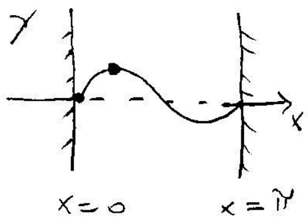
12ª LISTA   EQ. ONDA $\boxed{1}$

$\boxed{1}$ $\boxed{y_{tt} = 4 y_{xx}}$, $0 < x < \pi$, $t > 0$

$y(0,t) = y(\pi,t) = 0$,

$y(x,0) = \frac{1}{10} \operatorname{Sen}(2x)$ E $y_t(x,0) = 0$.

SOLUÇÃO :



DAS INFORMAÇÕES ACIMA :

- $a^2 = 4$
- $L = \pi$ (COMPRIMENTO DA CORDA)
- $\left\{ \begin{array}{l} y(0,t) = 0 \\ y(\pi,t) = 0 \end{array} \right\}$ PONTOS DAS EXTREMIDADES FIXOS
- $y_t(x,0) = 0 = V(t=0)$ ⇒ VELOCIDADE INICIAL NULA
- $y(x,0) = \frac{1}{10} \operatorname{Sen}(2x)$ ⇒ POSIÇÃO INICIAL DA CORDA $(t=0)$

$$y(x,t) = \sum_{n=1} C_n \cdot \operatorname{Sen}\left(\frac{n\pi x}{L}\right) \cdot \cos\left(\frac{n\pi a t}{L}\right) \quad ; \boxed{1}$$

$$C_n = \frac{2}{L} \int_0^L f(x) \cdot \operatorname{Sen}\left(\frac{n\pi x}{L}\right) dx \quad ; \boxed{2}$$

ESCREVENDO $\boxed{2}$ PARA ESSE PROBLEMA :

$$C_n = \frac{2}{\pi} \int_0^\pi \frac{1}{10} \operatorname{Sen}(2x) \cdot \operatorname{Sen}\left(\frac{n\pi x}{\pi}\right) dx \quad ; \boxed{3}$$

$$\frac{10 \cdot \pi}{2} \cdot C_n = \int_0^\pi \operatorname{Sen}(2x) \cdot \operatorname{Sen}(nx)\, dx \quad ; \boxed{4}$$

VIMOS QUE:

$$\int_{-\ell}^{\ell} \operatorname{sen}\left(\frac{m \cdot \pi \cdot x}{\ell}\right) \cdot \operatorname{sen}\left(\frac{n \cdot \pi \cdot x}{\ell}\right) dx = \begin{cases} 0, & m \neq n \\ \ell, & m = n \end{cases}$$

$\therefore$ DEVEMOS TER $n = 2$ EM $\boxed{4}$:

$$\frac{10}{2} \cdot \pi \cdot C_2 = \underbrace{\int_0^{\pi} \operatorname{sen}(2x) \cdot \operatorname{sen}(2x)\, dx}\ ; \quad \boxed{5}$$

$$\frac{10 \cdot \pi \cdot C_2}{2} = \frac{\pi}{2} \implies C_2 = \frac{\pi}{2} \cdot \frac{2}{\pi} \cdot \frac{1}{10}\ ; \quad \therefore C_2 = \frac{1}{10} \quad \boxed{6}$$

A SOLUÇÃO É: (EQ. $\boxed{1}$) $\begin{cases} n = 2 \\ a = 2 \end{cases}$

$$y(x,t) = C_2 \operatorname{sen}\left(\frac{2\pi x}{\pi}\right) \cdot \cos\left(\frac{2\pi 2 t}{\pi}\right);$$

$$\boxed{y(x,t) = \frac{1}{10} \operatorname{sen}(2x) \cos(4 t)} \quad \boxed{7}$$

$\ell = \pi$ E $n = 2$

$$\int_{-\pi}^{\pi} \operatorname{sen}(m x) \cdot \operatorname{sen}(n \cdot x)\, dx = 2 \int_0^{\pi} \operatorname{sen}^2(2x)\, dx$$

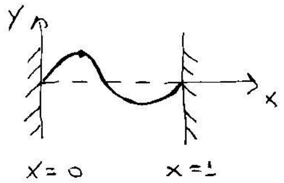
$$\frac{\pi}{2} = \int_0^{\pi} \operatorname{sen}^2(2x)\, dx$$

2 $y_{tt} = y_{xx}$; $0 < x < 1$, $t > 0$

$y(0,t) = y(1,t) = 0$

$y(x,0) = \frac{1}{10}\,\mathrm{sen}(\pi x) - \frac{1}{20}\,\mathrm{sen}(3\pi x)$; $y_t(x,0) = 0$

TEMOS:



- $a = 1$ E $L = 1$
- EXTREMIDADES FIXAS
- VELOCIDADE INICIAL NULA $(V = 0)$

SOLUÇÃO: $\left\{ \begin{array}{l} y(x,t) \\ C_n \end{array} \right.$

CÁLCULO C_n:

$$C_n = \frac{2}{L}\int_0^L f(x)\cdot \mathrm{sen}\left(\frac{n\pi x}{L}\right)dx \; ; \qquad (1)$$

$$C_n = \frac{2}{1}\int_0^1 \left[\frac{1}{10}\,\mathrm{sen}(\pi x) - \frac{1}{20}\,\mathrm{sen}(3\pi x)\right]\cdot \mathrm{sen}\left(\frac{n\pi x}{1}\right)dx \; ; \qquad (2)$$

$$\frac{C_n}{2} = \frac{1}{10}\int_0^1 \mathrm{sen}(\pi x)\cdot \mathrm{sen}(n\pi x)\,dx - \frac{1}{20}\int_0^1 \mathrm{sen}(3\pi x)\cdot \mathrm{sen}(n\pi x)\,dx \; ; \qquad (3)$$

$\therefore\; n = 1 \;\rightarrow\; C_1$ $\qquad$ $n = 3 \;\rightarrow\; C_3$

$$y(x,t) = C_1\,\mathrm{sen}\left(\frac{1\cdot\pi\cdot x}{1}\right)\cdot\cos\left(\frac{1\cdot\pi\cdot 1\cdot t}{1}\right) + C_3\,\mathrm{sen}\left(\frac{3\pi x}{1}\right)\cdot\cos\left(\frac{3\cdot\pi\cdot 1\cdot t}{1}\right) \qquad (4)$$

CÁLCULO DE C_1 $(n=1)$:

$$C_1 = \frac{2}{1}\int_0^1 \frac{1}{10} \operatorname{Sen}(1.\pi x)\operatorname{Sen}(1.\pi.x)\,dx \; ; \quad (5)$$

$$C_1 = \frac{2}{10}\int_0^1 \operatorname{Sen}(1.\pi.x).\operatorname{Sen}(1.\pi.x)\,dx$$

$$C_1 = \frac{2}{10}\cdot\frac{1}{2} \Rightarrow \boxed{C_1 = \frac{1}{10}} \quad (6)$$

CÁLCULO DE C_3 $(n=3)$:

$$C_3 = \frac{2}{1}\left(-\frac{1}{20}\right)\int_0^1 \operatorname{Sen}(3.\pi.x)\operatorname{Sen}(3.\pi.x)\,dx \; ; \quad (7)$$

$$C_3 = 2\left(-\frac{1}{20}\right)\cdot\frac{1}{2} \Rightarrow \boxed{C_3 = -\frac{1}{20}} \quad (8)$$

(6) E (8) EM (4):

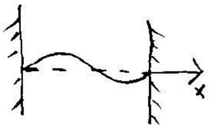
$$y(x,t) = \frac{1}{10}\operatorname{Sen}(\pi x).\cos(\pi.t) - \frac{1}{20}\operatorname{Sen}(3\pi x).\cos(3\pi t) \; ; \quad (9)$$

[3] $4\,y_{tt} = y_{xx}$, $0 < x < 2$, $t > 0$; [5]

$y(0,t) = y(2,t) = 0$;

$y(x,0) = \frac{1}{5}\,\mathrm{Sen}(\pi x)\cdot\cos(\pi x)$;

$y_t(x,0) = 0$



$x = 0$ $\quad$ $x = L = 2$

• EXTREMIDADES FIXAS
• VELOCIDADE INICIAL NULA.

$$C_n = \frac{2}{L}\int_0^L f(x)\,\mathrm{Sen}\left(\frac{n\pi x}{L}\right)dx \;;$$

$$C_n = \frac{2}{2}\int_0^2 \left[\frac{1}{5}\,\mathrm{Sen}(\pi x)\cdot\cos(\pi x)\right]\mathrm{Sen}\left(\frac{n\pi}{2}x\right)dx \;;$$

$$5\,C_n = \int_0^2 \left[\frac{1}{2}\cdot\mathrm{Sen}(2\pi x)\right]\cdot\mathrm{Sen}\left(\frac{n\pi}{2}x\right)dx \;; \Rightarrow \therefore n = 4.$$

$$5\cdot 2\cdot C_4 = \int_0^2 \mathrm{Sen}(2\pi x)\cdot\mathrm{Sen}(2\pi x)\,dx$$

$$10\cdot C_4 = \frac{2}{2} \Rightarrow \boxed{C_4 = \frac{1}{10}}\;!$$

SOLUÇÃO:

$$\boxed{y(x,t) = \sum_{n=1} C_n\,\mathrm{Sen}\left(\frac{n\pi x}{L}\right)\cdot\cos\left(\frac{n\pi a t}{L}\right)}\;;$$

$2\cdot\mathrm{Sen}(a)\cdot\cos(a) = \mathrm{Sen}(2\cdot a)$

ENTÃO:

$$y(x,t) = C_4 \operatorname{sen}\left(\frac{4\pi x}{2}\right) \cdot \cos\left(\frac{4\pi}{2} \cdot \frac{1}{2} \cdot t\right);$$

$$\boxed{y(x,t) = \frac{1}{10} \operatorname{sen}(2\pi x) \cdot \cos(\pi \cdot t)}$$

EQ. DADA: $4 y_{tt} = y_{xx}$

$$y_{tt} = \frac{1}{4} y_{xx}$$

$$y_{tt} = a^2 y_{xx}$$

$$\therefore \; a = \frac{1}{2}$$