Homework 8

1. Suppose that X_1, X_2, X_3 is a random sample from a population $X \sim N(0, \sigma^2)$. Find the distributions of the following statistic: $X_1 + X_2 + X_3$;

2. It is known that the bulbs (电灯泡) made by a company obey a normal distribution $X \sim N(1000, \sigma^2)$ (unit: hour). Now, a random sample of size 9 is obtained and also its corresponding mean value and variance. Unfortunately, the record of the mean value missed due to some reason and only the variance of the random sample was left, $S^2 = 100^2$. Calculate the probability $P(\overline{X} > 1062)$. ($t_{0.05}(8) = 1.86$)

3. Suppose that $X_1, X_2, \dots, X_{100}$ is a random sample from a population $X \sim N(60, 15^2)$. Compute the probability $P\left(\left|\overline{X} - 60\right| > 3\right)$. ($\Phi(2) = 0.9772$)

4. Suppose that $X_1, X_2, \dots, X_n$ is a random sample from a population $X \sim N(\mu, \sigma^2)$, with μ be known and σ^2 unknown. Which one of the following is not a statistic? (   )

   A. $\sum_{i=1}^{n}(X_i - \mu)^2$;   B. $\frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$;   C. $\sum_{i=1}^{n}\frac{X_i}{\sigma}$;   D. $\min_i\{X_i\}$

5. Suppose that random variables $X \sim N(\mu, 1^2)$, and $Y \sim \chi^2(n)$. Let $T = \frac{X - \mu}{\sqrt{Y}}\sqrt{n}$. Then, which one of the following statements is right? (   )

   A. $T \sim t(n-1)$;   B. $T \sim t(n)$;   C. T has a normal distribution;   D. $T \sim F(1, n)$

6. Suppose that $X_1, X_2, \dots, X_{10}$ is a random sample from a population $X \sim N(\mu, 0.5^2)$.

   If $\mu = 0$, calculate the probability $P\left(\sum_{i=1}^{10} X_i \geq 4\right)$. ($\sqrt{10} = 3.1623$, $\Phi(2.53) = 0.9943$)

7. Suppose that $X_1, X_2, \dots, X_4$ is a random sample from a population X. Given the following four point estimators for the mean of the population X as

   (A). $\frac{1}{3}X_1 + \frac{1}{6}X_2 + \frac{1}{3}X_3 + \frac{1}{6}X_4$.   (B). $\frac{1}{2}X_1 + \frac{1}{3}X_2 + \frac{1}{12}X_3 + \frac{1}{12}X_4$

   (C). $\frac{1}{3}X_1 + \frac{1}{6}X_2 + \frac{1}{19}X_3 + \frac{7}{8}X_4$.   (D). $\frac{1}{4}X_1 + \frac{1}{4}X_2 + \frac{1}{4}X_3 + \frac{1}{4}X_4$

   Whether the above estimators are unbiased or not? Which one is the most efficient estimator for the mean of the population X? (Hint: note that the mean of the population X is $E(X)$, while the mean of the random sample is denoted by $\overline{X} = \frac{1}{4}\sum_{i=1}^{4} X_i$. The most efficient estimator should have the smallest variance).

8. Assume that a population X has a possible pdf as $f(x;\theta) = \begin{cases} (\theta+1)x^{\theta}, 0 < x < 1 \\ 0, \text{otherwise} \end{cases}$, where $\theta > 1$ is unknown. Suppose $X_1, X_2, \dots, X_n$ is a random sample from X with observed values as x_1, x_2, $\dots$, x_n. Find the moment estimator and maximum likelihood estimator of θ.