

$u = 2x$

$\frac{du}{2} = dx$

$= \frac{1}{32}\left(-u^4 \cdot \cos u - \int -4u^3 \cos u \, du\right)$

$= \frac{1}{32}\left(-u^4 \cos u - \left(-4\left(u^3 sen\, u - \int 3u^2 sen\, u\, du\right)\right)\right)$

$*\begin{cases} I = u^4 \\ dI = 4u^3 \\ dv = sen\, u\, du \\ v = -\cos u \end{cases}$

$= \frac{1}{32}\left(-u^4 \cos u - \left(-4\left(u^3 sen\, u - 3\left(-u^2 \cdot \cos u - \int -2\cos u\, du\right)\right)\right)\right)$

$= \frac{1}{32}\left(-u^4 \cos u - \left(-4\left(u^3 sen\, u - 3\left(-u^2 \cos u + 2\left(u\, sen\, u + \cos u\right)\right)\right)\right)\right)$

$= \frac{1}{32}\left(-(2x)^4 \cos(2x) - \left(-4\left((2x)^3 sen(2x) - 3\left(-(2x)^2 \cos(2x) + 2\left((2x)\, sen(2x) + \cos(2x)\right)\right)\right)\right)\right) + C$

• $-4\int u^3 \cos u\, du$

$-4\left(u^3 sen\, u - \int 3u^2 sen\, u\, du\right)$

• $3\int u^2 sen\, u\, du$

$3\left(-u^2 \cos u - \int -2u \cos u\, du\right)$

• $-2\int u \cos u\, du$

$-2\left(u.sen\, u - \int sen\, u\, du\right)$

$-2\left(u.sen\, u + \cos u\right) + C$

b) $\int \left(1 - \frac{1}{t}\right)^3 \cdot \frac{1}{t^2}\, dt = \int \frac{1}{t^2} - \frac{3}{t^3} + \frac{3}{t^4} - \frac{1}{t^5}\, dt$

$= \int \frac{1}{t^2}\, dt - \int \frac{3}{t^3}\, dt + \int \frac{3}{t^4}\, dt - \int \frac{1}{t^5}\, dt$

$= \int t^{-2}\, dt - 3\int t^{-3}\, dt + 3\int t^{-4}\, dt - \int t^{-5}\, dt$

$= -t^{-1} - 3\left(-\frac{1}{2}t^{-2}\right) + \frac{\cancel{3}t^{-3}}{\cancel{3}} - \left(-\frac{t^{-4}}{4}\right) + C$

$= -\frac{1}{t} + \frac{3}{2t^2} - \frac{1}{t^3} + \frac{1}{4t^4} + C$

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c) $\int x^3 \arctan(x)\, dx = \dfrac{\arctan(x)\, x^4}{4} - \int \dfrac{x^4}{4} \cdot \dfrac{1}{1+x^2}\, dx$

$= \dfrac{x^4 \arctan(x)}{4} - \dfrac{1}{4}\int \dfrac{x^4}{1+x^2}\, dx$

$u = \arctan(x)$

$du = \dfrac{1}{1+x^2}\, dx$

$= \dfrac{x^4 \arctan(x)}{4} - \dfrac{1}{4}\int \dfrac{\cancel{(x^2+1)}(x^2-1)}{\cancel{1+x^2}} + \dfrac{1}{1+x^2}$

$= \dfrac{x^4 \arctan(x)}{4} - \dfrac{1}{4}\int \dfrac{1}{1+x^2} + \int (x^2-1)\, dx$

$dv = x^3\, dx$

$v = \dfrac{x^4}{4}$

$= \dfrac{x^4 \arctan(x)}{4} - \dfrac{1}{4}\left(\arctan(x) - \dfrac{x^3}{3} - x\right)$

$= \dfrac{x^4 \arctan(x)}{4} - \dfrac{\arctan(x)}{4} - \dfrac{x^3}{12} - \dfrac{x}{4} + C$

$x^4 = (x^2+1)(x^2-1)+1$

d) $\int \cos^{2/3}(2t)\, sen(2t)\, dt = \frac{1}{2}\int \cos^{2/3}(u) \cdot sen(u) \cdot du$

$= -\frac{1}{2}\int v^{2/3}\, dv$

$u = 2t$

$du = 2\, dt$

$\frac{du}{2} = dt$

$= -\frac{1}{2}\left(\frac{3}{5} v^{5/3}\right) + C$

$= \frac{-3}{10} \cos^{5/3}(2t) + C$

$v = \cos u$

$dv = -\, sen \cdot du$

$-dv = sen \cdot du$

$$e) \int \tan^6(x)\,dx = \int \tan^4(x)\tan^2(x)\,dx$$

$$= \int \tan^4(x)(\sec^2 x - 1)\,dx$$

$\tan^2(x) = \sec^2(x) - 1$

$$= \int \tan^4(x)\sec^2(x)\,dx - \int \tan^4(x)\,dx$$

$u = \tan x$

$du = \sec^2 x$

$$= \int u^4\,du - \int \tan^4(x)\,dx$$

$$= \frac{\tan^5(x)}{5} - \frac{\tan^3(x)}{3} + \tan x - x + C$$

$$f) \int \frac{dx}{x^2\sqrt{x^2+4}} = \int \frac{2\sec^2(u)\,du}{(2\tan(u))^2\sqrt{(2\tan(u))^2+4}}$$

$$= \int \frac{\cancel{2}\sec^2(u)}{\cancel{4}\tan^2(u)\sqrt{4(\tan^2(u)+1)}}\,du$$

$x = 2\tan(u)$

$dx = 2\sec^2(u)$

$$= \int \frac{\sec^2(u)}{2\tan^2(u)\cdot 2\sqrt{\sec^2(u)}}\,du$$

$\tan^2(u)+1 = \sec^2(u)$

$$= \int \frac{\sec^{\cancel{2}}(u)}{4\tan^2(u)\cdot \cancel{\sec(u)}}$$

$\sec(u) = \frac{1}{\cos(u)}$

$$= \frac{1}{4}\int \frac{\sec(u)}{\tan^2(u)}\,du$$

$$= \frac{1}{4}\int \frac{1}{\cancel{\cos(u)}}\cdot\frac{\cos^{\cancel{2}}(u)}{\sin^2(u)}$$

$\operatorname{tg}(u) = \frac{\sin(u)}{\cos(u)}$

$$= \frac{1}{4}\int \frac{\cos u}{\sin^2 u}\,du$$

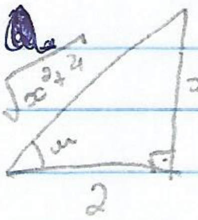
$v = \sin u$

$dv = \cos u$

$$= \frac{1}{4}\int \frac{1}{v^2}\,dv$$

$$= \frac{1}{4}\cdot\left(-\frac{1}{v}\right) + C$$

$$= -\frac{1}{4}\cdot\frac{1}{\sin u} + C$$



$x = 2 \tan u$

$\frac{x}{2} = \tan u$

$\operatorname{sen} u = \frac{x}{\sqrt{x^2-1}}$

$$= \frac{-1}{4} \cdot \frac{1}{\operatorname{sen} u} + C$$

$$= -\frac{1}{4} \cdot \frac{1}{\frac{x}{\sqrt{x^2-1}}} + C$$

$$= -\frac{1}{4} \cdot \frac{\sqrt{x^2-1}}{x} + C$$

$$= \frac{-\sqrt{x^2-1}}{4x} + C$$

2- $f(x) = \cos(2x)$ $[0, 5\pi]$

$g(x) = \operatorname{sen}(3x)$

$$\int_0^{5\pi} \operatorname{sen}(3x) - \cos(2x)\, dx = \frac{-\cos(3x)}{3} - \frac{\operatorname{sen}(2x)}{2} \Big|_0^{5\pi}$$

$$= \frac{\cos(3 \cdot 5\pi)}{3} - \cancelto{0}{\frac{\operatorname{sen}(2 \cdot 5\pi)}{2}} - \left(\frac{-\cos(3 \cdot 0)}{3} - \cancelto{0}{\frac{\operatorname{sen}(2 \cdot 0)}{2}} \right)$$

$$= \frac{1}{3} - \left(\frac{-1}{3} \right) = \frac{2}{3} \text{ u.a}$$

3_ $y = x^2 + 1 \qquad 1 \leq x \leq 3 \qquad V = \int_a^b A(x)\,dx$

$$V = \pi \int_1^3 (x^2+1)^2 \, dx$$

$$V = \pi \int_1^3 x^4 + 2x^2 + 1 \, dx$$

$$V = \pi \left(\frac{x^5}{5} + 2\frac{x^3}{3} + x \right) \Big|_1^3$$

$$V = \pi \left[\left(\frac{243}{5} + 18 + 3 \right) - \left(\frac{1}{5} + \frac{2}{3} + 1 \right) \right]$$

$$V = \pi \left[\frac{243}{5} + 20 - \frac{1}{5} - \frac{2}{3} \right]$$

$$V = \pi \left[\frac{729 + 300 - 3 - 10}{15} \right]$$

$$V = \frac{1016}{15}\pi \quad u.v.$$