Discrete Math II

Matteo Costagliola

Homework 2

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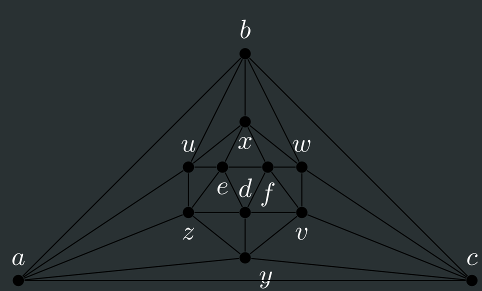
Problem 1. Find planar representations for each of the planar graphs in Figure 1.78.

Figure 1: Planar representations of graphs in figure 1.78

Problem 2. Draw a planar graph in which every vertex has degree exactly 5.



Problem 3. Fary and Wagner proved independently that every planar graph has a planar representation in which every edge is a straight line segment. Find such a representation for the graph in Figure 1.80.

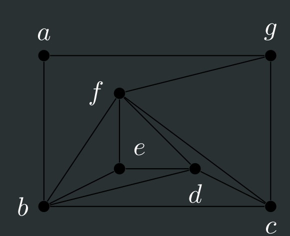


Figure 2: Planar Representation with line segment edges

Problem 4. If planar graphs G_1 and G_2 each have n vertices, q edges, and r regions, must the graphs be isomorphic?
Counterexample

Figure 3: Two graphs with 5 vertices, 4 edges, and 1 region that are no isomorphic

note:

Suppose there exists a planar graph such that each vertex has exactly degree 5. Then $\sum_{v\in V}\deg(v) = 5|V|$. By the handshaking lemma — $\sum_{v\in V}\deg(v) = 2|E| = 5|V|$, thus $|E| = \frac{5|V|}{2}$. Using Euler's formula, $V - E + R = 2$, where E must be an integer, this means that $|V|$ must be even for $|E|$ to be an integer. By theorem 1.33, a planar graph with at least $|V| \geq 3$ vertices and $|E|$ edges must satisfy $|E| \leq 3|V| - 6$. Substituting $|E| = \frac{5|V|}{2}$ into the inequality we see that $\frac{5|V|}{2} \leq 3|V| - 6$. Multiplying by 2 yields $5|V| \leq 6|V| - 12$. Then rearranging the inequality gives us $|V| \geq 12$. Revisiting Euler's formula and using the formula for the number of edges we found earlier, $|E| = \frac{5|V|}{2}$, using $|V| = 12$ we see that $E = \frac{5(12)}{2} = 30$. Solving for the number of regions, R, gives $R = 2 - V + E = 2 - 12 + 30 = 20$. However a graph cannot have a negative number of regions.