MS 114 In-class Problems

October 14, 2025

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The table below is taken from the website of the U.S. Department of Labor. It shows the minimum wage for each decade from 1950 to 2010. The figures are adjusted for inflation and expressed in constant 2012 dollars.

$y =$ Year	$m =$ Minimum wage
1950	\$7.01
1960	\$7.59
1970	\$9.28
1980	\$8.46
1990	\$6.66
2000	\$6.90
2010	\$7.67

The table below shows the highest grossing movies of the given year. The amount is the domestic box office gross, in millions of dollars. Let $G(y)$ be the gross of the highest grossing movies of that given year.

Year	Movie	Amount (millions)
2006	Pirates of the Caribbean: Dead Man's Chest	423.32
2007	Spider-Man 3	336.53
2008	The Dark Knight	533.35
2009	Avatar	760.51
2010	Toy Story 3	415.00
2011	Harry Potter and the Deathly Hallows: Part 2	381.01
2012	The Avengers	623.28
2013	The Hunger Games: Catching Fire	424.67
2014	American Sniper	350.13

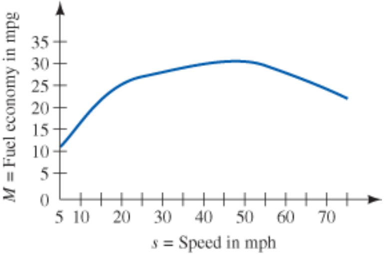
A chart from Dick's Sporting Goods gives the recommended bat length B in inches for a man weighing between 161 and 170 pounds as a function of his height h in inches. The table is partially reproduced below.

h = Height	B = Bat length
45–48	30
49–52	31
53–56	31
57–60	32
61–64	32
65–68	33
69–72	33
73+	33

Chapter 1 Section 3

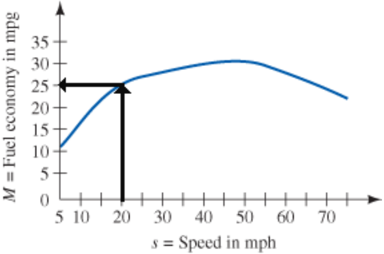
Many factors affect fuel economy, but a website maintained by the U.S. government warns that "gas mileage usually decreases rapidly at speeds above miles per hour. . . ."

The site includes the graph below (Fig 1.7 in our text book). It shows that the fuel economy M as a function of the speed s, so that $M = M(s)$.



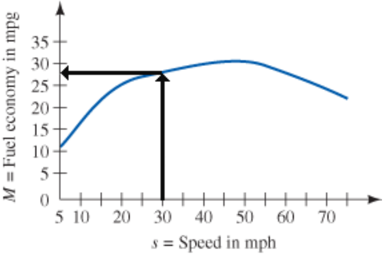
It is customary to describe this as a graph of M versus s or as a graph of M against s. These phrases indicate that the horizonal axis corresponds to s and the vertical axis corresponds to M.

What is the fuel economy at 20 mph?
Find the value 20 on the horizontal axis then go vertical until you hit the graph. Next go horizontal from that point until you hit the vertical axis.



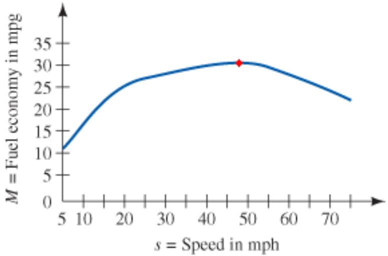
We can see that this hits the vertical axis at 25.
Written in functional notation we have $M(20) = 25$.

What if we repeat this at $s = 30$?



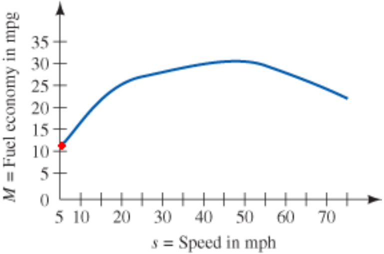
Now it is harder to see where you are hitting the vertical axis, but it looks to be close to 28, so $M(30) = 28$.
The graphical representation of a function may only allow us to make approximations of the function values. But they also allow us to see important features of the graph that cannot be seen from an equation.

One of these is the maximum value of the function.



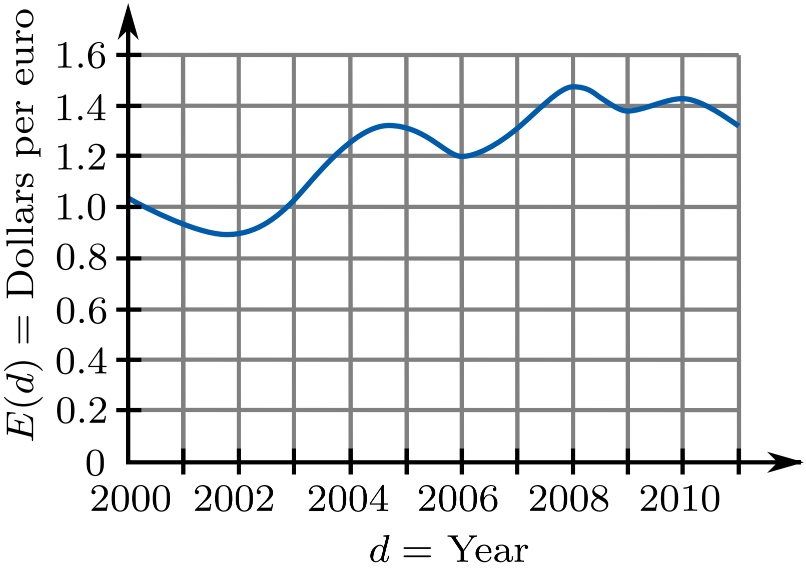
From this we can see that the maximum fuel economy is around 30 mpg and this occurs at around 50 mph.
When a function increases to a point and then decreases from that point that will always give us a maximum.
When a function decreases to a point and then increases that point will be a minimum.

Another place where a max or min can occur is at the endpoints of the function.

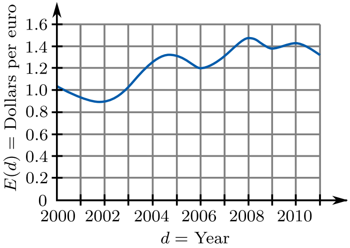


We see that the function has a minimum value of around 11 mpg at a speed of 5 mph.

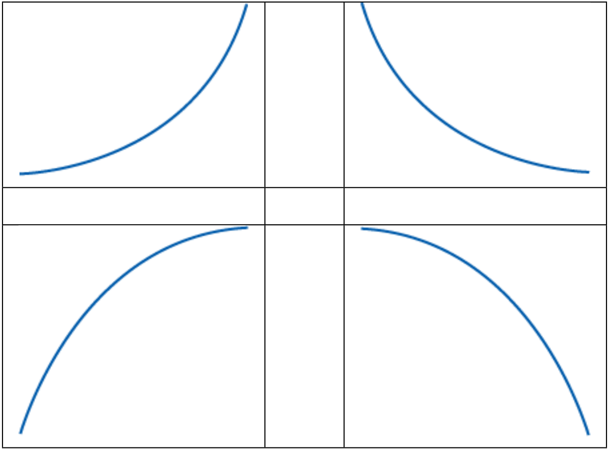
The value $E = E(d)$, in U.S. dollars, of the euro as a function of the date d is shown in the graph below.



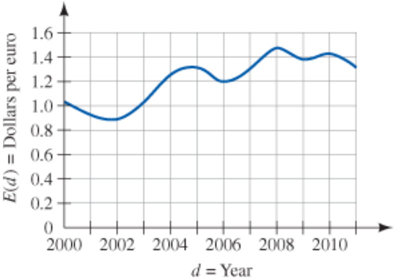
The value $E = E(d)$, in U.S. dollars, of the euro as a function of the date d is shown in the graph below.



1. Explain the meaning of $E(2003)$ and estimate its value.
2. From 2000 through 2011, what was the largest value the euro attained? When did that happen?
3. What was the average yearly increase in the value of the euro from 2006 to 2009?
4. During which one-year period was the graph increasing most rapidly?
5. As an American investor, would you have made money if you bought euros in 2002 and sold them in 2008?



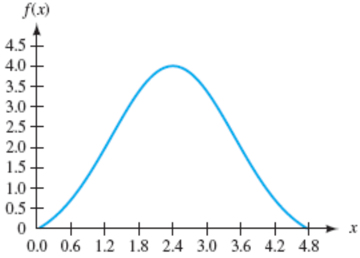
The value $E = E(d)$, in U.S. dollars, of the euro as a function of the date d is shown in the graph below.



1. From 2000 to late 2003, is the graph concave up or concave down?

2. Explain in practical terms what the concavity means about the value of the euro during this period.

1.3.SB.005 The following is the graph of a function $f = f(x)$.

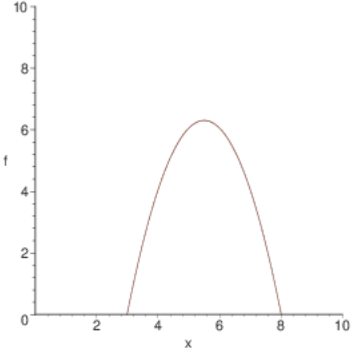


1. What is the value of $f(0.6)$?

2. What is the smallest value of x for which $f(x) = 1.5$?

3. What is the largest value of x for which $f(x) = 2.5$?

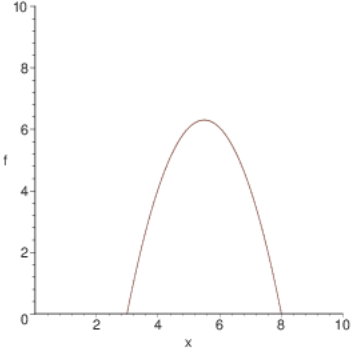
1.3.SB.007* The following is the graph of a function $f = f(x)$. Where does the graph reach a maximum, and what is that maximum value?



1. $x =$

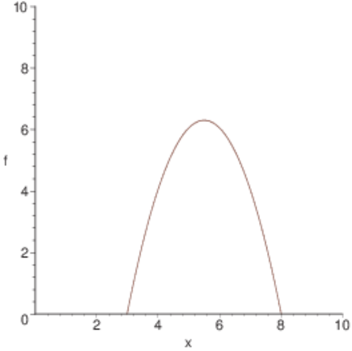
2. $f(x) =$

1.3.SB.008 The following is the graph of a function $f = f(x)$. Where is the graph increasing?



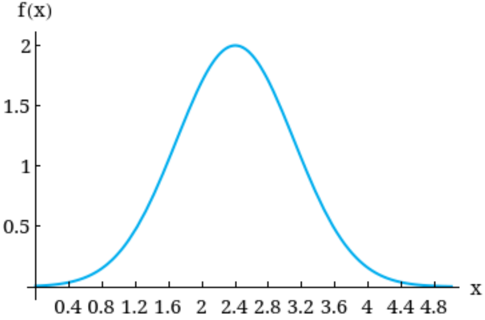
- $3.0 < x < 5.5$
- $x > 3.0$
- $x < 5.5$
- $x < 3.0 \cup x > 5.5$
- $-\infty < x < \infty$

1.3.SB.009 The following is the graph of a function $f = f(x)$. Where is the graph decreasing?



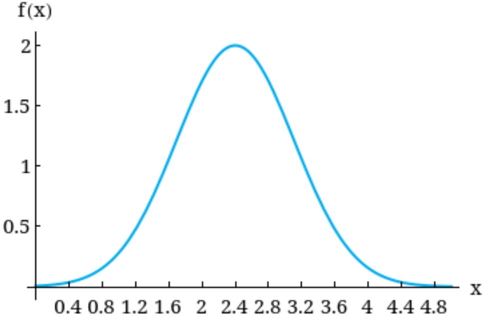
- $5.5 < x < 8.0$
- $x < 8.0$
- $x > 5.5$
- $x < 5.5 \cup x > 8.0$
- $-\infty < x < \infty$

1.3.SB.011* The following is the graph of a function $f = f(x)$. What is the concavity of the graph between $x = 0.4$ and $x = 1.6$?



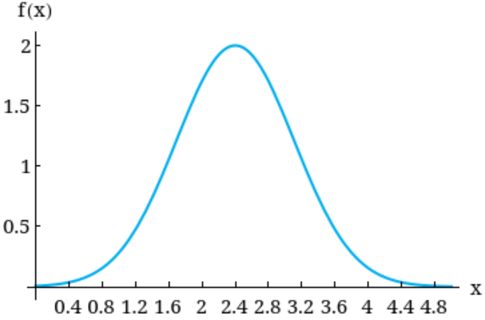
- concave up
- concave down

1.3.SB.012* The following is the graph of a function $f = f(x)$. What is the concavity of the graph between $x = 3.2$ and $x = 4.8$?



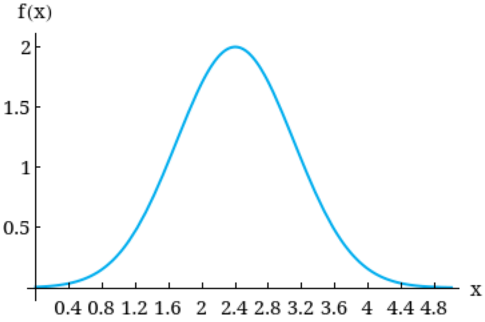
- concave up
- concave down

1.3.SB.013* The following is the graph of a function $f = f(x)$. What is the concavity of the graph between $x = 2.0$ and $x = 2.8$?



- concave up
- concave down

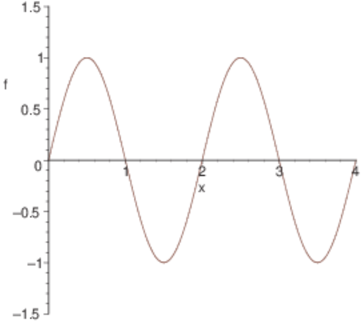
1.3.SB.014* The following is the graph of a function $f = f(x)$. Where on the graph are there points of inflection?



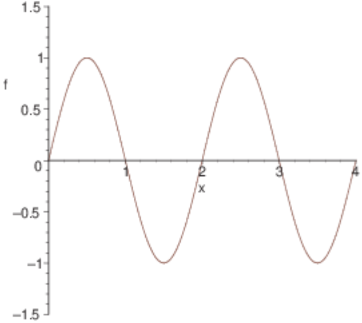
1. $x =$ (smaller x-value)
2. $x =$ (larger x-value)

1.3.SB.017 The following is the graph of a function $f = f(x)$. At what values of x does the graph reach

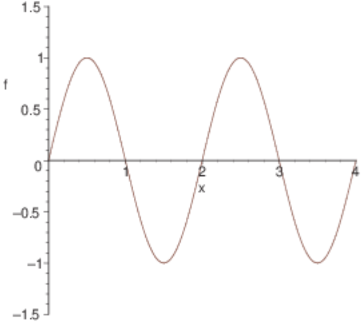
1. a maximum value?
2. a minimum value?



1.3.SB.023* The following is the graph of a function $f = f(x)$. Where is the graph increasing and concave up?



1.3.SB.019* The following is the graph of a function $f = f(x)$. Find the x-value of the inflection points of $f(x)$.



Chapter 1 Section 4

Suppose there are initially 2000 bacteria in a petri dish. The bacteria reproduce by cell division, and each hour the number of bacteria doubles. This is a verbal description of a function $N = N(t)$, where N is the number of bacteria present at time t.

It is common in situations like this to begin at time $t = 0$.

Thus, $N(0)$ is the number of bacteria we started with, 2000.

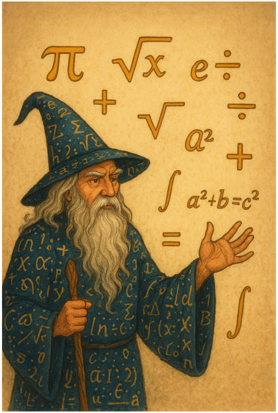
$N(0) =$

$N(1) =$

$N(2) =$

$N(3) =$

Finding $N(6)$ or any larger integer wouldn't be difficult, though tedious, how would we find the value for $t = 4.5$, $N(4.5) =$? Using "mathemagics"



we get the function

$$N(t) = 2000 \cdot 2^t$$

Now given

$$N(t) = 2000 \cdot 2^t$$

$N(0) =$

$N(1) =$

$N(2) =$

$N(3) =$

and now we can find:

$N(4.5) =$

A rental car agency charges \$15 per day plus \$0.27 per mile to rent a car. Determine a function that can be used to calculate the cost of daily car rentals.

- Step 1 Identify the function and the things on which it depends, and write the relationships you know in a formula using words.
- Step 2 Select and record letter names for the function and for each of the variables involved, and state their units.
- Step 3 Replace the words in Step 1 by the letters identified in step 2 and appropriate information from the verbal description.

A rental car agency charges \$15 per day plus \$0.27 per mile to rent a car. Determine a function that can be used to calculate the cost of daily car rentals.

Step 1

Step 2

Step 3

Ex 1 Extreme Protection, Inc. manufactures helmets for skiing and snowboarding. The fixed costs for one model of helmet are \$4700 per month. Materials and labor for each helmet of this model are \$50, and the company sells this helmet to dealers for \$70 each. (Let x represent the number of helmets sold. Let C and R be measured in dollars.)

1. For this helmet, write the function for monthly total costs $C(x)$.

2. Write the function for total revenue $R(x)$.

Ex 2 A manufacturer of DVD players has weekly fixed costs of \$1,520 and variable costs of \$13.50 per unit for one particular model. The company sells this model to dealers for \$18.50 each.

1. For this model DVD player, write the function for weekly total costs $C(x)$.

2. Write the function for total revenue $R(x)$.

Ex 3 The federal tax code allows some items used for business purposes to be *depreciated*. That is, their taxable value decreases over time. A new van used in your delivery business has a taxable value of $22,000. The tax code allows you to depreciate this van by $2300 per year. Find a formula that gives the taxable value T, in dollars, of the van after n years of depreciation.

Ex 4 The total investment a jeweler has in a gem-quality diamond is the price paid for the rough stone plus the amount paid to work the stone. Suppose the gem cutter earns 40 per hour. Find the function that calculates the total investment of working a stone. If a particular the stone costs \$320 what would the investment be?

Chapter 3 Section 1

Figure 3.1

A ramp on a retaining wall

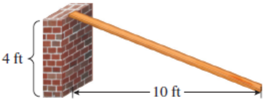


Figure 3.2

Representing the ramp line on coordinate axes

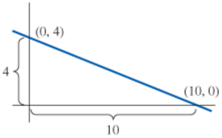


Figure 3.3

The roof of a building

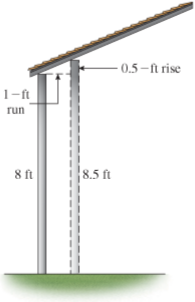


Figure 3.5

Extending the roof line to its peak

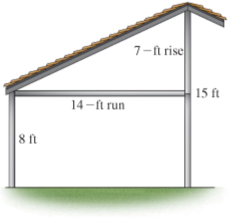


Figure 3.6

Finding the horizontal intercept

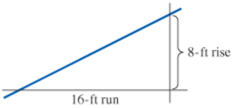


Figure 3.9

A circus tent

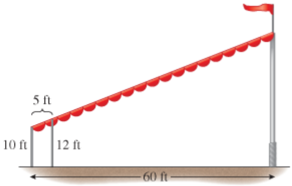
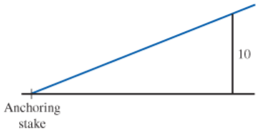


Figure 3.10

An anchor rope



Chapter 3 Section 2

Suppose the CEO of a company wants to have a dinner catered for employees. He finds that he must pay a dining hall rental fee of \$275 and an additional \$28 for each meal served.
Then the cost $C = C(n)$, in dollars, is a function of the number n of meals served. If an unexpected guest arrives for dinner, then the CEO will have to pay an additional \$28.
Suppose, for example, that the caterer, having anticipated that 35 people would attend the dinner, sent the CEO a bill for \$1255. But 48 people actually attended the dinner. What should the total price of the dinner be under these circumstances?

Ex 3.3 Oklahoma Income Tax The amount of income tax $T = T(I)$, in dollars, owed to the state of Oklahoma is a linear function of the taxable income I, in dollars, at least over a suitably restricted range of incomes.
According to the Oklahoma Income Tax table for the year 2015, a single Oklahoma resident taxpayer with a taxable income of $15,000 owes $579 in Oklahoma income tax. In functional notation, this is $T(15,000) = 579$. If the taxable income is $15,500, then the table shows a tax liability of $605.

Ex 3.3 Oklahoma Income Tax Part 1

$T = T(I)$, $T(15,000) = 579$. If the taxable income is $15,500, then the table shows a tax liability of $605.

Calculate the rate of change in T with respect to I, and explain in practical terms what it means.

Ex 3.3 Oklahoma Income Tax Part 2

$T = T(I)$, $T(15,000) = 579$. If the taxable income is \$15,500, then the table shows a tax liability of \$605.

How much does the taxpayer owe if the taxable income is \$15,350?

Ex 3.3 Oklahoma Income Tax Part 1 Function

$T = T(I)$, $T(15,000) = 579$. If the taxable income is \$15,500, then the table shows a tax liability of \$605.

Calculate the rate of change in T with respect to I, and explain in practical terms what it means.

Ex 3.3 Oklahoma Income Tax Part 2 Function

$T = T(I)$, $T(15,000) = 579$. If the taxable income is \$15,500, then the table shows a tax liability of \$605.

How much does the taxpayer owe if the taxable income is \$15,350?

If we look more carefully at the catered dinner, we can write a formula for the total cost $C = C(n)$ when there are n dinner guests:

$$\text{Cost} = \text{Cost of food} + \text{Rent}$$

Ex 3.4 Selling Jewelry at an Art Fair

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Ex 3.4 Selling Jewelry at an Art Fair Part 1

Suppose you pay $192 to rent a booth for selling necklaces at an art fair. The necklaces sell for $32 each.

Explain why the function that shows your net income (revenue from sales minus rental fee) as a function of the number of necklaces sold is a linear function.

Ex 3.4 Selling Jewelry at an Art Fair Part 2

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Write a formula for this function.

Ex 3.4 Selling Jewelry at an Art Fair Part 3

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Use functional notation to show your net income if you sell 25 necklaces, and then calculate that value.

Ex 3.4 Selling Jewelry at an Art Fair Part 4, 5, 6

Suppose you pay \$192 to rent a booth for selling necklaces at an art fair. The necklaces sell for \$32 each.

Plot

Initial value

Horizontal intercept

Getting a linear equation if you know the slope and initial value:
In Example 3.4, we were effectively told the slope, 32, of the linear function P and its initial value, -192.

Getting a linear equation if you know the slope and one data point:

Suppose that we are given the information for Example 3.4 in a different way: We are told that the price for each necklace is \$32 and that if we sell $n = 8$ necklaces, we will have a net income of $P = 64$ dollars.

Getting a linear equation from two data points:
Suppose the information in Example 3.4 were given as two data points.
For example, suppose that we make a net income of $P = 64$ dollars when we sell $n = 8$ necklaces and a net income of $P = 160$ dollars when $n = 11$ necklaces are sold.

In general, if you are given two data points and you need to find the linear function that they determine, then you proceed in two steps.
First, use the formula

$$m = \frac{\text{change in function}}{\text{change in variable}}$$

to compute the slope.
Next, use the slope you found and put either of the given data points into the equation

$$y \ = \text{Slope} \cdot x + \text{Initial value}$$

to solve for the initial value.

Ex 3.5 Changing Celsius to Fahrenheit

The temperature $F = F(C)$ in Fahrenheit is a linear function of the temperature C in Celsius.

A lab assistant placed a Fahrenheit thermometer beside a Celsius thermometer and observed the following:

When the Celsius thermometer reads 30 degrees ($C = 30$), the Fahrenheit thermometer reads 86 degrees ($F = 86$).

When the Celsius thermometer reads 40 degrees, the Fahrenheit thermometer reads 104 degrees.

Ex 3.5 Changing Celsius to Fahrenheit Part 1

The temperature $F = F(C)$ in Fahrenheit is a linear function of the temperature C in Celsius. When $C = 30$ then $F = 86$ and when $C = 40$ then $F = 104$ degrees.

Use a formula to express F as a linear function of C.

Ex 3.5 Changing Celsius to Fahrenheit Part 2

The temperature $F = F(C)$ in Fahrenheit is a linear function of the temperature C in Celsius. When $C = 30$ then $F = 86$ and when $C = 40$ then $F = 104$ degrees.
At sea level, water boils at 212 degrees Fahrenheit. What temperature in degrees Celsius makes water boil?

Ex 3.5 Changing Celsius to Fahrenheit Part 3

The temperature $F = F(C)$ in Fahrenheit is a linear function of the temperature C in Celsius. When $C = 30$ then $F = 86$ and when $C = 40$ then $F = 104$ degrees.

Explain in practical terms what the slope means in this setting.

Ex 3.2.005 If you take a brisk walk on a flat surface, you will burn about 258 calories per hour. You have just finished a hard workout that used 700 calories.

1. Find a formula that gives the total calories burned if you finish your workout with a walk of h hours.

2. How long do you need to walk at the end of your workout in order to burn a total of 1100 calories?

Ex 3.2.015 A study of average driver speed on rural highways by A. Taragin found a linear relationship between average speed S, in miles per hour, and the amount of curvature D, in degrees, of the road. On a straight road ($D = 0$), the average speed was found to be 46.26 miles per hour. This was found to decrease by 0.746 mile per hour for each additional degree of curvature.

1. Find a linear formula relating speed to curvature.

2. Express using functional notation the speed for a road with a curvature of 10 degrees, and then calculate that value.

Chapter 3 Section 3

Information about physical and social phenomena is frequently obtained by gathering data or sampling.

Once data are gathered, an important key to further analysis is to produce a *mathematical model* describing the data.

A model is a function that

1. represents the data either exactly or approximately and
2. incorporates patterns in the data.

In many cases, such a model takes the form of a linear function.

Testing Data for Linearity

Suppose you have a set of data, how can you try to check if the data is linear?

One of the most important events in the development of modern physics was Galileo's description of how objects fall.
In about 1590, he conducted experiments in which he dropped objects and attempted to measure their downward velocities $V = V(t)$ as they fell.
Here we measure velocity V in feet per second and time t as the number of seconds after release.
If Galileo had been able to nullify air resistance, and if he had been able to measure velocity without any experimental error at all, he might have recorded the following table of values for a rock.

t in seconds	0	1	2	3	4	5
V feet per second	0	32	64	96	128	160

Recall that for a linear process the rate of change is constant. So let us look at the change in time vs the change in velocity.

t in seconds	0		1		2		3		4		5
Δt											
V feet per second	0		32		64		96		128		160
ΔV											

Note we have a constant change in time of 1 second and a constant change in velocity (ROC) of 32 feet per second. The data is **Linear!**

Now to find the linear function that describes this data.
Since it is linear we know that $V = mt + b$.
What is the slope?

What about b?

Linear Models

This linear function serves as a *mathematical model* for the experimentally gathered data, and it gives us more information than is apparent from the data table alone.

Question:

What is the velocity at time $t = 3.6$ seconds?

t in seconds	0	1	2	3	4	5
V feet per second	0	32	64	96	128	160

The key physical observation that Galileo made was that falling objects have constant acceleration, 32 feet per second per second. He did additional experiments to show that if air resistance is ignored, then this acceleration does not depend on the weight or size of the object.

The same table of values would result, and the same acceleration would be calculated, whether the experiment was done with a pebble or with a cannonball.

According to tradition, Galileo conducted some of his experiments in public, dropping objects from the top of the leaning tower of Pisa.

His observations got him into serious trouble with the authorities because they conflicted with the accepted premise of Aristotle that heavier objects would fall faster than lighter ones.

If instead of measuring velocity, we had measured the distance $D = D(t)$ that the rock fell, we would have obtained the following data.

t in seconds	0	1	2	3	4	5
D feet	0	16	64	144	256	400

Ex 3.6 Sampling Voter Registration In this hypothetical experiment, a political analyst compiled data on the number of registered voters in Payne County, Oklahoma, each year from 2011 through 2016. For each of the following possible data tables that the analyst might have obtained, determine whether the data can be modeled with a linear function. If so, find such a formula and predict the number of registered voters in Payne County in the year 2018.

1.

Date	2011	2012	2013	2014	2015	2016
Registered voters	28321	28542	29466	30381	30397	31144

2.

Date	2011	2012	2013	2014	2015	2016
Registered voters	28321	28783	29245	29707	30169	30631

Ex 3.6 Sampling Voter Registration Data Set 1 For the following possible data table, determine whether the data can be modeled with a linear function. If so, find such a formula and predict the number of registered voters in Payne County in the year 2018.

Date	2011	2012	2013	2014	2015	2016
Registered voters	28321	28542	29466	30381	30397	31144

Ex 3.6 Sampling Voter Registration Data Set 2 For the following possible data table, determine whether the data can be modeled with a linear function. If so, find such a formula and predict the number of registered voters in Payne County in the year 2018.

Date	2011	2012	2013	2014	2015	2016
Registered voters	28321	28783	29245	29707	30169	30631

Ex 3.6 Sampling Voter Registration Data Set 2 For the following possible data table, determine whether the data can be modeled with a linear function. If so, find such a formula and predict the number of registered voters in Payne County in the year 2018.

d	0	1	2	3	4	5
Date	2011	2012	2013	2014	2015	2016
Registered voters	28321	28783	29245	29707	30169	30631

Graphing Discrete Data

Calculating differences can always tell you whether data are linear, but many times it is advantageous to view such data graphically.

Lets use Desmos to graph our data:

d	R
0	28321
1	28542
2	29466
3	30381
4	30397
5	31144

d	R
0	28321
1	28783
2	29245
3	29707
4	30169
5	30631

Ex 3.7 Newton's Second Law of Motion

Newton's second law of motion shows how force on an object, measured in newtons, is related to acceleration of the object, measured in meters per second per second. The following experiment might be conducted in order to discover Newton's second law. Objects of various masses, measured in kilograms, were given an acceleration of 5 meters per second per second, and the associated forces were measured and recorded in the table below.

Mass	1	1.3	1.6	1.9	2.2
Force	5	6.5	8	9.5	11

Ex 3.7 Newton's Second Law of Motion Part 1

Mass	1	1.3	1.6	1.9	2.2
Force	5	6.5	8	9.5	11

Check differences to show that these are linear data.

Ex 3.7 Newton's Second Law of Motion Part 2

Mass	1	1.3	1.6	1.9	2.2
Force	5	6.5	8	9.5	11

Find the slope of a linear model for the data, and explain in practical terms what the slope means.

Ex 3.7 Newton's Second Law of Motion Part 3

Mass	1	1.3	1.6	1.9	2.2
Force	5	6.5	8	9.5	11

Construct a linear model for the data.

Ex 3.7 Newton's Second Law of Motion Part 4

Mass	1	1.3	1.6	1.9	2.2
Force	5	6.5	8	9.5	11

What force does your model show for an object of mass 1.43 kilograms that is accelerating at 5 meters per second?

Ex 3.7 Newton's Second Law of Motion Part 5

Mass	1	1.3	1.6	1.9	2.2
Force	5	6.5	8	9.5	11

Make a graph showing the data, and overlay it with the graph of the linear model you made in part 3.

Chapter 3 Section 4

Linear Regression

In real life, rarely is information gathered that fits any simple formula perfectly.

In cases such as government spending, many factors influence the budget, including the political make-up of the legislature.

In the case of scientific experiments, variations may be due to *experimental error*, the inability of the data gatherer to obtain exact measurements; there may also be elements of chance involved.

Under these circumstances, it may be necessary to obtain an approximate rather than an exact mathematical model.

The Regression Line

To illustrate this idea, let's look at federal Medicare expenditures, in billions of dollars, in the United States as reported by the Centers for Medicare and Medicaid Services and recorded in the following table.

Date	2009	2010	2011	2012	2013
Expenditures in billions	498.8	520.5	546.1	569.2	586.3

Let's check for linearity.

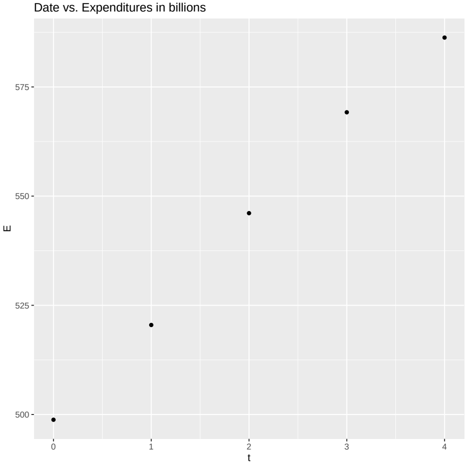
Date	2009		2010		2011		2012		2013
Expenditures in billions	498.8		520.5		546.1		569.2		586.3
ΔE		21.7		25.6		23.1		17.1	

It is not linear, but how far from linear is it?

Let's alter the table by letting t be the number of years since 2009:

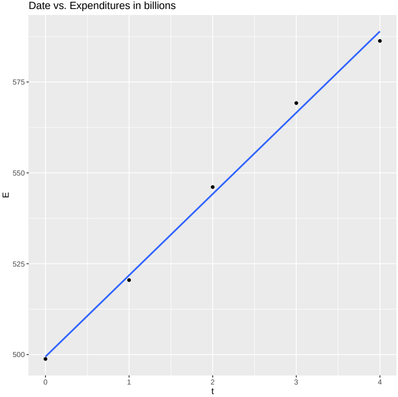
t	0	1	2	3	4
E	498.8	520.5	546.1	569.2	586.3

If we plot these points in a scatter plot we get:



This looks close to linear. It turns out there we can find a line that "best fits" this data. It is called the least-squares fit or linear regression line.

This is shown below:



So even though there is no line that perfectly fits the data we can find a line that best approximates the data.

Desmos graph

Let's graph this data, draw the regression line, and find it's equation. To do this we will use `https://www.desmos.com/calculator` with data:

$$\begin{array}{ll} 0, & 498.8 \\ 1, & 520.5 \\ 2, & 546.1 \\ 3, & 569.2 \\ 4, & 586.3 \end{array}$$

Desmos graph

Let's graph this data, draw the regression line, and find it's equation. To do this we will use `https://www.desmos.com/calculator` with data:

$$\begin{array}{ll} 0, & 498.8 \\ 1, & 520.5 \\ 2, & 546.1 \\ 3, & 569.2 \\ 4, & 586.3 \end{array}$$

This gives the regression line model for E as a function of t:

$$y = 22.37x + 499.44$$

$$E = 22.37t + 499.44$$

Note: *It is important to remember that even though we have written it as an equality, the equation* $y = 22.37x + 499.44$ *in fact is a model that only approximates the relationship between* t *and* E *given by the data table.*

For example, the initial value according to the linear model $y = 22.37x + 499.44$ is 499.44, whereas the entry in the table for $t = 0$ is $E = 498.8$.

We will use the equals sign as above, but you should be aware that in this setting, many people would prefer to replace it by an approximation symbol, $\approx$, or to use different letters for the regression equation. In statistics this is indicated by putting a "hat" on E, $\widehat{E}$.

Uses of the Regression Line:

Slope and Trends

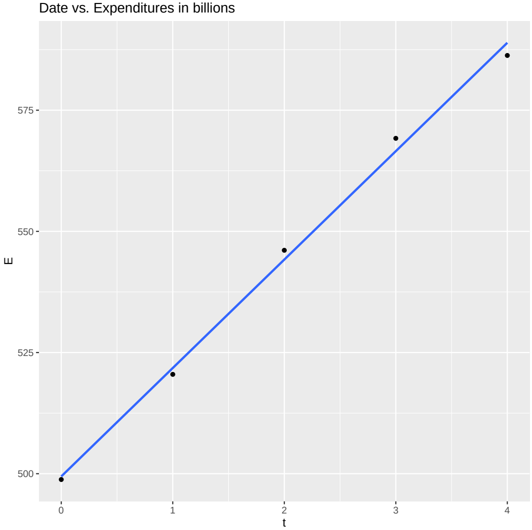
The most useful feature of the regression line is its slope, which in many cases provides the key to understanding data.

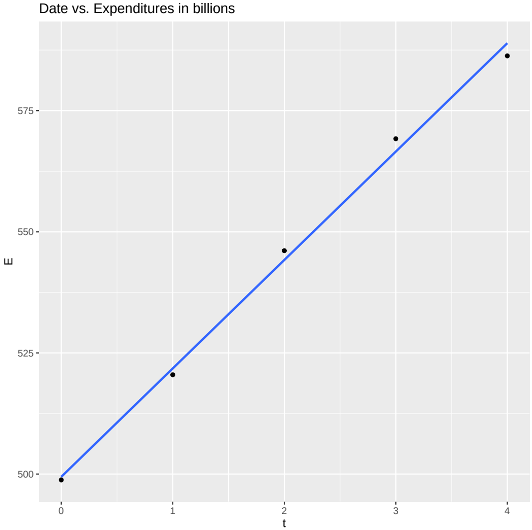
The most useful feature of the regression line is its slope, which in many cases provides the key to understanding data.

For Medicare spending, the slope, 22.37 billion dollars per year, of the regression line tells us that during the period from 2009 to 2013, Medicare spending grew by about 22.37 billion dollars per year. It tells how the data are changing.

A plot that shows the regression line with the data can be useful in analyzing trends.

For example, in the figure below, the fourth data point (corresponding to 2012) lies slightly above the regression line, whereas the fifth data point (corresponding to 2013) lies slightly below it.





Spending was definitely higher in 2013 than in 2012, but it could be argued, on the basis of the position of the data points relative to the line, that in 2012, spending was slightly ahead of the trend, whereas in 2013, it was slightly behind.

Question: What level of spending does the regression line $y = 22.37x + 499.44$ predict for 2014?

Consulting the source for our data, we find that Medicare expenditures in 2014 were in fact $E = 618.7$ billion dollars.

In this case, the projection given by the regression line was not exact but was within about 1% of the actual value.

Let's try to use the regression line to go the other way—that is, to estimate money spent before 2009.

In particular, we want to determine the level of Medicare expenditures in 2005. That is 4 years before 2009, so we want the value of E when $t\ = 2005 - 2009 = -4$.

Once again, we refer to the source and see that the actual expenditures for Medicare in 2005 were 339.7 billion dollars.

In this case, the value given by the regression line is a bad estimate of the real value.

The regression line is a powerful tool for analysis of certain kinds of data, but caution in its use is essential.

We saw above that the regression line gave a decent estimate for 2014 expenditures but a bad one for 2005.

In general, using the regression line to extrapolate beyond the limits of the data is risky, and the risk increases dramatically for long-range extrapolations.

What the regression line really shows is the linear trend established by the data.

Thus, for our 2014 projection, it would be appropriate to say, "If the trend established over the preceding five years had persisted, then Medicare spending in 2014 would have been about 611.29 billion dollars."

A check of the 2014 data showed that this trend did indeed persist into 2014.

An appropriate statement for our 2005 analysis might be "If the trend established from 2009 to 2013 had been valid since 2005, then expenditures in 2005 would have been about 409.96 billion dollars."

Because the actual expenditure was much less, we might proceed by gathering more data to see exactly how spending changed and then conduct historical, political, and economic investigations into why it changed.

On the other hand, the regression line can clearly show the trend of almost linear data and can appropriately be used to determine whether trends persist.

If information is available that indicates that linear trends are persisting, then the regression line can be used to make forecasts.

Question: The following table shows the amount $M = M(t)$ of money, in billions of dollars, spent by the United States on national defense. In the time row, $t = 0$ corresponds to 2004, $t = 1$ refers to 2005, and so on.

$t =$ years since 2004	0	1	2	3	4
$M =$ billons of dollars	455.8	495.3	521.8	551.3	616.1

1. Plot the data points. Does it appear that it is appropriate to approximate these data with a straight line?
2. Find the equation of the regression line for M as a function of t, and add the graph of this line to your data plot.

Question: $t = 0$ corresponds to 2004

t = years since 2004	0	1	2	3	4
M = billons of dollars	455.8	495.3	521.8	551.3	616.1

Part 1: Plot the points. Does it appear that it is appropriate to approximate these data with a straight line?

https://www.desmos.com/calculator

$0, \quad 455.8$

$1, \quad 495.3$

$2, \quad 521.8$

$3, \quad 551.3$

$4, \quad 616.1$

Question: $t = 0$ corresponds to 2004

$t =$ years since 2004	0	1	2	3	4
$M =$ billons of dollars	455.8	495.3	521.8	551.3	616.1

Part 2: Find the equation of the regression line for M as a function of t, and add the graph of this line to your data plot.

Question: $t = 0$ corresponds to 2004

$t =$ years since 2004	0	1	2	3	4
$M =$ billons of dollars	455.8	495.3	521.8	551.3	616.1

Part 3: Explain in practical terms the meaning of the slope of the regression line model we found in part 2.

Question: $t = 0$ corresponds to 2004

$t =$ years since 2004	0	1	2	3	4
$M =$ billons of dollars	455.8	495.3	521.8	551.3	616.1

Part 4: Use the regression equation to estimate military spending by the United States in 2009. The actual military expenditures in 2009 were \$661.0 billion. Did the trend established in the mid-2000s persist until 2009?

Chapter 3 Section 5

Systems of Equations

- Many physical problems can be described by a system of two equations in two unknowns, and often the desired information is found by solving the system of equations.?
- As we shall see, this involves nothing more than finding the intersection of two lines, and we already know how to do that, since we can find the intersection of any two graphs.?

Let's look at an example, but first a quick prep on concentrations for a solution.

Suppose we have 3 mL of Albuterol Sulfate Inhalation Solution at 0.083%.

How many milliliters of Albuterol Sulfate is actually in the solution?

$$0.083\% \equiv 0.00083$$

so there are

$$3\text{mL} \cdot 0.00083 = 0.002\text{mL}$$

A nurse has two solutions that contain different concentrations of a certain medication. One is a 12.5% concentration, and the other is a 5% concentration. How many cubic centimeters of each should she mix to obtain 20 cubic centimeters of an 8% concentration?

Let's let x represent the amount of 12.5% solution and y represent the amount of 5% solution.

Since we want 20 cc of solution we have the equation

$$x + y = 20$$

Now for the medication. The final solution consists of 20 cc at 8% so there will be

$$20 \text{ cc} \cdot 0.08 = 1.6 \text{ cc}$$

The first concentration is 12.5% and x cc so that is

$$x \text{ cc} \cdot 0.125$$

The second concentration is 5% and y cc so that is

$$y \text{ cc} \cdot 0.05$$

Dropping the units that means that our second equation is

$$0.125x + 0.05y = 1.6$$

So the system of equations we need to solve are

$$\begin{array}{ccccc} 0.125x & + & 0.05y & = & 1.6 \\ x & + & y & = & 20 \end{array}$$

So how do we solve this?

Graphical Solutions of Systems of Equations

Let's start with one we can do by hand.

$$\begin{cases} x + y = 2 \\ x - 2y = -1 \end{cases}$$

Going back to our concentration example:

$$\begin{array}{rcrcl} 0.125x & + & 0.05y & = & 1.6 \\ x & + & y & = & 20 \end{array}$$

Let's solve graphically in Desmos.

To show the full method, let's look at a simple example.

Example

We have \$900 to spend on the repair of a gravel drive. We want to make the repairs using a mix of coarse gravel priced at \$28 per ton and fine gravel priced at \$32 per ton. To make a good driving surface, we need three times as much fine gravel as coarse gravel. How much of each will our budget allow us to buy?

Blank

We have \$56 to spend on pizzas and drinks for a picnic. Pizzas cost \$12 each, and drinks cost \$0.50 each. Four times as many drinks as pizzas are needed. How many pizzas and how many drinks will our budget allow us to buy?

Algebraic Solutions

There are three methods of solving systems of equations:

1. Graphical
2. Substitution
3. Elimination

Substitution

Example 1S

Solve using substitution:

$$\begin{cases} x + y = 2 \\ x - 2y = -1 \end{cases}$$

Example 2S

Solve using substitution:

$$\begin{cases} 3u + z = 0 \\ u - 2z = -7 \end{cases}$$

Example 3S

Solve using substitution:

$$\begin{cases} \begin{array}{rcrcr} a & - & 4b & = & 24 \\ 3a & + & 6b & = & -72 \end{array} \end{cases}$$

Elimination

Example 1E

Solve using elimination:

$$\begin{cases} x & + & y & = & 2 \\ x & - & 2y & = & -1 \end{cases}$$

Example 2E

Solve using elimination:

$$\begin{cases} 3u & + & z & = & 0 \\ u & - & 2z & = & -7 \end{cases}$$

Example 3E

Solve using elimination:

$$\begin{cases} a - 4b = 24 \\ 3a + 6b = -72 \end{cases}$$

In class 1

Solve using substitution and then elimination and check graphically:

$$\left\{ \begin{array}{rcrcr} x & - & y & = & 5 \\ 2x & + & y & = & 19 \end{array} \right.$$

In class 2

Solve using substitution and then elimination and check graphically:

$$\begin{cases} 3a - 2b = 24 \\ a + 4b = -6 \end{cases}$$