FRAÇÕES PARCIAIS - F.P.

RESOLVER: $\int \frac{8x-9}{x^2-x-6}\,dx = \int \frac{8x-9}{(x-3)(x+2)}\,dx$

F.P. $\Rightarrow$ EXPRESSAR UMA FUNÇÃO RACIONAL $R(x) = P(x)/Q(x)$ NUMA SOMA DE FRAÇÕES.

CASO 1 - FATORES LINEARES DISTINTOS

$$\frac{8x-9}{(x-3).(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} \Rightarrow \text{ACHAR} \begin{cases} A = ? \\ B = ? \end{cases}$$

$$\frac{8x-9}{(x-3).(x+2)} = \frac{A(x+2)+B(x-3)}{(x-3).(x+2)} = \frac{Ax+2A+Bx-3B}{(x-3).(x+2)},$$

SE DENOMINADORES SÃO IGUAIS $\Rightarrow$ NUMERADORES SÃO IGUAIS:

$$8x-9 = (A+B).x + (2A-3B) \Rightarrow \begin{cases} A+B=8 & \times(-2) \\ 2A-3B=-9 \end{cases}$$

$$\begin{cases} -2A-2B=-16 \\ 2A-3B=-9 \end{cases}$$
$$-5B=-25 \Rightarrow \boxed{B=5} \Rightarrow \boxed{A=3}$$

ENTÃO:

$$\frac{8x-9}{(x-3).(x+2)} = \frac{3}{x-3} + \frac{5}{x+2} \Rightarrow$$

SUBSTITUINDO NA INTEGRAL:

$$\boxed{\int \frac{8x-9}{(x-3).(x+2)}\,dx = 3\int \frac{1}{x-3}\,dx + 5\int \frac{1}{x+2}\,dx}$$

EXS :

- $$\frac{1}{(x-1).(x+2).(x+4)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+4}$$

- $$\frac{8x-9}{(x-3).(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

- $$\frac{x-1}{x.(x-2).(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$$

- $$\frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$ $\boxed{a^2+b^2=(a-b).(a+b)}$

TRANSFORMADA INVERSA DE LAPLACE : $X \Leftrightarrow S$

- $$\frac{1}{(s-1)(s+2)(s+4)} = \frac{A}{s-1} + \frac{B}{s+2} + \frac{C}{s+4}$$

- $$\frac{8s-9}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2}$$

- $$\frac{s-1}{s.(s-2)(s+1)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+1}$$

- $$\frac{1}{s^2-4} = \frac{1}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$$

EX 1 - RESOLVA: $\begin{cases} \dfrac{dy}{dt} - 3y = e^{2.t} \\ y(0) = 1 . \end{cases}$

COM $y = f(t) \Rightarrow \left[\begin{array}{l} f'(t) - 3f(t) = e^{2.t} \\ f(0) = 1 \end{array} \right.$

APLICANDO T.L.:

$$\mathcal{L}\{f'(t)\} - 3\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{2.t}\};$$

$$s.\mathcal{L}\{f(t)\} - f(0) - 3\mathcal{L}\{f(t)\} = \frac{1}{s-2};$$

$$\mathcal{L}\{f(t)\}.(s-3) - 1 = \frac{1}{s-2};$$

$$\mathcal{L}\{f(t)\}.(s-3) = \frac{1}{s-2} + 1 = \frac{1+s-2}{s-2};$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s-3}\left[\frac{s-1}{s-2}\right];$$

$$\boxed{\mathcal{L}\{f(t)\} = \frac{s-1}{\underbrace{(s-2).(s-3)}}} \quad \mathrm{I}$$

F.P. ⇒ EXPRESSA EM TERMOS DE SOMA.

F.P.:

$$\frac{s-1}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3} = \frac{A(s-3)+B(s-2)}{(s-2).(s-3)};$$

$$s - 1 = (A+B)s + (-3A - 2B);$$

$$\begin{cases} A+B=1 \\ -3A-2B=-1 \end{cases} \Rightarrow \begin{cases} A+B=1 \\ 3A+2B=1 \end{cases} \quad (x)-3 \quad \begin{array}{r} -3A-3B=-3 \\ +\quad 3A+2B=1 \\ \hline -B=-2 \end{array}$$

$$\therefore \boxed{B=2} \Rightarrow A+B=1 \Rightarrow \boxed{A=-1}$$

RETORNANDO A E B EM $\boxed{I}$:

$$\boxed{\mathcal{L}\{f(t)\} = \frac{S-1}{(S-2).(S-3)} = \frac{-1}{S-2} + \frac{2}{S-3}} \quad \boxed{II}$$

COMO: $\mathcal{L}^{-1}\{F(s)\} = f(t)$:

$$\left.\begin{array}{l} \mathcal{L}^{-1}\left\{\frac{-1}{S-2}\right\} = -1.\mathcal{L}^{-1}\left\{\frac{1}{S-2}\right\} = -e^{2.t} \\ \mathcal{L}^{-1}\left\{\frac{2}{S-3}\right\} = 2\,\mathcal{L}^{-1}\left\{\frac{1}{S-3}\right\} = 2\,e^{3.t} \end{array}\right\} \text{EM } \boxed{II}$$

$$\boxed{f(t) = -e^{2t} + 2\,e^{3.t}}$$

$$\boxed{\begin{array}{l} \mathcal{L}\{e^{at}\} = \frac{1}{S-a} \\ \therefore \mathcal{L}^{-1}\left\{\frac{1}{S-a}\right\} = e^{at} \end{array}}$$