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Homework -3
Sub: Probability and Statistic

1.1 Here $\int(x,y) = \begin{cases} Ae^{-(3x+4y)}, x > 0,\ y > 0 \\ 0 \qquad\qquad otherwise \end{cases}$

Find the value of A: continuous joint PDF to find the value of A consider:-

$\oiint_0^\infty \int(x,y)\, dxdy = 1$

$=> \oiint_0^\infty Ae^{-(3x+4y)}\, dx\, dy = 1$

$=> A \oiint_0^\infty e^{-3x}\, e^{-4y}\ dx\, dy = 1$ ………………………………1

dx and dy separately

$\int_0^\infty e^{-3x}\ dx$ ………………………………………………………........2

Let u = -3x

- ⇨ du = - 3dx
- ⇨ dx = - $\frac{du}{3}$

Here x = 0 x = ∞

Therefor u = 0 u = ∞

Substituting dx. ∞ - 3x

= $-\frac{1}{3}\int_0^\infty e^u\, du$

=$-\frac{1}{3}[e^\infty - \ e^0]$

=$-\frac{1}{3}[e^\infty - \ e^0]$

= $-\frac{1}{3}[0 - \ 1]$

$= -\frac{1}{3}$

Therefore $\int_0^{\infty} e^{-3x}\, dx \ = \frac{1}{3}$

The same way to get $\int_0^{\infty} e^{-4y}\, dy \ = \frac{1}{4}$ ……………………….3

Substituting 3 ∞ 2 to 1

⇨ A. $\frac{1}{3} . \frac{1}{4} \ = 1$

∴ A = 12

1.2 compute the probability p(0≤x< 1, 0≤y< 2)

Here x = 0 to 0.9

y = 0 to 1.9

And A = 12

$\therefore \int_0^{1.9} \quad \int_0^{0.9} 12\, e^{-(3x+4y)}\, dx\, dy$

$= 12 \int_0^{1.9} \quad \int_0^{0.9} e^{-3x} e^{-4y}\, dx\, dy$ ………………………………….2(i)

Compute separately

$$\int_0^{0.9} e^{-3x} dx$$

Let u = - 3x

$\therefore dx \ = \ \frac{du}{3}$

Here x = 0 to x = 0.9

u = 0 to u = -2.7

So $-\frac{1}{3} \int_0^{-2.7} e^{u} du$

$= -\frac{1}{3} [e^{-2.7} - \ e^{0}]$

$= -\frac{1}{3} [e^{-2.7} - \ 1]$

$= -\frac{1}{3}[0.0672 - 1]$

$= 0.3109$

$\therefore \int_0^{0.9} e^{-3x}\,dx = -\frac{1}{3}[e^{-2.7} - 1]$ ……………………………………….2(ii)

And $\int_0^{1.9} e^{-4y}\,dy = -\frac{1}{4}[e^{7.6} - 1]$ ………………………………….....2(iii)

Substitute the value from 2(ii) and 2(iii) in 2(i)

Then p(0≤x< 1, 0≤y< 2) $= 12 . \left[-\frac{1}{2}(e^{-2.7} - 1)\right] . \left[-\frac{1}{4}(e^{-7.6} - 1)\right]$

= 0.932328 ans

1.3 Marginal PDF

$$\int_x (x) = 12 \int_0^{1.9} (e^{-3x} . e^{-4y})dy$$

$$= 12\, e^{-3x} \int_0^{1.9} e^{xy} dy$$

$$= 12\, e^{-3x} \left[-\frac{1}{4}(e^{-7.6} - 1)\right] \text{ [substitution the value from 2(iii)]}$$

$$= -\frac{12}{4} e^{-3x}(e^{-7.6} - 1)$$

$$= -3e^{-3x}(e^{-7.6} - 1)$$

And $\int_y (y) = 12 \int_0^{0.9} (e^{-3x} . e^{-xy})dx$

$$= 12e^{-4y} \int_0^{0.9} (e^{-3x})dx$$

$$= 12e^{-4y} \left[-\frac{1}{3}(e^{-2.7} - 1)\right] \; [subtitution\ the\ value\ from\ 2(iii)]$$

$= -4e^{-4y}(e^{-2.7} - 1)$ ans

1.4 find E(x), E(y) ∞ E(xy)

Here E(x) = $\int_x x \int_x (x)dx$

$= \int_0^{0.9} (x)\ [-3e^{-3x}(e^{-7.6} - 1)]$

$= \int_0^{0.9} x[(-3)e^{-3x}(-0.9995)]$

$= \int_0^{0.9} x\ [2.9985 * e^{-3x}]$

$= 2.9985 * \int_0^{0.9} x\ e^{-3x}$ ……………………3(i)

= 2.9985*0.08348

= 0.2503

$$\int \varphi\, dg = \varphi g - \int g\, dy$$

Where $\varphi$ = x, dg = $e^{-3x}\ dx$

dy = dx , g = $-\frac{1}{3}e^{-3x}$

so $\left[(1 - \frac{1}{3}e^{-3x} * x)\right]$ $0.9\ to\ 0$ $+ \frac{1}{3}\int_0^{0.9} e^{-3x}\ dx$

E(y) $= \int_y y \int_y (y)dy$

$= \int_0^{1.9} (y)\ [-4e^{-4y}(e^{-2.7} - 1)]$ dy

$= \int_0^{1.9} (y)\ [-4e^{-4y}(-0.9328)]$ dy

$= \int_0^{1.9} (y * 3.7312 * e^{-4y})$ dy

$= 3.7312 * \int_0^{1.9} (y * \ e^{-4y})\ $ dy ................................3(ii)

= 3.7312*0.062331

= 0.2322

E(xy) $= \int_x \quad \int_y \ xy \int xy\ (x, y)\ dydx$ – E(x)E(y)

$= \int_0^{0.9} \quad \int_0^{1.9} xy\ 12\ \ e^{-(3x+4y)} dydx$ – E(x)E(y)

$= 12 \int_0^{0.9} \quad \int_0^{1.9} xy\ \ e^{-3x} e^{-4y} dydx$ – E(x)E(y)

= [12*(0.08348)*(0.062231)] – (0.2503)(0.2322) [from 3(ii) and 3(i) ]

= 0.0042

**1.5 x and y independent or not ?**

i.e correction $P_{xy} = \frac{E(xy)}{6(x)6(y)}$

| | Value | comment |
|---|---|---|
| $P_{xy}$ | Close to 1 | Somehow related |
| $P_{xy}$ | Close to -1 | Strongly related |
| $P_{xy}$ | Close to 0 | independant |

$\vee ar(x) \ = \int_x \ x^2 \ \int_x \ (x)dx \ - \ [E(x)]^2$

$\int_x \ (x) = \ -3\ e^{-3x} (e^{-7.6} - 1)$

$= \int_0^{0.9} - 3\ e^{-3x} (e^{-7.6} - 1)\ x^2\ dx$

= 2.9985 * 0.0375

= 0.1124

Var (x) = 0.1124 - $(0.2503)^2$

= 0.0498

∴ 6x = $\sqrt{0.498}$

$$Var(y) = \int_y y^2 \int_y (y)dy - [E(y)]^2$$

$$= \int_0^{1.9} y^2[-4e^{-4y}(e^{-2.7} - 1)] \, dy - [E(y)]^2$$

$$= 3.3712 \int_0^{1.9} (y^2 * 4e^{-4y}) \, dy - [E(y)]^2$$

$$= 3.3712 * 0.03066 - [E(y)]^2$$

= 0.114398592 – 0.05391684

= 0.0605

Var (y) = 0.0605

∴ 6y = $\sqrt{0.0605}$

Now

$$P_{xy} = \frac{0.0042}{\sqrt{0.0498} * \sqrt{0.0605}}$$

= 0.0765

∴ $P_{xy}$ is close to zero so x and y are independent.