Cálculo I
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Lista de exercícios – Integral (parte 1)

Questão 1. Calcule as seguintes integrais:

a) $\displaystyle\int (x+4)(2x+1)\,dx$

b) $\displaystyle\int \frac{x^3 - 2\sqrt{x}}{x}\,dx$

c) $\displaystyle\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right)\,dx$

d) $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

e) $\displaystyle\int_{-1}^{2} (x - 2|x|)\,dx$

Questão 2. Calcule as seguintes integrais pela regra da substituição:

a) $\displaystyle\int x\,\mathrm{sen}(x^2)\,dx$

b) $\displaystyle\int x^2 e^{x^3}\,dx$

c) $\displaystyle\int (3x-2)^{20}\,dx$

d) $\displaystyle\int (x+1)\sqrt{2x+x^2}\,dx$

e) $\displaystyle\int e^x\sqrt{1+e^x}\,dx$

f) $\displaystyle\int \frac{\mathrm{sen}(\ln x)}{x}\,dx$

g) $\displaystyle\int e^{\mathrm{tg}\,x}\sec^2 x\,dx$

h) $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

Questão 3. Calcule as seguintes integrais pela regra da integração por partes (lembre-se do LIATE):

a) $\displaystyle\int \mathrm{sen}^2 x\,dx$

b) $\displaystyle\int xe^{-2x}\,dx$

c) $\displaystyle\int \mathrm{arcsen}\,x\,dx^*$

d) $\displaystyle\int (\ln x)^2\,dx$

e) $\displaystyle\int e^{2\theta}\,\mathrm{sen}\,3\theta\,d\theta$

f) $\displaystyle\int \frac{\ln y}{\sqrt{y}}\,dy$

g) $\displaystyle\int x\,\mathrm{tg}^2 x\,dx$

h) $\displaystyle\int (x^2+1)e^{-x}\,dx$

Questão 4. Calcule as seguintes integrais, deduzindo quais regras usar:

a) $\displaystyle\int e^x\,\mathrm{sen}(e^x)\,dx$

b) $\displaystyle\int (3x+1)^{\sqrt{2}}\,dx$

c) $\displaystyle\int \frac{\mathrm{sen}\,x + \sec x}{\mathrm{tg}\,x}\,dx$

d) $\displaystyle\int e^{x+e^x}\,dx$

e) $\displaystyle\int x\ln(1+x)\,dx$

f) $\displaystyle\int t^3 e^{-t^2}\,dt$

*Dica: calcule $\displaystyle\int 1\cdot\mathrm{arcsen}\,x\,dx$.

Gabarito

1a) $\dfrac{2x^3}{3} + \dfrac{9x^2}{2} + 4x + C$

1b) $\dfrac{x^3}{3} - 4\sqrt{x} + C$

1c) $\dfrac{x^2}{4} - 2\ln x\Big|_1^2 = \dfrac{3}{4} - 2\ln 2$

1d) $\theta + \operatorname{tg}\theta\Big|_0^{\pi/4} = \dfrac{\pi}{4} + 1$

1e) $\int_{-1}^{0}(x+2x)dx + \int_{0}^{2}(x-2x)dx = -\dfrac{7}{2}$

2a) $-\dfrac{1}{2}\cos(x^2) + C$

2b) $\dfrac{e^{x^3}}{3} + C$

2c) $\dfrac{(3x-2)^{21}}{63} + C$

2d) $\dfrac{(x^2+2x)^{3/2}}{3} + C$

2e) $\dfrac{2}{3}(e^x+1)^{3/2} + C$

2f) $-\cos(\ln x) + C$

2g) $e^{\operatorname{tg} x} + C$

2h) $\operatorname{arctg} x + \dfrac{1}{2}\ln|1+x^2| + C$

3a) $\dfrac{x - \operatorname{sen} x\cos x}{2} + C$

3b) $-\dfrac{xe^{-2x}}{2} - \dfrac{e^{-2x}}{4} + C$

3c) $\sqrt{1-x^2} + x\operatorname{arcsen} x + C$

3d) $x(\ln x)^2 - 2x\ln x + 2x + C$

3e) $\dfrac{e^{2\theta}(2\operatorname{sen} 3\theta - 3\cos 3\theta)}{13} + C$

3f) $2\sqrt{y}\ln y - 4\sqrt{y} + C$

3g) $x\operatorname{tg} x - \dfrac{x^2}{2} + \ln|\cos x| + C$

3h) $e^{-x}(-x^2 - 2x - 3) + C$

4a) $-\cos(e^x) + C$

4b) $\dfrac{(3x+1)^{\sqrt{2}+1}}{3(\sqrt{2}+1)} + C$

4c) $\operatorname{sen} x + \ln\left(\operatorname{tg}\left(\dfrac{x}{2}\right)\right) + C$

4d) $e^{e^x} + C$

4e) $\left(\dfrac{(x+1)^2}{2} - x - 1\right)\ln(x+1) + (x+1) - \dfrac{(x+1)^2}{4} + C$

4f) $-\dfrac{e^{-t^2}}{2}(t^2+1) + C$