Lista 2 – Álgebra Linear

Exercício 16. Calcular a inversa da seguinte matriz

$$A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & -\frac{4}{5} & \frac{1}{10} \end{bmatrix}$$

Solução.

1º. Vamos calcular o determinante da matriz A

$$\det(A) = \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \begin{vmatrix} 1 & 1 & -2 \\ 1 & 1 & \frac{1}{2} \\ 1 & -4 & \frac{1}{2} \end{vmatrix}$$

após aplicar as seguintes operações elementares $L_2 \to L_2 - L_1$ e $L_3 \to L_3 - L_1$ nas linhas da matriz, temos que:

$$\det(A) = \frac{1}{125} \begin{vmatrix} 1 & 1 & -2 \\ 0 & 0 & \frac{5}{2} \\ 0 & -5 & \frac{5}{2} \end{vmatrix}$$ $L_2 \leftrightarrow L_3$ permutando as linhas 2 e 3 segue

$$\det(A) = -\frac{1}{125} \begin{vmatrix} 1 & 1 & -2 \\ 0 & -5 & \frac{5}{2} \\ 0 & 0 & \frac{5}{2} \end{vmatrix}$$ matriz triangular superior. Logo:

$$\det(A) = -\frac{1}{125}\,(1)(-5)\left(\frac{5}{2}\right) = \frac{1}{10}$$

Portanto

$$\det(A) = \frac{1}{10}$$

2º. Vamos calcular a inversa da matriz

$$A = \begin{bmatrix} 1/5 & 1/5 & -2/5 \\ 1/5 & 1/5 & 1/10 \\ 1/5 & -4/5 & 1/10 \end{bmatrix}$$

$$\left[\begin{array}{cccccc} 1/5 & 1/5 & -2/5 & 1 & 0 & 0 \\ 1/5 & 1/5 & 1/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} L_1 \to 5\,L_1 \\ L_2 \to 5\,L_2 \\ L_3 \to 5\,L_3 \end{array} \left[\begin{array}{cccccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 1 & 1 & 1/2 & 0 & 5 & 0 \\ 1 & -4 & 1/2 & 0 & 0 & 5 \end{array}\right] \begin{array}{l} L_2 \to L_2 - L_1 \\ L_3 \to L_3 - L_1 \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \end{array}\right] L_2 \leftrightarrow L_3 \left[\begin{array}{cccccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -5 & 5/2 & -5 & 0 & 5 \\ 0 & 0 & 5/2 & -5 & 5 & 0 \end{array}\right] \begin{array}{l} L_2 \to -\frac{1}{5}L_2 \\ L_3 \to \frac{2}{5}L_3 \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right] L_1 \to L_1 - L_2 \left[\begin{array}{cccccc} 1 & 0 & -3/2 & 4 & 0 & 1 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} L_1 \to L_1 + 3/2L_3 \\ L_2 \to L_2 + 1/2L_1 \end{array} \left[\begin{array}{cccccc} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right] = [\,I \quad A^{-1}]$$

Logo $$A^{-1} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$$

Lista 3 – Álgebra Linear

Exercício 14. Calcular o determinante da seguinte matriz reduzindo a matriz à forma escalonada por linhas.

$$A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{bmatrix}$$

Solução

$$det(A) = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 5 & -9 & 6 & 3 \\ -1 & 2 & -6 & -2 \\ 2 & 8 & 6 & 1 \end{vmatrix} \quad \begin{matrix} L_2 \to L_2 - 5\,L_1 \\ L_3 \to L_3 + L_1 \\ L_4 \to \ L_4 \ - 2L_1 \\ \ \end{matrix} \quad \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 12 & 0 & -1 \end{vmatrix} \quad \begin{matrix} L_1 \to L_1 + 2\,L_2 \\ L_4 \to \ L_4 \ - 12L_2 \\ \ \\ \ \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & -15 & -3 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 108 & 23 \end{vmatrix} \quad L_4 \to \ L_4 + 36\,L_3 \quad \begin{vmatrix} 1 & 0 & -15 & -3 \\ 0 & 1 & -9 & -2 \\ 0 & 0 & -3 & -1 \\ 0 & 0 & 0 & -13 \end{vmatrix}$$

Matriz Triangular Superior

Logo

det(A) = (1)(1)(-3)(-13) = 39