Homework 10

1. Study the following knowledge points by heart, and copy them to your answer sheet: (50′)

   Standards on hypothesis testing for a parameter in a normal population $X \sim N(\mu, \sigma^2)$.

parameters to be tested	conditions	H_0	H_1	the rejection region for H_0	involved statistic
μ	σ^2 is known	$\mu = \mu_0$	$\mu \neq \mu_0$	$\lvert Z\rvert > z_{\alpha/2}$	$Z = \dfrac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$
		$\mu \leq (\text{or} =)\mu_0$	$\mu > \mu_0$	$Z > z_\alpha$	
		$\mu \geq (\text{or} =)\mu_0$	$\mu < \mu_0$	$Z < -z_\alpha$	
μ	σ^2 is unknown	$\mu = \mu_0$	$\mu \neq \mu_0$	$\lvert t\rvert > t_{\alpha/2}(n-1)$	$t = \dfrac{\overline{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1)$
		$\mu \leq (\text{or} =)\mu_0$	$\mu > \mu_0$	$t > t_\alpha(n-1)$	
		$\mu \geq (\text{or} =)\mu_0$	$\mu < \mu_0$	$t < -t_\alpha(n-1)$	
σ^2	μ is known	$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$\chi^2 > \chi^2_{\alpha/2}(n)$ or $\chi^2 < \chi^2_{1-\alpha/2}(n)$	$\chi^2 = \dfrac{\sum_{i=1}^n (X_i - \mu)^2}{\sigma_0^2} \sim \chi^2(n)$
		$\sigma^2 \leq (\text{or} =)\sigma_0^2$	$\sigma^2 > \sigma_0^2$	$\chi^2 > \chi^2_\alpha(n)$	
		$\sigma^2 \geq (\text{or} =)\sigma_0^2$	$\sigma^2 < \sigma_0^2$	$\chi^2 < \chi^2_{1-\alpha}(n)$	
σ^2	μ is unknown	$\sigma^2 = \sigma_0^2$	$\sigma^2 \neq \sigma_0^2$	$\chi^2 > \chi^2_{\alpha/2}(n-1)$ or $\chi^2 < \chi^2_{1-\alpha/2}(n-1)$	$\chi^2 = \dfrac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1)$
		$\sigma^2 \leq (\text{or} =)\sigma_0^2$	$\sigma^2 > \sigma_0^2$	$\chi^2 > \chi^2_\alpha(n-1)$	
		$\sigma^2 \geq (\text{or} =)\sigma_0^2$	$\sigma^2 < \sigma_0^2$	$\chi^2 < \chi^2_{1-\alpha}(n-1)$	

   when the signs " $\leq$ " and " $\geq$ " in H_0 are replaced by " $=$ ", the rejection regions for H_0 keep unchanged.

2. Suppose that the measurement accuracy of a range finder is $\sigma = 10m$. Now, a set of 9 measurements about the distance of an object which is 500m away is obtained with sample mean $\overline{X} = 510m$. Given the significance level $\alpha = 0.05$, whether the range finder has systems errors or not? (Hint: $\sigma = 10m$ is known, and test if $\mu = 500$ or not. $\Phi(1.96) = 0.975$)? (20′)

3. Suppose that the heights of a kind animal in a region obey a normal distribution. Now 36 animals are randomly selected, and the measured mean height is $\overline{X} = 67.5$, the sample variance $S^2 = 13^2$. Given the significance level $\alpha = 0.05$. (It is known that the critical points $t_{0.025}(35) = 2.0301$, $t_{0.025}(36) = 2.2081$, $t_{0.05}(35) = 1.6896$, $\chi^2_{0.025}(35) = 53.203$, $\chi^2_{1-0.025}(35) = 20.569$). (30′)

   (1) Can we say that the mean height of this kind of animal in this region is 71.5?

   (2) Can we say that the variance of this kind of animals' heights in this region is 12^2?