MS 113 In-class Problems

August 19, 2025

Table of Contents

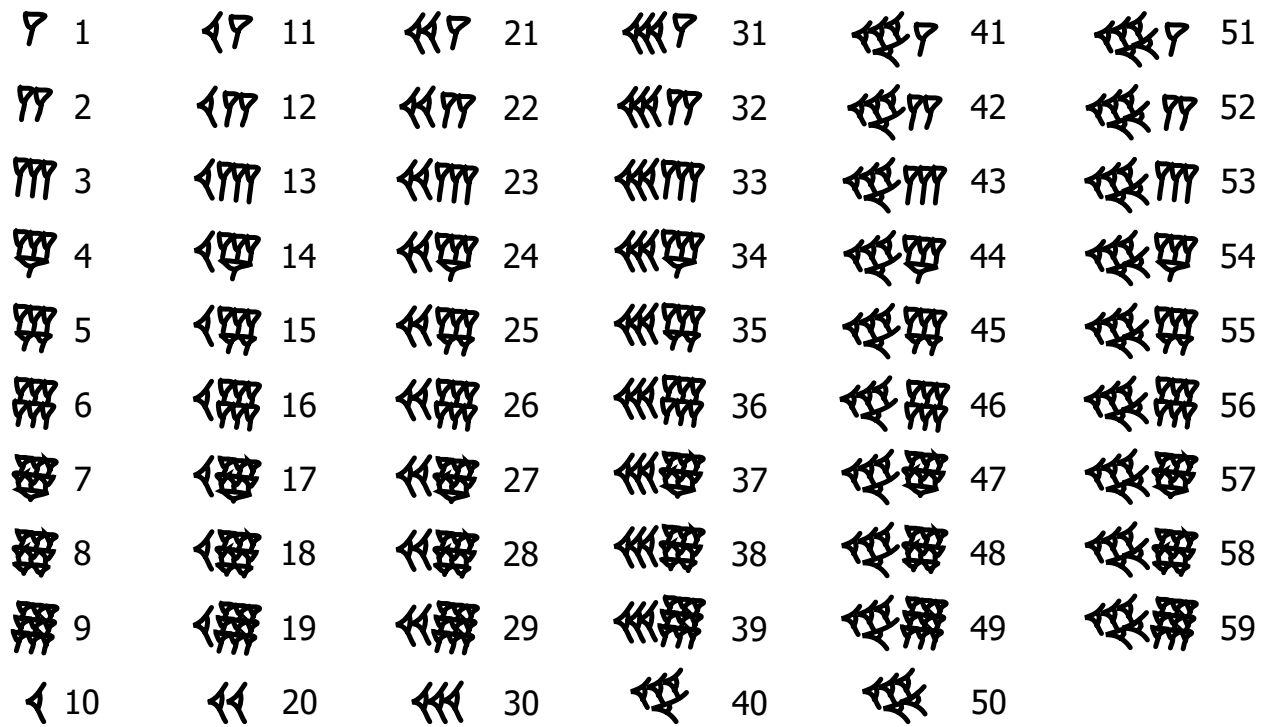
Chapter 5 Section 1

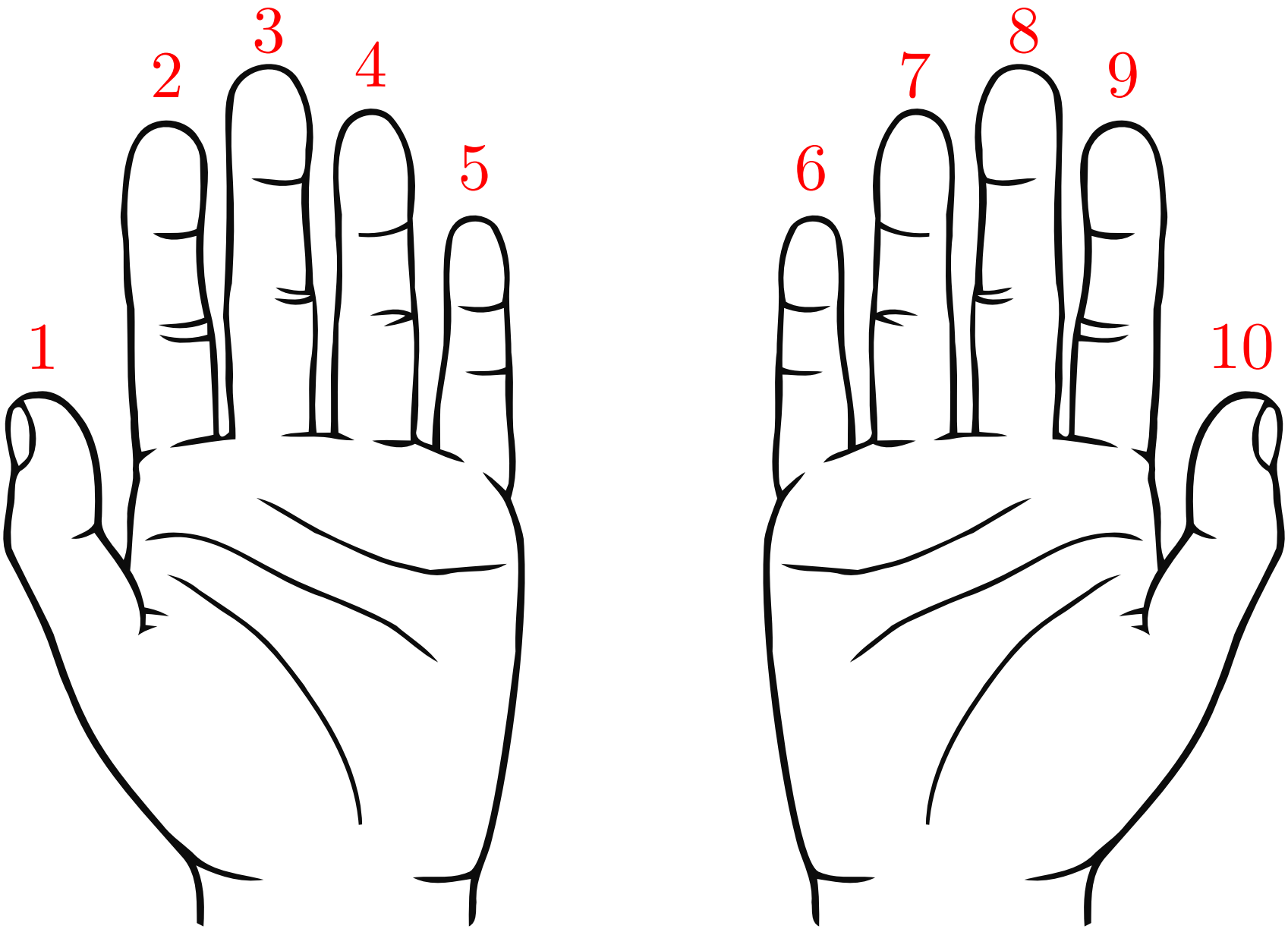
360 is divisible by many numbers:

$$1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24,$$

$$30, 36, 40, 45, 60, 72, 90, 120, 180, 360$$

Babylonians used a sexagismal (base 60) number system and did some of the earliest recorded astronomy and trigonometry.
They wrote using a logo-syllabic system called cuneiform:

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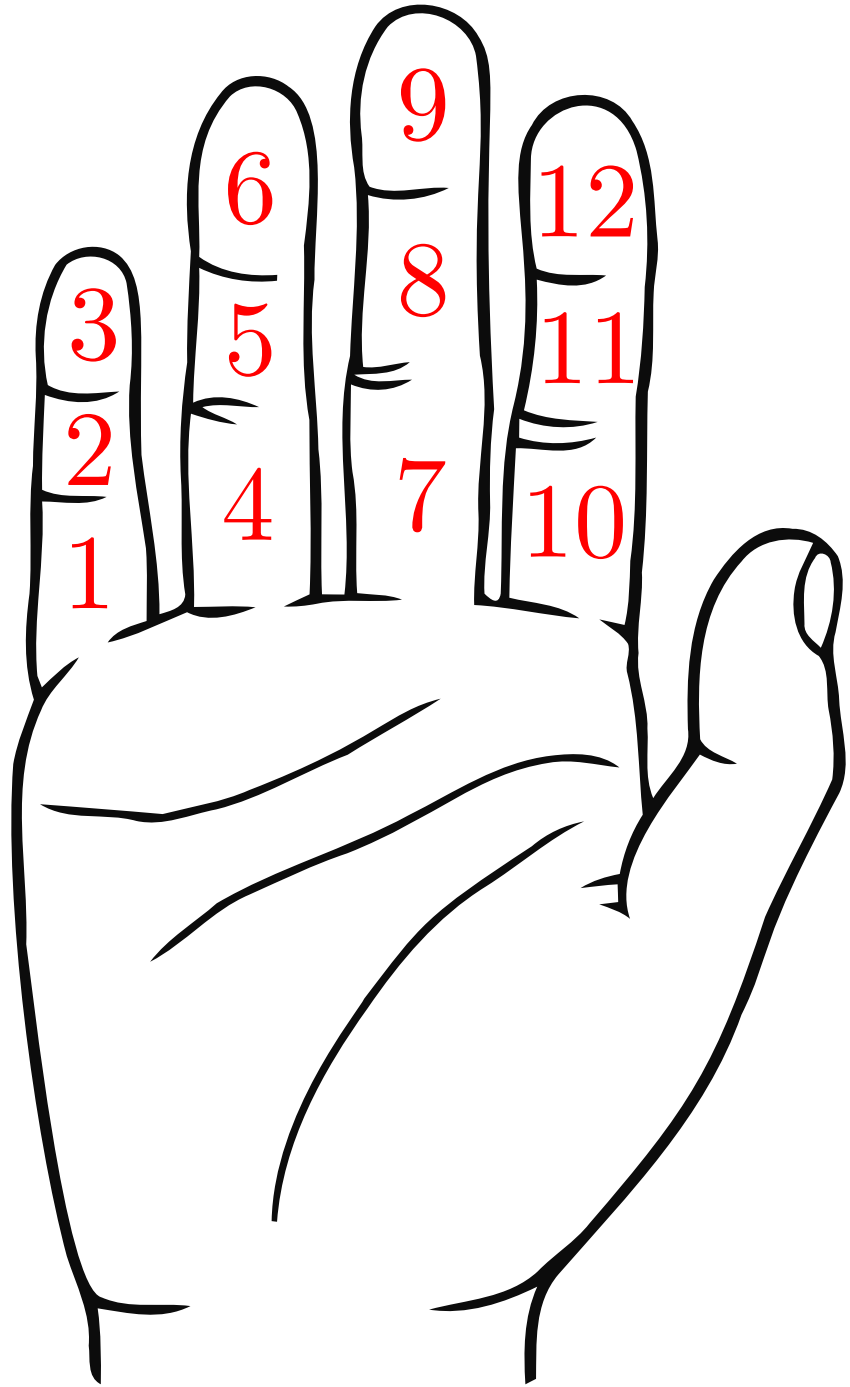
9
6
12
8
3
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11
2
4
7
10
1

DMS

Minute The degree can be further subdivided into minutes:

$$1' = \frac{1}{60}^\circ$$

Second The minute can be further subdivided into seconds:

$$1'' = \frac{1}{60}' = \frac{1}{3600}^\circ$$

So

$$\begin{aligned} 1^\circ &\equiv 60' \\ 1' &\equiv 60'' \\ 1^\circ &\equiv 3600'' \end{aligned}$$

Minute $<$ **Middle French** *minute* $<$ **post-classical Latin** *minuta* or *minutum*.

Second $<$ **French** *seconde* $<$ **medieval Latin** *secunda*.

"St Augustine refers to *minuta* and to *minutae minutarum* 'minutes of minutes', i.e. seconds ... as terms in use by *mathematici*."

"... used elliptically for *secunda minuta*, lit. 'second minute', i.e. the result of the second operation of sexagesimal division; the result of the first such operation (now called 'minute' simply) being the 'first' or 'prime minute' or 'prime"'

Oxford English Dictionary, s.v. "minute (n.1)," March 2024, https://doi.org/10.1093/OED/1094508711.
Oxford English Dictionary, s.v. "second (n.1)," December 2023, https://doi.org/10.1093/OED/7821975804.

Convert 6 ft to inches.

Convert 3 min to seconds.

Convert $3\dfrac{\text{ft}}{\text{min}}$ to $\dfrac{\text{in}}{\text{s}}$.

Convert 2° to minutes.

Convert $3'$ to seconds.

Etymology of angle adjectives

right Seems to be influenced from classical Latin *rect* meaning to be 90°.

acute From Latin *acūtus* meaning sharp.

obtuse From classical Latin *obtūsus* meaning blunt, dull, stupid. So most likely dull in comparison with acute's meaning of sharp.

reflex Not sure, but one meaning of classical Latin *reflexus* is curved back. So maybe this?

Gradian

$$1 \text{ grad} \equiv \frac{1}{400} \text{ rev}$$

So

$$400 \text{ grad} \equiv 1 \text{ rev} \equiv 360^\circ$$

Therefore a right angle:

$$90^\circ \equiv 100 \text{ grad}$$

It's the original proposed unit of measure for angles in the metric system (which became SI), but was replaced with radians. (The metric system was proposed during the French revolution.)

Mostly used in surveying and lasers in USA.

Find the gradian measure of the angle with the given degree measure.

$$135^\circ$$

Find the gradian measure of the angle with the given degree measure.

$$180^\circ$$

Find the degree measure of the angle with the given gradian measure.

300 grad

Find the degree measure of the angle with the given gradian measure.

50 grad

Dimensional Analysis - a little bit

Suppose you have a rectangle with length $l = 2$ in and width $w = 5$ in.

Can the following be the formula for the area of the rectangle?

$$A = w$$

Why not?

What about

$$A = lwh?$$

Why or why not?

Find the radian measure of the angle with the given degree measure.

$$45^\circ$$

Find the radian measure of the angle with the given degree measure.

$$-30^\circ$$

Find the radian measure of the angle with the given degree measure.

$$60^\circ$$

Find the radian measure of the angle with the given degree measure.

$$90^\circ$$

Find the radian measure of the angle with the given degree measure.

$$180^\circ$$

Find the radian measure of the angle with the given degree measure.

$$270^\circ$$

Find the radian measure of the angle with the given degree measure.

$$360^\circ$$

Find the degree measure of the angle with the given radian measure.

$$\frac{7\pi}{4}$$

Find the degree measure of the angle with the given radian measure.

$$\frac{5\pi}{6}$$

Find the degree measure of the angle with the given radian measure.

$$-\frac{\pi}{2}$$

Find the degree measure of the angle with the given radian measure. (Round your answer to one decimal place.)

$$6$$

Find the degree measure of the angle with the given radian measure. (Round your answer to one decimal place.)

$$1$$

Find the degree measure of the angle with the given radian measure. (Round your answer to one decimal place.)

$$-2.3$$

Danger

When writing an angle if you do not write the degree symbol it is interpreted as a radian!

So 2 is 2 rad and 2° is 2 deg.

Starting at $\frac{\pi}{2}$ count to 2π beyond it by $\frac{\pi}{2}$ths.

Starting at $\frac{\pi}{3}$ count to 2π beyond it by $\frac{\pi}{2}$ths.

Starting at $\frac{\pi}{6}$ count to 2π beyond it by $\frac{\pi}{6}$ths.

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle.

$$45^\circ$$

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle.

$$-30^\circ$$

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle.

$$400^\circ$$

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle.

$$\frac{3\pi}{4}$$

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle.

$$5$$

The measures of two angles in standard position are given. Determine whether the angles are coterminal.

$$70^\circ, 430^\circ$$

The measures of two angles in standard position are given. Determine whether the angles are coterminal.

$$\frac{5\pi}{6}, \frac{19\pi}{6}$$

Find an angle between 0° and 360° that is coterminal with the given angle.

$$-1190^\circ$$

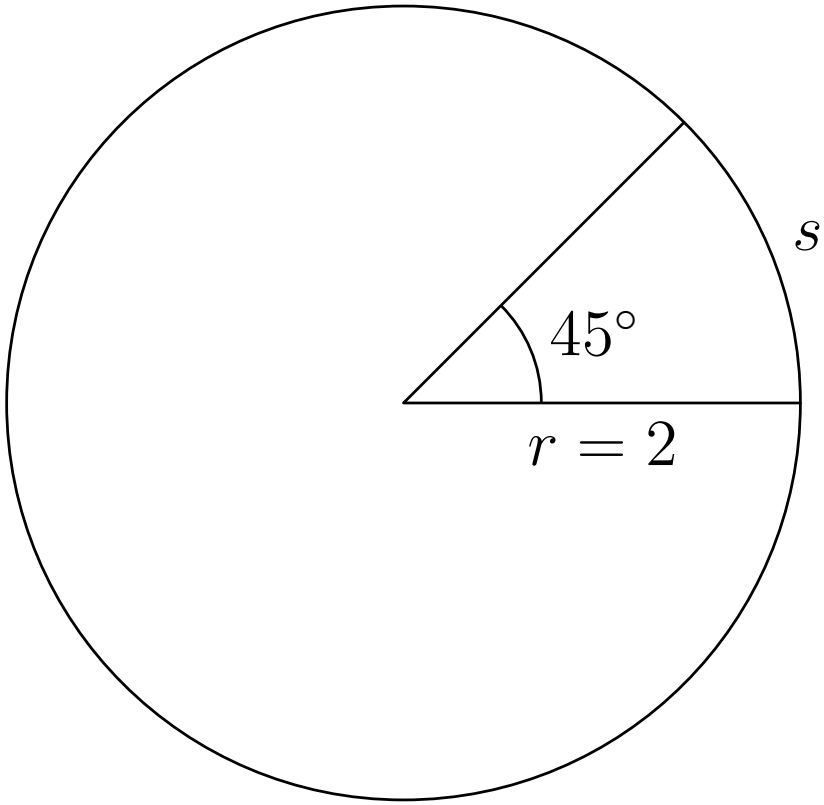
Find an angle between 0 and 2π that is coterminal with the given angle.

$$\frac{21\pi}{4}$$

Find an angle between 0 and 2π that is coterminal with the given angle.

$$7$$

Find the length s of the circular arc.



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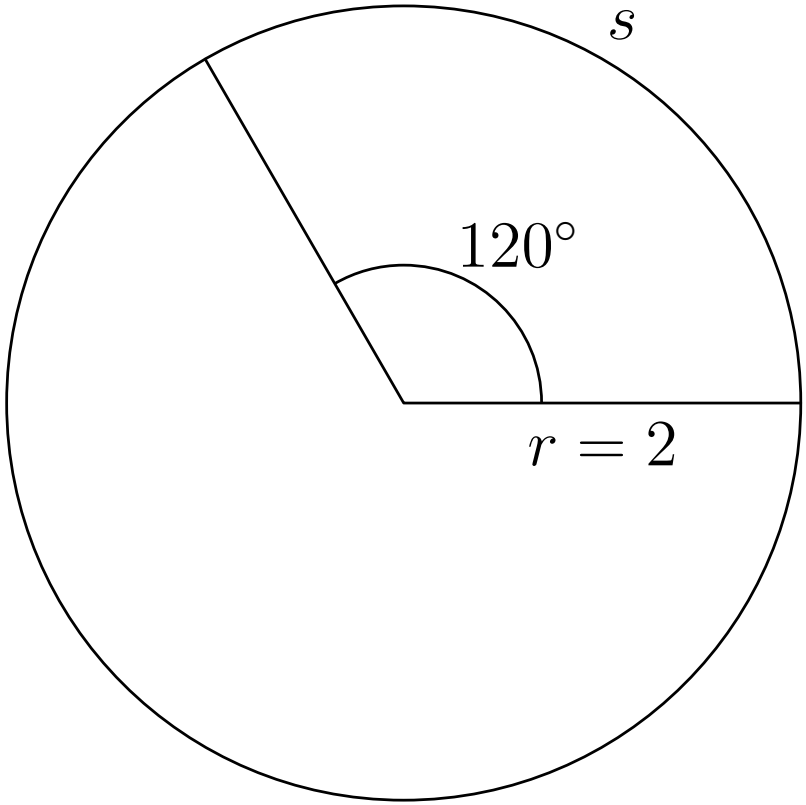

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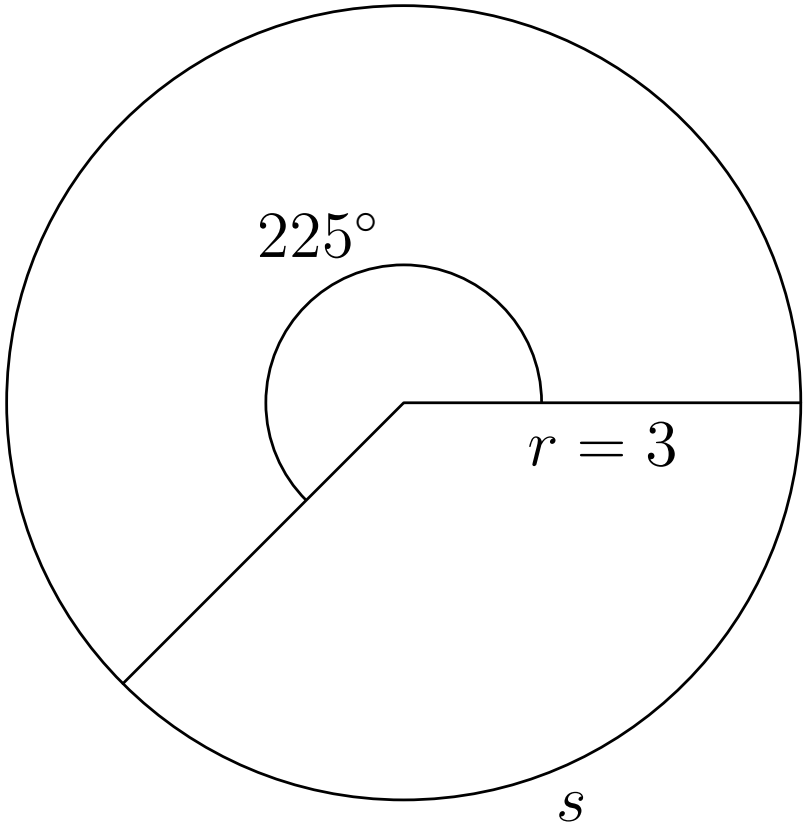

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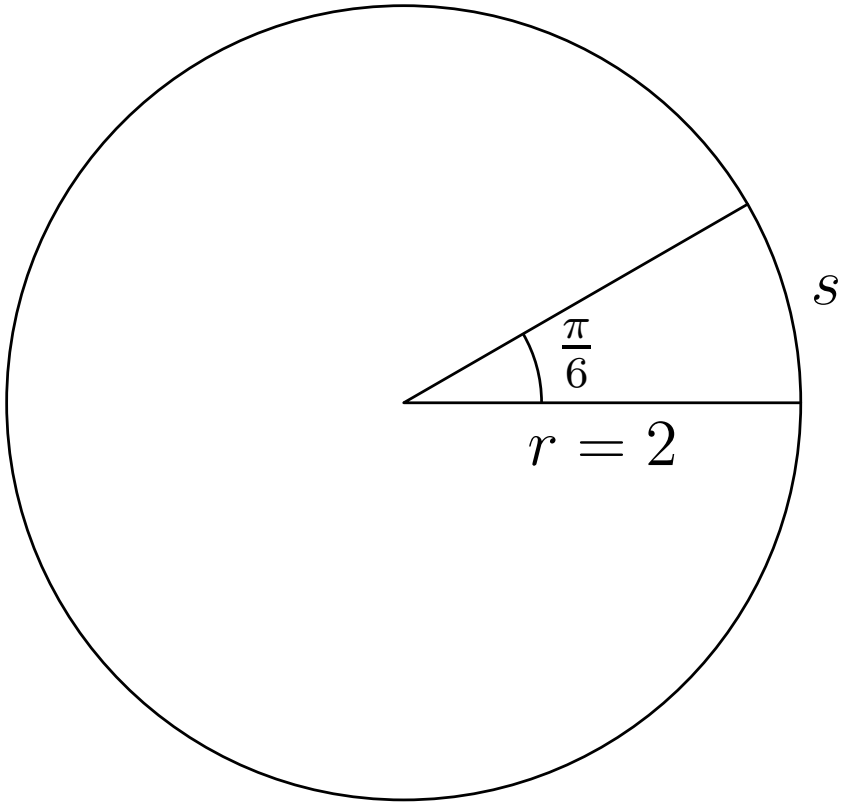
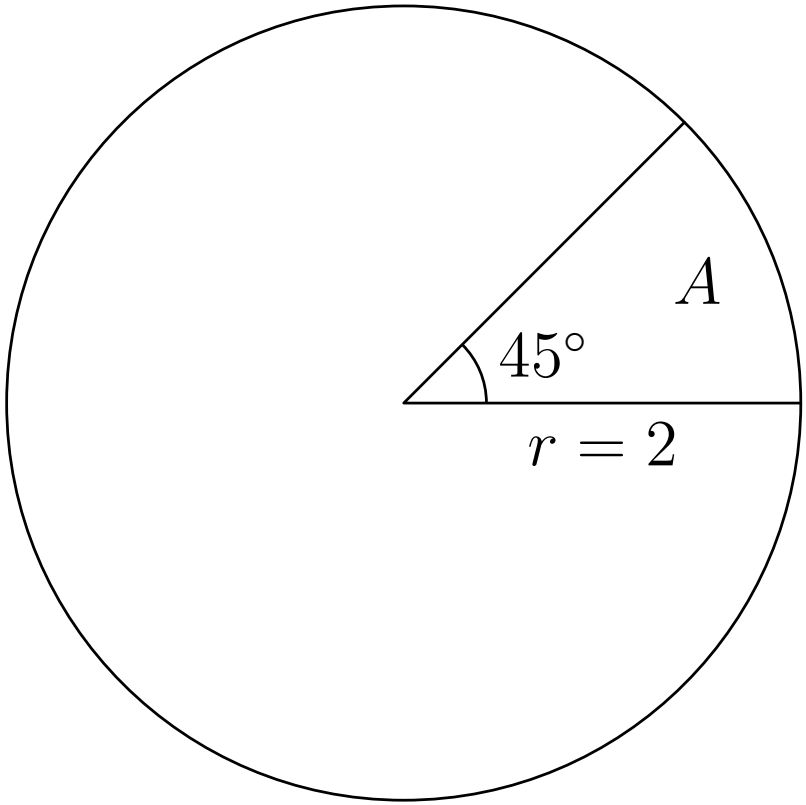


Find the area A of the sector.



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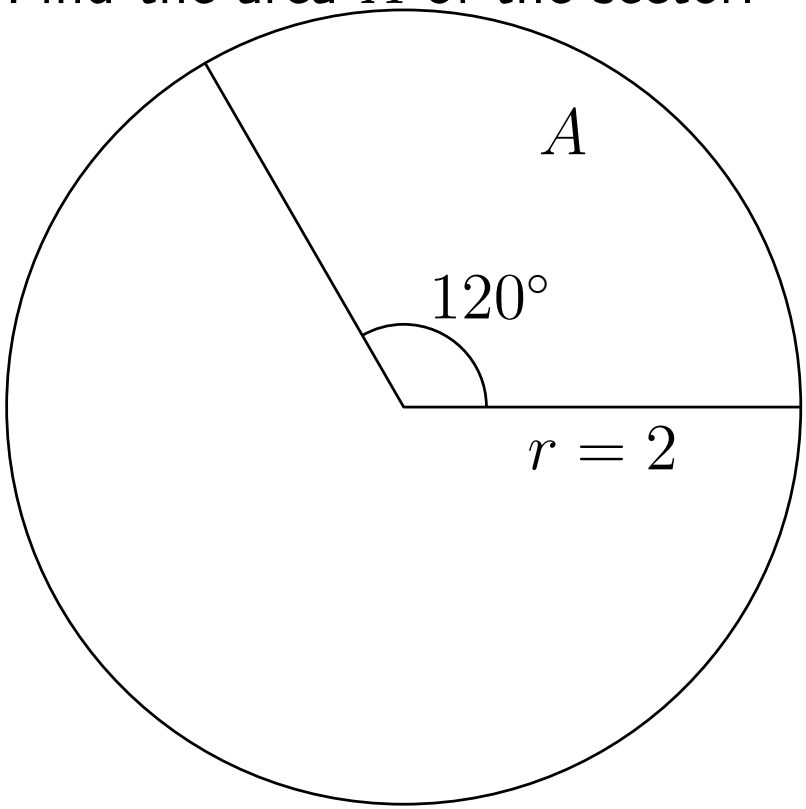

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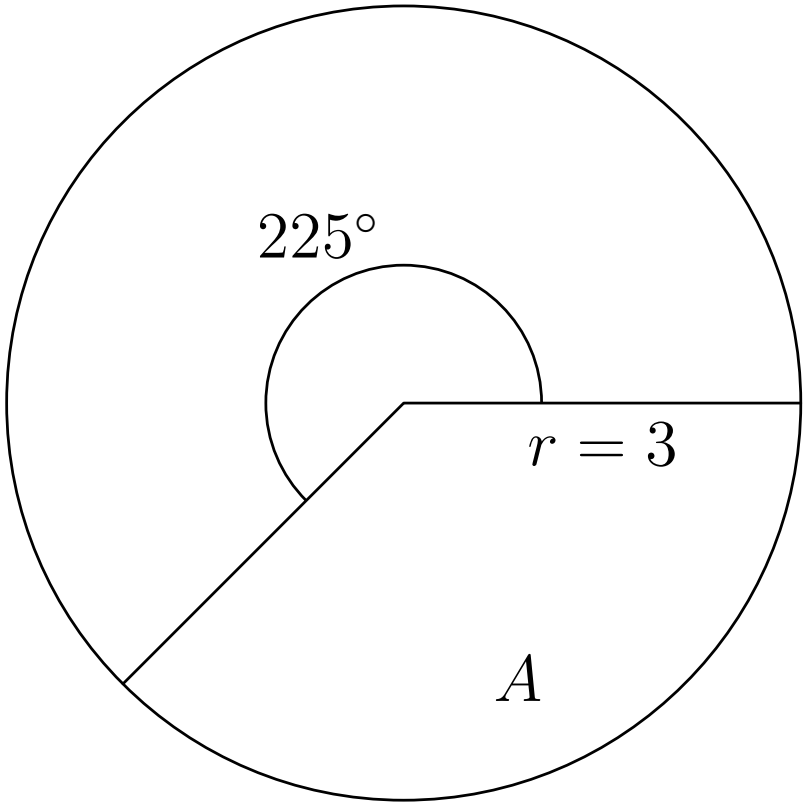
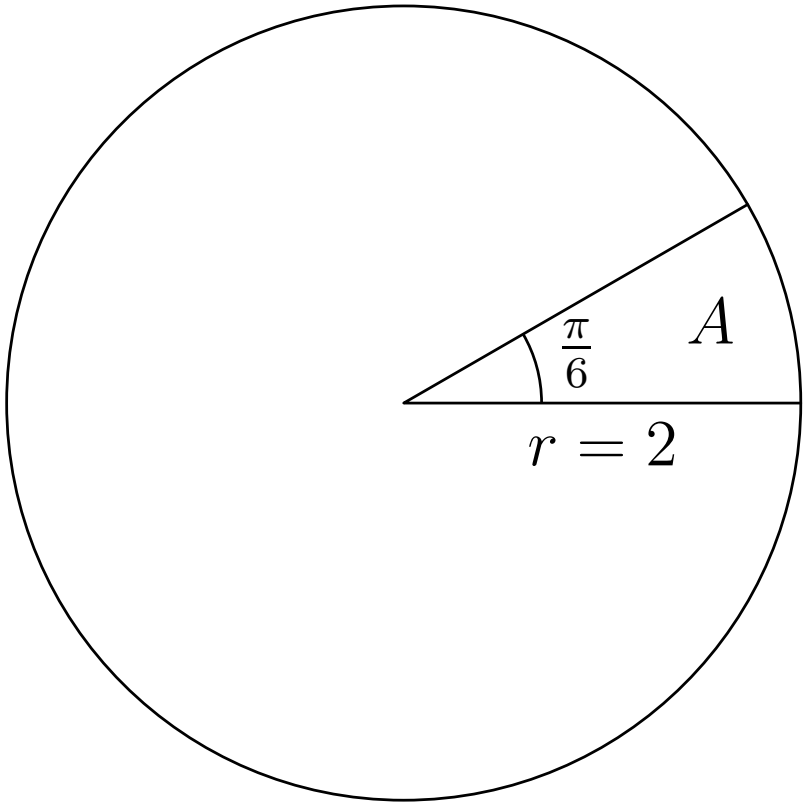


Find the area A of the sector.

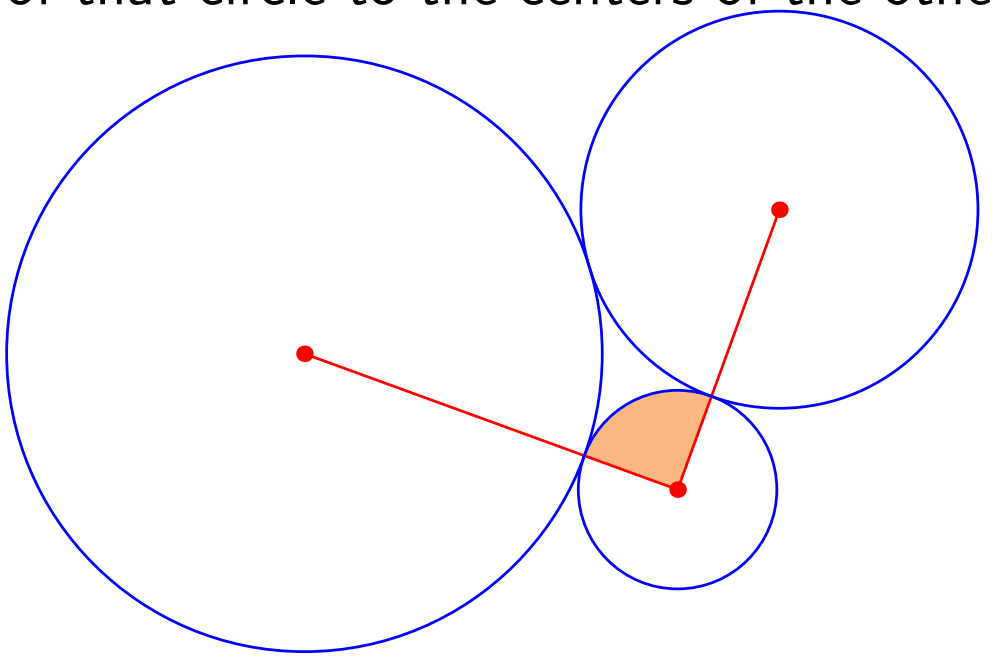


- If the central angle is $\theta = \dfrac{\pi}{2}$ and the arc length is $s = 3$, find the radius r.

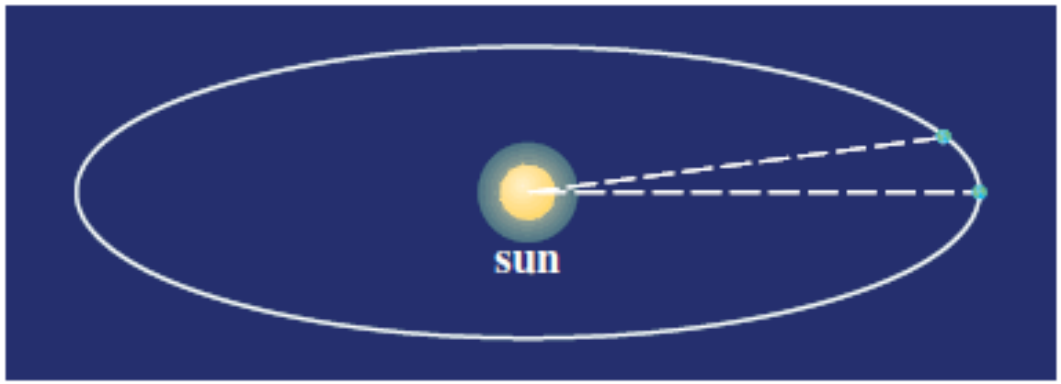
- If the radius is $r = 5$ and the arc length is $s = 10$, find the central angle θ.

- If the central angle is $\theta = \pi$ and the sector area is $A = 6\pi$, find the radius r.

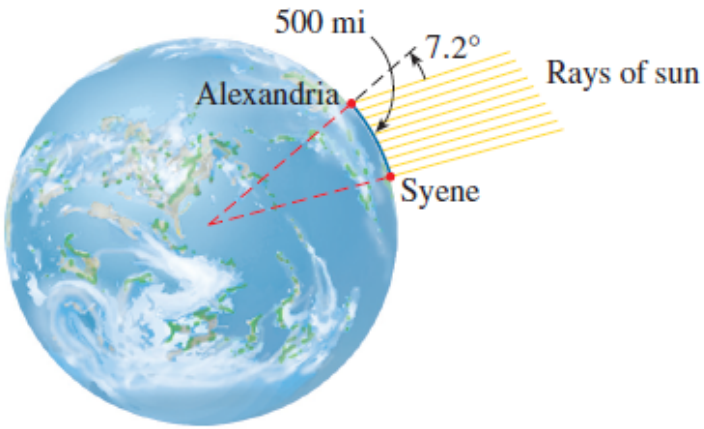
5.1.071 Three circles with radii 1, 2, and 3 ft are externally tangent to one another, as shown in the figure. Find the area of the sector of the circle of radius 1 that is cut off by the line segments joining the center of that circle to the centers of the other two circles.



5.1.079 Find the distance that the earth travels in two days in its path around the sun. Assume that a year has 365 days and that the path of the earth around the sun is a circle of radius 93 million miles.



5.1.080 The Greek mathematician Eratosthenes (ca. 276–195 B.C.) measured the circumference of the earth from the following observations. He noticed that on a certain day the sun shone directly down a deep well in Syene (modern Aswan). At the same time in Alexandria, 500 mi north (on the same meridian), the rays of the sun shone at an angle of $7.2°$ to the zenith.



1. Find the radius of the earth. (Round to nearest ten miles.)
2. Find the circumference of the earth.

Chapter 5 Section 2

Definition 1 (Trigonometry)

[from: modern Latin (1595) $<$ Greek (triangle) $+$ (-metry)]

You are doing trigonometry if

1. you can find a standard quantitative measure of the inclination of one line to another.
2. you have a capacity for calculating the lengths of line segments.

from The Mathematics of the Heavens and the Earth, Glen van Brummelen

What is the equation for a circle centered at the origin (h, k) and a radius of r?

Given a circle with center $C(0, 0)$ and radius $r = 5$:
What is the equation?

$$x^2 + y^2 = 25$$

Is the point $(3, 4)$ on the circle?

Given a circle with center $C(0, 0)$ and radius $r = 13$:

$$x^2 + y^2 = 169$$

If $y = 12$, find x.

Things you should know

- Pythagorean theorem
- Similar triangles
- Sum of the interior angles of a triangle
- Leg length/angle size correspondence

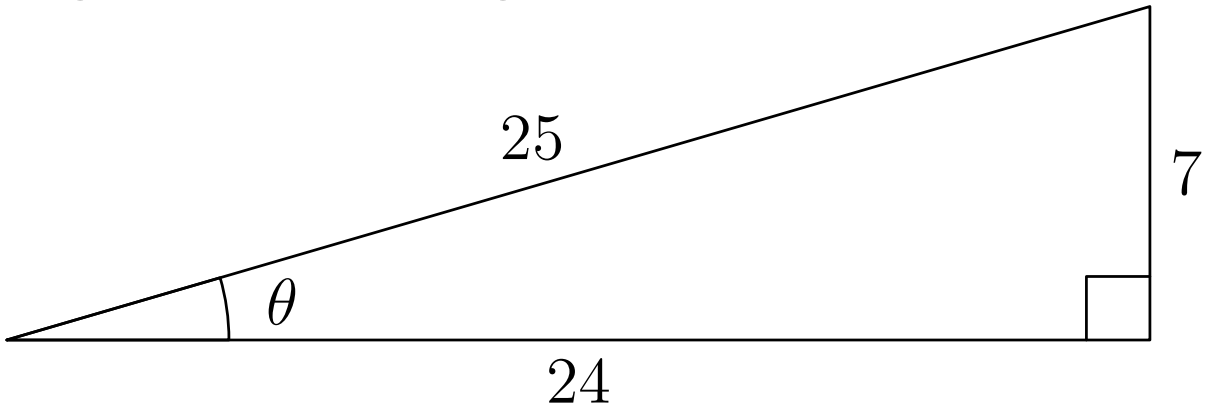
-fixes and stems

trigonometry triangle + -metry (measure)

isosceles iso- (same) + sceles (leg)

equilateral equi- (equal) + lateral (side)

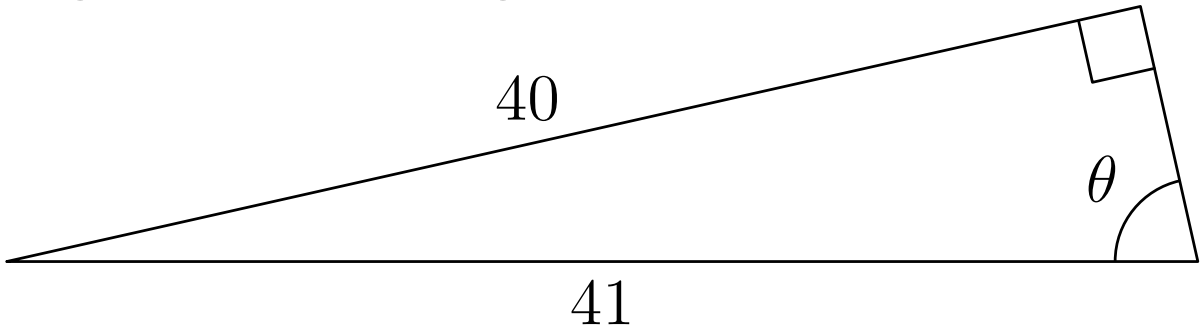
5.2.004 Find the exact values of the six trigonometric ratios of the angle θ in the triangle.



a) $\sin\theta =$ b) $\cos\theta =$ c) $\tan\theta =$

d) $\csc\theta =$ e) $\sec\theta =$ f) $\cot\theta =$

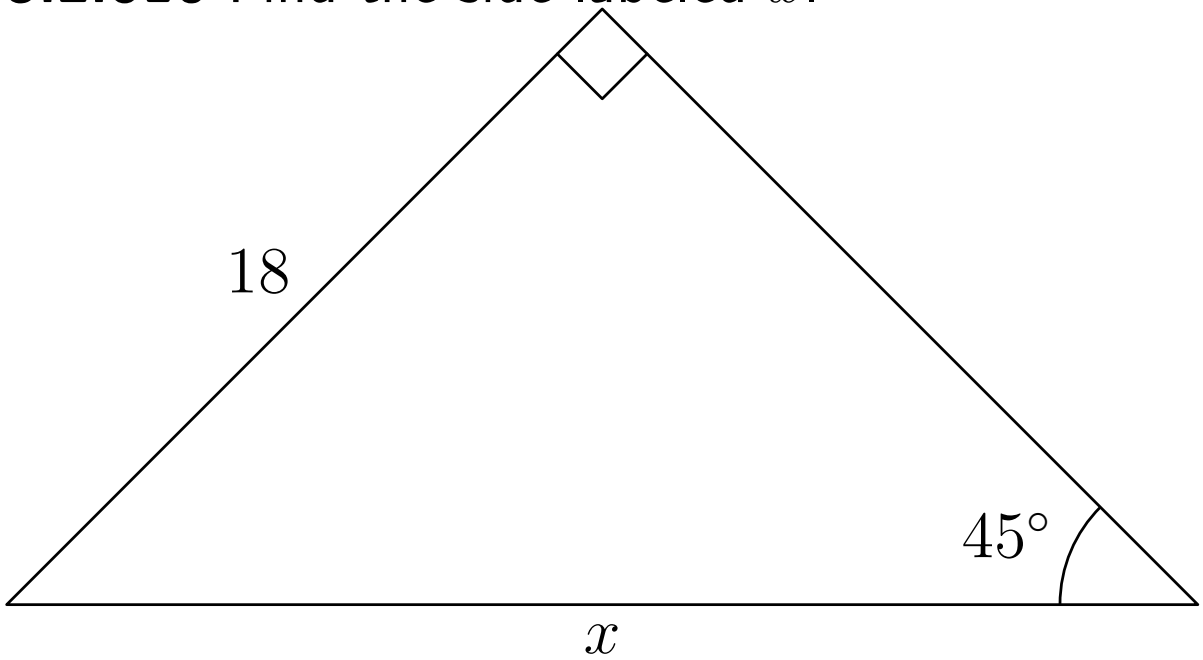
5.2.005 Find the exact values of the six trigonometric ratios of the angle θ in the triangle.



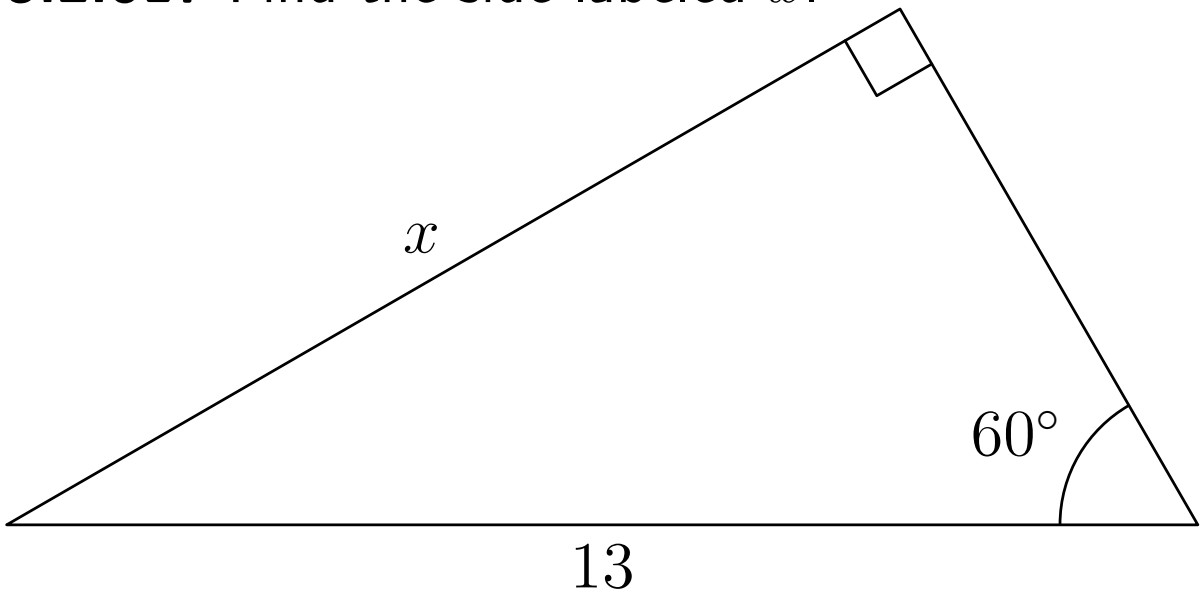
a) $\sin\theta =$ b) $\cos\theta =$ c) $\tan\theta =$

d) $\csc\theta =$ e) $\sec\theta =$ f) $\cot\theta =$

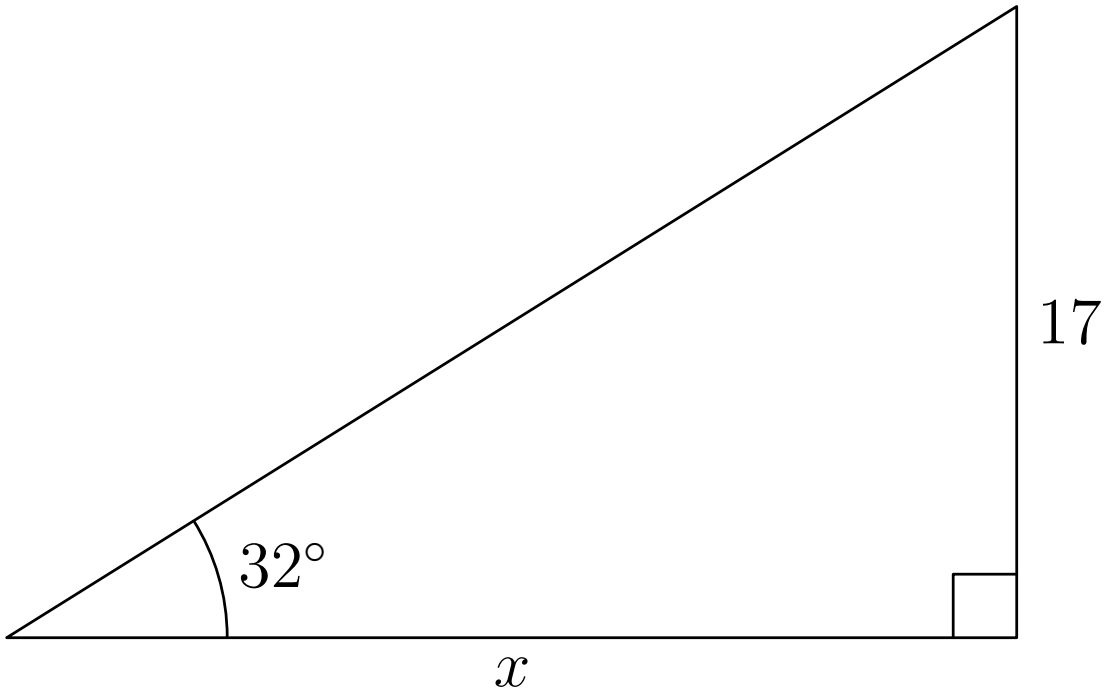
5.2.016 Find the side labeled x.



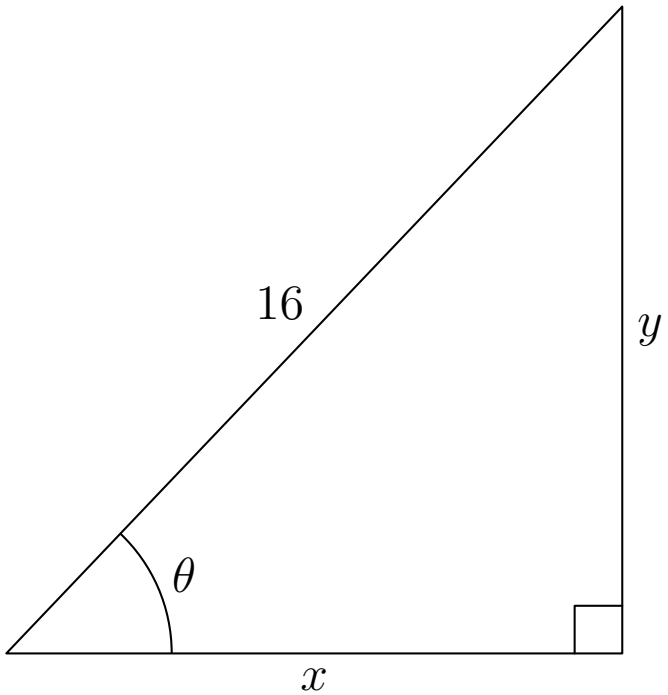
5.2.017 Find the side labeled x.



5.2.019 Find the side labeled x. State your answer rounded to 5 decimal places.



5.2.021 Express x and y in terms of trigonometric ratios of θ. (Express your answer in terms of θ only.)



5.2.023 Sketch a triangle that has acute angle θ.

$$\tan(\theta) = \frac{4}{7}$$

Then find

a) $\sin\theta =$

b) $\cos\theta =$

5.2.026 Sketch a triangle that has acute angle θ.

$$\tan(\theta) = \sqrt{3}$$

Then find

a) $\sin\theta =$

b) $\cos\theta =$

5.2.029 Evaluate the expression without using a calculator.

$$\sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right)$$

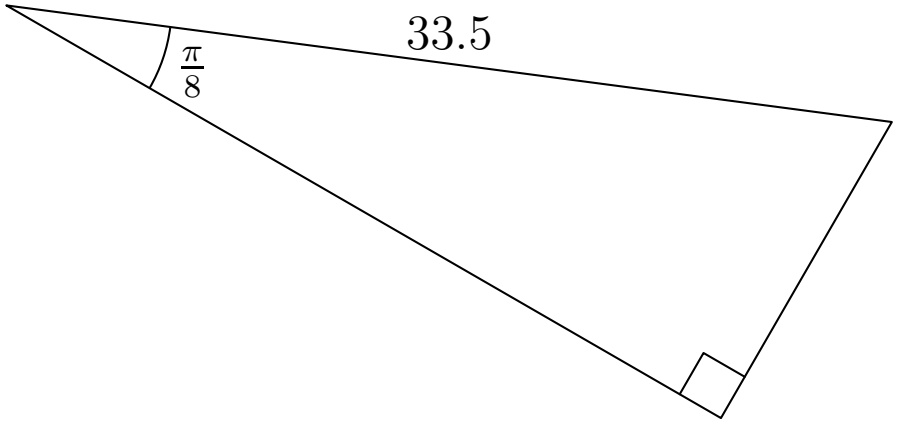
5.2.032 Evaluate the expression without using a calculator.

$$(\sin(30^\circ))^2 + (\cos(30^\circ))^2$$

5.2.035 Evaluate the expression without using a calculator.

$$\left(\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{6}\right)\right)^2$$

5.2.041 Solve the right triangle.

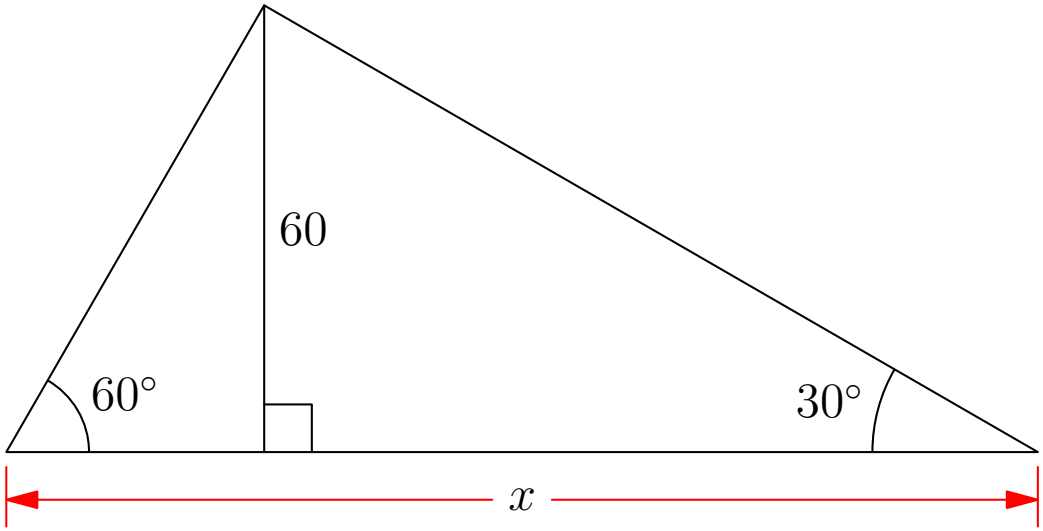


1. Find the length of the side opposite to the given angle. (Round your answer to two decimal places.)

2. Find the length of the side adjacent to the given angle. (Round your answer to two decimal places.)

3. Find the other acute angle.

5.2.047 Find x rounded to one decimal place.




YO DAWG I HEARD YOU LOVE TRIANGLES
SO WE PUT A TRIANGLE IN YOUR TRIANGLE SO YOU CAN LOVE TRIANGLES WHILE YOU LOVE TRIANGLES
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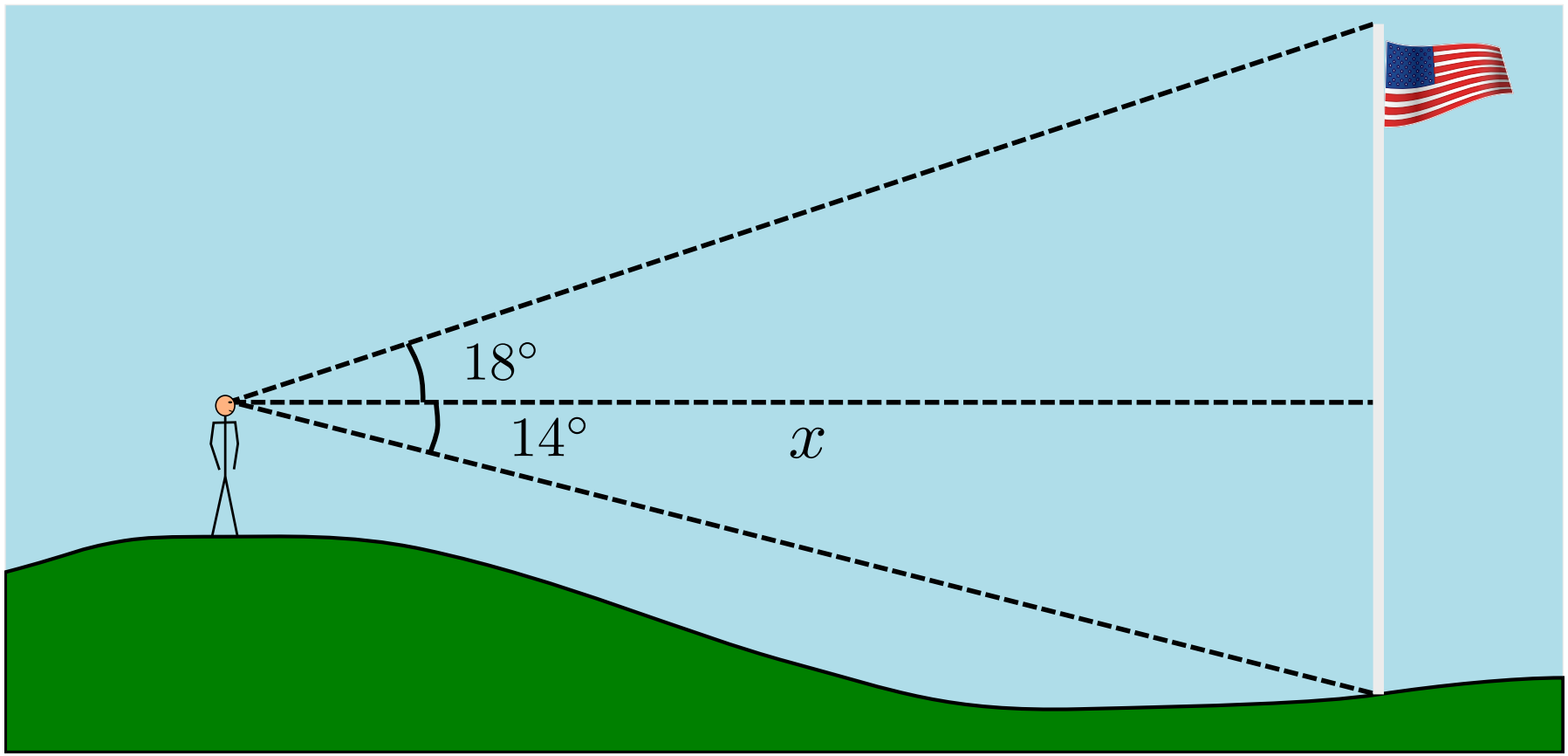
5.2.053 The angle of elevation to the top of a very tall Building is found to be 7° from the ground at a distance of 1 mi from the base of the building. Using this information, find the height of the building. (Round your answer to the nearest foot.)

5.2.056 From the top of a 170 ft lighthouse, the angle of depression to a ship in the ocean is 27°. How far is the ship from the base of the lighthouse? (Round your answer to the nearest foot.)

5.2.058 A 450 ft guy wire is attached to the top of a communications tower. If the wire makes an angle of 70° with the ground, how tall is the communications tower? (Round your answer to the nearest foot.)

5.2.060 A woman standing on a hill sees a flagpole that she knows is 35 ft tall. The angle of depression to the bottom of the pole is $14°$, and the angle of elevation to the top of the pole is $18°$. Find her distance x from the pole. (Round your answer to one decimal place.)



5.2.062 An airplane is flying at an elevation of 5150 ft, directly above a straight highway. Two motorists are driving cars on the highway on opposite sides of the plane. The angle of depression to one car is 32°, and that to the other is 57°. How far apart are the cars? (Round your answer to the nearest foot.)

5.2.067 When the moon is exactly half full, the earth, moon, and sun form a right angle (see the figure). At that time the angle formed by the sun, earth, and moon (using the earth as its vertex) is measured to be $89.85°$. If the distance from the earth to the moon is $240,000$ mi, estimate the distance from the earth to the sun. (Round your answer to one decimal place.)