

Homework 3

1. Filling in the blanks.

- (1) If the probability for the success in a Bernoulli trial is p , and the probability that the success appears at least one time in three independent repeated trials is $\frac{19}{27}$, then $p =$ _____?
- (2) Assume that random X has the following probability distribution $X \sim \begin{bmatrix} 0 & 1 & 3 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}$, then $P(X \leq 2) =$ _____.
- (3) Assume that random variable X has probability distribution $X \sim \begin{bmatrix} 1 & 2 & 3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}$, then its accumulative distribution function $F(X) =$ _____.
- (4) Suppose the accumulative distribution function of a random variable X is given as $F(x) = \begin{cases} 0, & x < -1 \\ 0.4, & -1 \leq x < 1 \\ 0.8, & 1 \leq x < 3 \\ 1, & x \geq 3 \end{cases}$, then the probability distribution of X is _____.
- (5) Let $X \sim b(2, p)$, $Y \sim b(3, p)$. If $P(X \geq 1) = \frac{5}{9}$, then $P(Y \geq 1) =$ _____.
- (6) Suppose that a random variable X obeys a Poisson distribution with parameter $\lambda > 0$. If $P(X = 1) = P(X = 2)$, then $\lambda =$ _____.

2. Single Choice.

- (1) Let $F_1(x)$ and $F_2(x)$ be the corresponding cumulative functions of two random variables. If $F(x) = aF_1(x) - bF_2(x)$ is a cumulative function of another random variable, then the values of a and b can be possibly selected as ().
- A. $a = 0.6, b = -0.4$; B. $a = b = \frac{2}{3}$; C. $a = -0.5, b = 1.5$; D. $a = 0.5, b = -1.5$

3. Calculations.

1. Suppose someone has only 6 bullets and shoots a target. Assume the probability that the target is hit for each shooting is p , and all the possible shootings are independent. The shooting will stop if the target is hit. Let X be the shooting times. Find the probability distribution of X .
2. Suppose a box contains 5 cards, which are numbered 1, 2, 3, 4 and 5. Now, select 3 of them randomly. Let X be the max number of these three labelled numbers. Find the probability distribution of X .
3. Suppose that a random variable X obeys the probability distribution $P(X = k) = \frac{a}{N}$, where $k = 1, 2, \dots, N$. Find the value of a .