6ª LISTA — EDO - EQS. LINEARES, HOMOG., DE 2ª ORDEM C/ COEFICIENTES CONSTANTES.

1) a) $\boxed{4 \cdot y'' + y' = 0}$

$a = 4$
$b = 1$
$c = 0$

$\Delta = b^2 - 4 \cdot a \cdot c$
$\Delta = 1 - 4 \cdot 4 \cdot 0$
$\Delta = 1 > 0$

$4 \cdot m^2 + m = 0 \Rightarrow m \cdot (4 \cdot m + 1) = 0 \Rightarrow$

$4m + 1 = 0$
ou
$m = 0$

$\Rightarrow \begin{cases} m = 0 \\ m = -\frac{1}{4} \end{cases}$

Sol. GERAL: $y = C_1 \cdot e^{0 \cdot x \to 1} + C_2 \cdot e^{-\frac{x}{4}} \Rightarrow \boxed{y = C_1 + C_2 \cdot e^{-\frac{x}{4}}}$

b) $\boxed{y'' - 36y = 0}$

$a = 1$
$b = 36$
$c = 0$

$a^2 - b^2 = (a-b) \cdot (a+b)$

$y'' + b \cdot y' + c y = 0$
$m^2 + bm + c = 0$

$\Delta = (36)^2 - 4 \cdot 0$
$\Delta = (36)^2$

$\boxed{m^2 - 36 = 0} \Rightarrow m^2 - 6^2 = 0 \Rightarrow (m-6) \cdot (m+6) = 0 \Rightarrow$

$\begin{cases} m = 6 \\ m = -6 \end{cases} \Rightarrow \boxed{y = C_1 \cdot e^{6 \cdot x} + C_2 \cdot e^{-6x}}$

c) $\boxed{y'' + 9 \cdot y = 0}$

$y'' + b y' + c y = 0 \Rightarrow m^2 + bm + c = 0$

$m^2 + 9 = 0 \Rightarrow m = \frac{-0 \pm \sqrt{-36}}{2} \Rightarrow m = \frac{\pm i \cdot 6}{2} \Rightarrow \begin{cases} m_1 = i \cdot 3 \\ m_2 = -i3 \end{cases}$

$\begin{cases} a = 0 \\ b = 3 \end{cases}$

$\Delta = 0 - 4 \cdot 9$
$\boxed{\Delta = -36}$

$y = e^{0 \cdot x} [C_1 \cdot \cos(3x) + C_2 \operatorname{sen}(3x)] \Rightarrow \boxed{y = C_1 \cdot \cos(3x) + C_2 \operatorname{sen}(3x)}$

d) $\boxed{y'' - y' - 6y = 0}$ $\Delta = 1 - 4 \cdot (-6) \Rightarrow \Delta = 1 + 24 \Rightarrow \Delta = 5$

$m^2 - m - 6 = 0 \Rightarrow (m+2)(m-3) = 0 \Rightarrow \begin{cases} m_1 = -2 \\ m_2 = 3 \end{cases}$

$m^2 - 3m + 2m - 6$

Sol. GERAL: $\boxed{y = C_1 \cdot e^{3 \cdot x} + C_2 \cdot e^{-2x}}$

$\Delta = 64 - 4(16) \Rightarrow \Delta = 64 - 64 \Rightarrow \Delta = 0$

e) $\boxed{\frac{d^2 y}{dx^2} + 8 \cdot \frac{dy}{dx} + 16 \cdot y = 0}$

$m^2 + 8m + 16 = 0 \Rightarrow m = \frac{-8 \pm \sqrt{0}}{2} \Rightarrow m = \frac{-8}{2} \Rightarrow \boxed{m = -4}$

$\therefore y = C_1 \cdot e^{-4 \cdot x} + C_2 \cdot x \cdot e^{-4x} \Rightarrow \boxed{y = (C_1 + C_2 \cdot x) e^{-4 \cdot x}}$

5) $y'' + 3.y' - 5.y = 0$ $\Delta = 9 - 4.(-5) = 9 + 20$, $\Delta = 29$

$m^2 + 3m - 5 = 0 \Rightarrow m = \frac{-3 \pm \sqrt{29}}{2} \Rightarrow \begin{cases} m_1 = \frac{-3 + \sqrt{29}}{2} \\ m_2 = \frac{-3 - \sqrt{29}}{2} \end{cases}$

$y = C_1 . e^{\left(\frac{-3+\sqrt{29}}{2}\right).x} + C_2 . e^{-\left(\frac{3+\sqrt{29}}{2}\right).x}$

6) $12.y'' - 5y' - 2y = 0$ $\Delta = 25 - 4(12)(-2) = 25 + 8(12)$, $\Delta = 25 + 96 \Rightarrow \Delta = 121$

$\sqrt{121} = 11$

$12.m^2 - 5.m - 2 = 0$

$m = \frac{-(-5) \pm \sqrt{121}}{2.(12)} \Rightarrow m = \frac{5 \pm 11}{24} \Rightarrow \begin{cases} m_1 = \frac{5+11}{24} \\ m_2 = \frac{5-11}{24} \end{cases}$

$\begin{cases} m_1 = \frac{16}{24} \Rightarrow m_1 = \frac{2}{3} \\ m_2 = -\frac{6}{24} \Rightarrow m_2 = \frac{-1}{4} \end{cases}$

$\therefore y = C_1 . e^{\frac{2}{3}.x} + C_2 . e^{-\frac{x}{4}}$

7) $y'' - 4.y' + 5.y = 0$ $\Delta = 16 - 4.5 \Rightarrow \Delta = 16 - 20 \Rightarrow \boxed{\Delta = -4}$

$m^2 - 4.m + 5 = 0 \Rightarrow m = \frac{-(-4) \pm \sqrt{-4}}{2} = \frac{4 \pm i 2}{2} \Rightarrow \begin{cases} m_1 = 2 + i \\ m_2 = 2 - i \end{cases}$ $a + ib$, $a = 2$, $b = 1$

$y = e^{2.x}[C_1 . \cos(x) + C_2 . \operatorname{sen}(x)]$

8) $3y'' + 2y' + y = 0$ $\Delta = 4 - 4.(3)1 = 4 - 12 \Rightarrow \Delta = -8$

$8 = 2^3 = 2.2^2$

$m = \frac{-2 \pm \sqrt{-8}}{2.3} \Rightarrow m = \frac{-2 \pm i\sqrt{8}}{6} \Rightarrow m = -\frac{2}{3} \pm \frac{2\sqrt{2}}{6}$

$m = -\frac{2}{3} \pm \frac{1}{3}\sqrt{2} \Rightarrow \begin{cases} m_1 = -2/3 + \sqrt{2}/3 \\ m_2 = -2/3 - \sqrt{2}/3 \end{cases}$ $a = -2/3$, $b = \sqrt{2}/3$ $y = e^{-2/3}\left[\cos\left(\frac{\sqrt{2}}{3}x\right) + \operatorname{sen}\left(\frac{\sqrt{2}}{3}x\right)\right]$

9) $2.y'' + 2.y' + y = 0$ $2.m^2 + 2m + 1 = 0 \Rightarrow \Delta = 4 - 4.2 \Rightarrow \Delta = 4 - 8 = -4$

$m = \frac{-2 \pm \sqrt{-4}}{2.2} = \frac{-2 \pm i 2}{4} = \frac{-1 \pm i}{2} \Rightarrow \begin{cases} m_1 = \frac{-1}{2} + i\frac{1}{2} \\ m_2 = -\frac{1}{2} - i\frac{1}{2} \end{cases}$ $a = -\frac{1}{2}$, $b = 1/2$

$y = e^{-\frac{x}{2}}\left[C_1 \cos\left(\frac{x}{2}\right) + C_2 \operatorname{sen}\left(\frac{x}{2}\right)\right]$