MS 120 PDF Extra Problems

Round to 6 decimal places.

1. (a) Is the following a pdf (probability density function)?

$$f(x) = x + 1; x \in [-1, 1]$$

   (b) If $f(x)$ in (a) is not a pdf, can we modify it to make it into one?

   (c) If either (a) or (b) is yes, find:

   i. $\Pr(-0.5 \leq X \leq 0.5) =$

   ii. $\Pr(-1 \leq X \leq 0.75) =$

   iii. $\Pr(0.75 \leq X \leq 2) =$

2. (a) Is the following a pdf (probability density function)?

$$f(x) = 3x^2; x \in [0, 4]$$

   (b) If $f(x)$ in (a) is not a pdf, can we modify it to make it into one?

   (c) If either (a) or (b) is yes, find:

   i. $\Pr(0 \leq X \leq 3) =$

   ii. $\Pr(3 \leq X \leq 4) =$

   iii. $\Pr(0.25 \leq X \leq 2.7) =$

3. (a) Is the following a pdf (probability density function)?

$$f(x) = \begin{cases} 2x & , 0 \leq x \leq 3 \\ -6x + 24 & , 3 < x \leq 4 \end{cases}; x \in [0, 4]$$

   (b) If $f(x)$ in (a) is not a pdf, can we modify it to make it into one?

   (c) If either (a) or (b) is yes, find:

   i. $\Pr(0 \leq X \leq 3) =$

   ii. $\Pr(3 \leq X \leq 4) =$

   iii. $\Pr(2.5 \leq X \leq 3.5) =$

4. (a) Is the following a pdf (probability density function)?

$$f(x) = -x + 5; x \in [0, 5]$$

   (b) If $f(x)$ in (a) is not a pdf, can we modify it to make it into one?

   (c) If either (a) or (b) is yes, find:

   i. $\Pr(0 \leq X \leq 3) =$

   ii. $\Pr(3 \leq X \leq 5) =$

   iii. $\Pr(1 \leq X \leq 2) =$

5. (a) Is the following a pdf (probability density function)?

$$f(x) = x^3; x \in [-1, 3]$$

   (b) If $f(x)$ in (a) is not a pdf, can we modify it to make it into one?

   (c) If either (a) or (b) is yes, find:

   i. $\Pr(-1 \leq X \leq 2) =$

   ii. $\Pr(2 \leq X \leq 3) =$

   iii. $\Pr(-0.5 \leq X \leq 1.25) =$

6. (a) Is the following a pdf (probability density function)?

$$f(x) = \frac{3\sqrt{x}}{16}; x \in [0, 4]$$

   (b) If $f(x)$ in (a) is not a pdf, can we modify it to make it into one?

   (c) If either (a) or (b) is yes, find:

   i. $\Pr(0 \leq X \leq 2) =$

   ii. $\Pr(1 \leq X \leq 3) =$

   iii. $\Pr(0.5 \leq X \leq 3.75) =$

MS 120 PDF Extra Problems Answers

1. (a) No, because the integral is $2 \neq 1$.
   (b)
   $$g(x) = (x+1)/2; x \in [-1, 1]$$
   (c) i. $\Pr(-0.5 \leq X \leq 0.5) = 0.500000$
   ii. $\Pr(-1 \leq X \leq 0.75) = 0.765625$
   iii. $\Pr(0.75 \leq X \leq 2) = 0.234375$

2. (a) No, because the integral is $64 \neq 1$.
   (b)
   $$f(x) = \frac{3}{64}x^2; x \in [0, 4]$$
   (c) i. $\Pr(0 \leq X \leq 3) = 0.421875$
   ii. $\Pr(3 \leq X \leq 4) = 0.578125$
   iii. $\Pr(0.25 \leq X \leq 2.7) = 0.307303$

3. (a) No, because the integral is $12 \neq 1$.
   (b)
   $$f(x) = \begin{cases} \frac{1}{6}x & , 0 \leq x \leq 3 \\ -\frac{1}{2}x + 2 & , 3 < x \leq 4 \end{cases}; x \in [0, 4]$$
   (c) i. $\Pr(0 \leq X \leq 3) = 0.75$
   ii. $\Pr(3 \leq X \leq 4) = 0.25$
   iii. $\Pr(2.5 \leq X \leq 3.5) = 0.416667$

4. (a) No, because the integral is $\frac{25}{2} \neq 1$.
   (b)
   $$f(x) = -\frac{2}{25}x + \frac{2}{5}; x \in [0, 5]$$
   (c) i. $\Pr(0 \leq X \leq 3) = 0.84$
   ii. $\Pr(3 \leq X \leq 5) = 0.16$
   iii. $\Pr(1 \leq X \leq 2) = 0.28$

5. (a) No, because $f(-1) = -1 < 0$.
   (b) No.
   (c) i. $\Pr(-1 \leq X \leq 2) = \text{DNE}$
   ii. $\Pr(2 \leq X \leq 3) = \text{DNE}$
   iii. $\Pr(-0.5 \leq X \leq 1.25) = \text{DNE}$

6. (a) Yes, because the integral is 1 and $f(x) \geq 0$ for every value of x in $[0, 4]$.
   (b) NA
   (c) i. $\Pr(0 \leq X \leq 2) = 0.353553$
   ii. $\Pr(1 \leq X \leq 3) = 0.524519$
   iii. $\Pr(0.5 \leq X \leq 3.75) = 0.863536$