LISTA - EQs NÃO-HOMOGÊNEAS - COEF. INDETERMINADOS (1)

a) $\boxed{y''+2.y'=\cos(2x)}$

$y_p = A\cos(2x)+B.\operatorname{sen}(2x) \rightarrow \boxed{y_p' = -2A\operatorname{sen}(2x)+2B\cos(2x)}$

$\boxed{y_p'' = -4A\cos(2x)-4B\operatorname{sen}(2x)}$

$-4.A.\cos(2x)-4B\operatorname{sen}(2x)+2[-2A\operatorname{sen}(2x)+2B\cos(2x)] = \cos(2x)+0.\operatorname{sen}(2x)$

$\cos(2x)[-4A+4B]+\operatorname{sen}(2x).[-4B-4A] = \cos(2x)+0.\operatorname{sen}(2x)$

$\begin{cases}-4.A+4.B=1\\ -4A-4B=0\end{cases} \Rightarrow \begin{array}{r}-4A+4B=1\\ +\ -4A-4B=0\\ \hline -8.A=1\end{array} \Rightarrow \boxed{A=-\frac{1}{8}}$

$-4.\left(-\frac{1}{8}\right)-4B=0$

$\frac{1}{2}-4B=0 \Rightarrow 4B=\frac{1}{2}$

$\boxed{B=\frac{1}{8}}$

$\therefore \boxed{y_p = -\frac{1}{8}.\cos(2x)+\frac{1}{8}.\operatorname{sen}(2x)}$

OBTENDO y_c. $\Rightarrow y''+2y'=0 \Rightarrow m^2+2m=0 \Rightarrow m.(m+2)=0$

$\begin{cases}m=0\\ m=-2\end{cases} \Rightarrow y_c = C_1.e^{0.x}+C_2.e^{-2x} \Rightarrow \boxed{y_c = C_1+C_2.e^{-2x}}$

sol. geral: $\boxed{y = C_1+C_2.e^{-2x}-\frac{1}{8}\cos(2x)+\frac{1}{8}\operatorname{sen}(2x)}$ ok

b) $\boxed{y''-y = x.e^{2x}}$

$\boxed{y_p = (A+B.x).e^{2x}} = A.e^{2x}+B.x.e^{2x}$

$y_p' = 2.A.e^{2x}+B(e^{2x}+x.2.e^{2x}) = 2A.e^{2x}+B.e^{2x}+2Bx.e^{2x}$

$\boxed{y_p' = (2A+B+2Bx)e^{2x}}$

$y_p'' = 4Ae^{2x}+2B.e^{2x}+2B(e^{2x}+x2e^{2x}) = (4A+2B)e^{2x}+(2B+4Bx).e^{2x}$

$y_p'' = (4A+2B+2B+4Bx)e^{2x} \Rightarrow \boxed{y_p'' = (4A+4B+4B.x)e^{2x}}$

SUBST. NA EQ.

$[4A+4B+4Bx-A-Bx]\cancel{e^{2x}} = x.\cancel{e^{2x}} \Rightarrow [3.A+4B,+3B.x] = x+0$

$\Rightarrow$

$$\begin{cases} 3A + 4B = 0 \\ 3B = 1 \end{cases} \Rightarrow \boxed{B = 1/3} \Rightarrow 3.A + 4.\frac{1}{3} = 0 \Rightarrow 3.A = -\frac{4}{3}$$

$$\boxed{A = -\frac{4}{9}}$$

$$\therefore \; y_p = \left(-\frac{4}{9} + \frac{1}{3}x\right) e^{2x} \Rightarrow \boxed{y_p = +\frac{1}{9}\left(3x - 4\right) \cdot e^{2x}}$$

$$y_c \Rightarrow \boxed{y'' - y = 0} \qquad m^2 - 1 = 0 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$$

$$\boxed{y_c = c_1 e^{x} + c_2 e^{-x}}$$

sol. geral Eq. NÃo-Hom. $\boxed{y = c_1 . e^{x} + c_2 . e^{-x} + \frac{1}{9}\left(3x - 4\right) e^{2x}}$ OK

c) $\boxed{y'' + 3.y' + 2.y = 6}$ Zill. 193.ex 1

$$\boxed{y_p = A} \Rightarrow y_p' = 0 \Rightarrow y'' = 0$$

$$0 + 3.0 + 2.A = 6 \Rightarrow \boxed{A = 3} \Rightarrow \boxed{y_p = 3}$$

$$y_c \Rightarrow \boxed{y'' + 3y' + 2.y = 0}, \quad m^2 + 3.m + 2 = 0 \Rightarrow (m+2).(m+1) = 0$$

$$\begin{cases} m = -1 \\ m = -2 \end{cases} \Rightarrow \boxed{y_c = c_1 . e^{-x} + c_2 . e^{-2x}}$$

sol. geral: $\boxed{y = c_1 . e^{-x} + c_2 . e^{-2x} + 3}$ OK

d) $y'' - 10.y' + 25.y = 30x + 3$ $\quad \frac{30}{25} = \frac{6}{5}$ ②

$\boxed{y_p = Ax + B} \Rightarrow \boxed{y'_p = A} \Rightarrow \boxed{y''_p = 0}$

$-10(A) + 25(Ax + B) = 30x + 3 \Rightarrow -10.A + 25Ax + 25B = 30x + 3$

$-10A + 25B + 25.A.x = 30.x + 3 \Rightarrow \begin{cases} -10.A + 25B = 3 \\ 25.A = 30 \end{cases} \Rightarrow \boxed{A = \frac{30}{25} = \frac{6}{5}}$

$-10\left(\frac{6}{5}\right) + 25B = 3 \Rightarrow -12 + 25B = 3 \Rightarrow B = \frac{15}{25} \Rightarrow \boxed{B = \frac{3}{5}}$

$\therefore \boxed{y_p = \frac{6}{5}x + \frac{3}{5}}$

$y_c \Rightarrow y'' - 10.y' + 25y = 0 \Rightarrow m^2 - 10.m + 25 = 0 \Rightarrow (m-5)(m-5) = 0$

$(m-5)^2 = 0 \Rightarrow \boxed{m_1 = m_2 = 5}$ $\quad \boxed{y_c = C_1.e^{5x} + C_2.x.e^{5x}}$

Sol. geral Eq. N Hom. $\Rightarrow \boxed{y = C_1.e^{5x} + C_2.x.e^{5x} + \frac{6}{5}x + \frac{3}{5}}$ ok

e) $\frac{1}{4}y'' + y' + y = x^2 - 2x$

$\boxed{y_p = Ax^2 + Bx + C} \Rightarrow \boxed{y'_p = 2Ax + B} \Rightarrow \boxed{y''_p = 2A}$

$\frac{1}{4}(2A) + 2Ax + B + Ax^2 + Bx + C = x^2 - 2x$

$\frac{A}{2} + B + C + (2A + B)x + Ax^2 = x^2 - 2.x \Rightarrow \begin{cases} \boxed{A = 1} \\ 2A + B = -2 \\ \frac{A}{2} + B + C = 0 \end{cases}$

$2(1) + B = -2 \Rightarrow 2 + B = -2 \Rightarrow B = -2 - 2 \Rightarrow \boxed{B = -4}$

$\frac{1}{2} - 4 + C = 0 \Rightarrow C = 4 - \frac{1}{2} \Rightarrow C = \frac{8-1}{2} \Rightarrow \boxed{C = 7/2}$

$\boxed{y_p = x^2 - 4x + \frac{7}{2}}$ $\quad y_c \Rightarrow \frac{1}{4}m^2 + m + 1 = 0 \Rightarrow \left(\frac{m}{2} + 1\right).\left(\frac{m}{2} + 1\right) = 0$

$\left(\frac{m+2}{2}\right)^2 = 0 \Rightarrow (m+2) = 0.4 \Rightarrow m + 2 = 0 \Rightarrow \boxed{m_1 = -2}$ $\quad m_1 = m_2 = m$

$y_c = C_1 e^{-2x} + C_2.x.e^{-2x}$ $\quad \boxed{y = C_1.e^{-2x} + C_2.x.e^{-2x} + x^2 - 4x + \frac{7}{2}}$ ok

Sol. Geral Eq.

f) $\boxed{y'' - 2y' - 3y = 2.e^{x} - 10.\operatorname{sen}(x)}$ ③

$g(x) = e^{x} - \operatorname{sen}(x) \Rightarrow \boxed{y_p = A.e^{x} + B.\operatorname{sen}(x) + C.\cos(x)}$

$\boxed{y_p' = A.e^{x} + B.\cos(x) - C\operatorname{sen}(x)}$

$\boxed{y_p'' = A.e^{x} - B\operatorname{sen}(x) - C\cos(x)}$

$A.e^{x} - B.\operatorname{sen}(x) - C.\cos(x) - 2A e^{x} - 2B.\cos(x) + 2C\operatorname{sen}(x) - 3Ae^{x} - 3B\operatorname{sen}(x) - 3C.\cos(x) = 2e^{x} - 10\operatorname{sen}(x)$

$(A - 2A - 3A).e^{x} + (-B + 2C - 3B)\operatorname{sen}(x) + (-C - 2B - 3C)\cos(x) = 2e^{x} - 10\operatorname{sen}(x)$

$-4A.e^{x} + (-4B + 2C).\operatorname{sen}(x) + (-2B - 4C)\cos(x) = 2.e^{x} - 10.\operatorname{sen}(x)$

$\begin{cases} -4A = 2 \\ -4B + 2C = -10 \\ -2B - 4C = 0 \quad \times(-2) \end{cases} \Rightarrow \begin{cases} \boxed{A = -1/2} \\ -4B + 2C = -10 \\ +4B + 8C = 0 \end{cases}$

$10C = -10 \Rightarrow \boxed{C = -1}$

$-2B + 4 = 0$

$-2B = -4$

$\boxed{B = 2}$

$\therefore \boxed{y_p = -\frac{1}{2}e^{x} + 2\operatorname{sen}(x) - \cos(x)}$

$y_c \Rightarrow y'' - 2y' - 3y = 0 \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow (m-3)(m+1) = 0$

$\begin{cases} m = 3 \\ m = -1 \end{cases} \Rightarrow \boxed{y_c = C_1.e^{3x} + C_2 e^{-x}}$

$\therefore y = y_c + y_p \Rightarrow \boxed{y = C_1.e^{3x} + C_2.e^{-x} - \frac{1}{2}e^{x} + 2\operatorname{sen}(x) - \cos(x)}$ ok

g) $\boxed{y'' - 4y' = 2 \cdot e^{3x}}$

(pq. derivada de e^{3x} é um múltiplo dele de e^{3x}, assim é razoável testar:)

$\boxed{y_p = A \cdot e^{3x}} \Rightarrow y_p' = 3A \cdot e^{3x} \Rightarrow y_p'' = 9 \cdot A \cdot e^{3x}$

Eq. $y'' - 4y' = 2 \cdot e^{3x} \Rightarrow 9 \cdot A \cdot e^{3x} - 4 \cdot 3 \cdot A\, e^{3x} = 2 \cdot e^{3x} \Rightarrow (9A - 12A) = 2 \Rightarrow$

$\Rightarrow -3A = 2 \Rightarrow \boxed{A = -2/3} \quad \therefore \boxed{y_p = -\frac{2}{3} e^{3x}}$

$y_c \Rightarrow y'' - 4y' = 0 \Rightarrow m^2 - 4m = 0 \Rightarrow m(m-4) = 0 \Rightarrow \begin{cases} m = 0 \\ \text{ou} \\ m = 4 \end{cases}$

$\boxed{y_c = c_1 e^{0 \cdot x} + c_2 e^{4x}}$

Sol geral Eq. Não homogênea:

$$\boxed{y = c_1 + c_2 e^{4x} - \frac{2}{3} \cdot e^{3x}}$$

h) $\boxed{y'' + 4.y = 3x^3}$ Howard p 123 - ex. 5

$y_p = A.x^3 + B.x^2 + C.x + D \Rightarrow y_p' = 3Ax^2 + 2Bx + C$

$y_p'' = 6AX + 2B$ $\quad 6.AX + 2B + 4(AX^3 + BX^2 + CX + D) = 3.X^3$

$6.A.X + 2B + 4AX^3 + 4BX^2 + 4CX + 4D = 3X^3$

$4AX^3 + 4BX^2 + (6A + 4C).X + (2B + 4D) = 3.X^3 \Rightarrow \begin{cases} 4A = 3 \\ 4B = 0 \\ 6A + 4C = 0 \\ 2B + 4D = 0 \end{cases}$

$A = 3/4$; $B = 0$, $C - \frac{9}{8}$ e $D = 0$

$6.\frac{3}{4} + 4C = 0$

$\frac{9}{2} + 4C \Rightarrow C = -\frac{9}{8}$

$\therefore \boxed{y_p = \frac{3}{4}x^3 - \frac{9}{8}x}$

$y_c = y'' + 4y = 0 \Rightarrow m^2 + 4 = 0 \Rightarrow m^2 = -4 \Rightarrow m = \sqrt{-4} = \sqrt{i^2 \cdot 4}$

$m = \pm i2 \Rightarrow \begin{cases} 2i \\ -2i \end{cases} \Rightarrow \begin{matrix} a = 0 \\ b = 2 \end{matrix} \Rightarrow y_c = e^{0.x}[C_1.\cos(2x) + C_2\,\text{Sen}(2x)]$

$\boxed{y_c = C_1.\cos(2x) + C_2\,\text{Sen}(2x)}$ Sol. Geral = $\boxed{C_1\cos(2x) + C_2\,\text{Sen}(2x) + \frac{3}{4}x^3 - \frac{9}{8}x}$