APLICAR O MÉTODO DE SEPARAÇÃO DE VARIÁVEIS PARA A EQUAÇÃO DO CALOR:

$$\boxed{\alpha^2 u_{xx} = u_t} \Rightarrow \boxed{\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}} \quad (1)$$

$\alpha^2$ → DIFUSIVIDADE TÉRMICA (CONSTANTE)

$$\boxed{u(x,t) = X(x) \cdot T(t)} \quad (2)$$

$$\alpha^2 \frac{\partial^2 (X \cdot T)}{\partial x^2} = \frac{\partial (X \cdot T)}{\partial t} \, ; \quad (3)$$

$$\alpha^2 T \frac{\partial^2 X}{\partial x^2} = X \frac{\partial T}{\partial t} \, ; \quad (4)$$

$$\frac{\alpha^2 T}{X \cdot T} \frac{d^2 X}{dx^2} = \frac{X}{X \cdot T} \frac{dT}{dt} \, ; \quad (5)$$

$$\frac{\alpha^2}{X} \frac{d^2 X}{dx^2} = \frac{1}{T} \frac{dT}{dt} \, ; \quad (6)$$

$$\frac{1}{X} \frac{d^2 X}{dx^2} = \frac{1}{\alpha^2 T} \frac{dT}{dt} \, ; \quad (7)$$

$-\lambda^2$ → CONSTANTE DE SEPARAÇÃO

$$\frac{1}{X} \frac{d^2 X}{dx^2} = -\lambda^2 \Rightarrow \boxed{\frac{d^2 X}{dx^2} + \lambda^2 \cdot X = 0} \quad (8)$$

$$\frac{1}{\alpha^2 T} \frac{dT}{dt} = -\lambda^2 \Rightarrow \boxed{\frac{dT}{dt} + \lambda^2 \alpha^2 \cdot T = 0} \quad (9)$$

SOLUÇÃO DA EQ. 8:

$$\boxed{X(x) = e^{m.x}} \Rightarrow (m^2+\lambda^2).e^{m.x} = 0 \Rightarrow m^2 = -\lambda^2 ;$$

$$\boxed{m_1 = i\lambda} \Rightarrow \quad X(x) = A\,e^{i\lambda x} + B\,e^{-i\lambda x} ; \qquad (10)$$

$$\boxed{m_2 = -i\lambda} \Rightarrow$$

$$\boxed{X(x) = C_1 \cos(\lambda.x) + C_2 \sin(\lambda.x)} \qquad (11)$$

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SOLUÇÃO DA EQ. 9

$$\frac{dT}{dt} + \lambda^2\alpha^2 T = 0 \Rightarrow \frac{dT}{dt} = -\lambda^2\alpha^2 T ; \qquad (12)$$

$$\frac{1}{T}\,dT = -\lambda^2\alpha^2\,dt \Rightarrow \int \frac{1}{T}\,dT = -\lambda^2\alpha^2 \int dt ;$$

$$\ln|T| = -\lambda^2\alpha^2 t + c \Rightarrow T = e^{c-\lambda^2\alpha^2.t} ;$$

$$T = \underbrace{e^{c}}_{D \to \text{cte}}.\,e^{-\lambda^2.\alpha^2.t} \Rightarrow \boxed{T(t) = D.e^{-\lambda^2\alpha^2.t}} \qquad (13)$$

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$$\boxed{u(x.t) = X(x).T(t)}$$