MS 204 In-class Problems

February 26, 2025

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Chapter 1 Section 1

Determine whether the statement describes a population or a sample.

The price of homes of all the employees at a software company.

- Population
- Sample

Determine whether the statement describes a population or a sample.

The heights of 5 out of the 32 eggplant plants at Mr. Lonardo's greenhouse.

- Population
- Sample

Identify the **population** being studied.

The number of times 10 out of 20 students on your floor order pizza in a week.

- The 20 students on your floor.
- All students who ordered pizza in a week.
- The 10 students on your floor.

Determine whether the statement describes a descriptive or inferential statistic.

A recent poll of 1443 luxury car owners in West Virginia showed that the average price of a luxury car in the U.S. is $48,900.

- Descriptive Statistic
- Inferential Statistic

Determine whether the statement describes a descriptive or inferential statistic.

The average price of a car at the new car dealership in town is $28,400.

- Descriptive Statistic
- Inferential Statistic

Determine if the numerical value describes a population parameter or a sample statistic.

A recent poll of 2935 corporate executives showed that the average price of their cars is $27,100.

- Population Parameter
- Sample Statistic

Determine if the numerical value describes a population parameter or a sample statistic.

The average price of a house in the new subdivision is $339,000.

- Population Parameter
- Sample Statistic

Identify the sample chosen for the study.

The number of times 4 out of 37 students on your floor order take-out in a week.

- The 4 students on your floor.
- All students who ordered take-out in a week.
- The 37 students on your floor.

Chapter 1 Section 2

Types of cars people own are an example of which type of data?

- Qualitative
- Quantitative
- Inferential
- Statistic

Football jersey numbers are an example of which type of data?

- Qualitative
- Quantitative
- Inferential
- Statistic

Goals scored during a soccer game are an example of which type of data?

- Qualitative
- Quantitative
- Inferential
- Statistic

Indicate the level of measurement for the data set described.

Monthly amounts of rain in Seattle over 10 years

- Interval
- Ratio
- Ordinal
- Nominal

Indicate the level of measurement for the data set described.

Categories of hurricanes that have hit the Atlantic coast

- Interval
- Ratio
- Ordinal
- Nominal

Classify data as discrete or continuous

Lengths of time it takes for new light bulbs to burn out are an example of which type of data?

- Discrete
- Continuous
- Neither

Classify data as discrete or continuous

Types of movies people go to see are an example of which type of data?

- Discrete
- Continuous
- Neither

Classify data as discrete or continuous

The numbers of each color of jelly beans in a jar (assuming they are all whole) are an example of which type of data?

- Discrete
- Continuous
- Neither

Chapter 1 Section 4

What is response bias and how can you avoid it?*

*This webpage seems to explain each type well, but I didn't read every sentence. I mainly put the link here for attributive purposes.

Chapter 2 Section 1

The following data describes grades of students in biology. Complete the frequency table for this data.

88.2,94.9,86.6,80.0,83.5,96.1,87.3,89.7,83.5,93.1,89.5,88.6,
95.2,96.7,86.8,96.8,95.1,89.0,88.2,94.9,86.6,80.0,83.5,96.1,
87.3,89.7,83.5,93.1,89.5,88.6,95.2,96.7,86.8,96.8,95.1,89.0

Determine the frequency of each class in the table shown.

Grades of Students in Biology	
Class	Frequency
77.0–80.9	
81.0–84.9	
85.0–88.9	
89.0–92.9	
93.0–96.9	

Consider the following frequency table representing the distribution of hours students spend on homework in a week.

Hours Students Spend on Homework in a Week	
Class	Frequency
19–28	3
29–38	11
39–48	15
49–58	6
59–68	9

Determine the class width of each class.

Consider the following frequency table representing the distribution of hours students spend on homework in a week.

Price of a Newspaper (in Dollars)	
Class	Frequency
0.34–0.42	11
0.43–0.51	12
0.52–0.60	14
0.61–0.69	10
0.70–0.78	10

Determine the class width of each class.

Consider the following frequency table representing the distribution of cost of a paperback book (in dollars).

Cost of a Paperback Book (in Dollars)	
Class	Frequency
5.7–6.1	6
6.2–6.6	13
6.7–7.1	12
7.2–7.6	14
7.7–8.1	1

1. Determine the relative frequency for the second class as a simplified fraction.

2. Determine the relative frequency for the fourth class as a simplified fraction.

Consider the following frequency table representing the distribution of hourly wages for first jobs of a certain population.

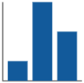
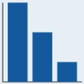
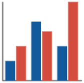
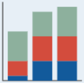
Hourly Wage at First Job	
Class	Frequency
6.1–7.1	2
7.2–8.2	9
8.3–9.3	9
9.4–10.4	13
10.5–11.5	9

1. Determine the cumulative frequency for the fifth class.

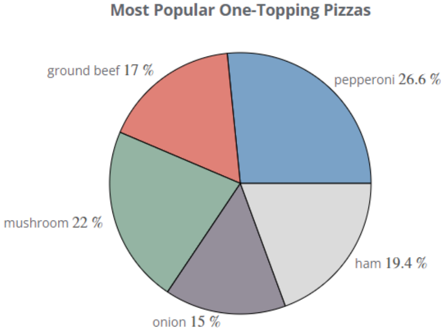
2. Determine the cumulative frequency for the third class.

Chapter 2 Section 2

Qualitative Data

Type of Graph		Description
Pie Chart		A pie chart shows how large each category is in relation to the whole; that is, it uses the relative frequencies from the frequency distribution to divide the "pie" into different-sized wedges. It can only be used to display qualitative data.
Bar Graph		In a bar graph, bars are used to represent the amount of data in each category; one axis displays the categories of qualitative data and the other axis displays the frequencies.
Pareto Chart		A Pareto chart is a bar graph with the bars in descending order of frequency. Pareto charts are typically used with nominal data.
Side-by-Side Bar Graph		A side-by-side bar graph is a bar graph that compares the same categories for different groups.
Stacked Bar Graph		A stacked bar graph is a bar graph that compares the same categories for different groups and shows category totals.

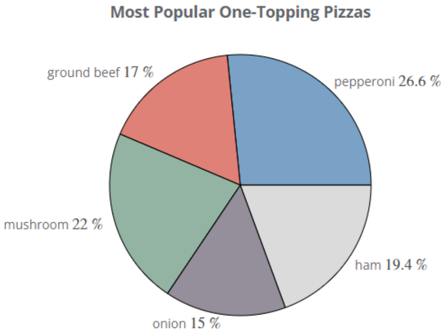
The Pizza Pie 'N Go sells about 2260 one-topping pizzas each month. The circle graph displays the most requested one-topping pizzas, by percentage, for one month.



Step 1 of 5: Find the number of pepperoni pizzas sold each month. Round your answer to the nearest integer.

Step 2 of 5: Find the number of ground beef pizzas sold each month. Round your answer to the nearest integer.

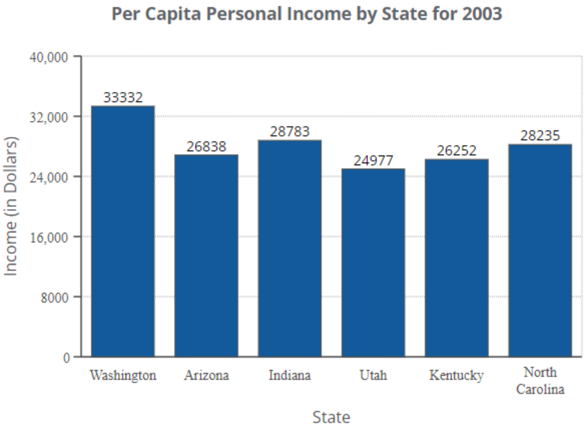
The Pizza Pie 'N Go sells about 2260 one-topping pizzas each month.



Step 3 of 5: Find the number of mushroom pizzas sold each month. Round your answer to the nearest integer.

Step 4 of 5: Find the number of onion pizzas sold each month. Round your answer to the nearest integer.

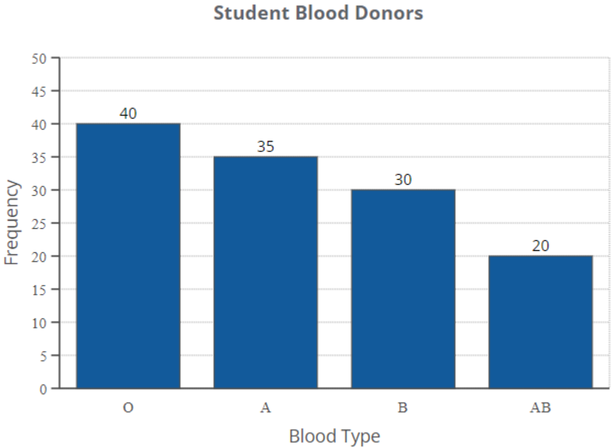
Step 5 of 5: Find the number of ham pizzas sold each month. Round your answer to the nearest integer.

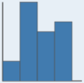
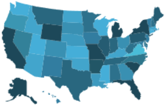
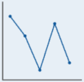


Step 1 of 2: Find the lowest per capita personal income for the six states shown.

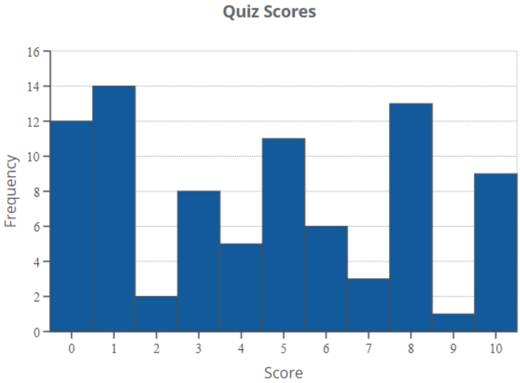
Step 2 of 2: Find the highest per capita personal income for the six states shown.

Consider the Pareto chart, which shows the number of student blood donors by their type for one day of a campus blood drive. How many students donated blood on that day?



<table>
<tr><th colspan="3">Quantitative Data</th></tr>
<tr><th colspan="2">Type of Graph</th><th>Definition</th></tr>
<tr><td>Histogram</td><td></td><td>A histogram is a bar graph of a frequency distribution of quantitative data; the horizontal axis is a number line.</td></tr>
<tr><td>Stem-and-Leaf Plot</td><td><table><tr><th>Stem</th><th>Leaves</th></tr><tr><td>32</td><td>0</td></tr><tr><td>33</td><td>7 7 7 8</td></tr><tr><td>34</td><td>0 0 0 0</td></tr></table></td><td>A stem-and-leaf plot retains the original data; the leaves are the last significant digit in each data value and the stems are the remaining digits.</td></tr>
<tr><td>Dot Plot</td><td></td><td>A dot plot retains the original data by plotting a dot above each data value on a number line.</td></tr>
<tr><td>Heat Map</td><td></td><td>A heat map depicts relative values of the data using shades of color.</td></tr>
<tr><td>Line Graph</td><td></td><td>A line graph uses straight lines to connect points plotted at the value of each measurement above the time it was taken.</td></tr>
</table>

The following histogram represents the distribution of scores on a ten point quiz.



Step 1 of 3: Which score has the highest frequency?

Step 2 of 3: What is the frequency corresponding to a score of 6?

Step 3 of 3: What is the total number of people who made a score between 0 and 2 inclusive?

The following stem-and-leaf plot represents the distribution of weights for a group of people.

Stem	Leaves					
8	0	3	6	6		
9	1	3				
10	1	2	4	4	5	9
11	1	2	5	7	8	
12	1	2	3	3	6	8
13	1	2	2	7	7	
14	8	8				
15	2	4	5	8	9	
16	4	5	5	6	9	

Key: $8|0 = 80$ pounds

Step 1 of 3: What is the weight of the lightest person in the group?

Step 2 of 3: How many people weigh in the range from 110 to 140 inclusive?

Step 3 of 3: What is the weight of the heaviest person in the range 80 to 89 inclusive?

The following data represent the test scores for 18 students in a class on their most recent test. Use the given data to determine the stems for this stem-and-leaf plot.

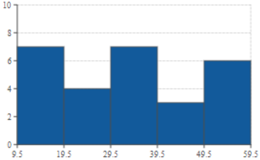
87	84	69	85	73	58
65	53	63	66	67	82
66	82	79	89	52	60

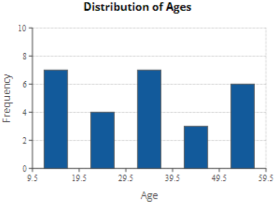
Test Scores by Student

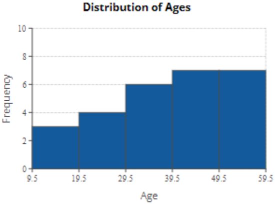
Stem	Leaves
______	2 3 8
______	0 3 5 6 6 7 9
______	2 9
______	2 2 4 5 7 9

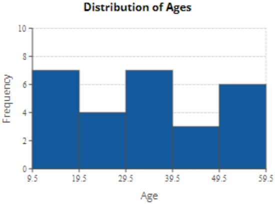
The following data represents the distribution of ages of a group of people. Determine the graph that correctly represents the data.

Age	10-19	20-29	30-39	40-49	50-59
Frequency	7	4	7	3	6







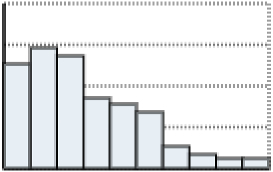


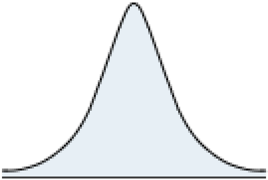
Chapter 2 Section 3

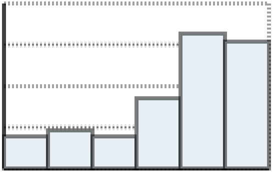
Select the graph that best illustrates the following distribution shape:

Uniform

a) 

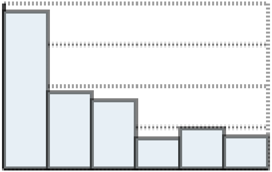
b) 

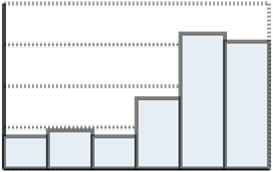
c) 

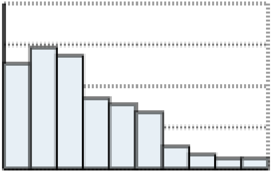
d) 

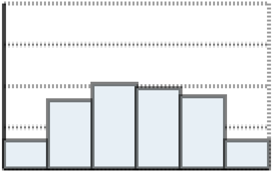
Select the graph that best illustrates the following distribution shape:

Symmetrical

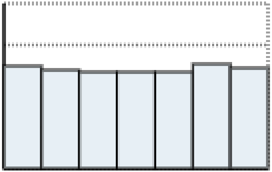
a) 

b) 

c) 

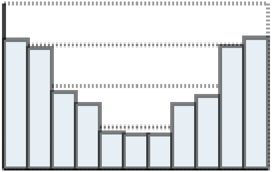
d) 

For the set of data displayed below, describe the most likely shape of its distribution.



- Skewed to the right
- Symmetrical, but not uniform
- Skewed to the left
- Uniform

For the set of data displayed below, describe the most likely shape of its distribution.

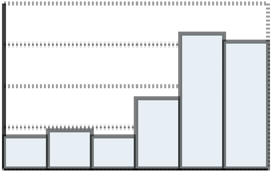


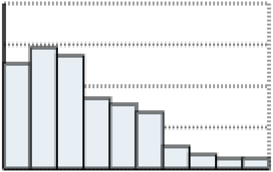
- Uniform
- Skewed to the left
- Symmetrical, but not uniform
- Skewed to the right

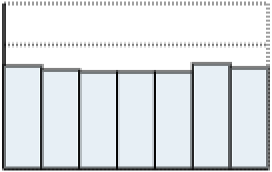
Select the graph that best illustrates the following distribution shape:

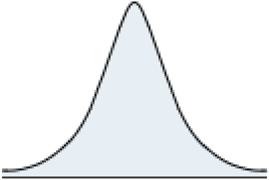
Skewed to the right

---

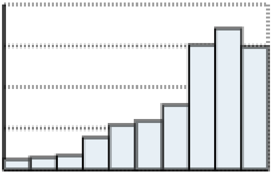
a) 

b) 

c) 

d) 

For the set of data displayed below, describe the most likely shape of its distribution.



- Symmetrical, but not uniform
- Uniform
- Skewed to the left
- Skewed to the right

Chapter 3 Section 1

For the data set shown in the table:

Data	
i	x_i
1	3
2	7
3	4

Find $\sum x_i$ written in two ways: as an unevaluated sum (it will have multiple terms) and then as an evaluated sum (a single term):

Unevaluated ______________________

Evaluated __________

Find the mean of:

1. $1, 2, 3$
2. $1, 2, 5, 8, 9$
3. $1, 1, 5, 9, 9$
4. $1, 1, 2, 8, 9, 9$
5. $1, 2, 5, 9, 13$
6. $1, 2, 4, 8, 100$

Find the mean of

$$50000, 30000, 45000, 33000, 47000, 51000, 6744000.$$

Consider the following data.

$$14, -10, 7, 13, 3, -3$$

Step 1 of 3: Determine the mean of the given data.

Step 2 of 3: Determine the median of the given data

Step 3 of 3: Determine if the data set is unimodal, bimodal, multimodal, or has no mode. Identify the mode(s), if any exist.

- No Mode
- Unimodal
- Bimodal
- Multimodal

Consider the following data.

$$-9, 11, 7, 11, 7, -9$$

Step 2 of 3: Determine the median of the given data.

Step 3 of 3: Determine if the data set is unimodal, bimodal, multimodal, or has no mode. Identify the mode(s), if any exist.

- No Mode
- Unimodal
- Bimodal
- Multimodal

Find the mode of

$$3, 2, 3, 1, 5, 1$$

For the following type of data set, would you be more interested in looking at the mean, median, or mode? State your reasoning.

The price for homes with similar floor plans in a new neighborhood

---

Correct measure of center:

a) mean   b) median   c) mode

Justification

- The prices for homes are quantitative data with outliers.
- The prices for homes are qualitative data.
- The prices for homes are quantitative data with no outliers.

A company has given you the task to research the storage cost for similarly sized spaces in downtown Houston. Would you be more interested in looking at the mean, median, or mode?

- mean
- median
- mode

A company has given you the task to research the color of a car preferred by the average male. Would you be more interested in looking at the mean, median, or mode?

- mean
- median
- mode

A company has given you the task to research the cost of cars at all the dealerships in town. Would you be more interested in looking at the mean, median, or mode?

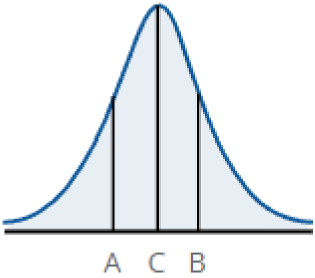
- mean
- median
- mode

Calculate the GPA of a student with the following grades:

A (17 hours), B (17 hours), F (10 hours).

Note that an A is equivalent to 4.0, a B is equivalent to a 3.0, a C is equivalent to a 2.0, a D is equivalent to a 1.0, and an F is equivalent to a 0. Round your answer to two decimal places.

For the graph shown, determine which letter represents the mean, the median, and the mode. Letters may be used more than once.

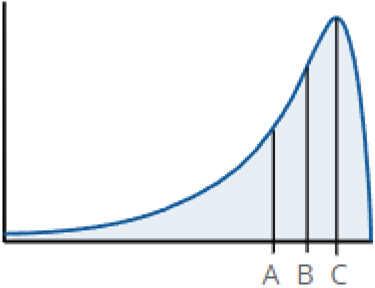


Mean =

Median =

Mode =

For the graph shown, determine which letter represents the mean, the median, and the mode. Letters may be used more than once.



Mean =

Median =

Mode =

Chapter 3 Section 2

Calculate the range, population variance, and population standard deviation for the following data set. If necessary, round to one more decimal place than the largest number of decimal places given in the data.

$$14, 18, 16$$

$$\begin{aligned} \text{Range} &= \underline{\hspace{2cm}} \\ \text{Population variance} &= \underline{\hspace{2cm}} \\ \text{Population standard deviation} &= \underline{\hspace{2cm}} \end{aligned}$$

Calculate the range, population variance, and population standard deviation for the following data set. If necessary, round to one more decimal place than the largest number of decimal places given in the data.

$$9, 9, 9, 9, 9, 9, 9, 9, 9, 9$$

$$\begin{aligned} \text{Range} &= \quad \_\_\_\_\_\_ \\ \text{Population variance} &= \quad \_\_\_\_\_\_ \\ \text{Population standard deviation} &= \quad \_\_\_\_\_\_ \end{aligned}$$

Use Excel:
Calculate the range, population variance, and population standard deviation for the following data set. If necessary, round to one more decimal place than the largest number of decimal places given in the data.

$$14, 18, 16, 5, 13, 9, 18, 16, 11, 17$$

$$\begin{aligned} \text{Range} &= \quad \_\_\_\_\_\_ \\ \text{Population variance} &= \quad \_\_\_\_\_\_ \\ \text{Population standard deviation} &= \quad \_\_\_\_\_\_ \end{aligned}$$

Donna is looking into investing a portion of her recent bonus into the stock market. While researching different companies, she discovers the following standard deviations of one year of daily stock closing prices.

Handy Prosthetics:	Standard deviation of stock prices = \$1.12
El Lobo Malo Incorporated:	Standard deviation of stock prices = \$9.63

Based on the data and assuming these trends continue, which company would give Donna a stable long-term investment?

- Handy Prosthetics; the smaller standard deviation indicates that Handy Prosthetics has a greater mean closing price than El Lobo Malo Incorporated.
- Handy Prosthetics; the smaller standard deviation indicates that Handy Prosthetics has a less variability in its closing prices than El Lobo Malo Incorporated.
- El Lobo Malo Incorporated the larger standard deviation indicates that El Lobo Malo Incorporated has a less variability in its closing prices than Handy Prosthetics.
- El Lobo Malo Incorporated the larger standard deviation indicates that El Lobo Malo Incorporated has a greater mean closing price than Handy Prosthetics.

Suppose that IQ scores have a bell-shaped distribution with a mean of 97 and a standard deviation of 17. Using the empirical rule, what percentage of IQ scores are between 46 and 148?

Suppose that grade point averages of undergraduate students at one university have a bell-shaped distribution with a mean of 2.52 and a standard deviation of 0.42. Using the empirical rule, what percentage of the students have grade point averages that are no more than 1.26? Please do not round your answer.

Chapter 3 Section 3

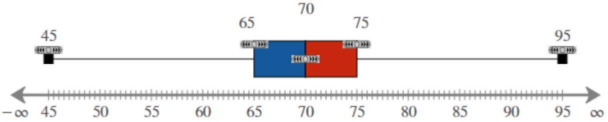
5 number summary

Calculate the 5 number summary for:

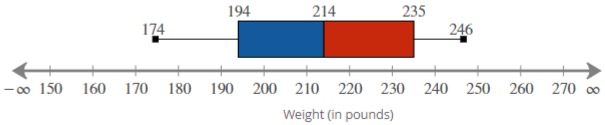
19
15
20
10
13
18
11
13
7
13
10
2
12
13
17
15
7
13
13
4

Copy Data

Goto problem in homework:
Construct a box plot from the given data. Use the approximation method.
Scores on a Statistics Test: $86, 79, 70, 91, 56, 48, 45, 81, 50, 89$
Draw the box plot by selecting each of the five movable parts to the appropriate position.



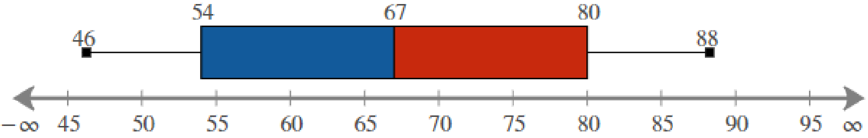
A high school has 52 players on the football team. The summary of the players' weights is given in the box plot. Approximately, what is the percentage of players weighing less than or equal to 194 pounds?



From:

- 194-235?
- 194-246?
- 174-246?

Given the following box plot, choose the best description of the distribution.



- The distribution of the data is skewed left.
- The distribution of the data is skewed right.
- The distribution of the data is symmetric.

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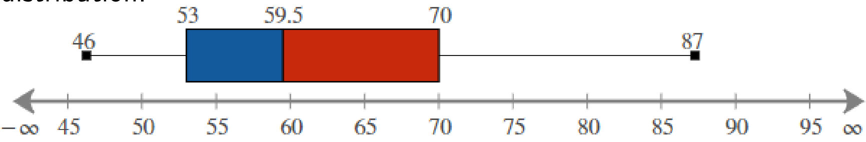
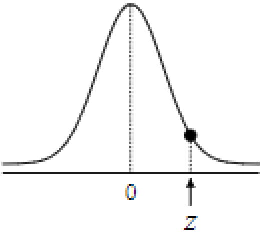


- The distribution of the data is skewed left.
- The distribution of the data is skewed right.
- The distribution of the data is symmetric.

Calculate the standard score of the given x value, $x = 59.6$, where $\mu = 65.5$, $\sigma = 3.7$. Round your answer to two decimal places.

Calculate the standard score of the given x value, $x = 22.8$, where $\overline{x} = 20.9$, $s = 3.6$. Round your answer to two decimal places.

Given the following graph, where the mean is marked, which value best represents the z-score shown?



- $z = 1.25$
- $z = 0$
- $z = -2.46$

Chapter 4 Section 1

Write out the sample space for the given experiment. Use the letter R to indicate red, G to indicate green, and B to indicate blue.

A die shows 3 different colors on it. Give the sample space for the next 2 rolls.

Experimental probability example

	A	B	C	D	E	F	G
1	3		3		Pips	Freq	Rel Freq
2	2		3		1	0	0
3	6		3		2	1	0.1
4	4		3		3	7	0.7
5	3		4		4	1	0.1
6	2		6		5	0	0
7	5		3		6	1	0.1
8	4		2				
9	5		3		Sum	10	
10	1		3				
11							

Code:

	A	B	C	D	E	F	G
1	=RANDBETWEEN(1,6)		3		Pips	Freq	Rel Freq
2	=RANDBETWEEN(1,6)		3		1	=COUNTIF(C:C, E2)	=F2/$F$9
3	=RANDBETWEEN(1,6)		3		2	=COUNTIF(C:C, E3)	=F3/$F$9
4	=RANDBETWEEN(1,6)		3		3	=COUNTIF(C:C, E4)	=F4/$F$9
5	=RANDBETWEEN(1,6)		4		4	=COUNTIF(C:C, E5)	=F5/$F$9
6	=RANDBETWEEN(1,6)		6		5	=COUNTIF(C:C, E6)	=F6/$F$9
7	=RANDBETWEEN(1,6)		3		6	=COUNTIF(C:C, E7)	=F7/$F$9
8	=RANDBETWEEN(1,6)		2				
9	=RANDBETWEEN(1,6)		3		Sum	=SUM(F2:F7)	
10	=RANDBETWEEN(1,6)		3				
11							

There are 219 identical plastic chips numbered 1 through 219 in a box. What is the probability of reaching into the box and randomly drawing the chip numbered 170? Express your answer as a simplified fraction or a decimal rounded to four decimal places.

There are 756 identical plastic chips numbered 1 through 756 in a box. What is the probability of reaching into the box and randomly drawing the chip number that is smaller than 570? Express your answer as a simplified fraction or a decimal rounded to four decimal places.

There are 569 identical plastic chips numbered 1 through 569 in a box. What is the probability of reaching into the box and randomly drawing the chip number that is greater than 220? Express your answer as a simplified fraction or a decimal rounded to four decimal places.

You decide to record the hair colors of people leaving a lecture at your school. What is the probability that the next person who leaves the lecture will have gray hair? Express your answer as a simplified fraction or a decimal rounded to four decimal places.

Blonde	Red	Brown	Black	Gray
20	45	21	44	33

What is the probability that a randomly selected person will have a birthday in November? Assume that this person was not born in a leap year. Express your answer as a simplified fraction or a decimal rounded to four decimal places.

A coin is tossed 6 times.
What is the probability of getting all heads? Express your answer as a simplified fraction or a decimal rounded to four decimal places.

A standard six-sided die is rolled.
What is the probability of rolling a number less than or equal to 5?
Express your answer as a simplified fraction or a decimal rounded to four decimal places.

What is the probability of rolling a sum of 9 on a standard pair of six-sided dice? Express your answer as a fraction or a decimal number rounded to three decimal places, if necessary.

$$
\begin{array}{cccccc}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\
(2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\
(3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\
(4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\
(5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\
(6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)
\end{array}
$$

Let C represent the event that you have cancer, C' that you do not have cancer, $+$ that the prostate cancer test came back positive, and $-$ that the prostate cancer test came back negative.

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	$+$	$-$	
C Yes	1688	187	1875
C' No	32381	65744	98125
	34069	65931	100000

Let C represent the event that you have cancer, C' that you do not have cancer, $+$ that the prostate cancer test came back positive, and $-$ that the prostate cancer test came back negative.

	$+$	$-$	
C Yes	1688	187	1875
C' No	32381	65744	98125
	34069	65931	100000

So

$$P(C) = \frac{1875}{100000} = \frac{3}{160} \approx .01875$$

$$P(+|C) = \frac{1688}{1875} = .9, \quad P(+|C') = \frac{32381}{98125} = .33$$

Let C represent the event that you have cancer, C' that you do not have cancer, $+$ that the prostate cancer test came back positive, and $-$ that the prostate cancer test came back negative.

	$+$	$-$	
C Yes	$.9$	$.1$	1
C' No	$.33$	$.67$	1

Let C represent the event that you have cancer, C' that you do not have cancer, $+$ that the prostate cancer test came back positive, and $-$ that the prostate cancer test came back negative.

	$+$	$-$	
C Yes	$.9$	$.1$	1
C' No	$.33$	$.67$	1

, $P(C) = \frac{1875}{100000} = \frac{3}{160}$

But what we really want to know is:

$$\begin{aligned} P(C|+) &= \frac{P(C)P(+|C)}{P(C)P(+|C) + P(C')P(+|C')} \\ &= \frac{\frac{3}{160}(.9)}{\frac{3}{160}(.9) + \left(1 - \frac{3}{160}\right)(.33)} \\ &= .0495 \end{aligned}$$

So the probability is 4.95% that you have cancer given that the test came back positive.

Let C represent the event that you have cancer, C' that you do not have cancer, $+$ that the prostate cancer test came back positive, and $-$ that the prostate cancer test came back negative.

	$+$	$-$	
C Yes	$.9$	$.1$	1
C' No	$.33$	$.67$	1

So when $P(C) \approx .01875$

$$P(C|+) = .0495$$

and the probability is 4.95% that you have cancer given that the test came back positive.
Now when a male is 60 years old or older, then $P(C) = .4$. This results in

$$P(C|+) = .645$$

or as a percentage 64.5%.

Monty Hall Problem

Birthday Paradox

Birthday Paradox - How many people would you need in a room so that there is a 50% chance that two of them have the same birthday?

Related question - How many people would you need in a room so that there is a 50% chance that someone has your birthday?

Chapter 4 Section 4

Evaluate the following expression.

$$6!$$

Evaluate the following expression.

$$_7P_3$$

Evaluate the following expression.

$$_{9}C_{7}$$

Evaluate the following expression.

$$\frac{13!}{5!(13-5)!}$$

Pascal's triangle I

$$1$$

Pascal's triangle II

$$\begin{array}{ccc} & 1 & \\ 1 & & 1 \end{array}$$

Pascal's triangle III

$$\begin{array}{ccccc} & & 1 & & \\ & 1 & & 1 & \\ 1 & & 2 & & 1 \end{array}$$

Pascal's triangle IV

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1
\end{array}
$$

Pascal's triangle V

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Pascal's triangle VI

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Pascal's triangle Combinations

n											
0						1					
1					1		1				
2				1		2		1			
3			1		3		3		1		
4		1		4		6		4		1	
5	1		5		10		10		5		1
0						${}_0C_0$					
1					${}_1C_0$		${}_1C_1$				
2				${}_2C_0$		${}_2C_1$		${}_2C_2$			
3			${}_3C_0$		${}_3C_1$		${}_3C_2$		${}_3C_3$		
4		${}_4C_0$		${}_4C_1$		${}_4C_2$		${}_4C_3$		${}_4C_4$	
5	${}_5C_0$		${}_5C_1$		${}_5C_2$		${}_5C_3$		${}_5C_4$		${}_5C_5$

Chapter 5 Section 1

Expected value example here.

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1. How many heads were flipped

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Let's flip a fair coin 3 times and use this to learn about random variables.
First thing is to determine what outcome are we interested in.

1. How many heads were flipped
2. How many tails were flipped

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First thing is to determine what outcome are we interested in.

1. How many heads were flipped
2. How many tails were flipped
3. How many heads minus how many tails were flipped

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2. How many tails were flipped
3. How many heads minus how many tails were flipped

We will look at how many heads were flipped.

Let is take a moment and look at an easier example and then we will return to the flip coin three times example.
Let us flip a fair coin once.

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Let X *count* the number of times we have seen a head for each flip.

Let is take a moment and look at an easier example and then we will return to the flip coin three times example.
Let us flip a fair coin once.
Sample space is $S = \{H, T\}$.
Let X *count* the number of times we have seen a head for each flip.

Outcome	Count
T	$X(\mathsf{T}) = 0$
H	$X(\mathsf{H}) = 1$

Back to three flips.

Back to three flips.

$$S =$$

Let's make a table of Outcomes, Count of heads, and the Random Variable of that outcome:

Outcome	# of H	RV

Why add this complexity?

Count	Outcomes	Probability
$X = 0$		$P(X = 0) =$
$X = 1$		$P(X = 1) =$
$X = 2$		$P(X = 2) =$
$X = 3$		$P(X = 3) =$

This is a probability distribution and one way of writing it is in table form:

x				
$P(X = x)$				

Definition 1 (Probability distribution, Discrete random variable)

A **probability distribution** is a function that gives the probabilities of occurrence of different possible outcomes for an experiment and must satisfy:

1. $0 \leq P(X = x) \leq 1$
2. $\sum P(X = x) = 1$

A **discrete random variable** has either a finite number of possible values or a countably infinite number.

Finite Number of Possible Values

Roll a 2D6 and let r.v. $X = R_1 + R_2$.

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Definition 2 (Poisson distribution)

From:
https://en.wikipedia.org/wiki/Poisson_distribution A **Poisson distribution** is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time if these events occur with a known constant mean rate and independently of the time since the last event.

Examples 1

- soldiers killed by horse-kicks each year in each corps in the Prussian cavalry. This example was used in a book by Ladislaus Bortkiewicz (1868–1931).
- yeast cells used when brewing Guinness beer. This example was used by William Sealy Gosset (1876–1937).
- phone calls arriving at a call center within a minute. This example was described by A.K. Erlang (1878–1929).
- goals in sports involving two competing teams.
- deaths per year in a given age group,
- jumps in a stock price in a given time interval,

Examples 2

- times a web server is accessed per minute (under an assumption of homogeneity,
- mutations in a given stretch of DNA after a certain amount of radiation,
- cells infected at a given multiplicity of infection,
- bacteria in a certain amount of liquid.
- photons arriving on a pixel circuit at a given illumination over a given time period,
- landing of V-1 flying bombs on London during World War II, investigated by R. D. Clarke in 1946.

Determine whether or not the distribution is a discrete probability distribution and select the reason why or why not.

x	-4	-3	-2
$P(X = x)$	0.55	0.39	0.06

Decide

- Yes
- No

Reason

- Since the probabilities lie inclusively between 0 and 1 and the sum of the probabilities is equal to 1.
- Since at least one of the probability values is greater than 1 or less than 0.
- Since the sum of the probabilities is not equal to 1.
- Since the sum of the probabilities is equal to 1.
- Since the probabilities lie inclusively between 0 and 1.

Consider the following data:

x	-4	-3	-2	-1	0
$P(X = x)$	0.2	0.2	0.2	0.2	0.2

Step 1 of 5: Find the expected value $E(X)$. Round your answer to one decimal place.

Step 2 of 5: Find the variance. Round your answer to one decimal place.

Step 3 of 5: Find the standard deviation. Round your answer to one decimal place.

Consider the following data:

x	-4	-3	-2	-1	0
$P(X = x)$	0.2	0.2	0.2	0.2	0.2

Step 4 of 5: Find the value of $P(X \leq -2)$. Round your answer to one decimal place.

Step 5 of 5: Find the value of $P(X < -1)$. Round your answer to one decimal place.

If you draw a card with a value of two or less from a standard deck of cards, I will pay you \$30. If not, you pay me \$6. (Aces are considered the highest card in the deck.)

Step 1 of 2: Find the expected value of the proposition. Round your answer to two decimal places. Losses must be expressed as negative values.

Step 2 of 2: If you played this game 794 times how much would you expect to win or lose? Round your answer to two decimal places. Losses must be expressed as negative values.

In the long run, which plan has the least amount of risk?

$Plan A$	
Payout	P(Payout)
$-\$50000$	0.17
$-\$10000$	0.31
$\$75000$	0.52

$Plan B$	
Payout	P(Payout)
$-\$20000$	0.68
$\$20000$	0.12
$\$80000$	0.2

Chapter 5 Section 2

You can skip to frame number **???add frame number later???**. Suppose we flip a fair coin three times and we are interested in the number of heads we flip (so our random variable X counts the number of heads). Recall that the sample space is:

$$S = \{TTT, TTH, THT, HTT, HHT, HTH, THH, HHH\}$$

Let us determine the probability of each outcome using the fact that our flips are independent. Let us start with TTT (no heads) and find $P(TTT)$. By independence and the fact that its a fair coin we get

$$P(TTT) = P(T)P(T)P(T) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

What about an event with one head: TTH?

$$P(TTH) = P(T)P(T)P(H) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

and we get the same probability for the other two permutations: $P(THT) = \dfrac{1}{8}$ and $P(HTT) = \dfrac{1}{8}$.

And two heads: HHT?

$$P(HHT) = P(H)P(H)P(T) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

and again the other two permutations gives the same probabilities: $P(HTH) = \dfrac{1}{8}$ and $P(HHT) = \dfrac{1}{8}$.

And finally three heads: HHH:

$$P(HHH) = P(H)P(H)P(H) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

Now let's look at $P(X = 0)$. Well $(X = 0) \equiv \{TTT\}$. So

$$P(X = 0) = P(TTT) = \frac{1}{8}$$

Now $(X = 1) \equiv \{TTH, THT, HTT\}$ and we get

$$\begin{aligned} P(X=1) &= P(TTH, THT, HTT) \\ &= P(TTH) + P(THT) + P(HTT) \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ &= 3 \cdot \frac{1}{8} \end{aligned}$$

If we are interested in two heads:
$(X = 2) \equiv \{HHT, HTH, THH\}$

$$\begin{aligned} P(X = 2) &= P(HHT, HTH, THH) \\ &= P(HHT) + P(HTH) + P(THH) \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ &= 3 \cdot \frac{1}{8} \end{aligned}$$

And finally three heads: $(X = 0) \equiv \{HHH\}$ gives

$$P(X = 3) = P(HHH) = \frac{1}{8}$$

What if its an unfair coin? Let's let $P(T) = .3$ and $P(H) = .7$ and run through the same steps as above. No heads:

$$P(TTT) = P(T)P(T)P(T) = 0.3 \cdot 0.3 \cdot 0.3 = (0.3)^3$$

One head:

$$P(HTT) = P(H)P(T)P(T) = 0.7 \cdot 0.3 \cdot 0.3 = 0.7(0.3)^2$$
$$P(THT) = P(T)P(H)P(T) = 0.3 \cdot 0.7 \cdot 0.3 = 0.7(0.3)^2$$
$$P(TTH) = P(T)P(T)P(H) = 0.3 \cdot 0.3 \cdot 0.7 = 0.7(0.3)^2$$

Two heads:

$$P(HHT) = P(H)P(H)P(T) = 0.7 \cdot 0.7 \cdot 0.3 = (0.7)^2 \cdot 0.3$$

$$P(HTH) = P(H)P(T)P(H) = 0.7 \cdot 0.3 \cdot 0.7 = (0.7)^2 \cdot 0.3$$

$$P(THH) = P(T)P(H)P(H) = 0.3 \cdot 0.7 \cdot 0.7 = (0.7)^2 \cdot 0.3$$

Three heads

$$P(HHH) = P(H)P(H)P(H) = 0.7 \cdot 0.7 \cdot 0.7 = (0.7)^3$$

So now calculating the probability of getting 0, 1, 2, or 3 heads is:

$$P(X=0) = P(TTT) = (0.3)^3$$

$$\begin{aligned} P(X=1) &= P(TTH, THT, HTT) \\ &= P(TTH) + P(THT) + P(HTT) \\ &= 0.7(0.3)^2 + 0.7(0.3)^2 + 0.7(0.3)^2 \\ &= 3(0.7)(0.3)^2 \end{aligned}$$

$$\begin{aligned} P(X=2) &= P(HHT, HTH, THH) \\ &= P(HHT) + P(HTH) + P(THH) \\ &= (0.7)^2 \cdot 0.3 + (0.7)^2 \cdot 0.3 + (0.7)^2 \cdot 0.3 \\ &= 3(0.7)^2(0.3) \end{aligned}$$

$$P(X=3) = P(HHH) = (0.7)^3$$

So now we can make out probability distribution:

x	0	1	2	3
$P(X = x)$	$1(0.3)^3$	$3(0.7)(0.3)^2$	$3(0.7)^2(0.3)$	$1(0.7)^3$

Doing the same steps we can find that the probability distributions for flipping a coin twice and four times are:

x	0	1	2
$P(X=x)$	$1(0.7)^0(0.3)^2$	$2(0.7)^1(0.3)^1$	$1(0.7)^2(0.3)^0$

x	0	1	2	3
$P(X=x)$	$1(0.7)^0(0.3)^3$	$3(0.7)^1(0.3)^2$	$3(0.7)^2(0.3)^1$	$1(0.7)^3(0.3)^0$

x	0	1	2	3	4
$P(X=x)$	$1(0.7)^0(0.3)^4$	$4(0.7)^1(0.3)^3$	$6(0.7)^2(0.3)^2$	$4(0.7)^3(0.3)^1$	$1(0.7)^4(0.3)^0$

I've add some information that we normally don't write to help recognize the patterns.
What patterns are we seeing?

First recalling that $P(H) = 0.7$ and $P(H) = 0.3$ we see that we have

Number of heads: x	0	1	2
Heads probability	$(0.7)^0$	$(0.7)^1$	$(0.7)^2$
Tails probability	$(0.3)^2$	$(0.3)^1$	$(0.3)^0$

Number of heads: x	0	1	2	3
Heads probabilitiy	$(0.7)^0$	$(0.7)^1$	$(0.7)^2$	$(0.7)^3$
Tails probability	$(0.3)^3$	$(0.3)^2$	$(0.3)^1$	$(0.3)^0$

Number of heads: x	0	1	2	3	4
Heads probabilitiy	$(0.7)^0$	$(0.7)^1$	$(0.7)^2$	$(0.7)^3$	$(0.7)^4$
Tails probability	$(0.3)^4$	$(0.3)^3$	$(0.3)^2$	$(0.3)^1$	$(0.3)^0$

So in general if we flip a coin n times and x is the number of heads we have the following pattern:

Number of heads: x	0	1	2	3	4	$\dots$	n
Heads probabilitiy	$(0.7)^0$	$(0.7)^1$	$(0.7)^2$	$(0.7)^3$	$(0.7)^4$	$\dots$	$(0.7)^n$
Tails probability	$(0.3)^{n-0}$	$(0.3)^{n-1}$	$(0.3)^{n-2}$	$(0.3)^{n-3}$	$(0.3)^{n-4}$	$\dots$	$(0.3)^{n-n} = (0.$

So in general if we flip a coin n times and x is the number of heads we have the following pattern:

Num heads: x	0	1	2	3	4	$\dots$	n
Heads prob	$(0.7)^0$	$(0.7)^1$	$(0.7)^2$	$(0.7)^3$	$(0.7)^4$	$\dots$	$(0.7)^n$
Tails prob	$(0.3)^{n-0}$	$(0.3)^{n-1}$	$(0.3)^{n-2}$	$(0.3)^{n-3}$	$(0.3)^{n-4}$	$\dots$	$(0.3)^{n-n} = (0.3)^0$

So far the formula for the probability of getting x heads when flipping a coin n times is:

$$P(X = x) = c_x(0.7)^x(0.3)^{n-x}$$

If we can just find a pattern for the coefficients c_x we will have a formula for calculating the probability of getting x heads when flipping a coin n times. Lets go back to calculations:

x	0	1	2
$P(X=x)$	$1(0.7)^0(0.3)^2$	$2(0.7)^1(0.3)^1$	$1(0.7)^2(0.3)^0$

x	0	1	2	3
$P(X=x)$	$1(0.7)(0.3)^3$	$3(0.7)(0.3)^2$	$3(0.7)^2(0.3)$	$1(0.7)^3(0.3)^0$

x	0	1	2	3	4
$P(X=x)$	$1(0.7)^0(0.3)^4$	$4(0.7)^1(0.3)^3$	$6(0.7)^2(0.3)^2$	$4(0.7)^3(0.3)^1$	$1(0.7)^4(0.3)^0$

and we see that the coefficients are:

n									
2			1		2		1		
3		1		3		3		1	
4	1		4		6		4		1

and we see that the coefficients are:

$$\begin{array}{c|ccccccccc}
n & & & & & & & & & \\
\hline
2 & & & 1 & & 2 & & 1 & & \\
3 & & 1 & & 3 & & 3 & & 1 & \\
4 & 1 & & 4 & & 6 & & 4 & & 1
\end{array}$$

Well that looks familiar:

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

And we have a formula for each of these values:

n									
2			1		2		1		
3		1		3		3		1	
4	1		4		6		4		1
2			${}_2C_0$		${}_2C_1$		${}_2C_2$		
3		${}_3C_0$		${}_3C_1$		${}_3C_2$		${}_3C_3$	
4	${}_4C_0$		${}_4C_1$		${}_4C_2$		${}_4C_3$		${}_4C_4$

And there are have it, the full formula for the probability of getting x heads when flipping a coin n times with $P(H) = 0.7$ and $P(T) = 0.3$ is

$$P(X = x) = {}_nC_x (0.7)^x (0.3)^{n-x}$$

So the fully general formula for the probability of getting x heads when flipping a coin n times with $P(H) = p$ and $P(T) = q = (1 - p)$ is

$$P(X = x) = {}_nC_x p^x q^{n-x}$$

Formula 1 (Probability for a Binomial Distribution)

For a binomial random variable X, the probability of obtaining x successes in n independent trials is given by

$$P(X = x) = {}_nC_x \cdot p^x (1-p)^{n-x}$$

where

- x *is the number of successes,*
- n *is the number of trails, and*
- p *is the probability of a success on any trial.*

Definition 3 (Binomial Distribution)

The binomial distribution is a discrete probability distribution with a fixed number of independent trials, where each trial has only two possible outcomes and one of these outcomes is counted. It has the following properties:

1. The experiment consists of a fixed number, n, of identical trials.
2. Each trial is independent of the others.
3. For each trial, there are only two possible outcomes. For counting purposes, one outcome is labeled a success, and the other a failure.
4. For every trial, the probability of getting a success is called p. The probability of getting a failure is then $1-p$.
5. The binomial random variable, X, counts the number of successes in n trials.
6. For a binomial distribution, the mean is given by $\mu = np$ and the variance is given by $\sigma^2 = np(1-p)$.

Why tables.

$$n = 4,\ p = 0.25,\ x = 3,\ P(X \leq 3) = ?$$

						p						
n	x	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
2	0	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0625	0.0400	0.0100
	1	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.4375	0.3600	0.1900
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.7290	0.5120	0.4219	0.3430	0.2160	0.1250	0.0640	0.0270	0.0156	0.0080	0.0010
	1	0.9720	0.8960	0.8438	0.7840	0.6480	0.5000	0.3520	0.2160	0.1562	0.1040	0.0280
	2	0.9990	0.9920	0.9844	0.9730	0.9360	0.8750	0.7840	0.6570	0.5781	0.4880	0.2710
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0039	0.0016	0.0001
	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0508	0.0272	0.0037
	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.2617	0.1808	0.0523
	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.6836	0.5904	0.3439
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
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n x 0.10 0.20 0.25 0.30 0.40 0.50 0.60 0.70 0.75 0.80 0.90

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	1	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.4375	0.3600	0.1900
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
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		p										

$n = 4$, $p = 0.25$, $x = 3$, $P(X \leq 3) = 0.9961$

						p						
n	x	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
2	0	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0625	0.0400	0.0100
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n	x	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90
						p						

Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the number of trials and the probability of obtaining a success. Round your answer to four decimal places.

$$P(X \leq 4), n = 7, p = 0.6$$

Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the number of trials and the probability of obtaining a success. Round your answer to four decimal places.

$$P(X < 5), n = 6, p = 0.7$$

Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the number of trials and the probability of obtaining a success. Round your answer to four decimal places.

$$P(X = 4), n = 6, p = 0.3$$

Assume the random variable X has a binomial distribution with the given probability of obtaining a success. Find the following probability, given the number of trials and the probability of obtaining a success. Round your answer to four decimal places.

$$P(X > 4), n = 6, p = 0.4$$

The random variable X is a binomial random variable with $n = 10$ and $p = 0.3$. What is the expected value of X? Do not round your answer.

The random variable X is a binomial random variable with $n = 8$ and $p = 0.4$. What is the standard deviation of X?

A researcher wishes to conduct a study of the color preferences of new car buyers. Suppose that 40% of this population prefers the color green. If 12 buyers are randomly selected, what is the probability that exactly 2 buyers would prefer green? Round your answer to four decimal places.

A real estate agent has 18 properties that she shows. She feels that there is a 30% chance of selling any one property during a week. The chance of selling any one property is independent of selling another property. Compute the probability of selling at least 5 properties in one week. Round your answer to four decimal places.

A quality control inspector has drawn a sample of 13 light bulbs from a recent production lot. If the number of defective bulbs is 1 or more, the lot fails inspection. Suppose 20% of the bulbs in the lot are defective. What is the probability that the lot will fail inspection? Round your answer to four decimal places.

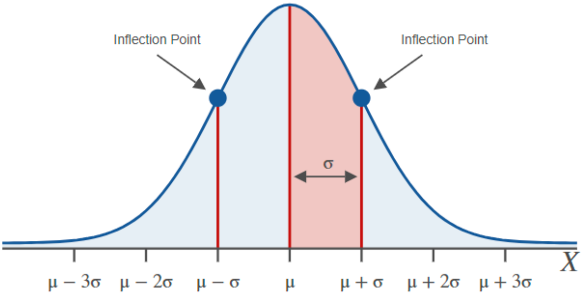
A certain insecticide kills 70% of all insects in laboratory experiments. A sample of 9 insects is exposed to the insecticide in a particular experiment. What is the probability that exactly 4 insects will survive? Round your answer to four decimal places.

Test 1 Spring 2025

Chapter 6 Section 1

Definition 4 (Continuous random variable)

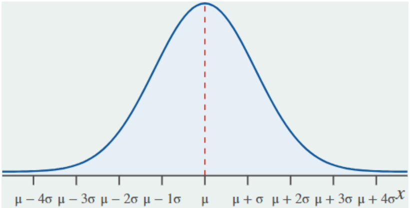
A continuous random variable is a continuous variable whose numeric value is determined by the outcome of a probability experiment.

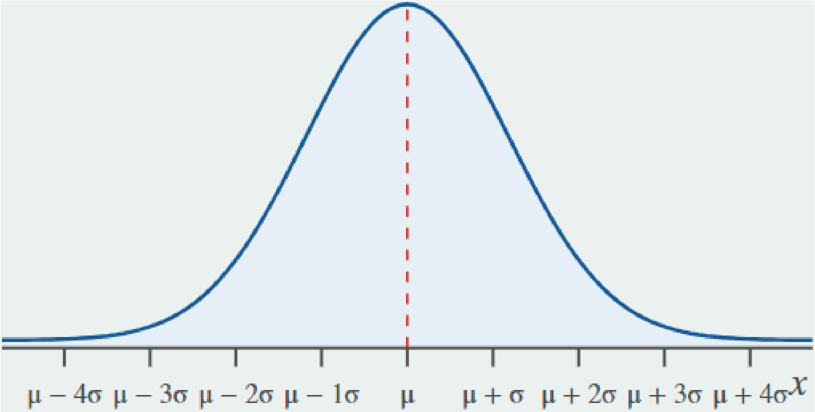

Inflection Point
Inflection Point
σ
X
μ − 3σ
μ − 2σ
μ − σ
μ
μ + σ
μ + 2σ
μ + 3σ

Definition 5 (Normal distribution)

A normal distribution is a probability distribution for a continuous random variable, X, defined completely by its mean and standard deviation, such that the following properties are true:

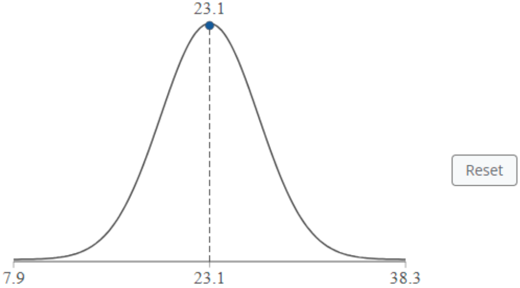
1. A normal distribution is bell-shaped and symmetric about its mean.
2. A normal distribution is completely defined by its mean, μ, and standard deviation, σ.
3. The total area under a normal distribution curve equals 1.
4. The x-axis is a horizontal asymptote for a normal distribution curve.




μ − 4σ
μ − 3σ
μ − 2σ
μ − 1σ
μ
μ + σ
μ + 2σ
μ + 3σ
μ + 4σ
x

Given $X = 11.6$, $\mu = 23.1$, and $\sigma = 2.4$, indicate on the curve where the given X value would be.

The point can be moved by dragging or using the arrow keys. Select the Reset button to reset the point.

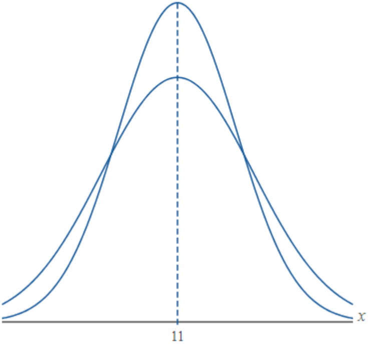


Decide which of the following statements are true.

- ☐ There are an unlimited number of normal distributions.
- ☐ For any normal distribution, only the mean and mode are equal. The median is different from the mean and mode.
- ☐ The line of symmetry for all normal distributions is $x = \mu$.
- ☐ The x-axis is a vertical asymptote for all normal distributions.

Calculate the standard score of the given X value, $x = 88.9$, where $\mu = 94.5$ and $\sigma = 95.1$. Round your answer to two decimal places.

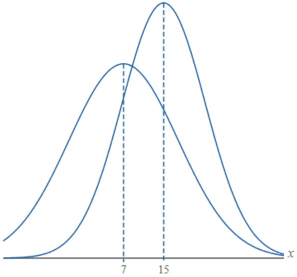
The following is a graph of two normal distributions plotted on the same x-axis.



Based on the graph above, which statement best describes the graph?

- The two distributions have equal means and different standard deviations.
- The two distributions have equal means and equal standard deviations.
- The two distributions have equal means and standard deviations that differ by 11 units.
- The two distributions have equal standard deviations and different means.

The following is a graph of two normal distributions plotted on the same x-axis.



Based on the graph above, which statement best describes the graph?

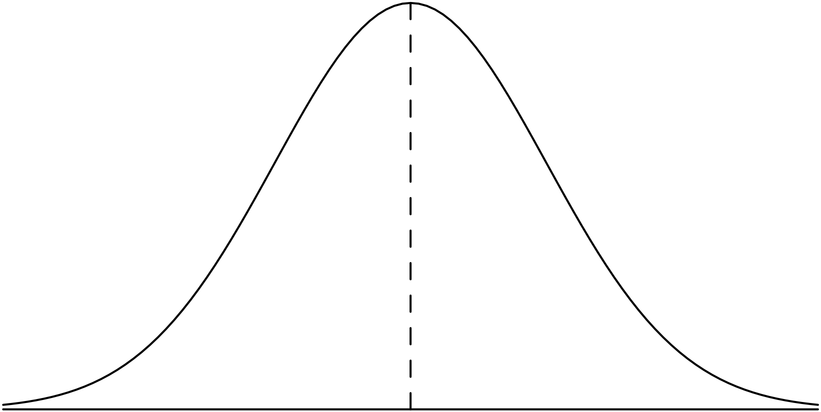
- The two distributions have equal means and standard deviations that differ by 8 units.
- The two distributions have means that differ by 8 units and equal standard deviations.
- The two distributions have means that differ by 8 units and different standard deviations.
- The two distributions have equal means and means standard deviations.

Chapter 6 Section 2

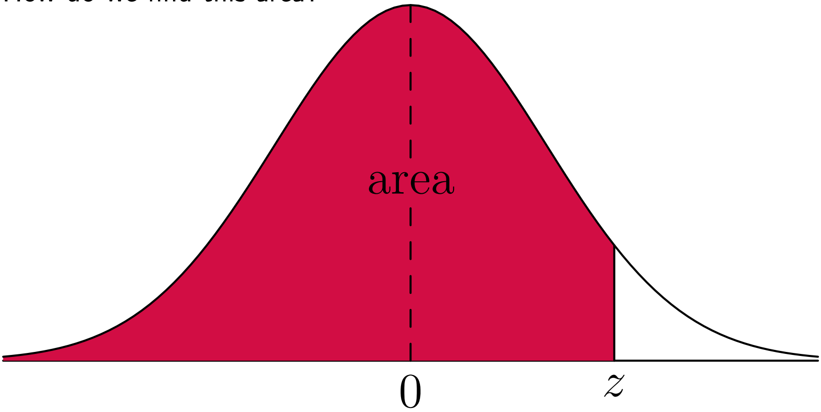
Draw Binomial with $n = 4, p = 0.5$.
Find

1. $P(X = 0) =$
2. $P(X = 1) =$
3. $P(X = 2) =$
4. $P(X = 3) =$
5. $P(X = 4) =$
6. $P(X \leq 0) =$
7. $P(X \leq 1) =$
8. $P(X \leq 2) =$
9. $P(X \leq 3) =$
10. $P(X \leq 4) =$

Probability is area under the curve:



How do we find this area?



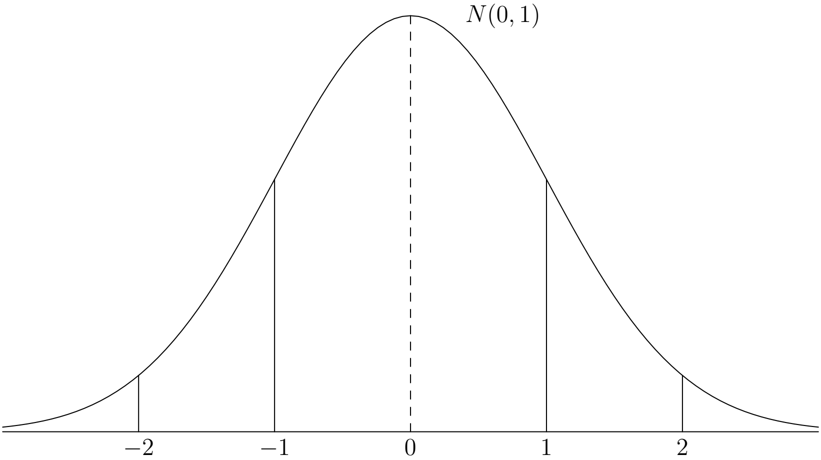
In the case of a normal distribution with mean μ and standard deviation σ its function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Definition 6 (Standard normal distribution)

The standard normal distribution is the normal distribution with $\mu = 0$ and $\sigma = 1$, such that the following properties are true:

1. The standard normal distribution is bell-shaped and symmetric about its mean.
2. The standard normal distribution is completely defined by its mean, $\mu = 0$, and standard deviation, $\sigma = 1$.
3. The total area under the standard normal distribution curve equals 1.
4. The x-axis is a horizontal asymptote for the standard normal distribution curve.


$N(0,1)$
-2
-1
0
1
2

Find the area under the standard normal curve to the left of $z = -1.67$. Round your answer to four decimal places, if necessary.

Find the area under the standard normal curve to the left of $z = -1.67$. Round your answer to four decimal places, if necessary.

$$z = -1.6 + 0.07^{\dagger}$$

[†] Technically this should say $-(1.6 + 0.07)$ or $-1.6 - 0.07$. Think of the $+$ as the word "and".

Find the area under the standard normal curve to the left of $z = -1.67$. Round your answer to four decimal places, if necessary.

$$z = -1.6 + 0.07 = \text{row value} + \text{column value}$$

Find the area under the standard normal curve to the left of $z = -1.67$. Round your answer to four decimal places, if necessary.

$$z = -1.6 + 0.07 = \text{row value} + \text{column value}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Find the area under the standard normal curve to the left of $z = -1.67$. Round your answer to four decimal places, if necessary.

$$z = -1.6 + 0.07 = \text{row value} + \text{column value}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Find the area under the standard normal curve to the right of $z = 1.38$. Round your answer to four decimal places, if necessary.

Find the area under the standard normal curve to the right of $z = 2.38$. Round your answer to four decimal places, if necessary.

Find the area under the standard normal curve to the right of $z = -2.37$. Round your answer to four decimal places, if necessary.

Find the area under the standard normal curve between $z = 1.38$ and $z = 2.92$. Round your answer to four decimal places, if necessary.

Find the area under the standard normal curve between $z = -1.44$ and $z = 1.44$. Round your answer to four decimal places, if necessary.

Find the area under the standard normal curve to the left of $z = -1.44$ and to the right of $z = 1.44$. Round your answer to four decimal places, if necessary.

Find the specified probability. Round your answer to four decimal places, if necessary.

$$P(0 < z < 2.03)$$

Use the z-score formula, $z = \dfrac{x - \mu}{\sigma}$, and the information below to find the mean μ. Round your answer to one decimal place, if necessary.

$$z = 1.75, \quad x = 10.9, \quad \sigma = 4.4$$

Chapter 6 Section 3

The life of light bulbs is distributed normally. The variance of the lifetime is 625 hours and the mean lifetime of a bulb is 510 hours. Find the probability of a bulb lasting for at most 550 hours. Round your answer to four decimal places.

A soft drink machine outputs a mean of 25 ounces per cup. The machine's output is normally distributed with a standard deviation of 4 ounces. What is the probability of filling a cup between 22 and 33 ounces? Round your answer to four decimal places.

The time spent waiting in the line is approximately normally distributed. The mean waiting time is 6 minutes and the standard deviation of the waiting time is 1 minutes. Find the probability that a person will wait for more than 8 minutes. Round your answer to four decimal places.

Trucks in a delivery fleet travel a mean of 120 miles per day with a standard deviation of 18 miles per day. The mileage per day is distributed normally. Find the probability that a truck drives between 150 and 156 miles in a day. Round your answer to four decimal places.

The weights of newborn baby boys born at a local hospital are believed to have a normal distribution with a mean weight of 3844 grams and a standard deviation of 612 grams. If a newborn baby boy born at the local hospital is randomly selected, find the probability that the weight will be less than 4456 grams. Round your answer to four decimal places.

Chapter 6 Section 4

ADD notes on how to do a “reverse table read”.

What value of z divides the standard normal distribution so that half the area is on one side and half is on the other? Round your answer to two decimal places.

Find the value of z such that 0.1401 of the area lies to the left of z. Round your answer to two decimal places.

Find the value of z such that 0.14 of the area lies to the left of z. Round your answer to two decimal places.

Find the value of z such that 0.03 of the area lies to the right of z. Round your answer to two decimal places.

Find the value of z such that 0.05 of the area lies to the right of z. Round your answer to two decimal places.

Find the value of z such that 0.8664 of the area lies between $-z$ and z. Round your answer to two decimal places.

Suppose SAT Writing scores are normally distributed with a mean of 497 and a standard deviation of 109. A university plans to award scholarships to students whose scores are in the top 8%. What is the minimum score required for the scholarship? Round your answer to the nearest whole number, if necessary.