MS 114 In-class Problems

September 30, 2025

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The table below is taken from the website of the U.S. Department of Labor. It shows the minimum wage for each decade from 1950 to 2010. The figures are adjusted for inflation and expressed in constant 2012 dollars.

$y =$ Year	$m =$ Minimum wage
1950	\$7.01
1960	\$7.59
1970	\$9.28
1980	\$8.46
1990	\$6.66
2000	\$6.90
2010	\$7.67

The table below shows the highest grossing movies of the given year. The amount is the domestic box office gross, in millions of dollars. Let $G(y)$ be the gross of the highest grossing movies of that given year.

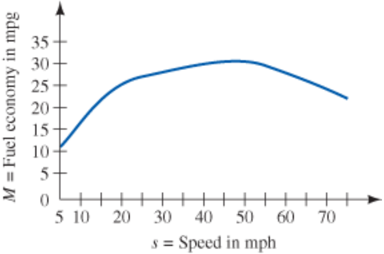
Year	Movie	Amount (millions)
2006	Pirates of the Caribbean: Dead Man's Chest	423.32
2007	Spider-Man 3	336.53
2008	The Dark Knight	533.35
2009	Avatar	760.51
2010	Toy Story 3	415.00
2011	Harry Potter and the Deathly Hallows: Part 2	381.01
2012	The Avengers	623.28
2013	The Hunger Games: Catching Fire	424.67
2014	American Sniper	350.13

A chart from Dick's Sporting Goods gives the recommended bat length B in inches for a man weighing between 161 and 170 pounds as a function of his height h in inches. The table is partially reproduced below.

h = Height	B = Bat length
45–48	30
49–52	31
53–56	31
57–60	32
61–64	32
65–68	33
69–72	33
73+	33

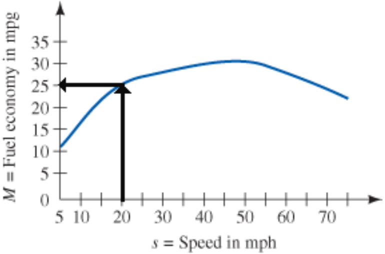
Chapter 1 Section 3

Many factors affect fuel economy, but a website maintained by the U.S. government warns that "gas mileage usually decreases rapidly at speeds above miles per hour...."
The site includes the graph below (Fig 1.7 in our text book). It shows that the fuel economy M as a function of the speed s, so that $M = M(s)$.



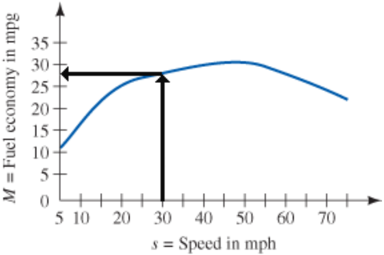
It is customary to describe this as a graph of M versus s or as a graph of M against s. These phrases indicate that the horizonal axis corresponds to s and the vertical axis corresponds to M.

What is the fuel economy at 20 mph?
Find the value 20 on the horizontal axis then go vertical until you hit the graph. Next go horizontal from that point until you hit the vertical axis.



We can see that this hits the vertical axis at 25.
Written in functional notation we have $M(20) = 25$.

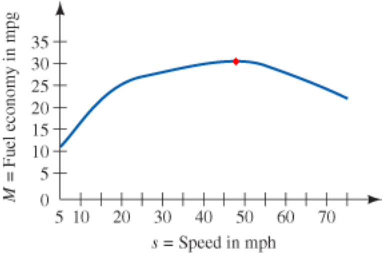
What if we repeat this at $s = 30$?



Now it is harder to see where you are hitting the vertical axis, but it looks to be close to 28, so $M(30) = 28$.

The graphical representation of a function may only allow us to make approximations of the function values. But they also allow us to see important features of the graph that cannot be seen from an equation.

One of these is the maximum value of the function.

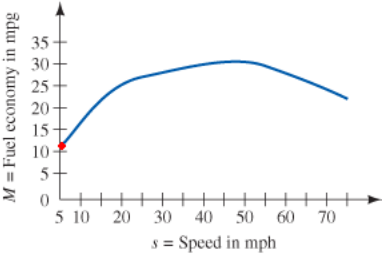


From this we can see that the maximum fuel economy is around 30 mpg and this occurs at around 50 mph.

When a function increases to a point and then decreases from that point that will always give us a maximum.

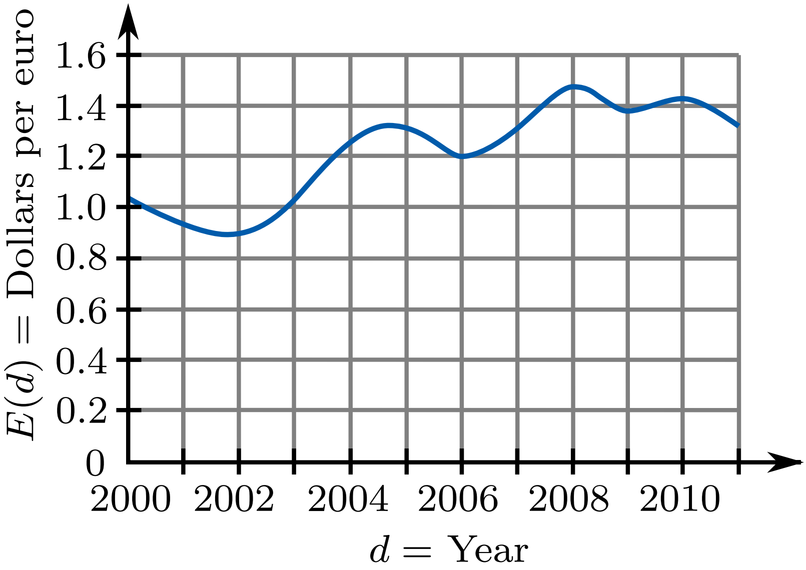
When a function decreases to a point and then increases that point will be a minimum.

Another place where a max or min can occur is at the endpoints of the function.

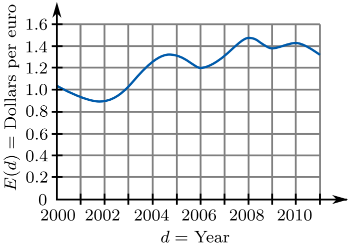


We see that the function has a minimum value of around 11 mpg at a speed of 5 mph.

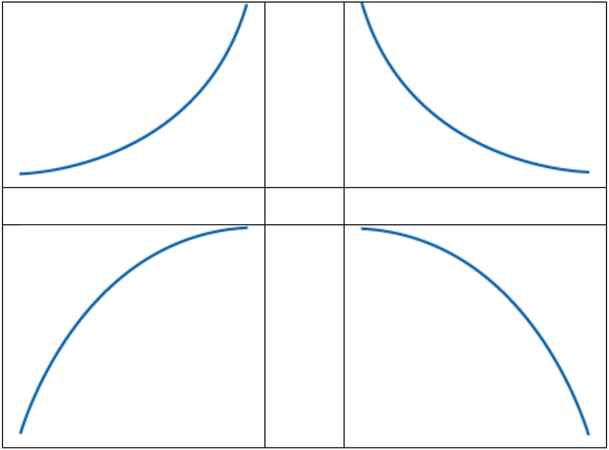
The value $E = E(d)$, in U.S. dollars, of the euro as a function of the date d is shown in the graph below.



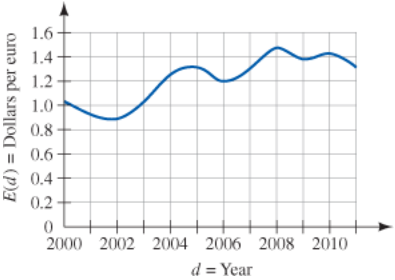
The value $E = E(d)$, in U.S. dollars, of the euro as a function of the date d is shown in the graph below.



1. Explain the meaning of $E(2003)$ and estimate its value.
2. From 2000 through 2011, what was the largest value the euro attained? When did that happen?
3. What was the average yearly increase in the value of the euro from 2006 to 2009?
4. During which one-year period was the graph increasing most rapidly?
5. As an American investor, would you have made money if you bought euros in 2002 and sold them in 2008?



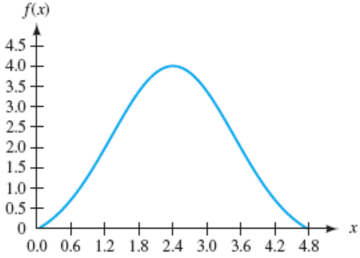
The value $E = E(d)$, in U.S. dollars, of the euro as a function of the date d is shown in the graph below.



1. From 2000 to late 2003, is the graph concave up or concave down?

2. Explain in practical terms what the concavity means about the value of the euro during this period.

1.3.SB.005 The following is the graph of a function $f = f(x)$.

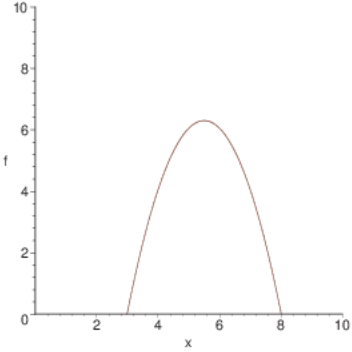


1. What is the value of $f(0.6)$?

2. What is the smallest value of x for which $f(x) = 1.5$?

3. What is the largest value of x for which $f(x) = 2.5$?

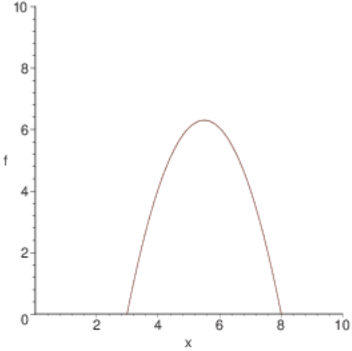
1.3.SB.007* The following is the graph of a function $f = f(x)$. Where does the graph reach a maximum, and what is that maximum value?



1. $x =$

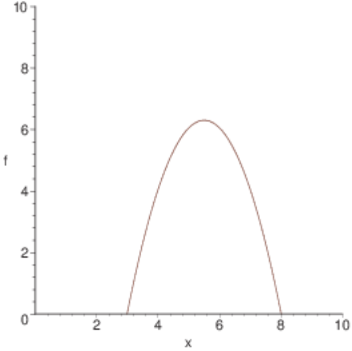
2. $f(x) =$

1.3.SB.008 The following is the graph of a function $f = f(x)$. Where is the graph increasing?



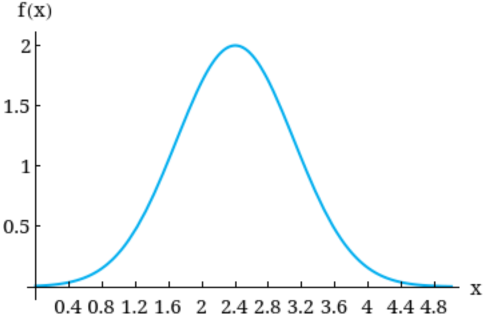
- $3.0 < x < 5.5$
- $x > 3.0$
- $x < 5.5$
- $x < 3.0 \cup x > 5.5$
- $-\infty < x < \infty$

1.3.SB.009 The following is the graph of a function $f = f(x)$. Where is the graph decreasing?



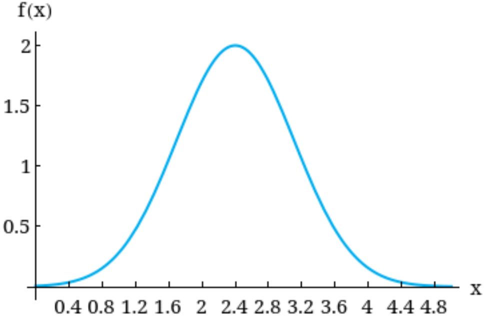
- $5.5 < x < 8.0$
- $x < 8.0$
- $x > 5.5$
- $x < 5.5 \cup x > 8.0$
- $-\infty < x < \infty$

1.3.SB.011* The following is the graph of a function $f = f(x)$. What is the concavity of the graph between $x = 0.4$ and $x = 1.6$?



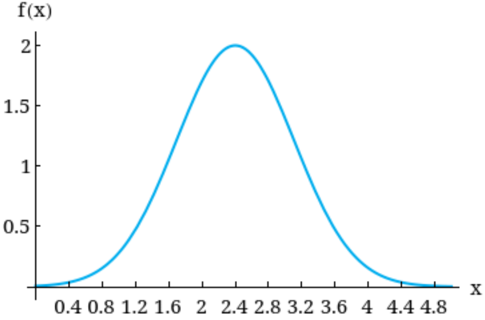
- concave up
- concave down

1.3.SB.012* The following is the graph of a function $f = f(x)$. What is the concavity of the graph between $x = 3.2$ and $x = 4.8$?



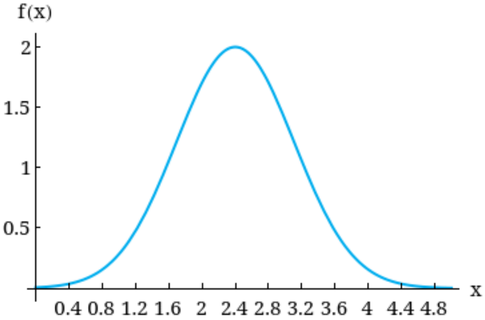
- concave up
- concave down

1.3.SB.013* The following is the graph of a function $f = f(x)$. What is the concavity of the graph between $x = 2.0$ and $x = 2.8$?



- concave up
- concave down

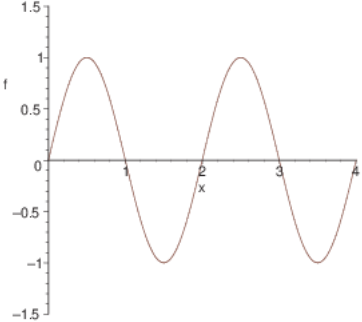
1.3.SB.014* The following is the graph of a function $f = f(x)$. Where on the graph are there points of inflection?



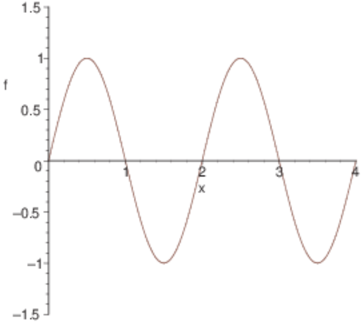
1. $x =$ (smaller x-value)
2. $x =$ (larger x-value)

1.3.SB.017 The following is the graph of a function $f = f(x)$. At what values of x does the graph reach

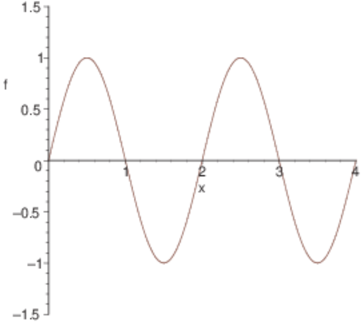
1. a maximum value?
2. a minimum value?



1.3.SB.023* The following is the graph of a function $f = f(x)$. Where is the graph increasing and concave up?



1.3.SB.019* The following is the graph of a function $f = f(x)$. Find the x-value of the inflection points of $f(x)$.



Chapter 1 Section 4

Suppose there are initially 2000 bacteria in a petri dish. The bacteria reproduce by cell division, and each hour the number of bacteria doubles. This is a verbal description of a function $N = N(t)$, where N is the number of bacteria present at time t.
It is common in situations like this to begin at time $t = 0$.
Thus, $N(0)$ is the number of bacteria we started with, 2000.

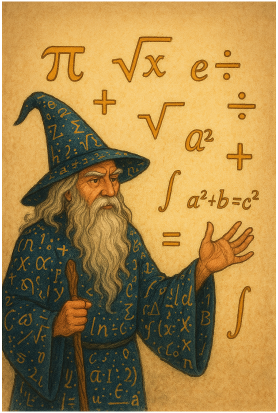
$N(0) =$

$N(1) =$

$N(2) =$

$N(3) =$

Finding $N(6)$ or any larger integer wouldn't be difficult, though tedious, how would we find the value for $t = 4.5$, $N(4.5) = ?$
Using "mathemagics"



we get the function

$$N(t) = 2000 \cdot 2^t$$

Now given

$$N(t) = 2000 \cdot 2^t$$

$N(0) =$

$N(1) =$

$N(2) =$

$N(3) =$

and now we can find:

$N(4.5) =$

A rental car agency charges \$15 per day plus \$0.27 per mile to rent a car. Determine a function that can be used to calculate the cost of daily car rentals.

- Step 1 Identify the function and the things on which it depends, and write the relationships you know in a formula using words.
- Step 2 Select and record letter names for the function and for each of the variables involved, and state their units.
- Step 3 Replace the words in Step 1 by the letters identified in step 2 and appropriate information from the verbal description.

A rental car agency charges \$15 per day plus \$0.27 per mile to rent a car. Determine a function that can be used to calculate the cost of daily car rentals.

Step 1

Step 2

Step 3

Ex 1 Extreme Protection, Inc. manufactures helmets for skiing and snowboarding. The fixed costs for one model of helmet are \$4700 per month. Materials and labor for each helmet of this model are \$50, and the company sells this helmet to dealers for \$70 each. (Let x represent the number of helmets sold. Let C and R be measured in dollars.)

1. For this helmet, write the function for monthly total costs $C(x)$.

2. Write the function for total revenue $R(x)$.

Ex 2 A manufacturer of DVD players has weekly fixed costs of \$1,520 and variable costs of \$13.50 per unit for one particular model. The company sells this model to dealers for \$18.50 each.

1. For this model DVD player, write the function for weekly total costs $C(x)$.

2. Write the function for total revenue $R(x)$.

Ex 3 The federal tax code allows some items used for business purposes to be *depreciated*. That is, their taxable value decreases over time. A new van used in your delivery business has a taxable value of \$22,000. The tax code allows you to depreciate this van by \$2300 per year. Find a formula that gives the taxable value T, in dollars, of the van after n years of depreciation.

Ex 4 The total investment a jeweler has in a gem-quality diamond is the price paid for the rough stone plus the amount paid to work the stone. Suppose the gem cutter earns 40 per hour. Find the function that calculates the total investment of working a stone. If a particular the stone costs \$320 what would the investment be?

Chapter 3 Section 1

Figure 3.1

A ramp on a retaining wall

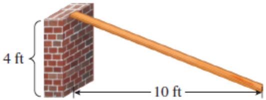


Figure 3.2

Representing the ramp line on coordinate axes

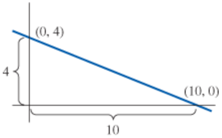


Figure 3.3

The roof of a building

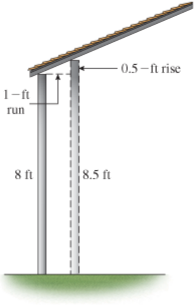


Figure 3.5

Extending the roof line to its peak

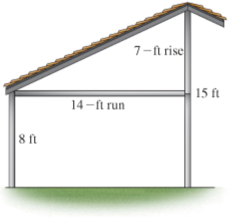


Figure 3.6

Finding the horizontal intercept

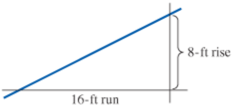


Figure 3.9

A circus tent

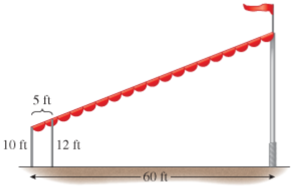
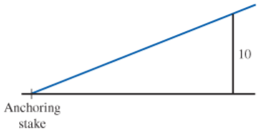


Figure 3.10

An anchor rope



Chapter 3 Section 2

Suppose the CEO of a company wants to have a dinner catered for employees. He finds that he must pay a dining hall rental fee of \$275 and an additional \$28 for each meal served.
Then the cost $C = C(n)$, in dollars, is a function of the number n of meals served. If an unexpected guest arrives for dinner, then the CEO will have to pay an additional \$28.
Suppose, for example, that the caterer, having anticipated that 35 people would attend the dinner, sent the CEO a bill for \$1255. But 48 people actually attended the dinner. What should the total price of the dinner be under these circumstances?

Ex 3.3 Oklahoma Income Tax The amount of income tax $T = T(I)$, in dollars, owed to the state of Oklahoma is a linear function of the taxable income I, in dollars, at least over a suitably restricted range of incomes.

According to the Oklahoma Income Tax table for the year 2015, a single Oklahoma resident taxpayer with a taxable income of $15,000 owes $579 in Oklahoma income tax. In functional notation, this is $T(15,000) = 579$. If the taxable income is $15,500, then the table shows a tax liability of $605.

Ex 3.3 Oklahoma Income Tax Part 1

$T = T(I)$, $T(15,000) = 579$. If the taxable income is \$15,500, then the table shows a tax liability of \$605.

Calculate the rate of change in T with respect to I, and explain in practical terms what it means.

Ex 3.3 Oklahoma Income Tax Part 2

$T = T(I)$, $T(15,000) = 579$. If the taxable income is \$15,500, then the table shows a tax liability of \$605.

How much does the taxpayer owe if the taxable income is \$15,350?

Ex 3.3 Oklahoma Income Tax Part 1 Function

$T = T(I)$, $T(15,000) = 579$. If the taxable income is $15,500, then the table shows a tax liability of $605.

Calculate the rate of change in T with respect to I, and explain in practical terms what it means.

Ex 3.3 Oklahoma Income Tax Part 2 Function

$T = T(I)$, $T(15,000) = 579$. If the taxable income is \$15,500, then the table shows a tax liability of \$605.

How much does the taxpayer owe if the taxable income is \$15,350?

If we look more carefully at the catered dinner, we can write a formula for the total cost $C = C(n)$ when there are n dinner guests:

$$\text{Cost} = \text{Cost of food} + \text{Rent}$$

Ex 3.4 Selling Jewelry at an Art Fair

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Ex 3.4 Selling Jewelry at an Art Fair Part 1

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Explain why the function that shows your net income (revenue from sales minus rental fee) as a function of the number of necklaces sold is a linear function.

Ex 3.4 Selling Jewelry at an Art Fair Part 2

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.
Write a formula for this function.

Ex 3.4 Selling Jewelry at an Art Fair Part 3

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Use functional notation to show your net income if you sell 25 necklaces, and then calculate that value.

Ex 3.4 Selling Jewelry at an Art Fair Part 4, 5, 6

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Plot

Initial value

Horizontal intercept

Getting a linear equation if you know the slope and initial value:
In Example 3.4, we were effectively told the slope, 32, of the linear function P and its initial value, -192.

Getting a linear equation if you know the slope and one data point:
Suppose that we are given the information for Example 3.4 in a different way: We are told that the price for each necklace is \$32 and that if we sell $n = 8$ necklaces, we will have a net income of $P = 64$ dollars.

Getting a linear equation from two data points:
Suppose the information in Example 3.4 were given as two data points.
For example, suppose that we make a net income of $P = 64$ dollars when we sell $n = 8$ necklaces and a net income of $P = 160$ dollars when $n = 11$ necklaces are sold.