MS 302 In-class Problems

May 15, 2025

Table of Contents

Chapter 2 Section 6
Conditional Probability, Independence, Product Rule

Suppose that I have a screen setup so that you cannot see me rolling a fair 6-sided die. I roll the die.
What is $P(2) =$?

Now suppose that I rolled the 6-sided die and tell you that I rolled an even number. Now what is $P(2) =$?

Definition 1 (Conditional probability of B given A)

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Ex 2.77 In the senior year of a high school graduating class of 100 students, 42studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history but neither mathematics nor psychology, 10 studied all three subjects, and 8 did not take any of the three. Randomly select a student from the class and find the probabilities of the following events.

(a) A person enrolled in psychology takes all three subjects.

(b) A person not taking psychology is taking both history and mathematics.

Ex 2.79 Statement In USA Today (Sept. 5, 1996), the results of a survey involving the use of sleepwear while traveling were listed as follows:

	Male	Female	Total
Underwear	0.220	0.024	0.244
Nightgown	0.002	0.180	0.182
Nothing	0.160	0.018	0.178
Pajamas	0.102	0.073	0.175
T-shirt	0.046	0.088	0.134
Other	0.084	0.003	0.087

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It is often convenient to also have column totals so...

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Total	0.614	0.386	1.000

Ex 2.79 Questions

(a) What is the probability that a traveler is a female who sleeps in the nude?

(b) What is the probability that a traveler is male?

(c) Assuming the traveler is male, what is the probability that he sleeps in pajamas?

(d) What is the probability that a traveler is male if the traveler sleeps in pajamas or a T-shirt?

Ex 2.80 The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and the filter need changing is 0.14.

(a) If the oil has to be changed, what is the probability that a new oil filter is needed?

(b) If a new oil filter is needed, what is the probability that the oil has to be changed?

Theorem 1

If events A and B can both occur in any exeriment, then

$$P(A \cap B) = P(B|A)P(A), \quad \textit{if } P(A) > 0.$$

Ex 2.81 The probability that a married man watches a certain television show is 0.4, and the probability that a married woman watches the show is 0.5. The probability that a man watches the show, given that his wife does, is 0.7. Find the probability that

(a) a married couple watches the show;

(b) a wife watches the show, given that her husband does;

(c) at least one member of a married couple will watch the show.

Roll a fair 6-sided die, let $A = \{2\}$, and $B = \{2, 4, 6\}$.

$$P(A) = \qquad P(B|A) =$$

Flip a fair coin twice. Let

$$H_1 = \text{head on 1st flip}, \qquad H_2 = \text{head on 2nd flip}$$

$$P(H_1) = \qquad P(H_2) = \qquad P(H_2|H_1) =$$

Definition 2 (Independent)

Events A and B are independent iff

$$P(B|A) = P(B) \quad \textit{or} \quad P(A|B) = P(A)$$

provided the conditional probability exists.

Otherwise they are dependent.

Theorem 2

Events A and B are independent iff

$$P(A \cap B) = P(A)P(B)$$

Ex 2.36 Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

Ex 2.38 A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Ex Suppose you are flipping a fair coin.

1. If you have flipped 4 heads what is the probability that the 5th flip is a head?

2. What is the probability of flipping 5 heads in a row?

Chapter 2 Section 7
Bayes' Rule

Suppose you get checked for prostrate (or breast) cancer.
You are told that the test's sensitivity is 90%.
This means that if you have cancer (C), then the test will return a positive result ($+$) 90% of the time

$$P(+|C) = 0.90$$

If the test comes back positive, how concerned should you be?

Ex pg 72 Suppose a town can be divided into the categories:

E: employed, $U = E'$: unemployed; M: male, F: female

as seen in the table:

	E	E'	
M	460	40	500
F	140	260	400
	600	300	900

We also know that 36 employed and 12 unemployed are members of the Rotary Club.

Let $A \equiv$ is a Rotary Club member, find $P(A)$.

Theorem 3

If events $B_1, B_2, \ldots, B_k$ are a partition of S such that $P(B_i) \neq 0\ \forall i$, then for any event A of S

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{n} P(B_i)P(A|B_i)$$

Ex 2.41 pg 74 In a certain assembly plant, three machines, B_1, B_2, and B_3, make 30%, 45%, and 25%, respectively, of the products. It is known from past experience that 2%, 3%, and 2% of the products made by each machine, respectively, are defective. Now, suppose that a finished product is randomly selected. What is the probability that it is defective?

Back to our cancer discussion.
We know the sensitivity,

$$P(+|C) = 0.90,$$

but that's not what we want to know.
We want to know is

$$P(C|+).$$

Theorem 4 (Bayes' Rule)

If $B_1, B_2, \ldots, B_k$ are a partition of S such that $P(B_i) \neq 0\ \forall i$, then for any event A of S such that $P(A) \neq 0$

$$P(B_r|A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A|B_r)}{\sum_{i=1}^{k} P(B_i)P(A|B_i)}$$

Ex 2.101 A paint-store chain produces and sells latex and semigloss paint. Based on long-range sales, the probability that a customer will purchase latex paint is 0.75. Of those that purchase latex paint, 60% also purchase rollers. But only 30% of semigloss paint buyers purchase rollers. A randomly selected buyer purchases a roller and a can of paint. What is the probability that the paint is latex?

Ex 2.99

Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If a customer complains that her package of film does not show the expiration date, what is the probability that it was inspected by John?

Ex 2.100 A regional telephone company operates three identical relay stations at different locations. During a one-year period, the number of malfunctions reported by each station and the causes are shown below.

Station	A	B	C
Problems with electricity supplied	2	1	1
Computer malfunction	4	3	2
Malfunctioning electrical equipment	5	4	2
Caused by other human errors	7	7	5

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

Finally, lets work the prostrate cancer problem.
Let C represent the event that you have cancer, C' that you do not have cancer, $+$ that the prostate cancer test came back positive, and $-$ that the prostate cancer test came back negative.

	$+$	$-$	
C Yes	1688	187	1875
C' No	32381	65744	98125
	34069	65931	100000

So

$$P(C) = \qquad , \quad P(C') =$$

$$P(+|C) = \qquad , \quad P(+|C') =$$

Now we know the sensitivity, $P(+|C)$, and the specificity, $P(+|C')$.

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$P(C|+) =$

Now when a male is 60 year old or older, then $P(C) = .4$.
This results in

$$P(C|+) = .645.$$