

Homework 5

1. Filling in the blanks (10').

- (1) Suppose that events A and B are mutually exclusive, and $P(A - B) = 0.5, P(B) = 0.3$, then $P(A) = \underline{\hspace{2cm}}$.
- (2) Suppose a random variable X has a probability density function as $f(x) = \begin{cases} A \sin x, & x \in [0, \pi] \\ 0, & \text{otherwise} \end{cases}$, then $A = \underline{\hspace{2cm}}?$

2. Single Choice (10').

- (1) Which of the following functions could be a probability density function of continuous random variables ().
- A. $f(x) = \begin{cases} \sin x, & x \in [\pi, \frac{3\pi}{2}] \\ 0, & \text{otherwise} \end{cases}$; B. $f(x) = \begin{cases} -\sin x, & x \in [\pi, \frac{3\pi}{2}] \\ 0, & \text{otherwise} \end{cases}$;
- C. $f(x) = \begin{cases} \cos x, & x \in [\pi, \frac{3\pi}{2}] \\ 0, & \text{otherwise} \end{cases}$; D. $f(x) = \begin{cases} 1 - \cos x, & x \in [\pi, \frac{3\pi}{2}] \\ 0, & \text{otherwise} \end{cases}$
- (2) Let $f(x)$ and $F(x)$ be the probability density function and cumulative distribution function of a continuous random variable X . Then, which of the following is correct? ()
- A. $0 \leq f(x) \leq 1$; B. $P(X = x) < F(x)$; C. $P(X \leq x) = F(x)$; D. $P(X = x) = f(x)$

3. Calculations(80').

1. Let $F(x) = 2A - 3B \arctan x, x \in (-\infty, \infty)$ be a cumulative distribution function of a continuous random variable X .
- (1) Find the values of A and B ; (2) Calculate the probability that X takes values on $[-\sqrt{3}, \sqrt{3}]$; (3) Find the probability density function $f(x)$ of X . (20')
2. Let $f(x) = \begin{cases} ax^b, & x \in (0, 1), a, b > 0 \\ 0, & \text{otherwise} \end{cases}$ be a probability density function of a continuous random variable X and $P(X > 0.5) = 0.75$. Find the values of a and b . (20')
3. Suppose that a continuous random variable X has a probability density functions as
- $$f(x) = \begin{cases} x, & 0 < x < 1; \\ k - x, & 1 \leq x \leq 2; \\ 0, & \text{otherwise.} \end{cases}$$
- Find (1) the value of k ; (2) the cumulative function $F(x)$ of X ;
- (3) $P(\frac{1}{2} \leq X \leq \frac{3}{2})$; (4) $E(X)$ and $Var(X)$. (20')
4. If a continuous random variable X has a probability density function as $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$, where $\lambda > 0$ is a constant. Find the expectation $E(X)$ and the variance $Var(X)$. (20')