称　Probability theory and statistics

考试时间 120 分钟　A 卷　共 4 页第 页

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一、选择题（本大题共 8 小题，每小题 3 分，总计 24 分；Choose the best answer from the four choices marked A, B, C and D for each of the following questions. 3 points for each question and totally 24 points ）

1. Suppose that events A and B are mutually exclusive, and $P(A)>0, P(B)>0$. Then, which one of the following statements is correct? (　　)

A. $P(A|B)=P(A)$　B. $P(AB)=0$　C. $P(A|B)=1$　D. $P(B|A)>0$

2. Suppose that A and B are two arbitrary events, then $P(A\bar{B})=$ (　　)

A. $P(B)-P(A)$　B. $P(B)-P(AB)$

C. $P(B)-P(A)+P(AB)$　D. $P(A)-P(AB)$

3. It is known that a random variable X has a probability distribution as $X\sim\begin{bmatrix}-8 & 0 & 7\\ 0.2 & 0.6 & 0.2\end{bmatrix}$, then the probability $P(X\le 6.9)$= (　　)

A. 0.2　B. 0.6　C. 0.7　D. 0.8

4. Assume that $P(A)=0.4$, $P(B)=0.6$. If events A and B are independent, then $P(A\cup B)$ =(　　)

A. 0.24　B. 0.2　C. 1　D. 0.76

5. Suppose that random variable X obeys a Poisson distribution with parameter $\lambda\,(\lambda>0)$. If $P(X=4) = P(X=5)$, then $\lambda=$ (　　)

A. $\frac{1}{2}$　B. 2　C. 5　D. 4

6. Suppose that a random variable X obeys a binomial distribution, that is $X\sim b(n,p)$. If the mean and variance of X are, $E(X)=10$, $Var(X)=5$, respectively. Then (　　)

A. $n=100,\ p=0.08$　B. $n=20,\ p=0.50$

C. $n=50,\ p=0.20$　D. $n=40,\ p=0.25$

7. Suppose that random variable $X\sim N(\mu,1)$ and $Y\sim\chi^2(n)$. ee t $T=\frac{X-\mu}{\sqrt{Y}}\sqrt{n}$. Then, which one of the following statements is correct? (　　)

A. $T\sim t(n-1)$　B. $T\sim t(n)$

C. T is a normal random variable　D. $T\sim F(1,n)$

8. Assume that X_1, X_2, and X_3 are three independent standard normal random variables, i.e. $X_i \sim N(0,1), i = 1,2,3$. ee t $Y = -X_1 + 2X_2 - 2X_3$, then the variance of Y is ( )

A. 0 B. 4 C. 9 D. 5

二、判断题（本大题共 5 小题，每小题 2 分，总计 10 分；Determine whether the following statements are true (T) or false (F). 2 points for each question and totally 10 points）

1. Suppose that A is an impossible event, then $P(A) = 0$. （ ）

2. Assume that random variables X and Y are independent, then they must be uncorrelated. （ ）

3. ee t A be an event with probability $P(A) = 1$, then A must be a certain event. （ ）

4. ee t A, B and C be three events. If $P(ABC) = P(A)P(B)P(C)$, then events A, B and C must be mutually independent. （ ）

5. Any random variable X has only one unique cumulative distribution function (CDF) $F(x)$. （ ）

三、填空题（本大题共 8 小题，每小题 2 分，总计 16 分 Fill in the blanks with correct answers，2 points for each blank and totally 16 points)）

1. e et $A, B,$ and C be three events, the event "exactly two of them occur" can be expressed as________.

2. Assume that X is a normal random variable, i.e. $X \sim N(-1, 4)$. Then the expected value $E(3X+1) =$______.

3. ee t $X \sim N(-2, 4)$, $Y \sim N(4, 4)$. If X and Y are independent, then the expected value the variance $Var(X+Y)$=________.

4. Assume that events A and B are independent, and $P(A) = 0.4$, $P(B) = 0.7$, then $P(AB) =$______.

5. Assume that X is a normal random variable, i.e. $X \sim N(-2, 4^2)$. Then the probability $P(X = -2) =$________.

6. Assume that X obeys a uniform distribution over an interval $[1, 3]$. ee t random variable $Y = 15X + 20$, then the correlation coefficient ρ_{XY} between X and Y is ______.

7. Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a population $X \sim N(\mu, \sigma^2)$, where the variance σ^2 is unknown. Then, a confidence interval for the unknown parameter μ with confidence level $1-\alpha$ is ____.

8. Suppose that $X_1, X_2, \ldots, X_6$ is a random sample from a population $X \sim N(0, 1)$. Then, the distribution of the statistic $\frac{X_1 + X_2 + \cdots + X_6}{6}$ is ______.

四、Calculations（总计 50 分；Totally 50 scores）

1. (**15 scores**) Assume that a two-dimensional random variable (X, Y) has the following joint probability distribution as

Y \ X	-2	0	3
-4	0.2	0.3	0.1
1	0.1	0.2	0.1

(1) Find the marginal probability distributions of X and Y, respectively,

(2) Find the probability distribution of $M = max(X, Y)$,

(3) Calculate the expected values $E(X)$ and $E(Y)$,

(4) Calculate the covariance $Cov(X, Y)$ between X and Y.

2. (**12 scores**) Suppose that a continuous random variable X has a probability density function as $f(x) = \begin{cases} kx, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$.

(1) Find the value of the constant k,

(2) Calculate the probability that X is between 0 and 0.5,

(3) Calculate the mean $E(X)$.

3. (**12 scores**) Assume that a population X has the following probability density function

$f(x)=\begin{cases}(\theta+1)x^{\theta}, & 0<x<1\\ 0, & \text{otherwise}\end{cases}$, where $\theta>-1$ is an unknown parameter. Given a random sample $X_1, X_2, \dots, X_n$, find the **moment** estimator of θ.

4. (**6 scores**) Assume that random variables X and Y have the following joint probability density function as $f(x,y)=\begin{cases}2-x-y, & 0\le x\le 1,\ 0\le y\le 1\\ 0, & \text{otherwise}\end{cases}$.

(1) Find the marginal probability density functions of X and Y,

(2) Calculate $E(X)$ and $E(Y)$.

5. (**5 scores**) Suppose that the weights of the males in a region obey a normal distribution, i.e. $X\sim N(\mu,\sigma^2)$. oow 36 males are randomly selected with the mean weight as $\bar{X}=67.5$, and sample variance $S^2=14^2$. Given the significance level $\alpha=0.05$, can we say that the variance of the males' weight in this region is $\sigma^2=16^2$? (It is known that $t_{0.025}(35)=2.0301$, $t_{0.025}(36)=2.2081$, $t_{0.05}(35)=1.6896$, $\chi^2_{0.025}(35)=53.203$; $\chi^2_{1-0.025}(35)=20.569$).