TRANSFORMADA DE DERIVADAS

RESOLVIENDO $\boxed{\mathcal{L}\{f'(t)\} = \int_0^{\infty} e^{-s.t}\{f'(t)\}\,dt}$

$$\mathcal{L}\{f'(t)\} = \int_0^{+\infty} e^{-s.t}\{f'(t)\}.\,dt \; ; \quad (1)$$

$$\boxed{u = e^{-st}} \Rightarrow \boxed{du = -s e^{-st}.\,dt \, ;} \quad (2)$$

$$dv = f'(t).\,dt \Rightarrow \int dv = \int f'(t).\,dt$$

$$\boxed{v = f(t)} \; ; \quad (3)$$

$$\mathcal{L}\{f'(t)\} = \left[e^{-s.t} f(t)\right]_0^{+\infty} - \int_0^{+\infty} f(t).(-s)e^{-s.t}.\,dt \; ; \quad (4)$$

$$\mathcal{L}\{f'(t)\} = \left[\underbrace{\frac{f(+\infty)}{e^{\infty}}}_{0} - \underbrace{\frac{f(0)}{e^{0}}}_{1}\right] + s \int_0^{+\infty} e^{-s.t}\{f(t)\}\,dt \; ; \quad (5)$$

$$\boxed{\mathcal{L}\{f'(t)\} = -f(0) + s.\,\mathcal{L}\{f(t)\}} \; ; \quad (6)$$

RESOLVENDO $\boxed{\mathcal{L}\{f''(t)\}}$

$$\mathcal{L}\{f''(t)\} = \int_0^{\infty} e^{-s.t} \{f''(t)\}\, dt \; ; \quad (1)$$

$$\boxed{u = e^{-s.t}} \Rightarrow \boxed{du = -s.e^{-s.t}} \quad (2)$$

$$\boxed{dv = f''(t)\, dt} \Rightarrow v = \int f''(t).dt \Rightarrow \boxed{V = f'(t)} \quad (3)$$

$$\mathcal{L}\{f''(t)\} = \left[e^{-s.t} . f'(t)\right]_0^{+\infty} - \int_0^{\infty} f'(t)(-s).e^{-s.t}\, dt \; ; \quad (4)$$

$$\mathcal{L}\{f''(t)\} = \left[\frac{f'(\infty)}{e^{\infty}} - \frac{f'(0)}{e^{0}}\right] + s.\int_0^{\infty} e^{-s.t} \{f'(t)\}\, dt \; ; \quad (5)$$

$$\mathcal{L}\{f''(t)\} = -f'(0) + s\,\underbrace{\mathcal{L}\{f'(t)\}}_{\text{ITEM ANTERIOR}} \; ; \quad (6)$$

$$\mathcal{L}\{f''(t)\} = -f'(0) + s\left[-f(0) + s.\mathcal{L}\{f(t)\}\right] \; ; \quad (7)$$

$$\boxed{\mathcal{L}\{f''(t)\} = s^2\, \mathcal{L}\{f(t)\} - s.f(0) - f'(0)} \; ; \quad (8)$$

OU $\boxed{\mathcal{L}\{f''(t)\} = s^2.\mathcal{L}\{f(t)\} - s^1.f(0) - s^0.f'(0)}$

TRANSFORMADA DE DERIVADAS

FINALIDADE: TÉCNICA DE SOLUÇÃO DE EQS. DIFERENCIAIS → P.V.I.

LEI DE FORMAÇÃO

$$\mathcal{L}\{f(t)\} = s^0 \mathcal{L}\{f(t)\} \quad (1)$$

$$\mathcal{L}\{f'(t)\} = s^1 \mathcal{L}\{f(t)\} - s^0 \cdot f(0) \quad (2)$$

$$\mathcal{L}\{f''(t)\} = s^2 \mathcal{L}\{f(t)\} - s^1 \cdot f(0) - s^0 \cdot f'(0) \quad (3)$$

$$\mathcal{L}\{f'''(t)\} = s^3 \mathcal{L}\{f(t)\} - s^2 \cdot f(0) - s^1 \cdot f'(0) - s^0 \cdot f''(0) \quad (4)$$

FORMA GERAL:

$$\mathcal{L}\{f^{n}(t)\} = s^n \mathcal{L}\{f(t)\} - s^{n-1} \cdot f(0) - \ldots\ldots - s^0 f^{n-1}(0) \quad (5)$$

- $\boxed{n=1}$

$$\boxed{\mathcal{L}\{f'(t)\} = s^1 \mathcal{L}\{f(t)\} - s^0 \cdot f(0)} \quad (6) \rightarrow (2)$$

- $\boxed{n=2}$

$$\boxed{\mathcal{L}\{f''(t)\} = s^2 \mathcal{L}\{f(t)\} - s^1 \cdot f(0) - s^0 f'(0)} \quad (7) \rightarrow (3)$$

SOLUÇÃO DE EQS. DIFERENCIAIS

$\boxed{T.L.}$ ⇒ USADA PARA RESOLVER P.V.I. DE EQS. LINEARES COM COEFICIENTES CONSTANTES.

$\boxed{\text{UTILIDADE}}$ ⇒ $\mathcal{L}\{f^{n}_{(t)}\}$ ESTÁ RELACIONADA DE MANEIRA SIMPLES À T.L. DE $f(t) \Rightarrow \mathcal{L}\{f_{(t)}\}$

APLICAÇÕES:

$\boxed{\text{EQ. 1ª ORDEM}}$

$\boxed{\text{EX1}}$ — RESOLVA O P.V.I. $\begin{cases} \dfrac{dy}{dt} - y = 0 \\ y(0) = 1 \end{cases}$

CONVENIENTE ESCREVER $\boxed{y = f(t)}$

ENTÃO $\begin{cases} f'_{(t)} - f_{(t)} = 0 \\ f(0) = 1 \end{cases}$, (1)

- APLICA T.L. EM (1)

$$\mathcal{L}\{f'_{(t)} - f_{(t)}\} = 0 \quad , \quad (2)$$

$$\mathcal{L}\{c_1 . f_1 + c_2 . f_2\} = c_1 \mathcal{L}\{f_1\} + c_2 \mathcal{L}\{f_2\}$$

→ LINEARIDADE ⇒ EM 2 :

$$\mathcal{L}\{f'(t)\} - \mathcal{L}\{f(t)\} = 0 \; ; \quad 3$$

$$\mathcal{L}\{f'(t)\} = s^1 \mathcal{L}\{f(t)\} - s^0 f(0)$$

$$s \mathcal{L}\{f(t)\} - f(0) - \mathcal{L}\{f(t)\} = 0 \; ; \quad 4$$

$$\mathcal{L}\{f(t)\} . [s - 1] - \underbrace{f(0)}_{1 \to \text{PVI}} = 0 \; ; \quad 5$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{1}{s-1}} \; , \quad 6$$

→ PROCURAMOS $f(t) = y$

QUAL A FUNÇÃO $y = f(t)$ CUJA T.L. FORNECE $\frac{1}{s-1}$?

DA TABELA ⇒ $\mathcal{L}\{e^{a.t}\} = \frac{1}{s-a}$

$\boxed{a = 1}$ EM 6 : $\mathcal{L}\{e^{1.t}\} = \frac{1}{s-1}$

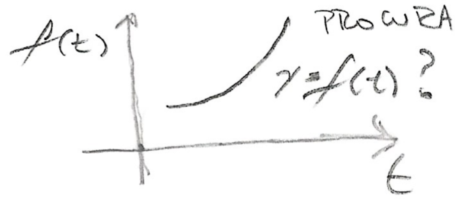
$$\mathcal{L}\{f(t)\} = \frac{1}{s-1}$$

$\therefore \boxed{f(t) = e^t}$ É A SOLUÇÃO DA EQ. 1

EX2 - RESOLVA $\boxed{y'' - 3y' = 3.e^{3.t}}$, COM $y(0) = 0$ E $y'(0) = 1$.

SOLUÇÃO:

$$y = f(t) \Rightarrow \begin{cases} f''(t) - 3f'(t) = 3.e^{3t} & (1) \\ f(0) = 0, \\ f'(0) = 1. \end{cases}$$



APLICA T.L. EM (1):

$$\mathcal{L}\{f''(t) - 3f'(t)\} = \mathcal{L}\{3e^{3t}\} \quad ; \quad (2)$$

↓ LINEARIDADE

$$\mathcal{L}\{f''(t)\} - 3\mathcal{L}\{f'(t)\} = 3.\mathcal{L}\{e^{3.t}\}; \quad (3)$$

$$\boxed{\begin{aligned} \mathcal{L}\{f'\} &= s^1\mathcal{L}\{f\} - s^0 f(0) \\ \mathcal{L}\{f''\} &= s^2\mathcal{L}\{f\} - s^1 f(0) - s^0 f'(0) \end{aligned}}$$

$$s^2\mathcal{L}\{f(t)\} - s\underbrace{f(0)}_{0} - \underbrace{f'(0)}_{1} - 3\left[s\mathcal{L}\{f\} - \underbrace{f(0)}_{0}\right] = 3\mathcal{L}\{e^{3t}\} \quad (4)$$

$$s^2\mathcal{L}\{f(t)\} - 1 - 3.s.\mathcal{L}\{f(t)\} = 3\mathcal{L}\{e^{3.t}\} \quad ; \quad (5)$$

$\mathcal{L}\{f(t)\} \Rightarrow$ EM EVIDÊNCIA

$$\mathcal{L}\{f(t)\}\cdot[s^2-3s]-1 = 3\,\mathcal{L}\{e^{3.t}\}\ ;\ (6)$$

TABELA

$$\mathcal{L}\{f(t)\}\cdot[s^2-3s]-1 = 3\left(\frac{1}{s-3}\right)\ ;\ (7)\qquad \boxed{\mathcal{L}\{e^{a.t}\}=\frac{1}{s-a}}$$

$$\mathcal{L}\{f(t)\}\cdot[s^2-3s] = \frac{3}{s-3}+1 = \frac{3+s-3}{s-3}\ ;\ (8)$$

$$\mathcal{L}\{f(t)\} = \frac{s}{(s^2-3s)(s-3)} = \frac{s}{s(s-3).(s-3)}\ ;\ (9)$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{1}{(s-3)^2}}\ (10)$$

QUAL A FUNÇÃO $f(t)$ CUJA T.L. FORNECE

$1/(s-3)^2$?

TABELA: $F(s) = \mathcal{L}\{t^n.e^{a.t}\} = \dfrac{n!}{(s-a)^{n+1}} \Rightarrow \begin{cases} s>a \\ n \in \mathbb{Z}_+^* \end{cases}$

$$(10) \longrightarrow \mathcal{L}\{t^{\bigcirc}.e^{\square.t}\} = \frac{1}{(s-\boxed{3})^{1+1}} \Rightarrow \begin{cases} a=3 \\ n=1 \end{cases}$$

$$\mathcal{L}\{t^1.e^{3.t}\} = \frac{1!}{(s-3)^2}$$

$$\therefore$$

$$\mathcal{L}\{f(t)\} = \frac{1}{(s-3)^2} \qquad \boxed{f(t) = t.e^{3.t}}$$

SOLUÇÃO PVI