EX. 2 - RESOLVA A EQUAÇÃO

$$\left[e^{2y} - y.\cos(x.y)\right]dx + \left[2x\,e^{2y} - x.\cos(x.y) + 2y\right]dy = 0 \qquad (1)$$

$$\boxed{M\,dx + N\,dy = 0} \qquad (2)$$

EQ. É EXATA SE: $\boxed{\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}}$ (3)

$$\frac{\partial M}{\partial y} = 2e^{2y} - \left[1.\cos(x.y) + y(-1)\,\mathrm{sen}(x.y)\,x\right]; \qquad (4)$$

$$\frac{\partial M}{\partial y} = 2e^{2y} - \left[\cos(x.y) - xy\,\mathrm{sen}(x.y)\right]; \qquad (5)$$

$$\frac{\partial N}{\partial x} = 2e^{2y} - \left[1.\cos(x.y) + x(-1)\,\mathrm{sen}(x.y).y\right]; \qquad (6)$$

$$\frac{\partial N}{\partial x} = 2e^{2y} - \left[\cos(x.y) - xy\,\mathrm{sen}(x.y)\right]. \qquad (7)$$

COMPARANDO (5) E (7) $\Rightarrow$ EDO É EXATA.

ENTÃO: $\boxed{\dfrac{\partial f}{\partial x} = M}$ E $\boxed{\dfrac{\partial f}{\partial y} = N}$

$$\frac{\partial f}{\partial x} = e^{2y} - y.\cos(x.y); \qquad (8)$$

PROCURA $f(x,y) = C$

$$f(x,y) = \int \left[e^{2y} - y\cos(x.y)\right]dx + g(y); \qquad (9)$$

$$f(x,y) = \int e^{2y}\,dx - \int y.\cos(x.y)\,dx + g(y); \qquad (10)$$

$$f(x,y) = e^{2y}\int dx - y\int \cos(x.y)\,dx + g(y) \;; \quad (11)$$

$$f(x,y) = e^{2y}.\,x - y\int \cos(u)\,\frac{1}{y}\,du + g(y); \quad (12)$$

$u = x.y;$

$\frac{du}{dx} = y;$

$\frac{1}{y}\,du = dx;$

$\to$ EM (11)

$$f(x,y) = x.\,e^{2y} - \underbrace{y.\frac{1}{y}}_{=1}\,\underbrace{Sen(u)}_{\text{VOLTA NA VARIÁVEL } x.} + g(y); \quad (13)$$

$$\boxed{f(x,y) = x.\,e^{2y} - Sen(xy) + g(y)} \quad (14)$$

$\to$ QUER: $f(x,y) \Rightarrow$ FALTA: $g(y)$?

SABEMOS $\boxed{\frac{\partial f}{\partial y} = N}$ $\Rightarrow$ DERIVANDO (14) (EM y):

$$\underbrace{\frac{\partial f}{\partial y}}_{=N} = \frac{\partial}{\partial y}\left\{x.\,e^{2y} - Sen(x.y) + g(y)\right\}; \quad (15)$$

$$2x.\,e^{2y} - x.\cos(x.y) + 2y = \frac{\partial}{\partial y}\left[x.\,e^{2y}\right] - \frac{\partial}{\partial y}\left[Sen(x.y)\right] + \frac{dg}{dy}; \quad (16)$$

$$2x\,e^{2y} - x.\cos(x\,y) + 2y = 2x\,e^{2y} - x\cos(x.y) + \frac{dg(y)}{dy}; \quad (17)$$

SIMPLIFICANDO (17) RESULTA

$$2y = \frac{dg}{dy}; \quad (18) \Rightarrow \text{AGORA} \int$$

$$dg = 2y\,dy \Rightarrow \int dg = 2\int y\,dy \;; \quad (19)$$

$$\therefore \boxed{g(y) = y^2} \quad (20)$$

SUBSTITUINDO (20) EM (14) TEMOS A FAMÍLIA DE CURVAS:

$$\boxed{x \cdot e^{2y} - \operatorname{sen}(x \cdot y) + y^2 = C} \quad (21)$$

EX - 3 - RESOLVA O P.V.I. :

$$\begin{cases} \left[\cos(x).\text{sen}(x) - x.y^2\right] dx + y(1-x^2)\, dy = 0 \ ; & (1) \\ y(0) = 2 & (2) \end{cases}$$

DE (1) :

$$M = \cos(x).\text{sen}(x) - xy^2 \Rightarrow \frac{\partial M}{\partial y} = -2xy \ ; \quad (3)$$

$$N = y - x^2 y \Rightarrow \frac{\partial N}{\partial x} = -2xy \ ; \quad (4)$$

DE (3) E (4) $\Rightarrow$ EQ. EXATA ; ENTÃO :

$$\frac{\partial f}{\partial x} = M \Rightarrow \boxed{f(x,y) = \int M(x,y)\, dx + g(y)} \quad (5)$$

$$f(x,y) = \int \left[\cos(x).\text{sen}(x) - x.y^2\right] dx + g(y) \ ; \quad (6)$$

$$f(x,y) = \int \cos(x).\text{sen}(x)\, dx - y^2 \int x\, dx + g(y) \ ; \quad (7)$$

$$f(x,y) = \int u\, du - y^2 \frac{x^2}{2} + g(y) \ ; \quad (8)$$

$u = \text{sen}(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\, dx$

$\hookrightarrow$ EM (7)

$$f(x,y) = \frac{u^2}{2} - \frac{x^2.y^2}{2} + g(y) \ ; \quad (9)$$

RETORNA EM $x.y$:

$$\boxed{f(x,y) = \frac{1}{2}\left\{\text{sen}^2(x) - x^2.y^2\right\} + g(y)} \quad (10)$$

$\hookrightarrow$ QUER : $\underline{f(x,y)}$ $\Rightarrow$ FALTA : $\underline{g(y)}$?

SABEMOS: $\boxed{\dfrac{\partial f}{\partial y} = N} \Rightarrow$ DERIVANDO (10):

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left\{\frac{1}{2}\left[\operatorname{sen}^2(x) - x^2 \cdot y^2\right] + g(y)\right\}; \qquad (11)$$

$\downarrow = N$

$$y - x^2 y = \frac{1}{2}\left\{\underbrace{\cancel{\frac{\partial}{\partial y}\left[\operatorname{sen}^2(x)\right]}}_{0} - \frac{\partial}{\partial y}\left[x^2 \cdot y^2\right]\right\} + \frac{\partial g(y)}{\partial y}; \qquad (12)$$

$$y - x^2 y = \frac{1}{2}\left\{-2x^2 y\right\} + \frac{dg(y)}{dy}; \qquad (13)$$

$$y \underbrace{- x^2 \cdot y = - x^2 y}_{\text{CANCELA}} + \frac{dg}{dy}; \qquad (14)$$

$$\frac{dg}{dy} = y \Rightarrow \int dg = \int y\,dy \Rightarrow \boxed{g(y) = \frac{y^2}{2}} \qquad (15)$$

(15) EM (10): $f(x,y) = \frac{1}{2}\left\{\operatorname{sen}^2(x) - x^2 \cdot y^2\right\} + \frac{y^2}{2}$; (16)

SOLUÇÃO (FAMÍLIA CURVAS): $\boxed{\frac{1}{2}\left\{\operatorname{sen}^2(x) - x^2 \cdot y^2 + y^2\right\} = C}$ (17)

PVI: $y(0) = 2 \Rightarrow \begin{cases} x = 0 \\ y = 2 \end{cases} \Rightarrow$ OBTER C (EM (17))

$$\frac{1}{2}\left\{\underbrace{\cancel{[\operatorname{sen}(0)]^2}}_{0} - \underbrace{\cancel{0 \cdot (2)^2}}_{0} + (2)^2\right\} = C \Rightarrow \boxed{C = \frac{4}{2} = 2}$$

C EM (17) $\boxed{\frac{1}{2}\left\{\operatorname{sen}^2(x) - x^2 \cdot y^2 + y^2\right\} = 2}$ (18)