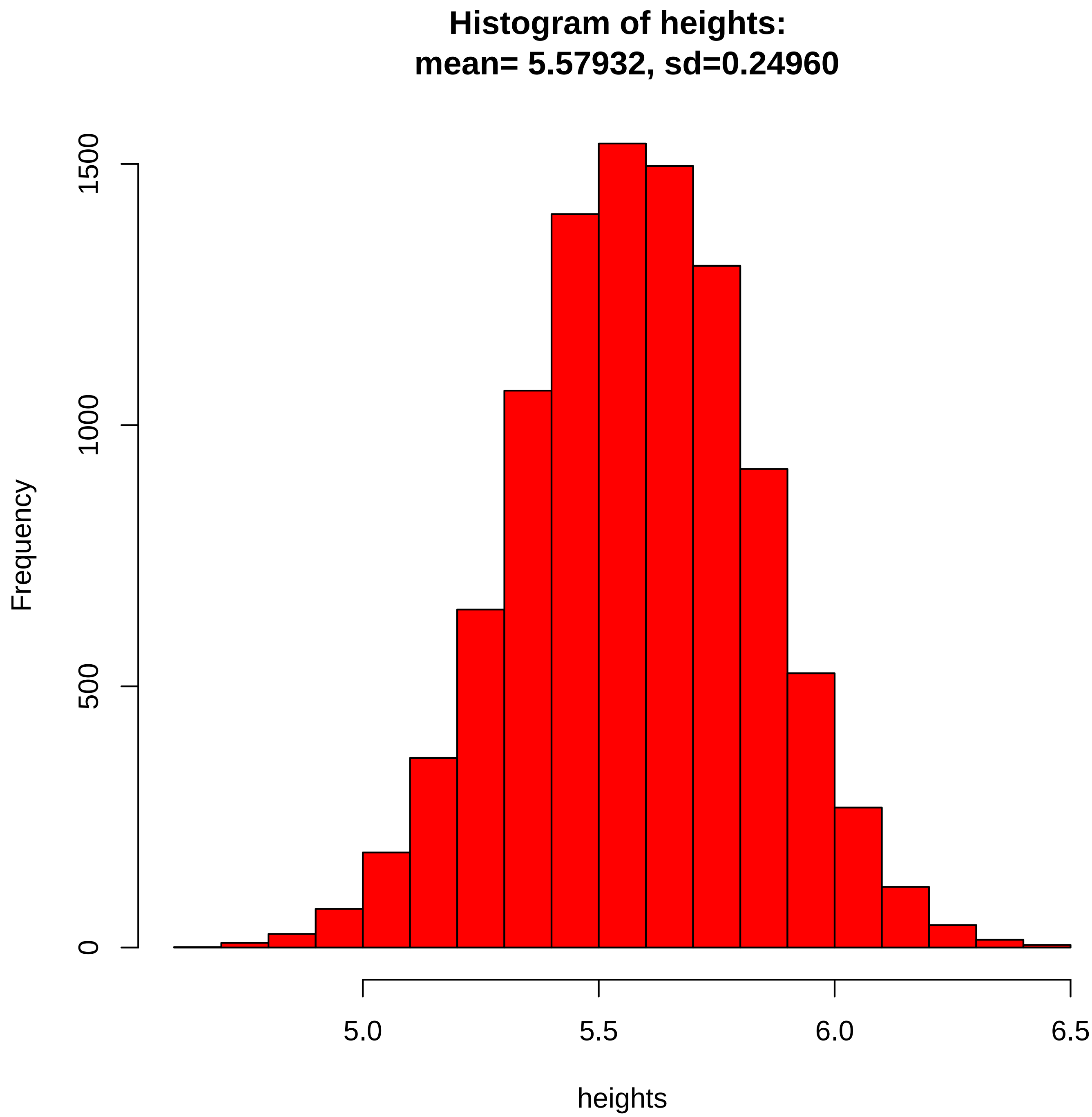
Let the random variable X be the heights of men with mean $\mu = 5.57932$ and standard deviation of $\sigma = .24960$. Suppose that there are 10000 heights in the population:

5.233732	5.589581	5.389242	5.633077	5.936384	5.766120	5.755057	5.522661	5.629273	5.881788
5.659584	5.224050	5.478727	5.828847	5.819704	5.809522	5.542258	5.274233	5.362794	5.319379
5.304091	5.691046	5.528762	5.998908	5.547169	5.530029	5.593728	5.409459	5.398074	5.364524
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5.570619	5.172144	5.624292	5.576875	5.481421	5.667891	5.799692	5.631164	5.358155	5.460696

The summary statistics for this population is:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Std Dev
4.692	5.410	5.580	5.579	5.751	6.467	0.24960

The histogram for this data set is:



Let us take a sample of size $n = 10$ from the population of heights:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
5.318737	5.920416	5.554704	5.415792	5.277280	5.496126	5.626964	5.366746	5.235289	5.207904

We can take the average of these values:

x_1	x_2	x_3	x_4	$\dots$	x_{10}	$\overline{x}$
5.318737	5.920416	5.554704	5.415792	$\dots$	5.207904	5.441996

Now let us repeat this process 1,000,000 times:

x_1	x_2	x_3	x_4	$\dots$	x_{10}	$\overline{x}$
5.318737	5.920416	5.554704	5.415792	$\dots$	5.207904	5.441996
6.035656	5.547326	5.307189	6.095374	$\dots$	6.004688	5.663764
5.632127	5.372547	5.694491	5.523235	$\dots$	5.557954	5.549524
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5.714549	5.700857	5.288253	5.363723	$\dots$	5.776595	5.587488

Since these means, $\overline{x}$, are random they can be considered as the values of a random variable. Let us call this random variable $\overline{X}$ and in this case it has the values:

$$5.441996, 5.663764, 5.549524, \dots, 5.587488$$

If we do this same process for each sample size

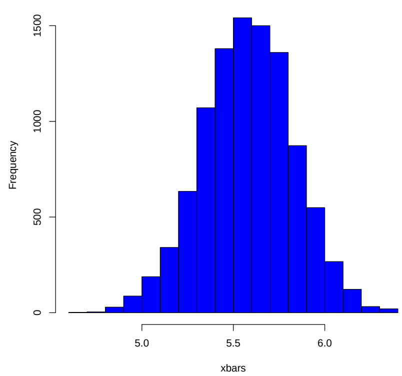
$$n = 1, 10, 25, 50, 75, 100, 500, 1000, 2500, 5000, 10000$$

and draw their histogram we get the figures on the next page. The sample mean ($\mu_{\overline{X}}$) and the sample standard deviation ($\sigma_{\overline{X}}$) for each sample size is

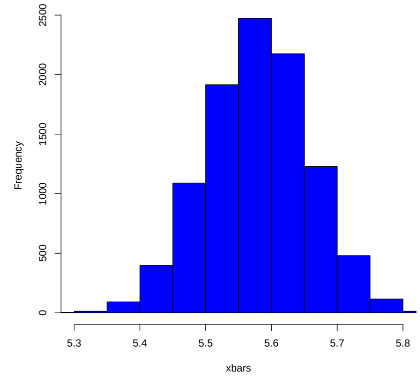
Sample size	$\mu_{\overline{X}}$	$\sigma_{\overline{X}}$
1	5.579931	0.249499947
10	5.579237	0.078466556
25	5.579184	0.050039821
50	5.579710	0.035372314
75	5.578922	0.028724527
100	5.579497	0.024999452
500	5.579379	0.010747983
1000	5.579292	0.007540395
2500	5.579346	0.004303151
5000	5.579302	0.002511437
7500	5.579323	0.001438324
10000	5.579316	0.000000000

Note that the sample means $\mu_{\overline{X}}$ hover around the population mean of $\mu = 5.57932$ for every sample size. But as the sample size increases the sample standard deviations $\sigma_{\overline{X}}$ approach 0. When the sample size becomes the population size of $n = 10000$ the sample mean is the same as the population mean and the sample standard deviation equals 0. We interpret these facts as saying that when we take a random sample its mean should be fairly close to the population mean and as the sample size increases the approximation gets better. Also note that the distribution for each sample size is also normally distributed.

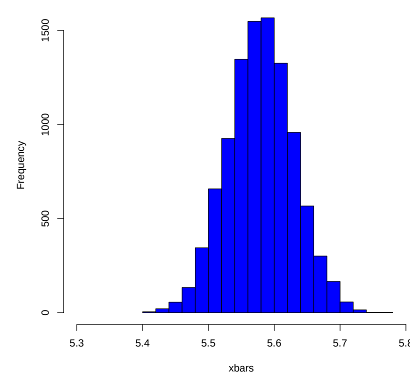
Histogram of xbars(heights): sample size = 1,
sample mean= 5.57993, sample sd=0.24950



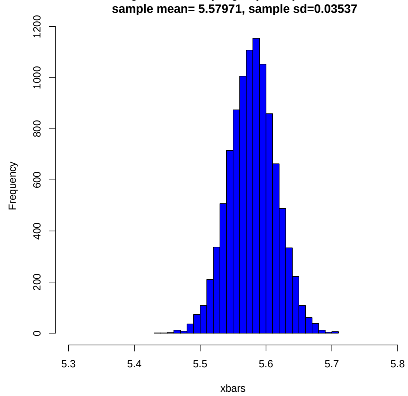
Histogram of xbars(heights): sample size = 10,
sample mean= 5.57924, sample sd=0.07847



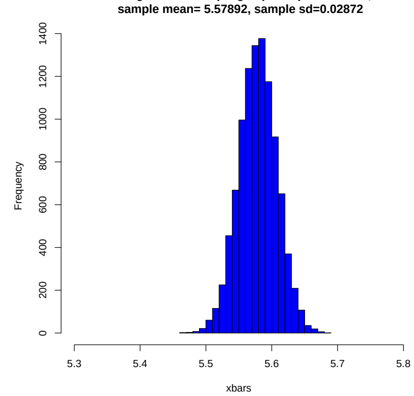
Histogram of xbars(heights): sample size = 25,
sample mean= 5.57918, sample sd=0.05004



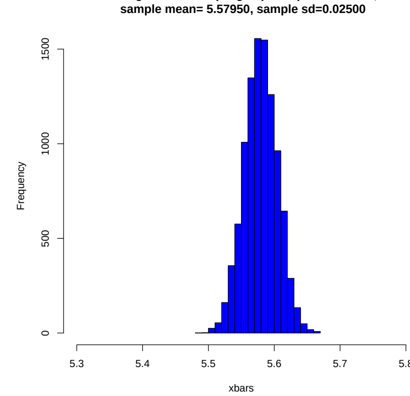
Histogram of xbars(heights): sample size = 50,
sample mean= 5.57971, sample sd=0.03537



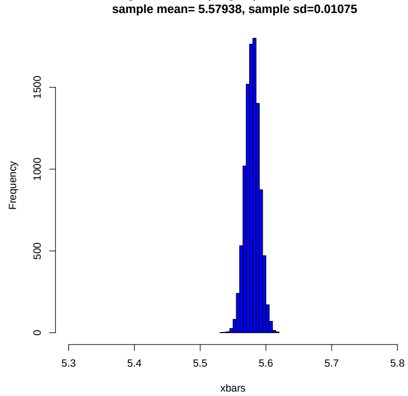
Histogram of xbars(heights): sample size = 75,
sample mean= 5.57892, sample sd=0.02872



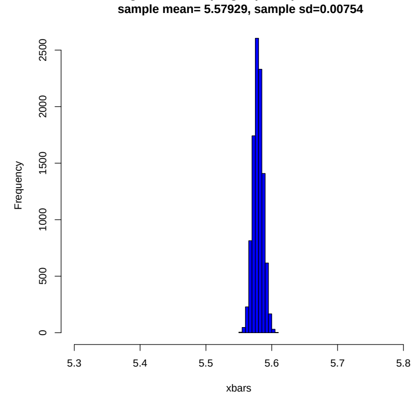
Histogram of xbars(heights): sample size = 100,
sample mean= 5.57950, sample sd=0.02500



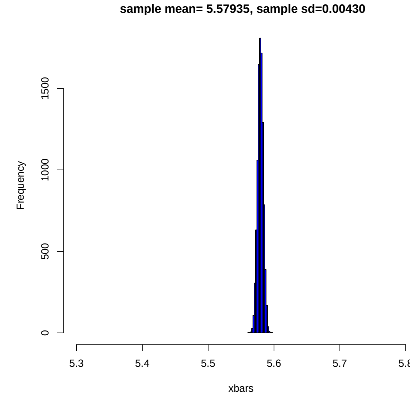
Histogram of xbars(heights): sample size = 500,
sample mean= 5.57938, sample sd=0.01075



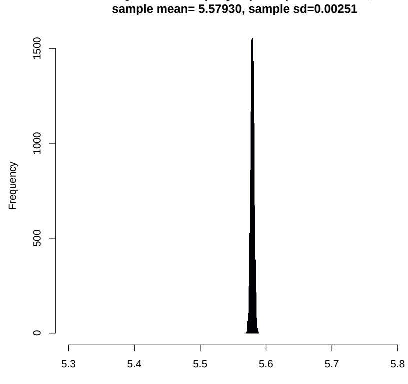
Histogram of xbars(heights): sample size = 1000,
sample mean= 5.57929, sample sd=0.00754



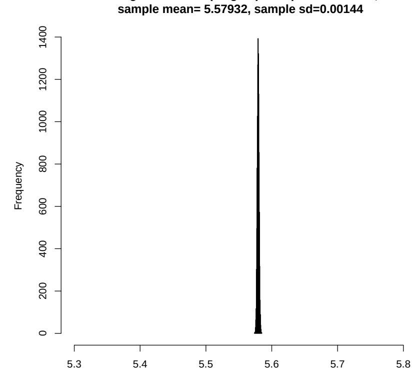
Histogram of xbars(heights): sample size = 2500,
sample mean= 5.57935, sample sd=0.00430



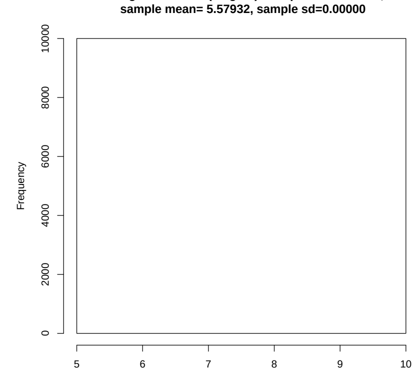
Histogram of xbars(heights): sample size = 5000,
sample mean= 5.57930, sample sd=0.00251

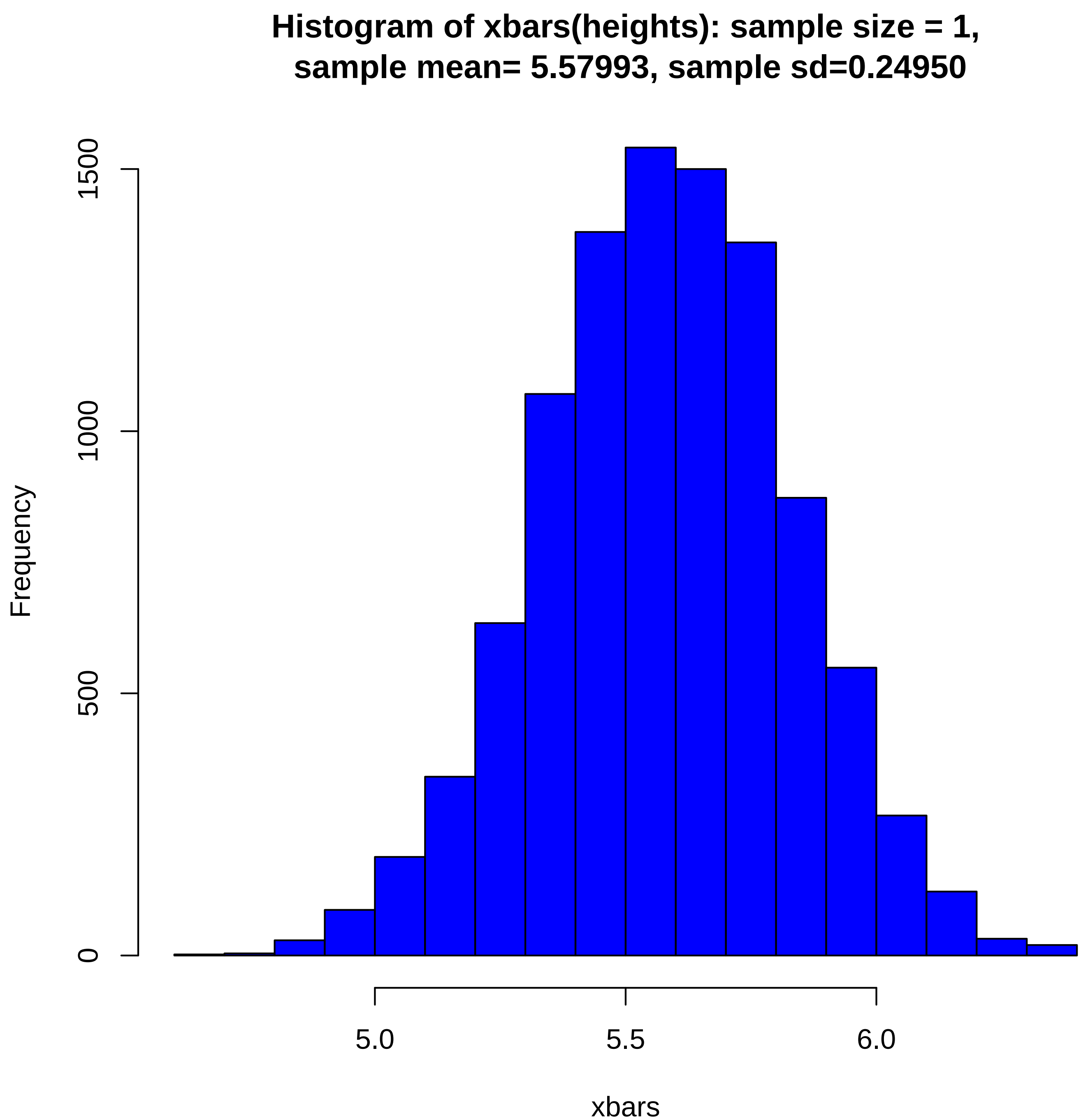


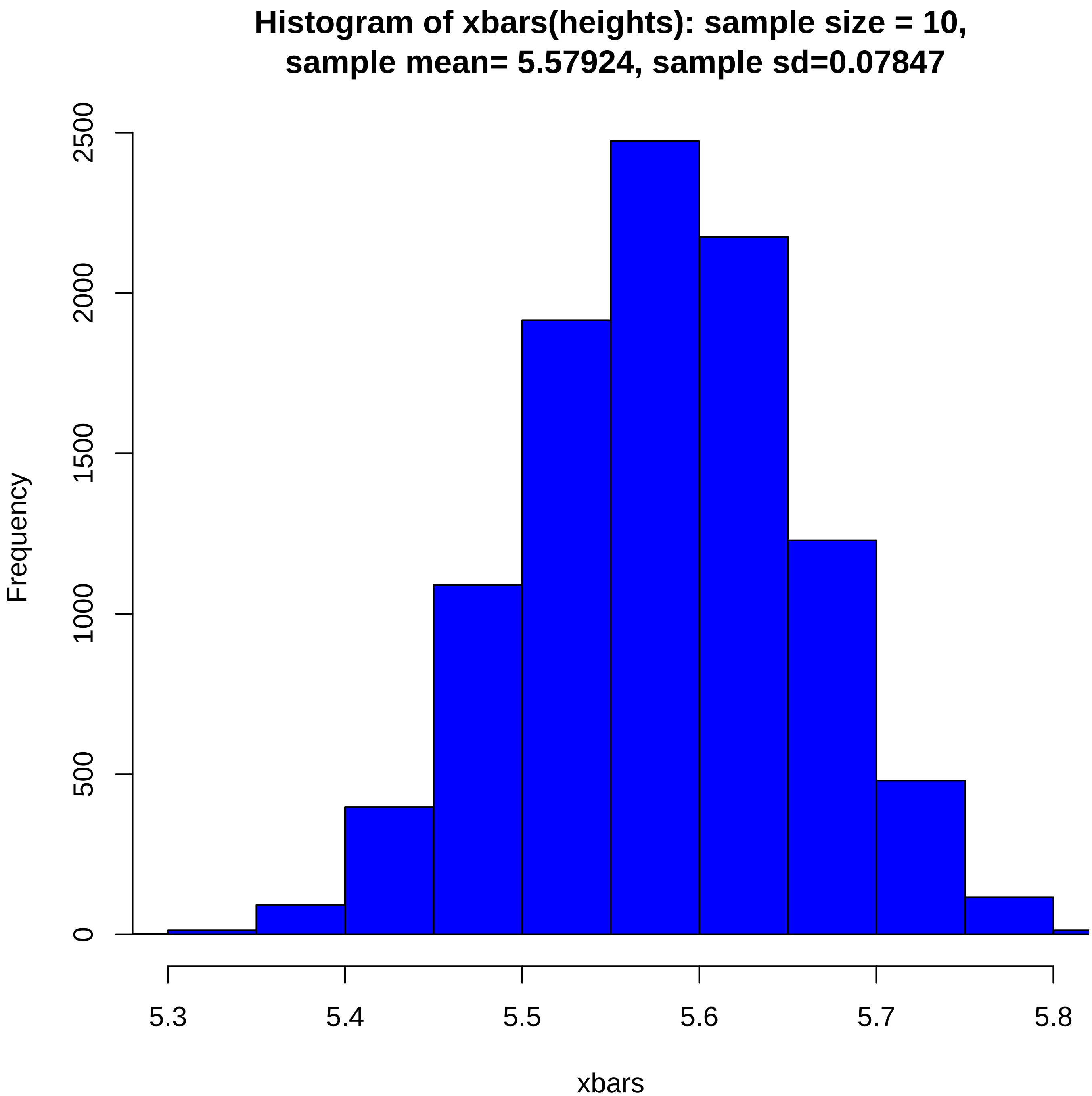
Histogram of xbars(heights): sample size = 7500,
sample mean= 5.57932, sample sd=0.00144

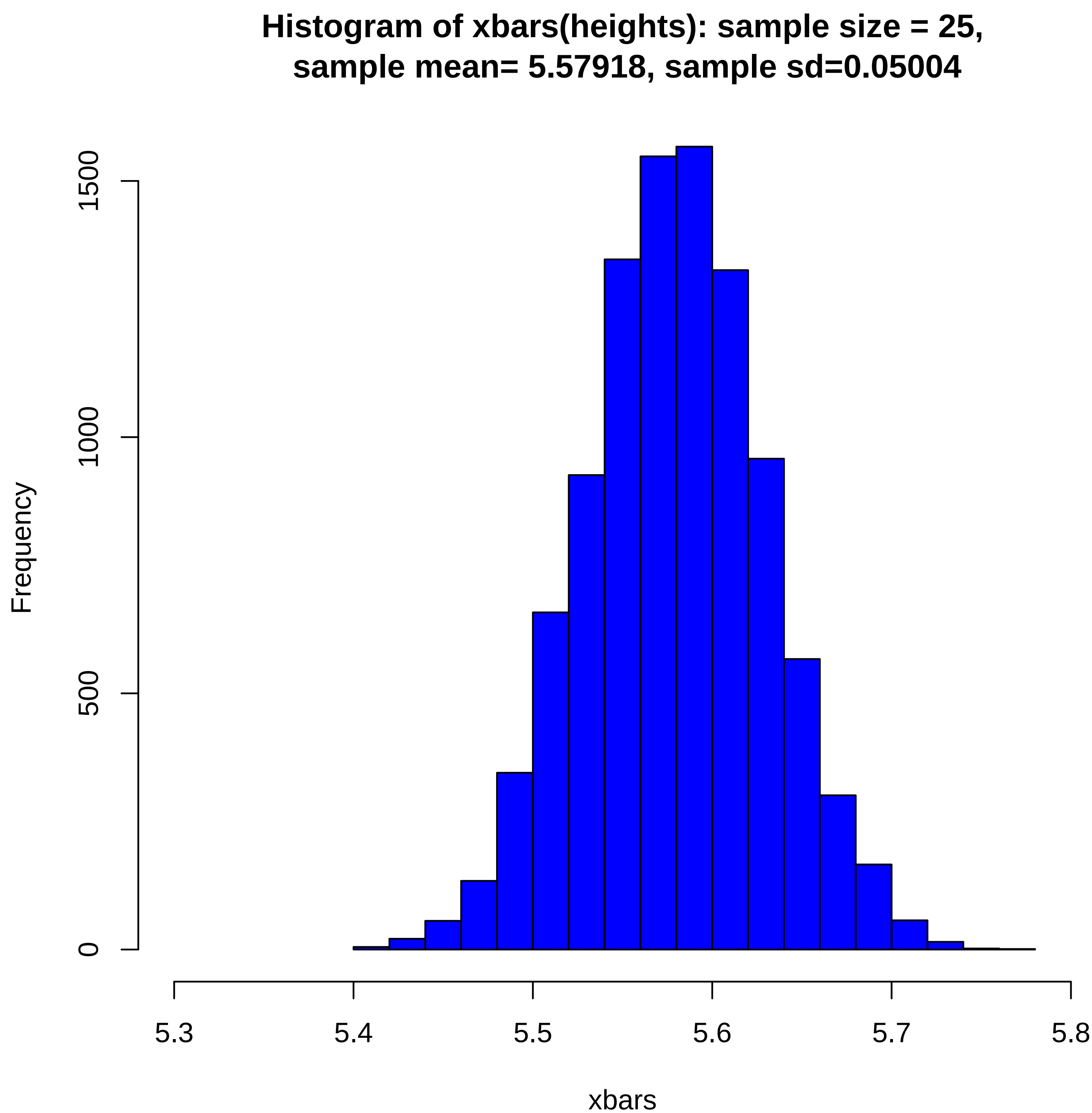


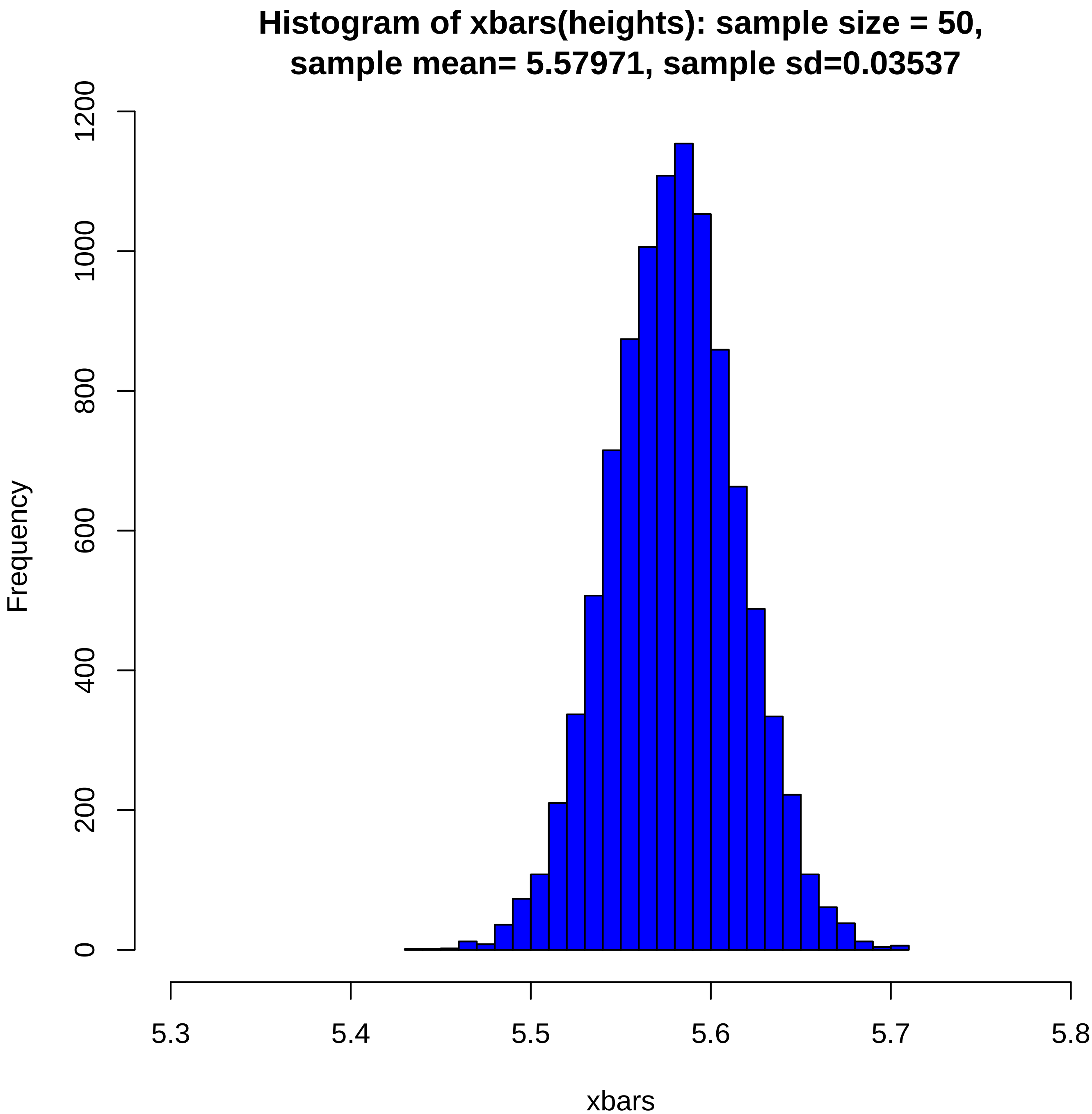
Histogram of xbars(heights): sample size = 10000,
sample mean= 5.57932, sample sd=0.00000

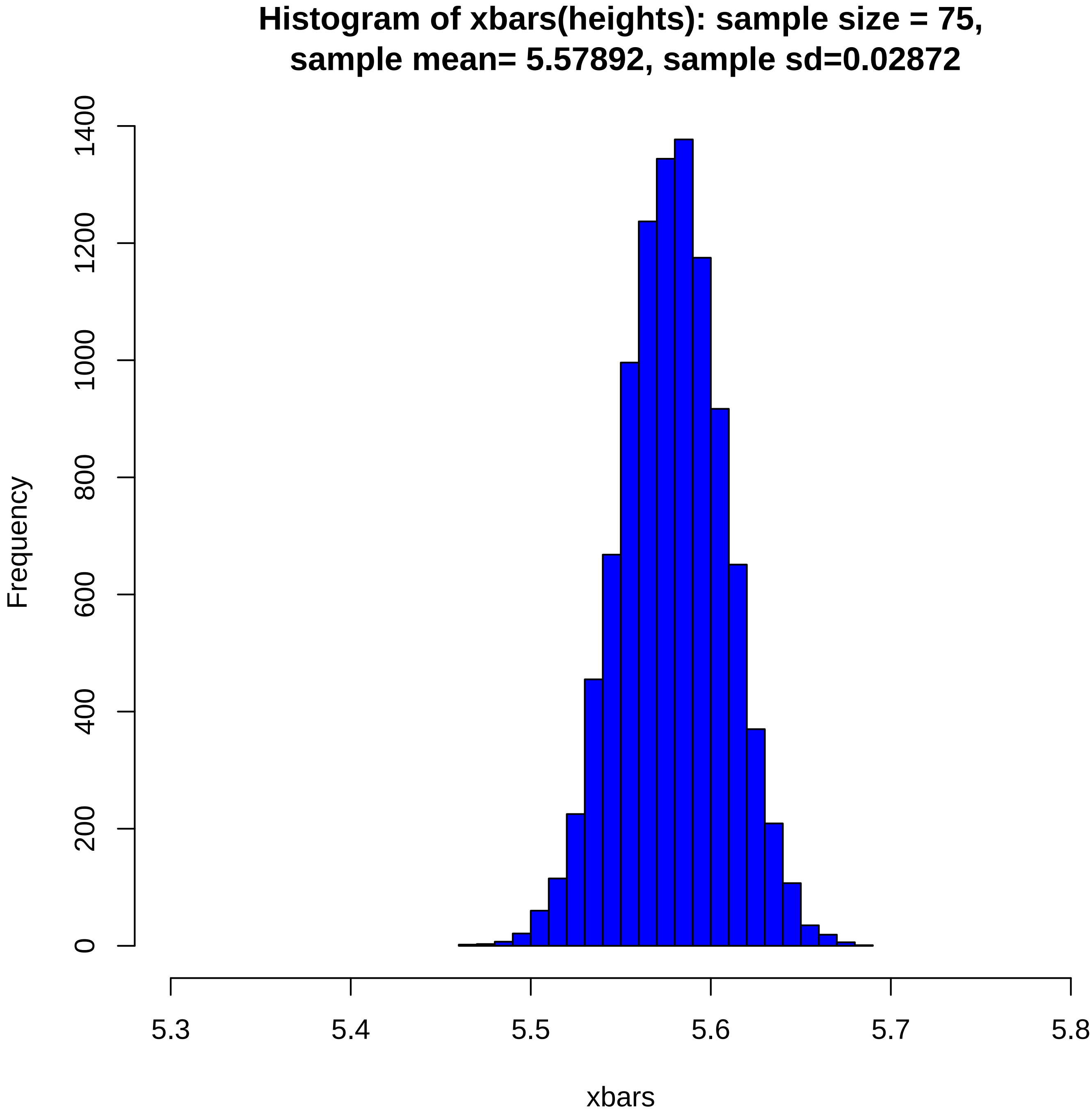


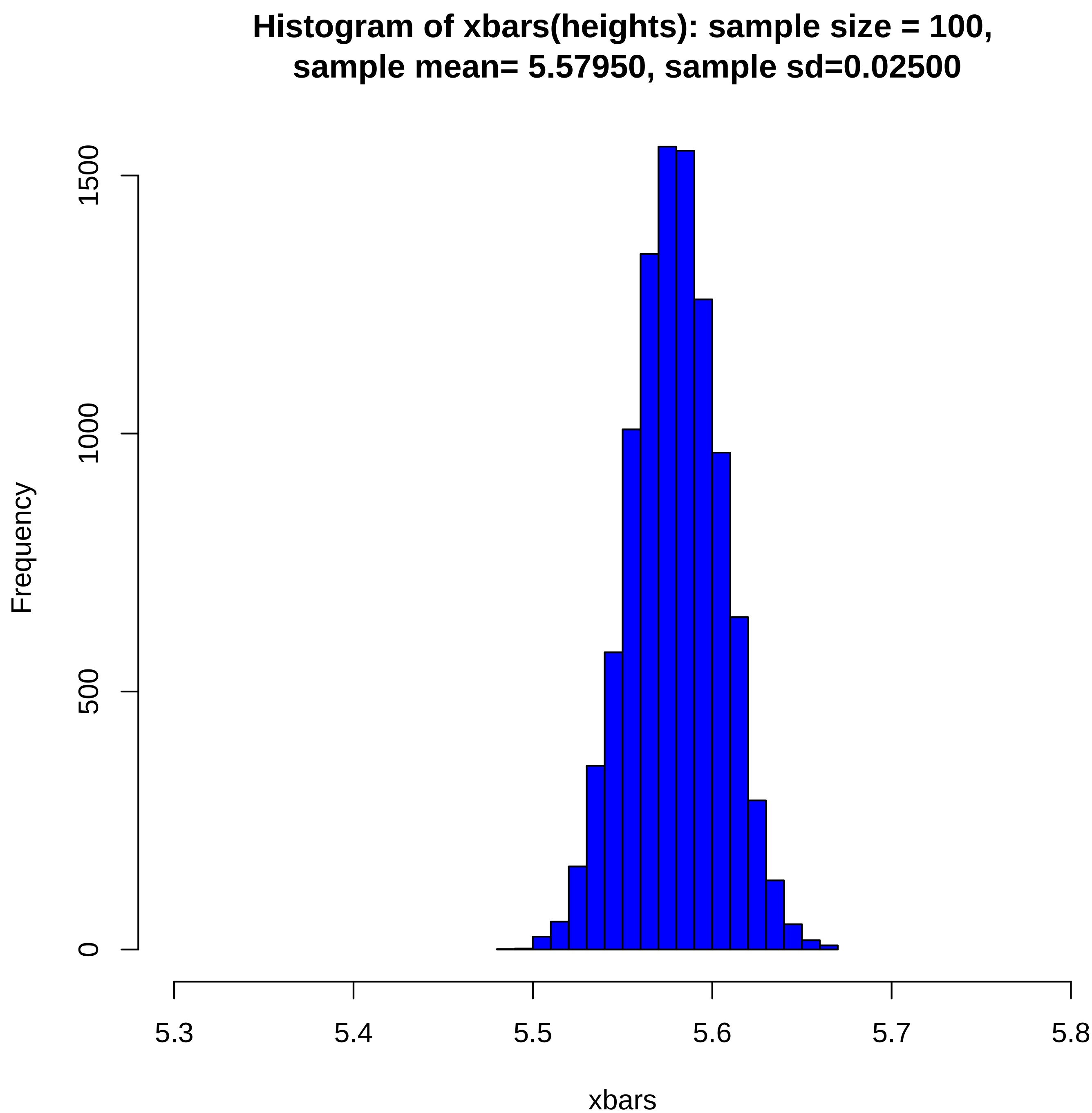

Histogram of xbars(heights): sample size = 1,
sample mean= 5.57993, sample sd=0.24950
Frequency
0
500
1000
1500
5.0
5.5
6.0
xbars


Histogram of xbars(heights): sample size = 10,
sample mean= 5.57924, sample sd=0.07847
Frequency
0
500
1000
1500
2000
2500
5.3
5.4
5.5
5.6
5.7
5.8
xbars

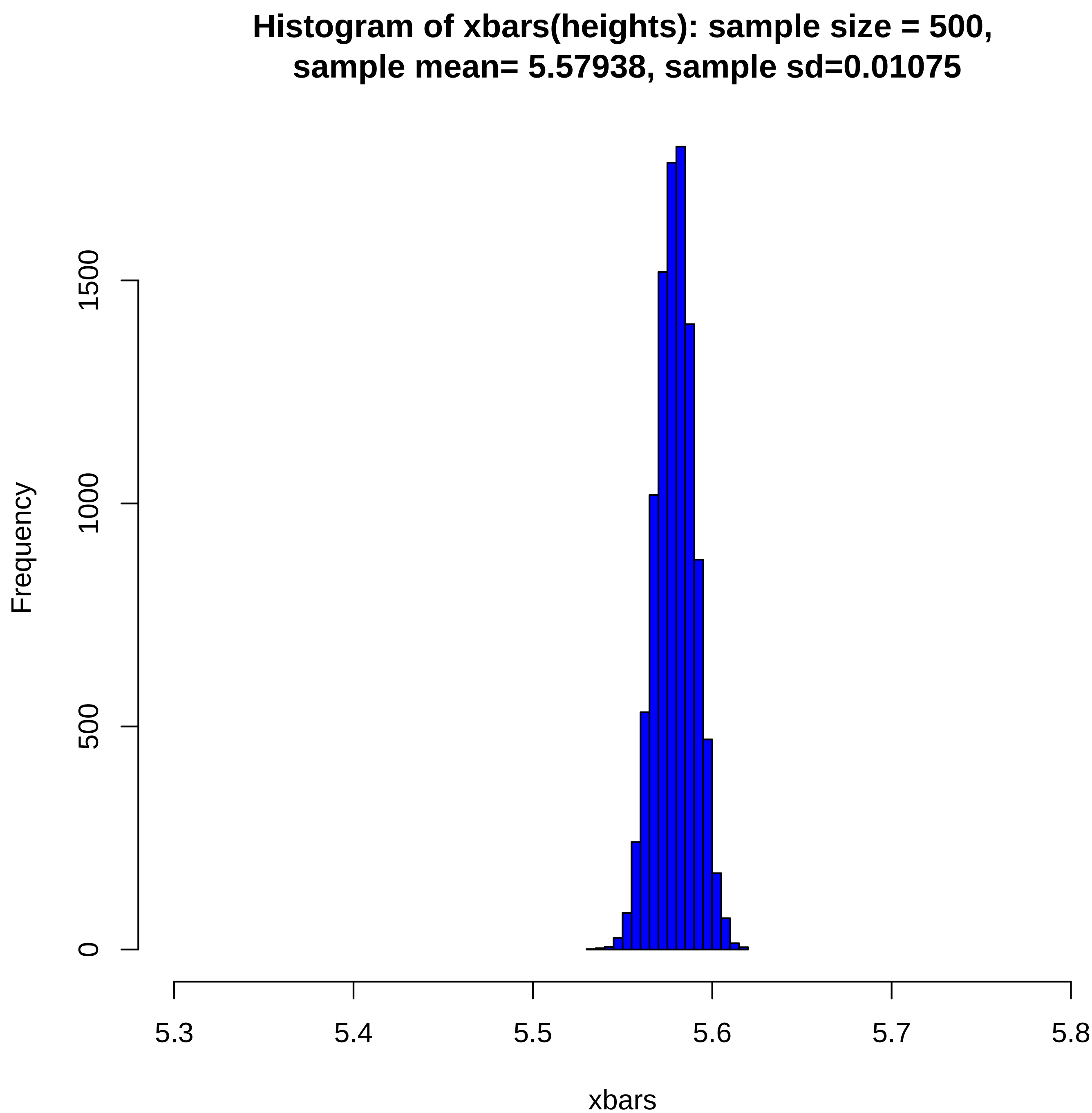

Histogram of xbars(heights): sample size = 25,
sample mean= 5.57918, sample sd=0.05004
Frequency
0
500
1000
1500
5.3
5.4
5.5
5.6
5.7
5.8
xbars

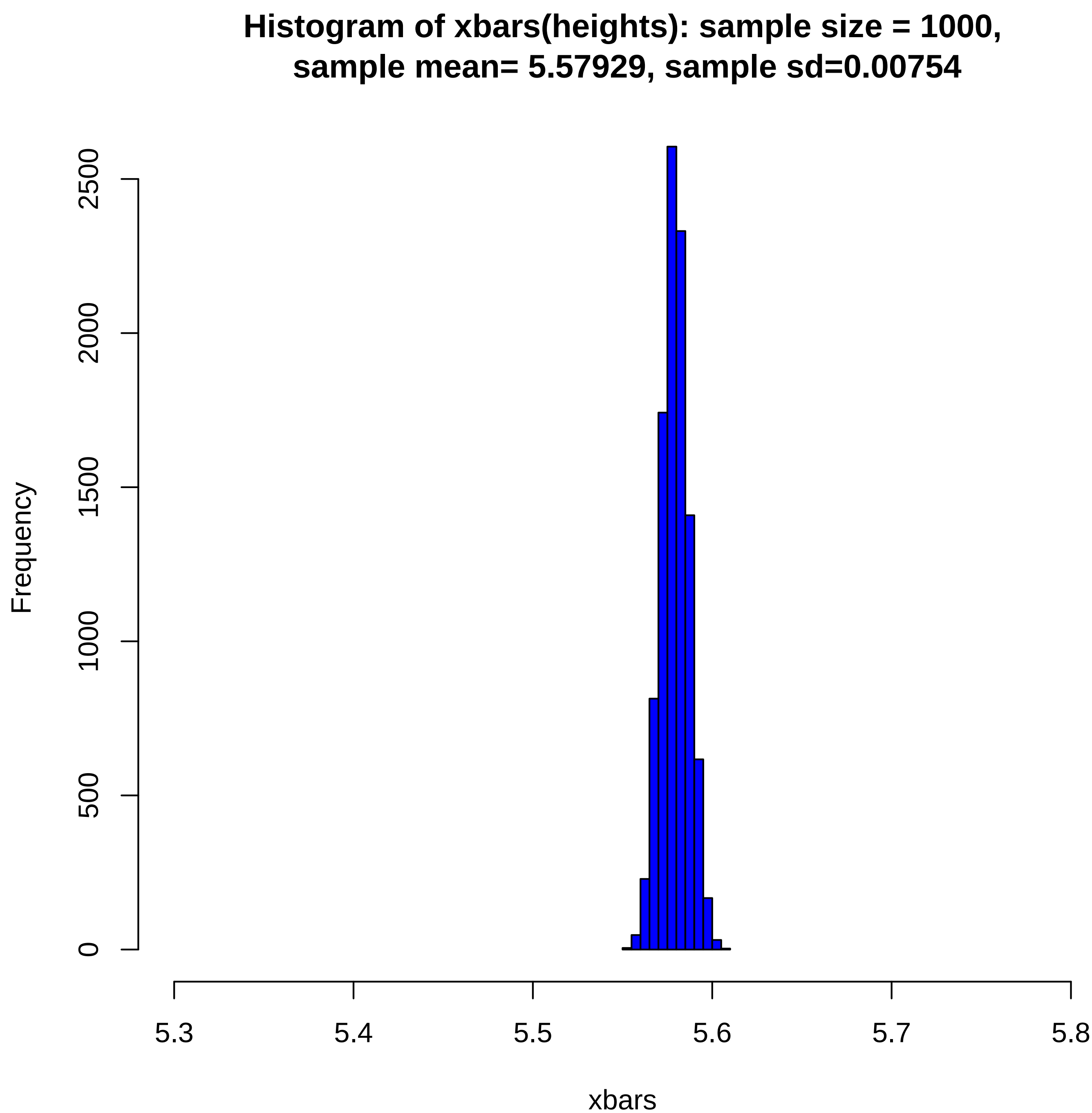

Histogram of xbars(heights): sample size = 50,
sample mean= 5.57971, sample sd=0.03537
Frequency
0
200
400
600
800
1000
1200
5.3
5.4
5.5
5.6
5.7
5.8
xbars


Histogram of xbars(heights): sample size = 75,
sample mean= 5.57892, sample sd=0.02872
Frequency
0
200
400
600
800
1000
1200
1400
5.3
5.4
5.5
5.6
5.7
5.8
xbars



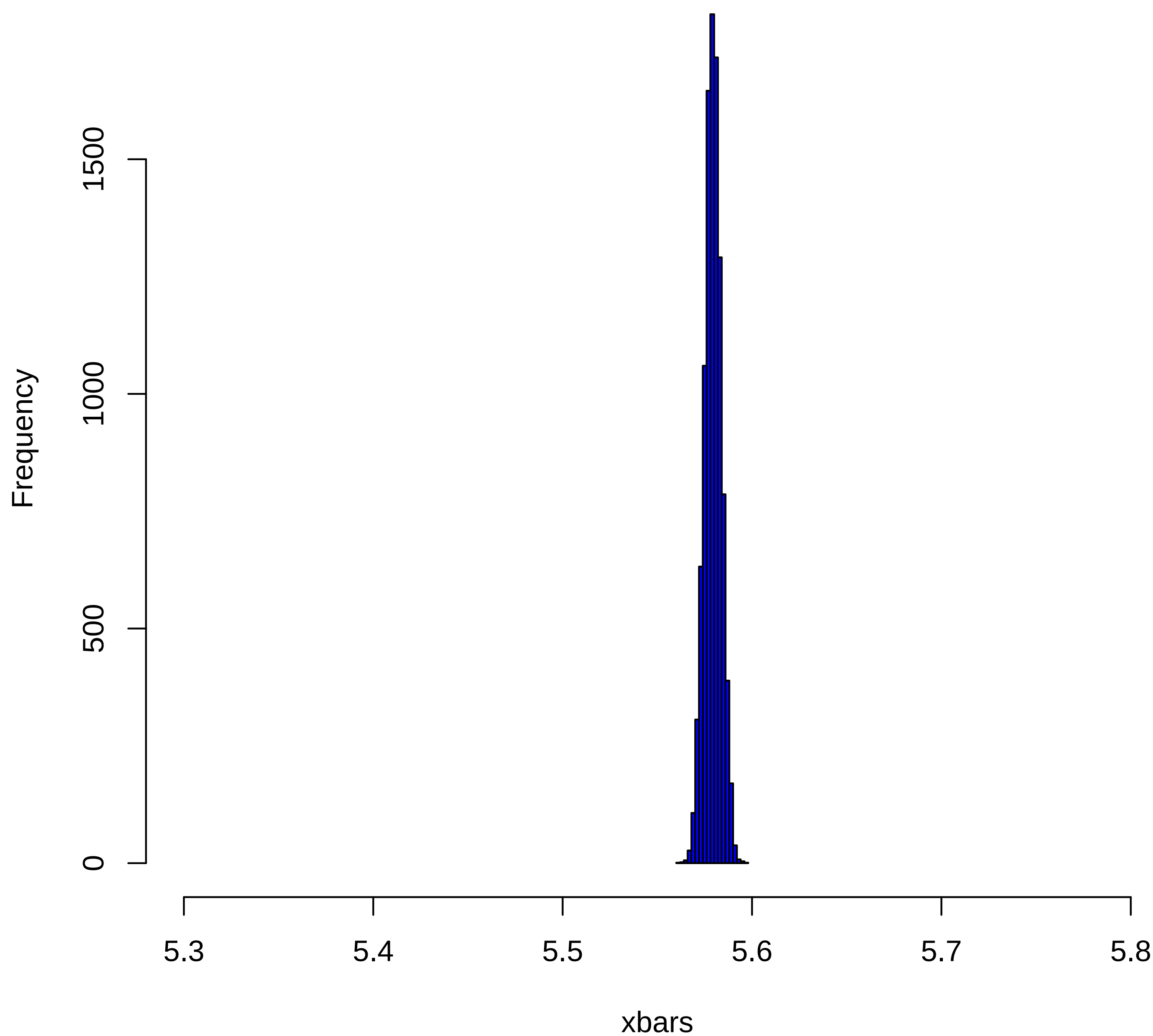
Histogram of xbars(heights): sample size = 100,
sample mean= 5.57950, sample sd=0.02500
Frequency
0
500
1000
1500
5.3
5.4
5.5
5.6
5.7
5.8
xbars

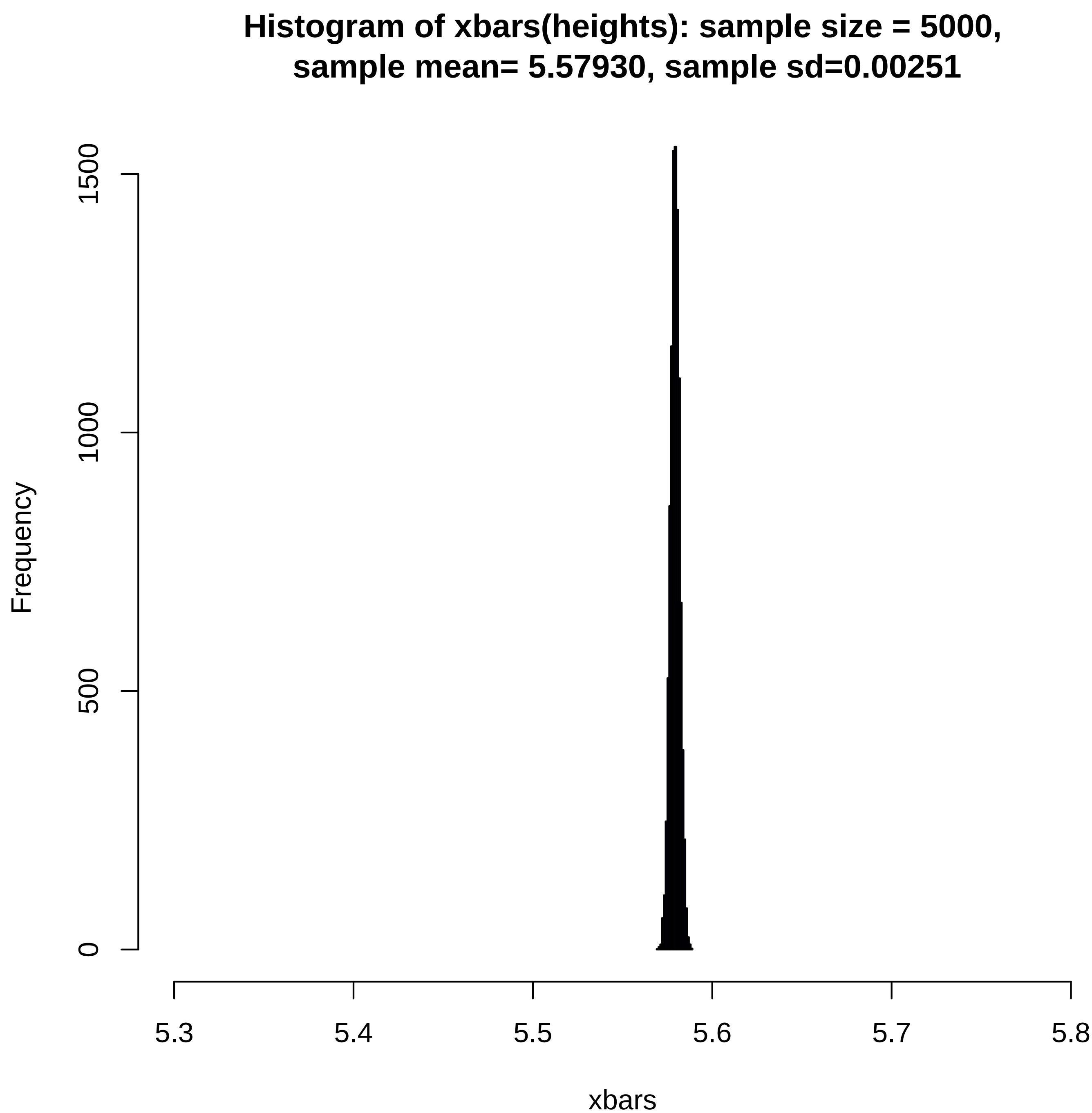

Histogram of xbars(heights): sample size = 500,
sample mean= 5.57938, sample sd=0.01075
Frequency
0
500
1000
1500
5.3
5.4
5.5
5.6
5.7
5.8
xbars

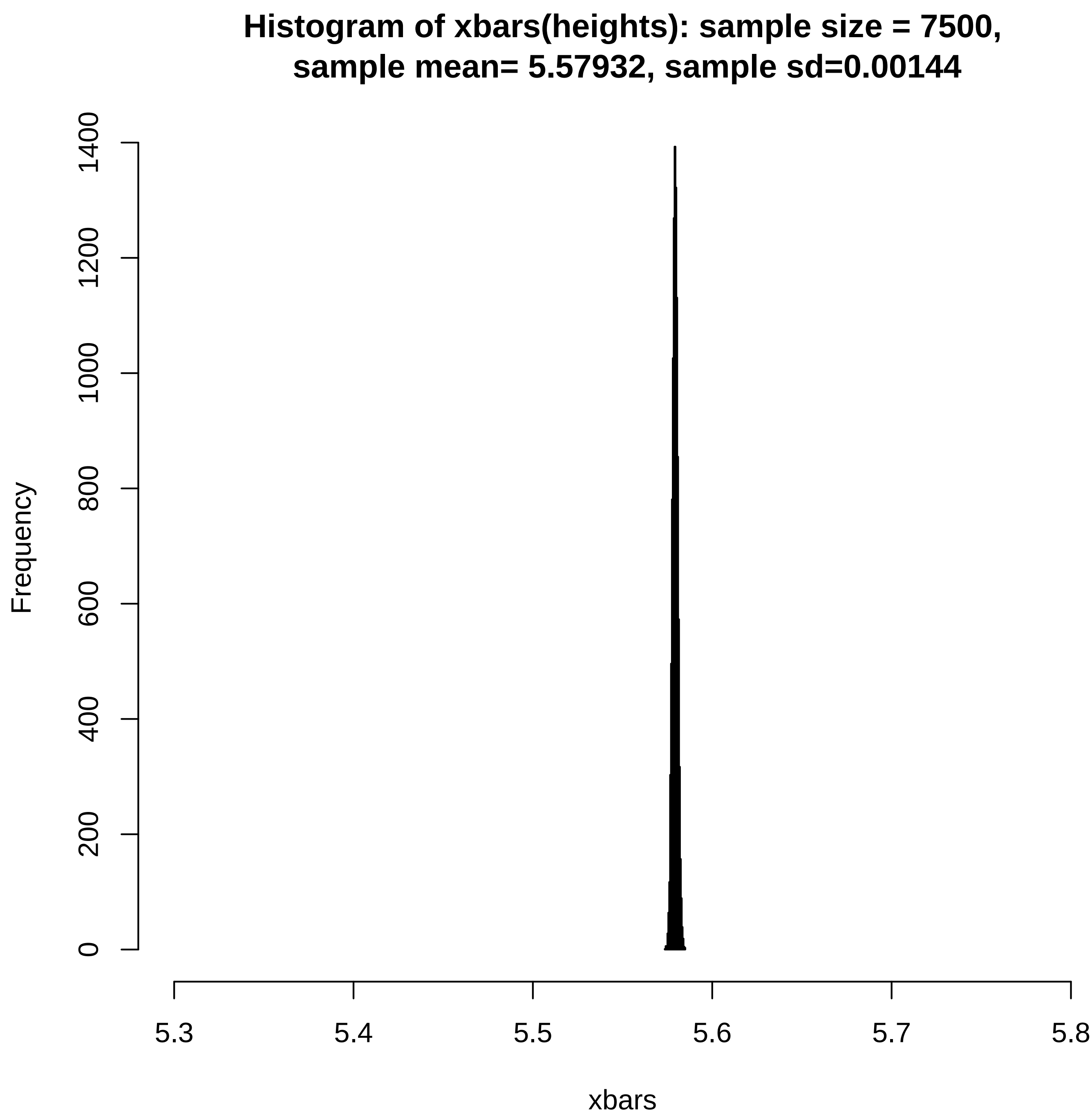


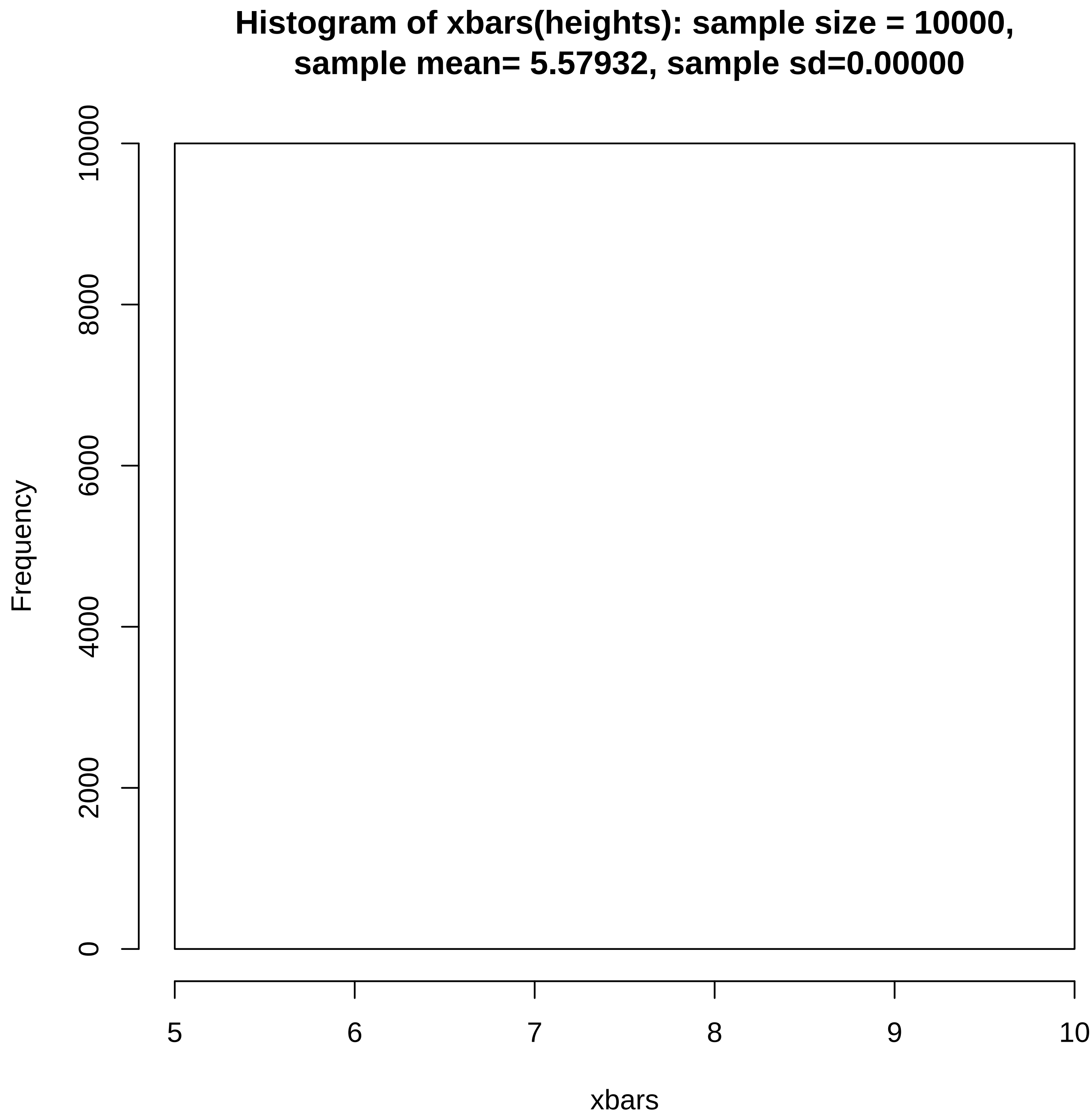
Histogram of xbars(heights): sample size = 1000,
sample mean= 5.57929, sample sd=0.00754
Frequency
0
500
1000
1500
2000
2500
5.3
5.4
5.5
5.6
5.7
5.8
xbars

Histogram of xbars(heights): sample size = 2500, sample mean= 5.57935, sample sd=0.00430

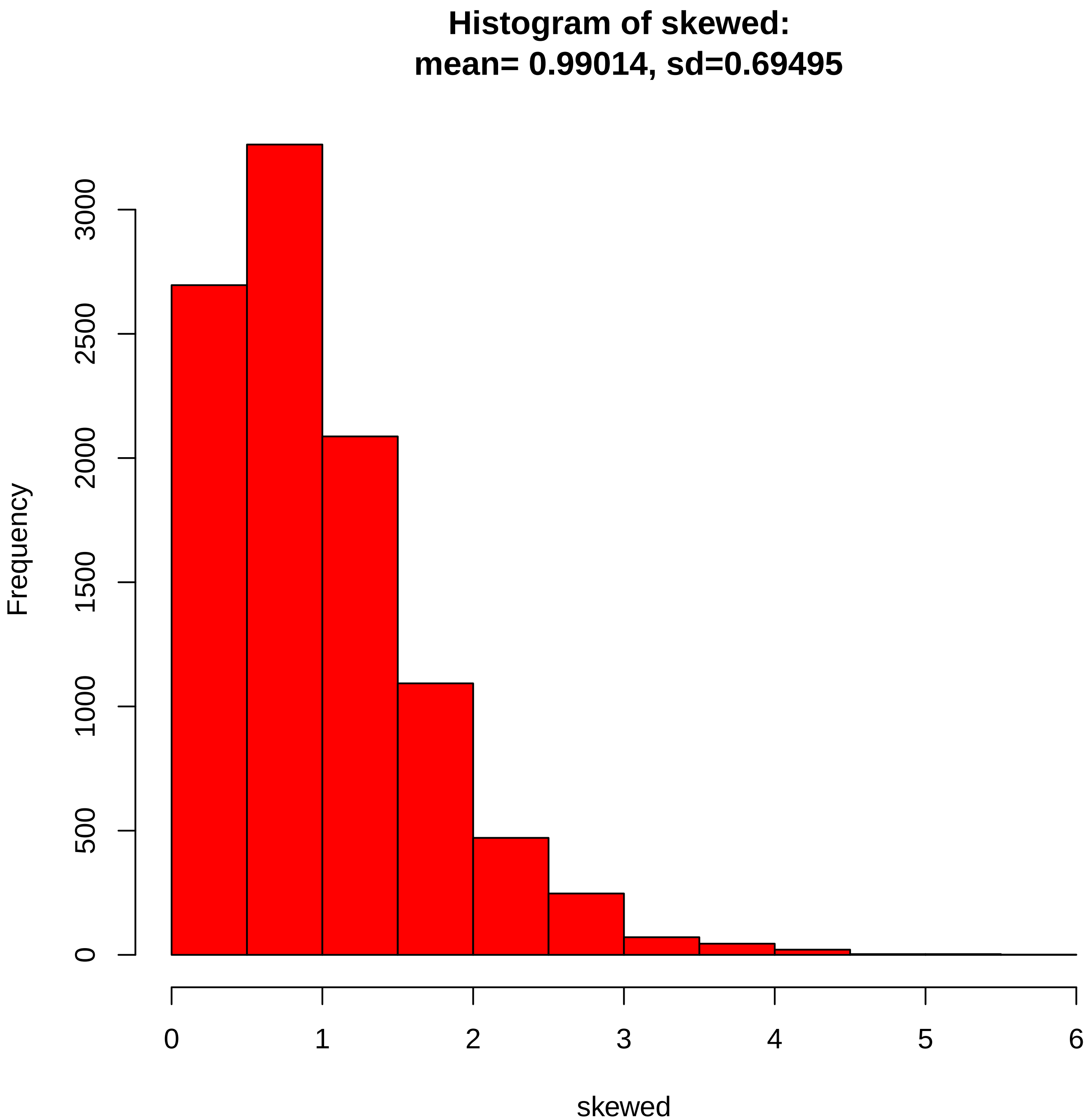



Histogram of xbars(heights): sample size = 5000,
sample mean= 5.57930, sample sd=0.00251
Frequency
0
500
1000
1500
5.3
5.4
5.5
5.6
5.7
5.8
xbars


Histogram of xbars(heights): sample size = 7500,
sample mean= 5.57932, sample sd=0.00144
Frequency
0
200
400
600
800
1000
1200
1400
5.3
5.4
5.5
5.6
5.7
5.8
xbars


Histogram of xbars(heights): sample size = 10000,
sample mean= 5.57932, sample sd=0.00000
Frequency
0
2000
4000
6000
8000
10000
5
6
7
8
9
10
xbars

Let us repeat this experiment, but this time we will begin with a distribution that is not normal but skewed right with mean $\mu = 0.99014$ and standard deviation $\sigma = 0.69495$:



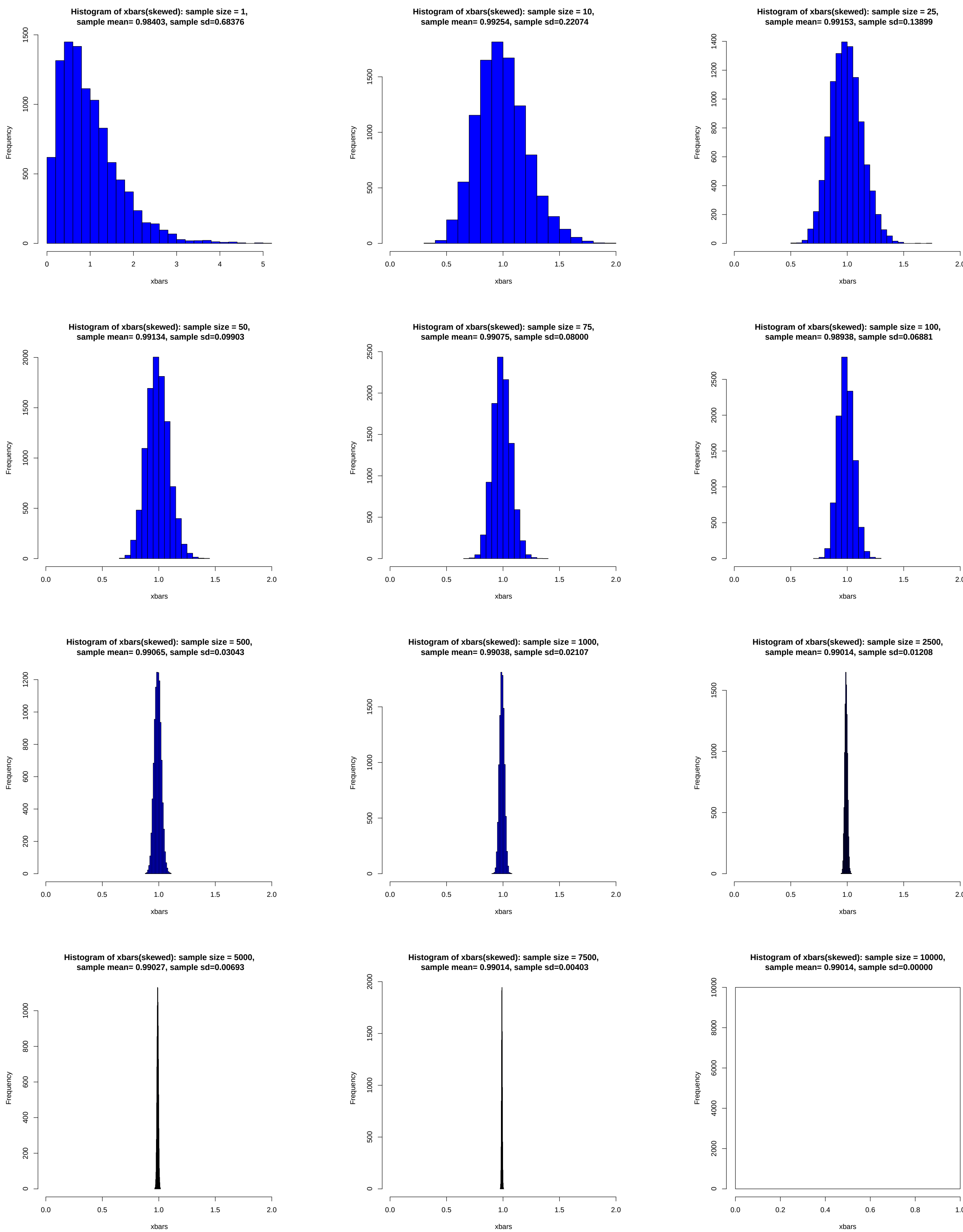
and with the following summary stats:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	Std Dev
0.0142	0.4763	0.8303	0.9901	1.3382	5.6951	0.69495

Finding samples of sizes

$$n = 1, 10, 25, 50, 75, 100, 500, 1000, 2500, 5000, 10000$$

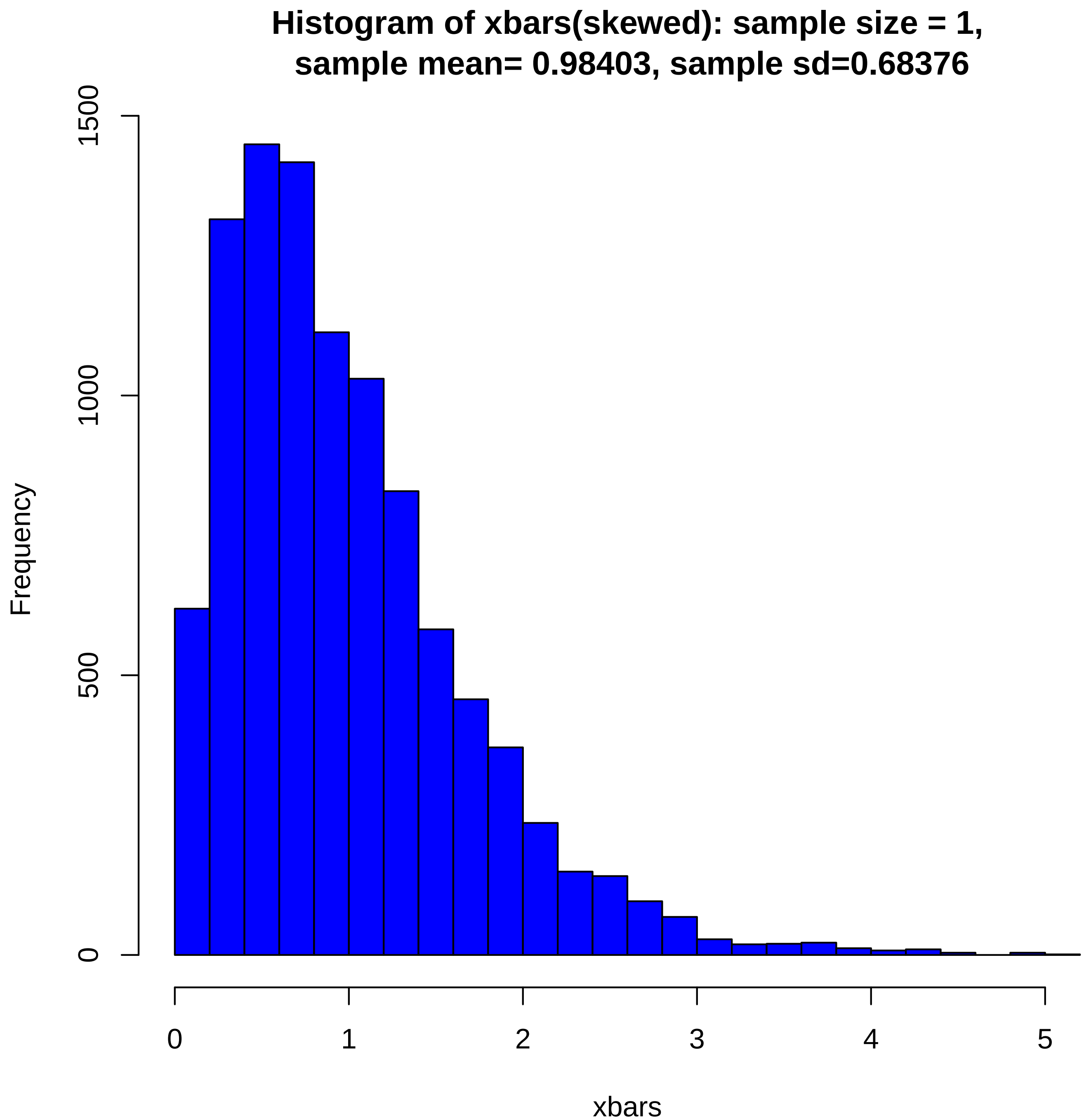
and graphing their histograms we get:

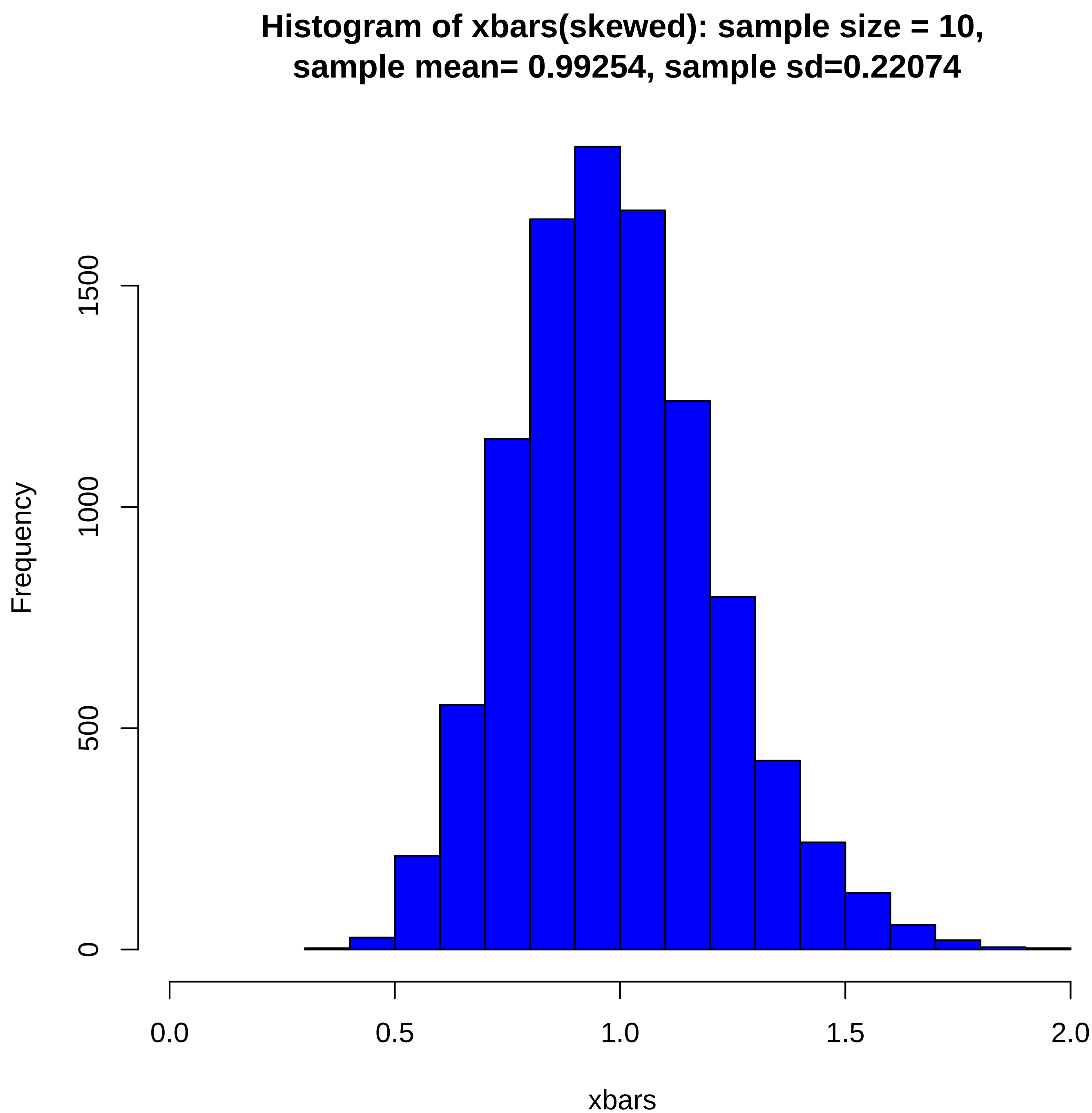

Histogram of xbars(skewed): sample size = 1, sample mean= 0.98403, sample sd=0.68376
Histogram of xbars(skewed): sample size = 10, sample mean= 0.99254, sample sd=0.22074
Histogram of xbars(skewed): sample size = 25, sample mean= 0.99153, sample sd=0.13899
Histogram of xbars(skewed): sample size = 50, sample mean= 0.99134, sample sd=0.09903
Histogram of xbars(skewed): sample size = 75, sample mean= 0.99075, sample sd=0.08000
Histogram of xbars(skewed): sample size = 100, sample mean= 0.98938, sample sd=0.06881
Histogram of xbars(skewed): sample size = 500, sample mean= 0.99065, sample sd=0.03043
Histogram of xbars(skewed): sample size = 1000, sample mean= 0.99038, sample sd=0.02107
Histogram of xbars(skewed): sample size = 2500, sample mean= 0.99014, sample sd=0.01208
Histogram of xbars(skewed): sample size = 5000, sample mean= 0.99027, sample sd=0.00693
Histogram of xbars(skewed): sample size = 7500, sample mean= 0.99014, sample sd=0.00403
Histogram of xbars(skewed): sample size = 10000, sample mean= 0.99014, sample sd=0.00000
Frequency
xbars

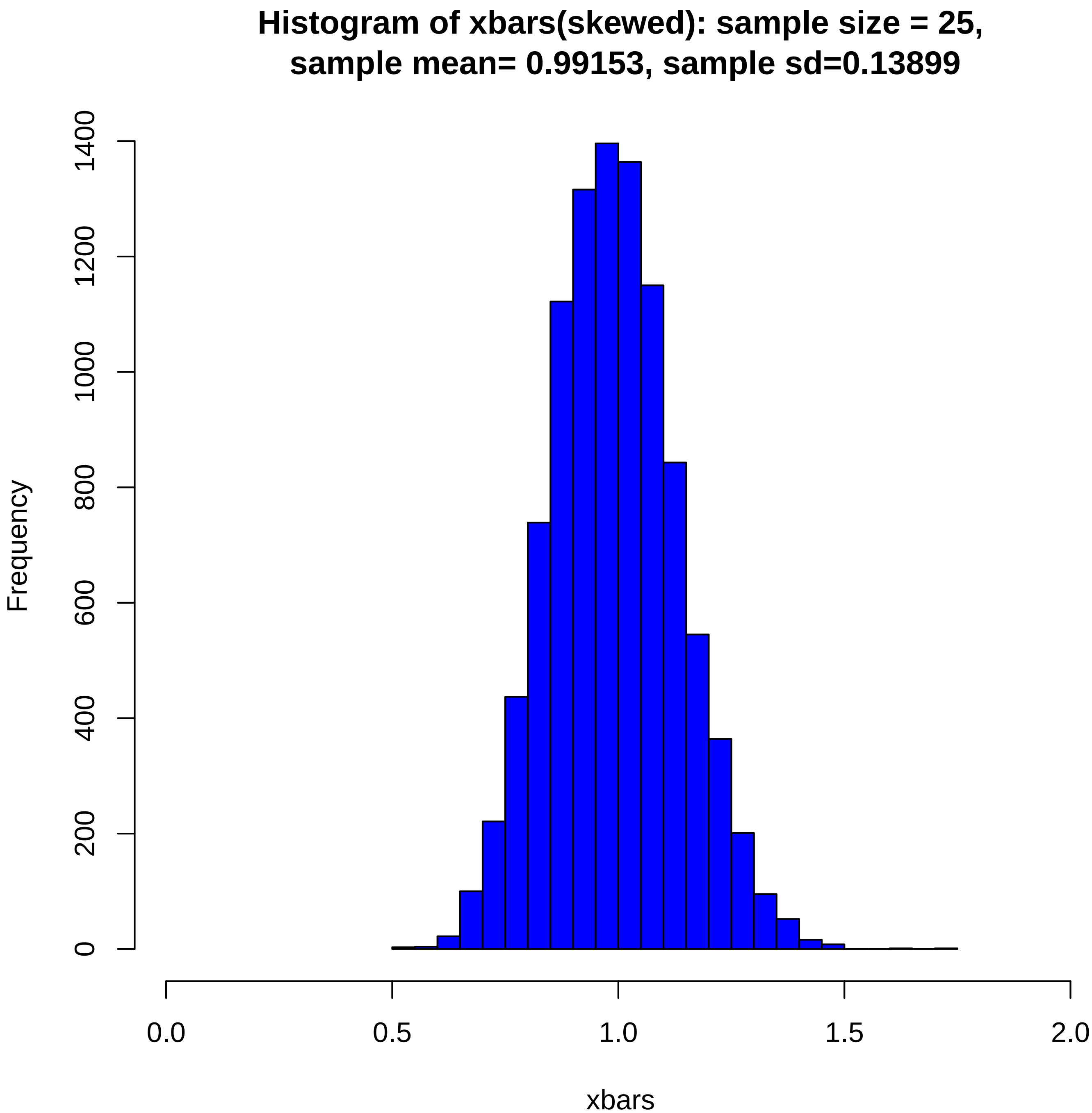
The first thing to notice is that even though our original distribution was not normal but skewed right the sampling distributions start to look more normal as the sample size increases. Next looking at the sample means $\mu_{\overline{X}} = 0.9901385$ and sample standard deviations $\sigma_{\overline{X}} = 0.69495$:

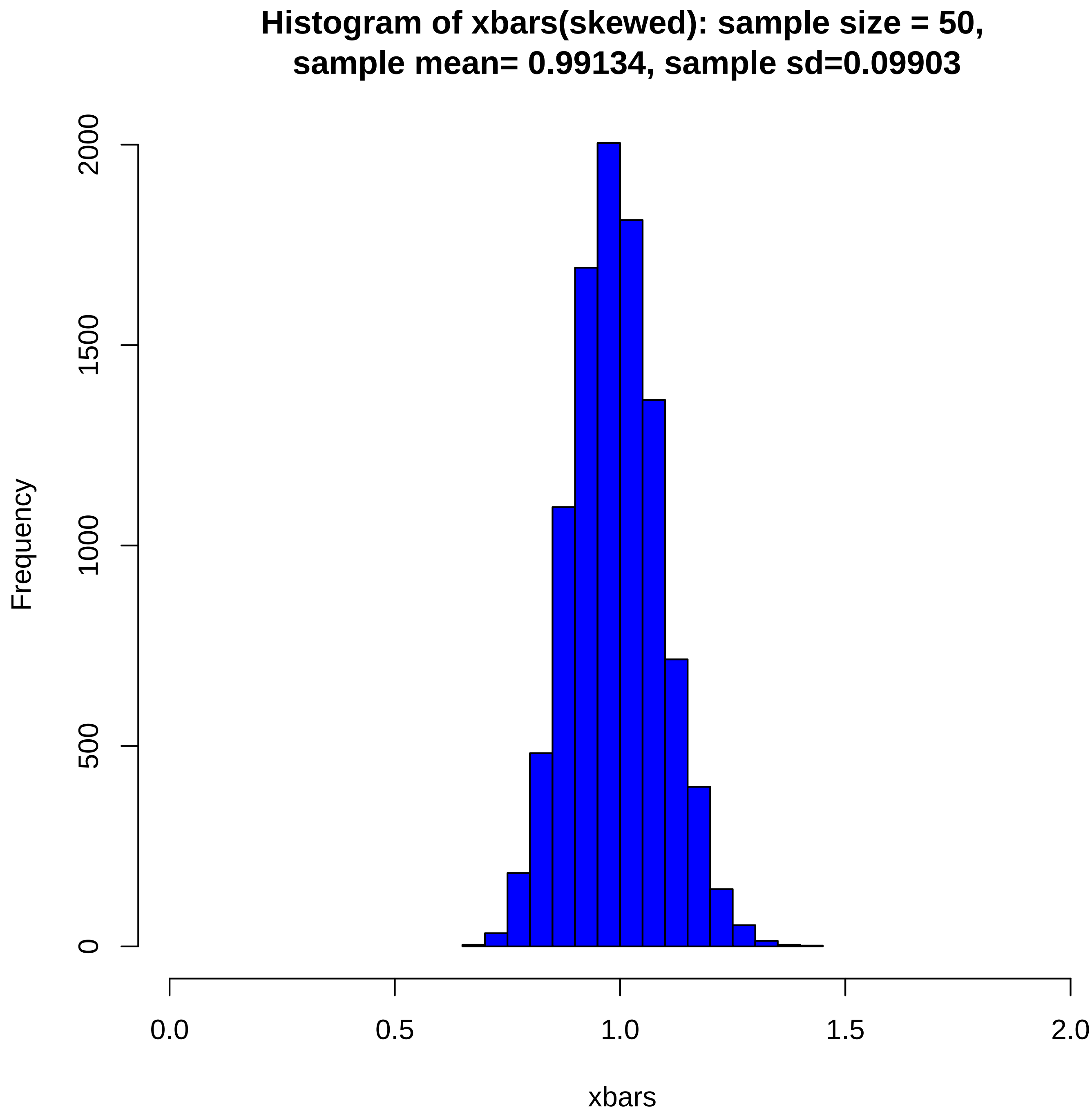
Sample size	$\mu_{\overline{X}}$	$\sigma_{\overline{X}}$
1	0.9840327	0.683760893
10	0.9925392	0.220744517
25	0.9915268	0.138985857
50	0.9913373	0.099026387
75	0.9907530	0.080001154
100	0.9893763	0.068806402
500	0.9906488	0.030432479
1000	0.9903801	0.021065101
2500	0.9901397	0.012081798
5000	0.9902687	0.006932911
7500	0.9901378	0.004033101
10000	0.9901385	0.000000000

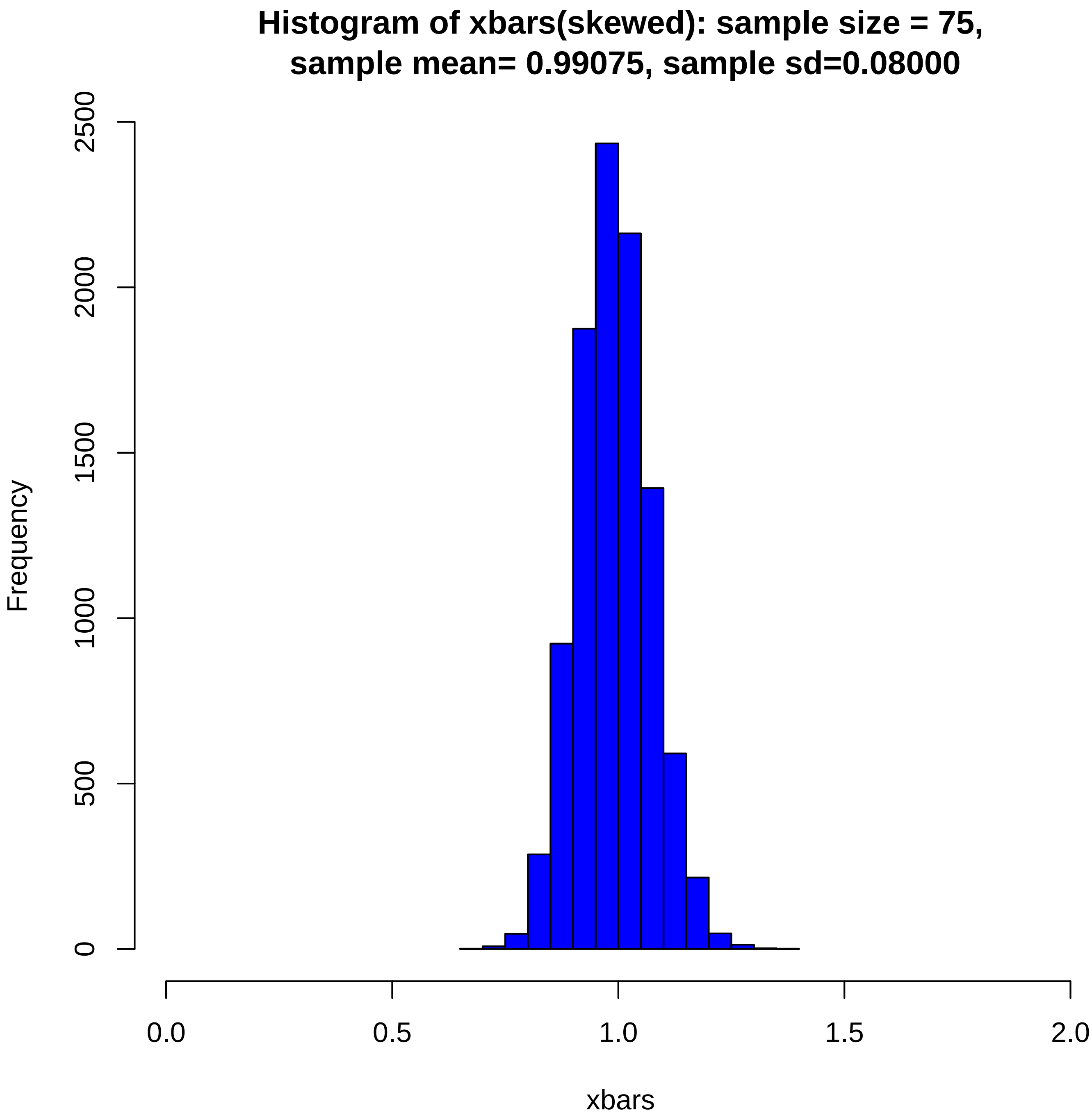
we notice that the sample means hover around the population mean of $\mu = 0.9901385$. And because the sample standard deviations are going to 0 as the sample size increases we know that the sample means become better approximations to the population mean. When the sample size becomes the population size the sample mean is exactly the population mean $\mu = \mu_{\overline{X}}$ and the sample standard deviation is $\sigma_{\overline{X}} = 0$.

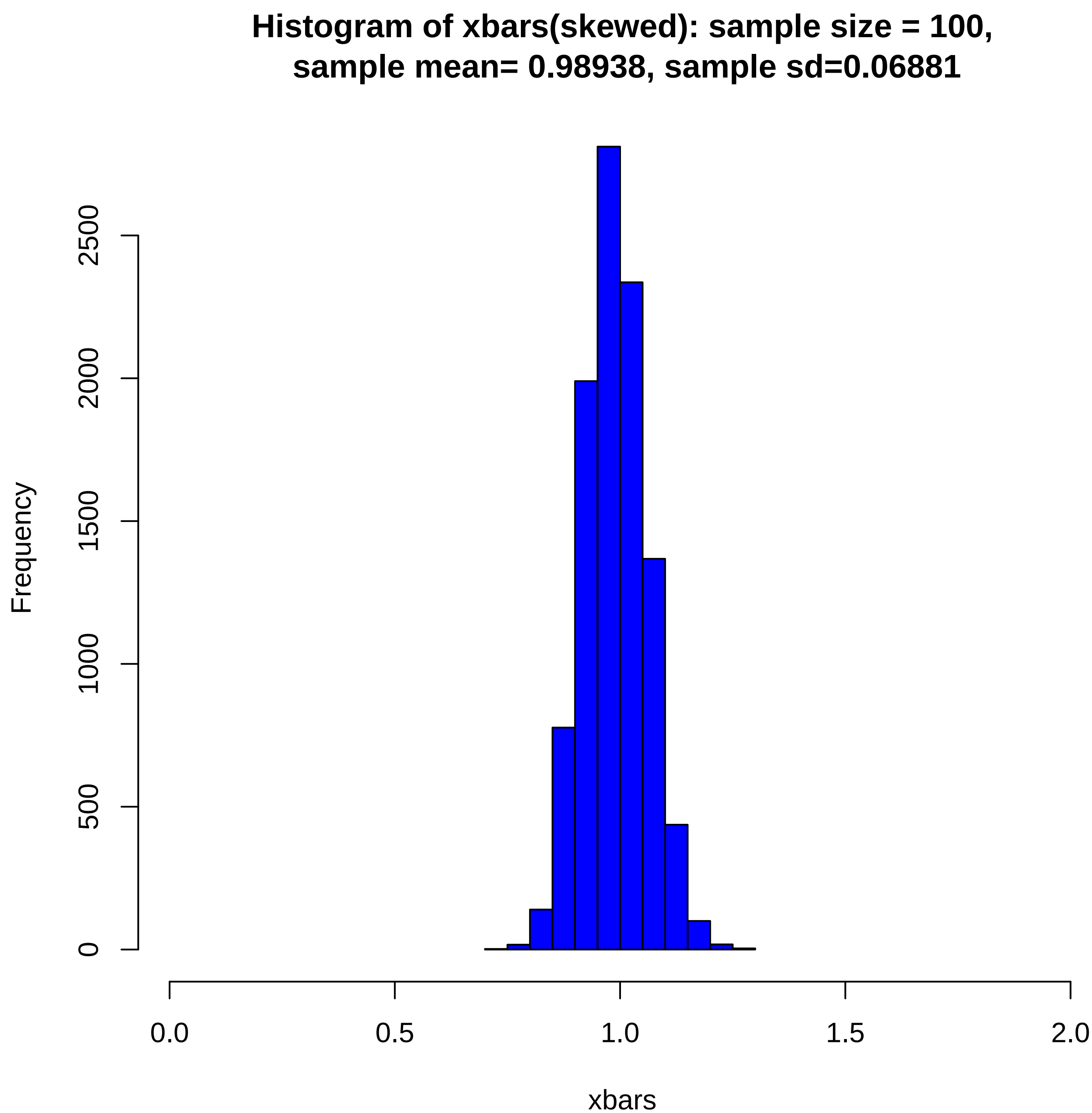

Histogram of xbars(skewed): sample size = 1,
sample mean= 0.98403, sample sd=0.68376
Frequency
0
500
1000
1500
0
1
2
3
4
5
xbars


Histogram of xbars(skewed): sample size = 10,
sample mean= 0.99254, sample sd=0.22074
Frequency
0
500
1000
1500
0.0
0.5
1.0
1.5
2.0
xbars

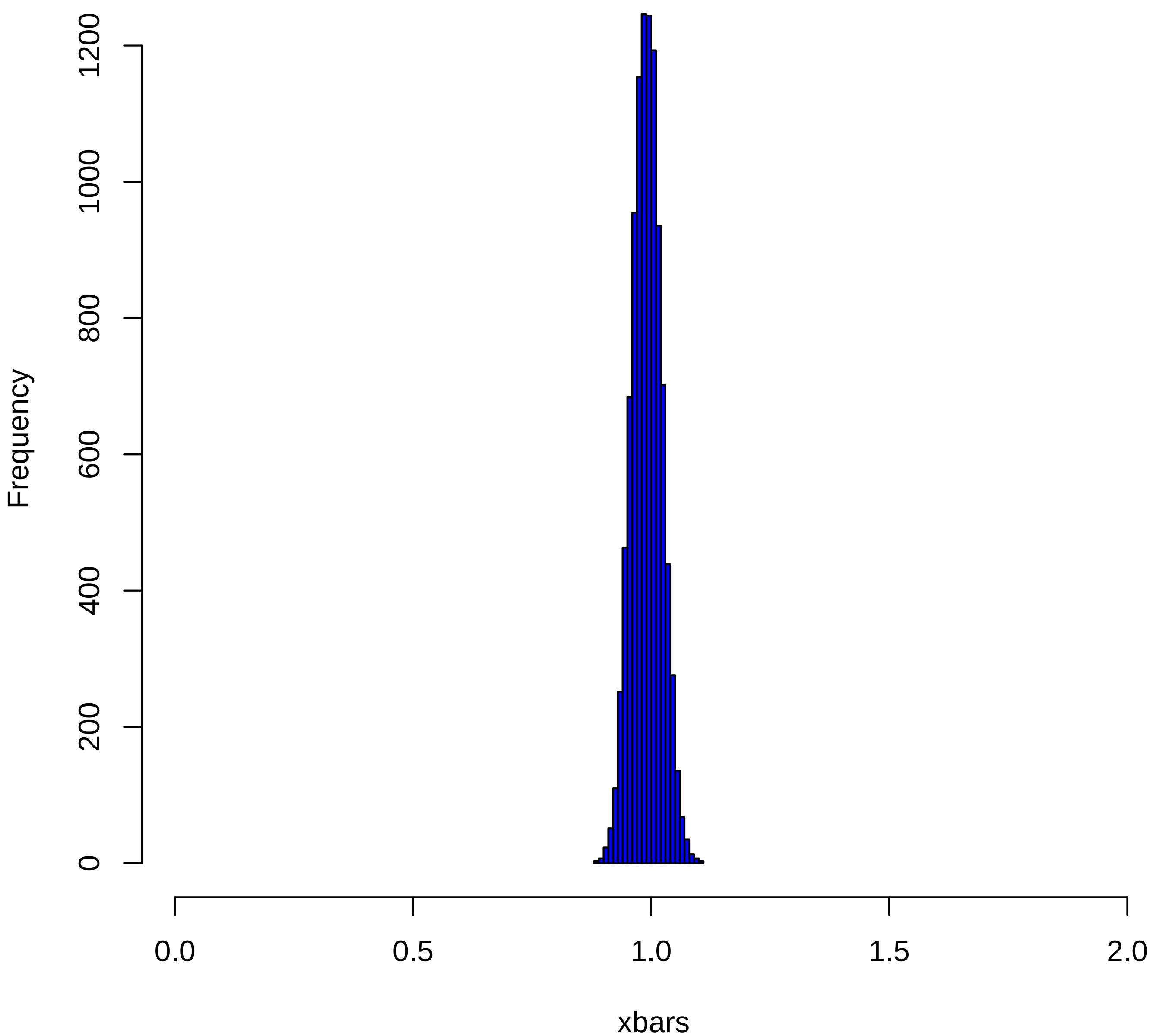

Histogram of xbars(skewed): sample size = 25,
sample mean= 0.99153, sample sd=0.13899
Frequency
0
200
400
600
800
1000
1200
1400
0.0
0.5
1.0
1.5
2.0
xbars


Histogram of xbars(skewed): sample size = 50,
sample mean= 0.99134, sample sd=0.09903
Frequency
0
500
1000
1500
2000
0.0
0.5
1.0
1.5
2.0
xbars

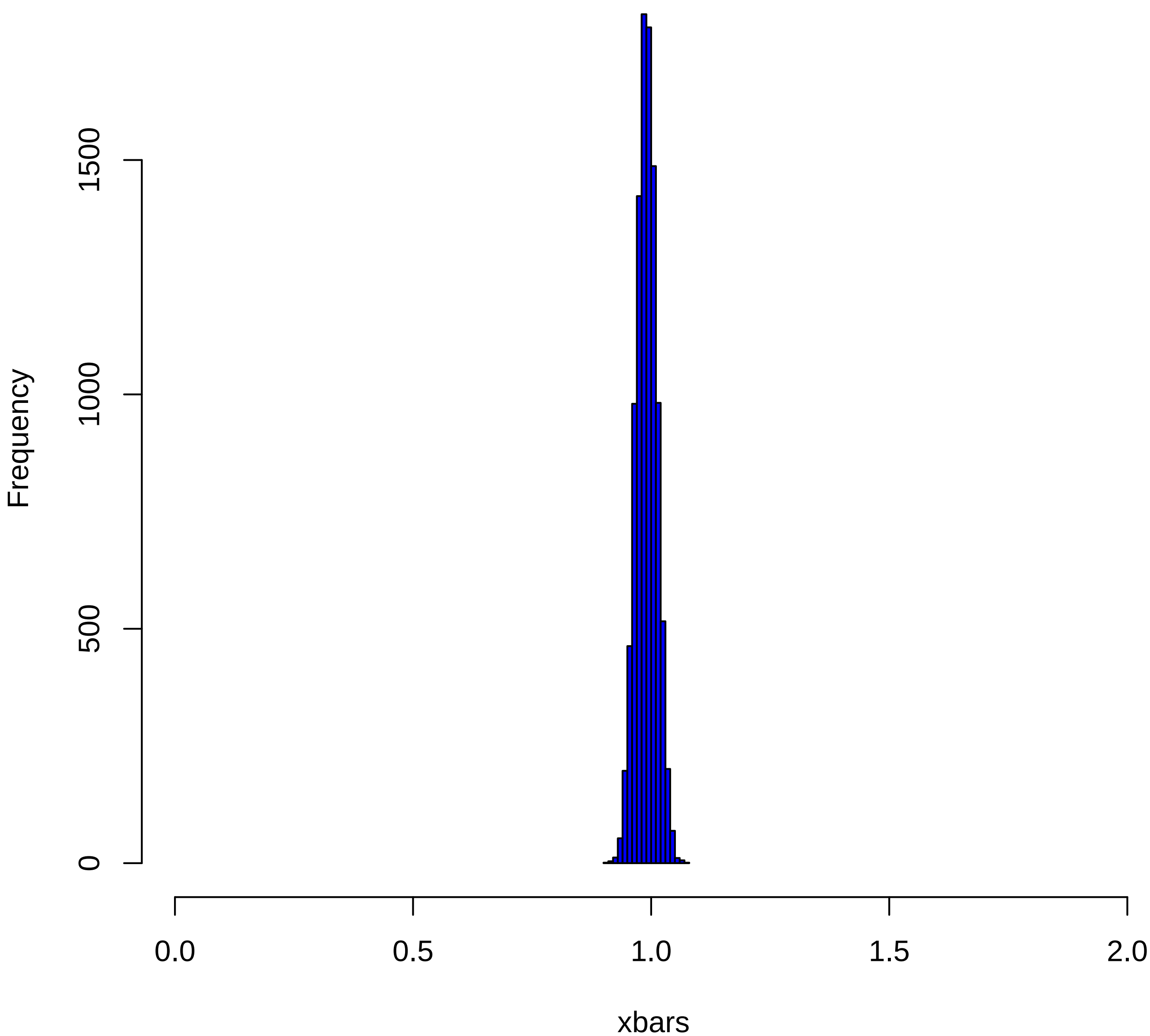

Histogram of xbars(skewed): sample size = 75,
sample mean= 0.99075, sample sd=0.08000
Frequency
0
500
1000
1500
2000
2500
0.0
0.5
1.0
1.5
2.0
xbars

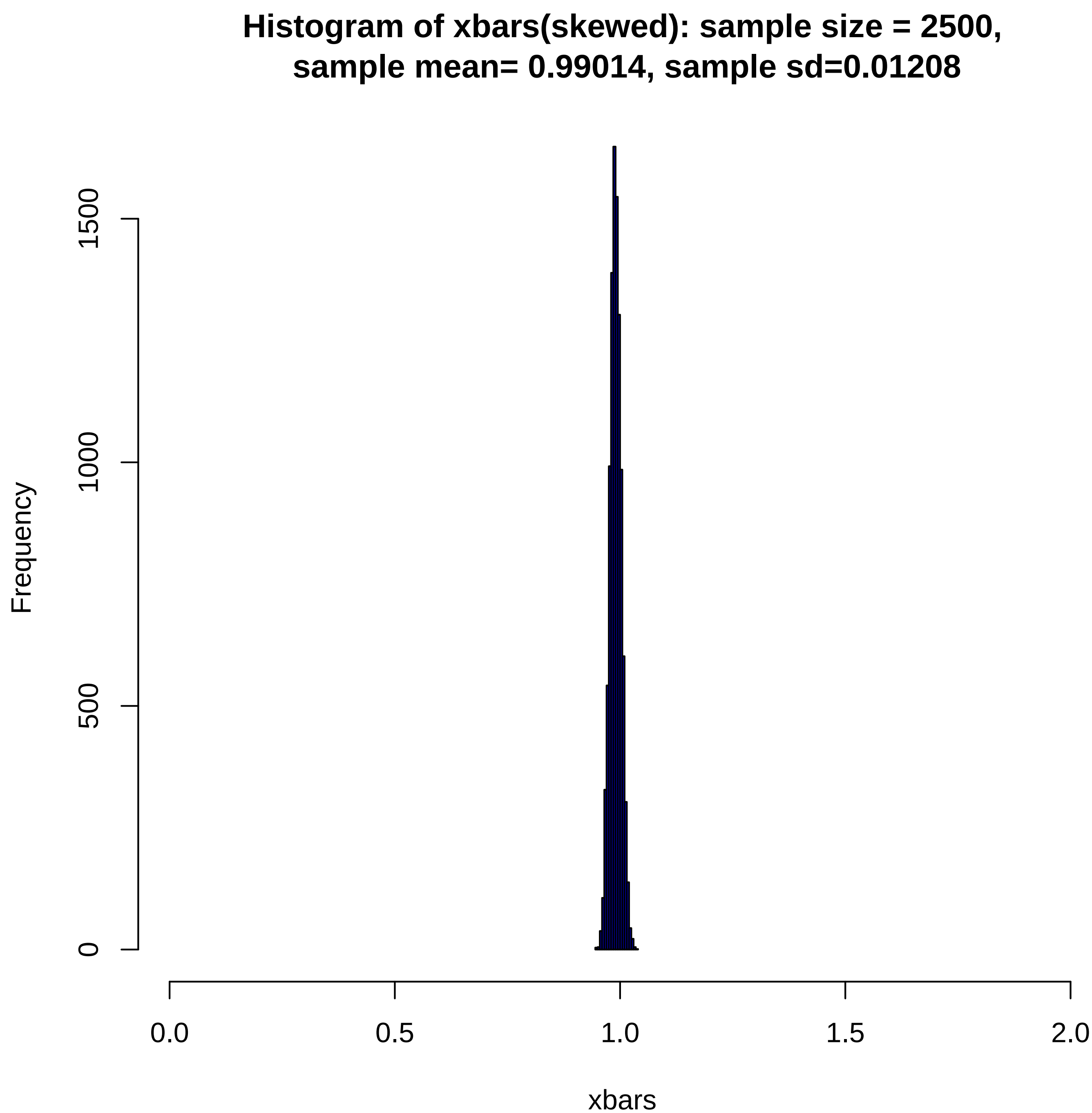

Histogram of xbars(skewed): sample size = 100,
sample mean= 0.98938, sample sd=0.06881
Frequency
0
500
1000
1500
2000
2500
0.0
0.5
1.0
1.5
2.0
xbars

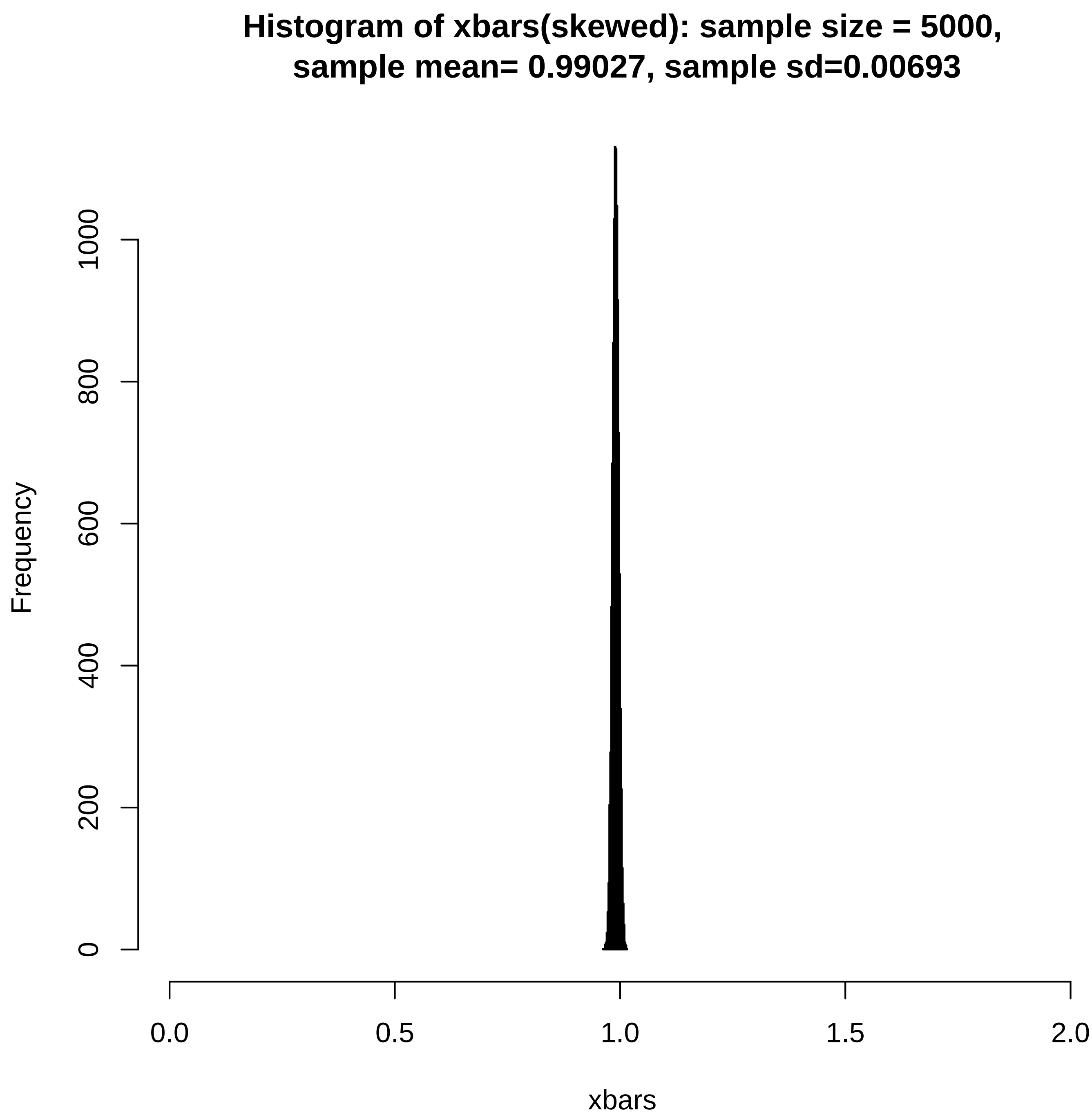
Histogram of xbars(skewed): sample size = 500, sample mean= 0.99065, sample sd=0.03043

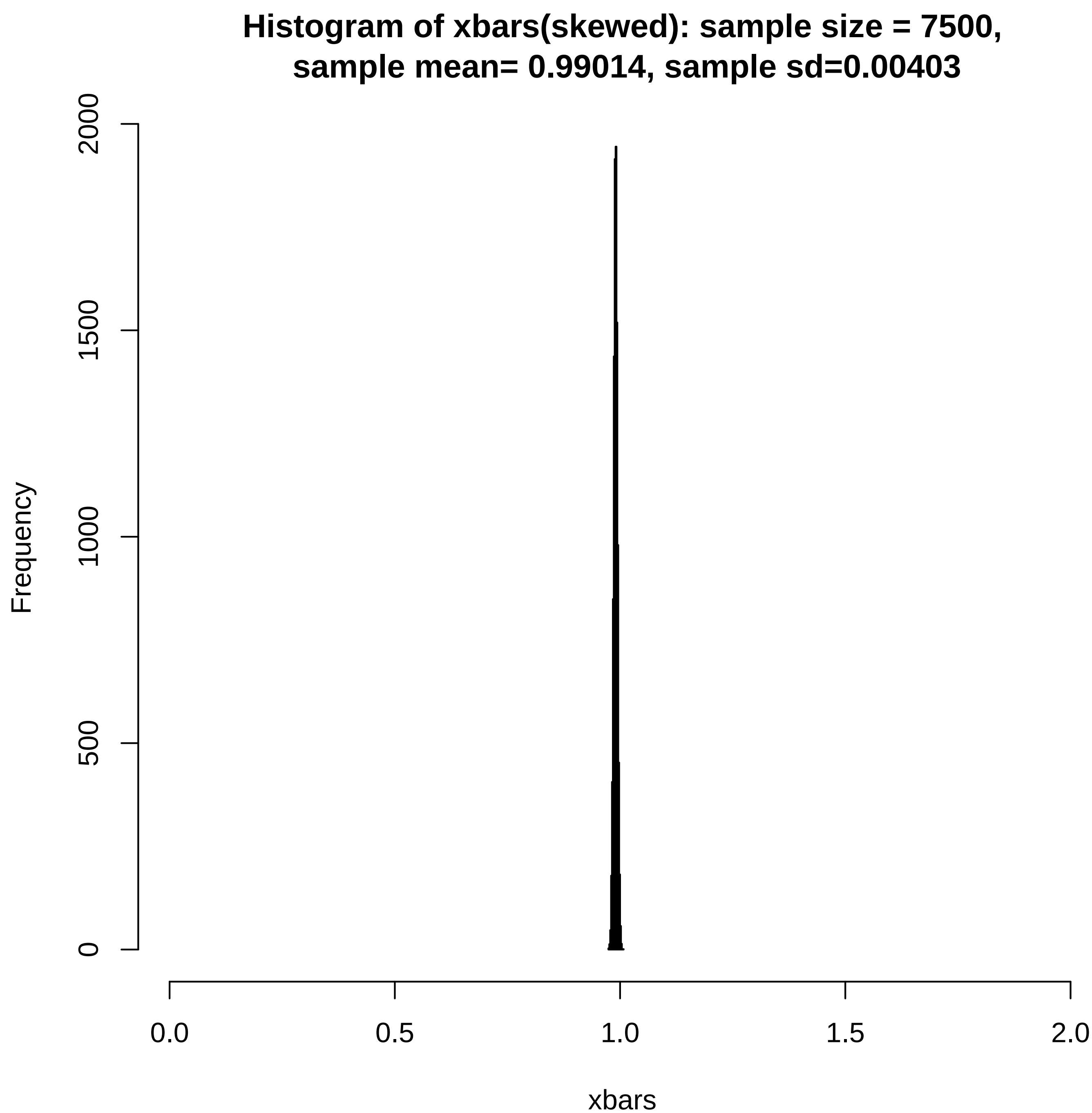


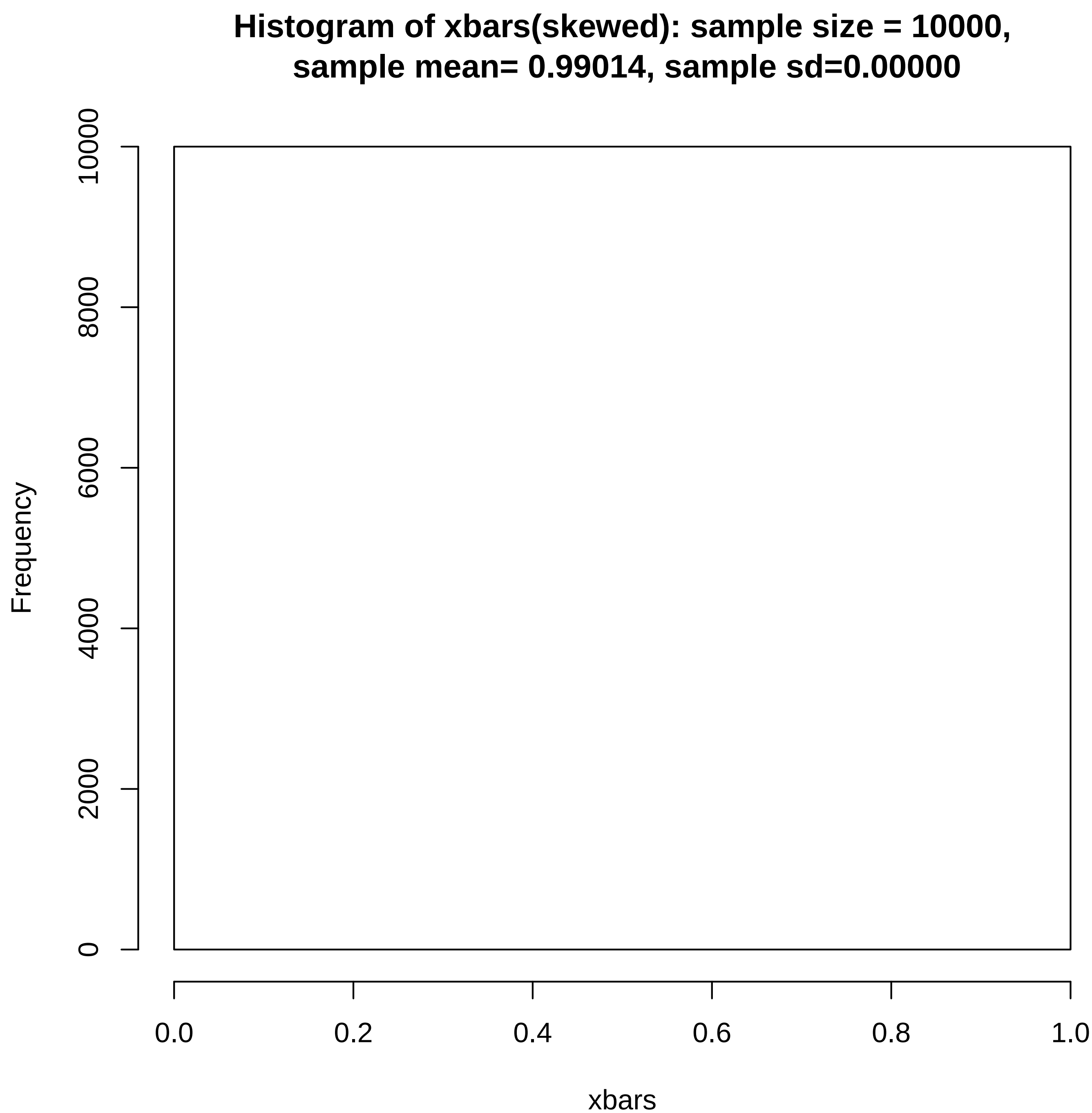
Histogram of xbars(skewed): sample size = 1000, sample mean= 0.99038, sample sd=0.02107




Histogram of xbars(skewed): sample size = 2500,
sample mean= 0.99014, sample sd=0.01208
Frequency
0
500
1000
1500
0.0
0.5
1.0
1.5
2.0
xbars


Histogram of xbars(skewed): sample size = 5000,
sample mean= 0.99027, sample sd=0.00693
Frequency
0
200
400
600
800
1000
0.0
0.5
1.0
1.5
2.0
xbars


Histogram of xbars(skewed): sample size = 7500,
sample mean= 0.99014, sample sd=0.00403
Frequency
0
500
1000
1500
2000
0.0
0.5
1.0
1.5
2.0
xbars


Histogram of xbars(skewed): sample size = 10000,
sample mean= 0.99014, sample sd=0.00000
Frequency
0
2000
4000
6000
8000
10000
0.0
0.2
0.4
0.6
0.8
1.0
xbars