Homework for Module 3 Part 1

Quiz, 9 questions

1 point

1. (Difficulty: ★) Write out the phase of the complex numbers $a_1 = 1 - \mathrm{j}$ and $a_2 = -1 - \mathrm{j}$.

Express the phase in degrees and separate the two phases by a single white space. Each phase should be a number in the range $[-180, 180]$.

-45 -135

1 point

2. (Difficulty: ★) Let $W_N^k = e^{-\mathrm{j}\frac{2\pi}{N}k}$ and $N > 1$. Then $W_N^{N/2}$ is equal to...

- (selected) -1
- 1
- $-\mathrm{j}$
- $e^{-\mathrm{j}(2\pi/N)+N}$

1 point

3. (Difficulty: ★) Which of the following signals (continuous- and discrete-time) are periodic signals?

Note that $t \in \mathbb{R}$ and $n \in \mathbb{Z}$.

- [x] $x(t) = t - \text{floor}(t)$.
- [x] $x[n] = 1$.
- [x] $x[n] = \sin(n)$.
- [x] $x(t) = \cos(2\pi f_0 t + \phi)$ with $f_0 \in \mathbb{R}$.
- [ ] $x[n] = e^{-jf_0 n} + e^{+jf_0 n}$, where $f_0 = \sqrt{2}$.

2 points

4. (Difficulty: ★ ★ ★) Choose the correct statements from the choices below.

- [x] If we apply the DFT twice to a signal $x[n]$, we obtain the signal itself scaled by N, i.e. $Nx[n]$.
- [x] Consider the length-N signal $x[n] = (-1)^n$ with N even. Then $X[k] = 0$ for all k except $k = N/2$
- [ ] Consider the length-N signal $x[n] = \cos(\frac{2\pi}{N} Ln + \phi)$, where N is even and $L = N/2$. Then $X[k] = \begin{cases} \frac{N}{2} e^{j\phi} & \text{for } k = L \\ 0 & \text{otherwise} \end{cases}$.

1
point

5. (Difficulty: ★) Consider the Fourier basis $\{\mathbf{w}^k\}_{k=0,\ldots,N-1}$, where $\mathbf{w}^k[n] = e^{-j\frac{2\pi}{N}nk}$ for $0 \leq n \leq N-1$.

Select the correct statement below.

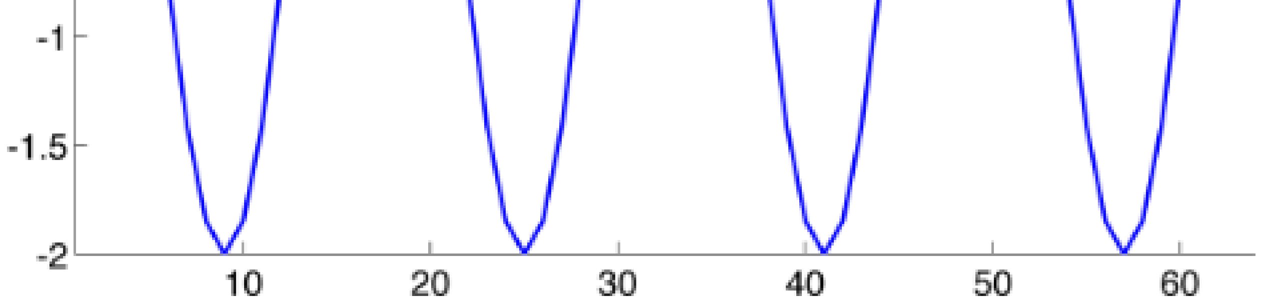
- The elements of the basis are orthogonal:

$$\langle \mathbf{w}^i, \mathbf{w}^j \rangle = \begin{cases} N & \text{for } i = j \\ 0 & \text{otherwise.} \end{cases}$$

- The elements of the basis are orthonormal:

$$\langle \mathbf{w}^i, \mathbf{w}^j \rangle = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{otherwise.} \end{cases}$$

- The orthogonality of the vectors depends on the length N of the elements of the basis.



(Difficulty: ★ ★) Consider the three sinusoids of length $N = 64$ as illustrated in the above figure; note that the signal values are shown from $n = 0$ to $n = 63$.

Call $y_1[n]$ the blue signal, $y_2[n]$ the green and $y_3[n]$ the red. Further, define $x[n] = y_1[n] + y_2[n] + y_3[n]$.

Choose the correct statements from the list below. Note that the capital letters indicate the DFT vectors.

- [ ] $\|x\|_2^2 = \|X\|_2^2 = 12800$
- [ ] $Y_2[k] = \begin{cases} 16j & \text{for } k = 8 \\ 16j & \text{for } k = 56 \\ 0 & \text{otherwise} \end{cases}$
- [ ] $Y_1[k] = \begin{cases} 2 & \text{for } k = 4, 60 \\ 0 & \text{otherwise} \end{cases}$
- [x] $Y_3[k] = \begin{cases} 64 & \text{for } k = 0 \\ 0 & \text{otherwise} \end{cases}$

1
point

7. (Difficulty: ★ ★ ★) Consider the length-N signal

$$x[n] = \cos\left(2\pi \frac{L}{M} n\right)$$

where M and L are integer parameter with $0 < L \leq N-1, 0 < M \leq N$.

Choose the correct statements among the choices below.

- Consider the circularly shifted signal $y[n] = x[(n-D) \mod N]$. In the Fourier domain, the two DFTs related by a modulation factor: $Y[k] = X[k]e^{-j2\pi k \frac{D}{N}}$.
- If $M = N$ and $2L < N$, the signal has exactly L periods for $0 \leq n < N$
- The DFT $X[k]$ has two elements different from zero if $N = M$ and $N \neq 2L$.
- In general, it will be easier to compute the norm of the signal $\|\mathbf{x}\|_2$ in the Fourier domain, using the Parseval's Identity.

1 point

8. (Difficulty: ★) Consider an orthogonal basis $\{\phi_i\}_{i=0,\ldots,N-1}$ for $\mathbb{R}^N$. Select the statements that hold for any vector $\mathbf{x} \in \mathbb{R}^N$.

- [ ] $\|\mathbf{x}\|_2^2 = \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2.$
- [x] $\|\mathbf{x}\|_2^2 = \frac{1}{P} \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2$ if and only if $\|\phi_i\|_2^2 = P \, \forall i.$
- [x] $\|\mathbf{x}\|_2^2 = \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2$ if and only if $\|\phi_i\|_2 = 1 \, \forall i.$
- [ ] $\|\mathbf{x}\|_2^2 = \frac{1}{P} \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2$

  if and only if $\|\phi_i\|_2 = P \, \forall i.$

Congratulations! You passed!

Next Item

1/1 points

1. (Difficulty: ★) Write out the phase of the complex numbers $a_1 = 1 - \mathrm{j}$ and $a_2 = -1 - \mathrm{j}$.

Express the phase in degrees and separate the two phases by a single white space. Each phase should be a number in the range $[-180, 180]$.

1/1 points

2. (Difficulty: ★) Let $W_N^k = e^{-\mathrm{j}\frac{2\pi}{N}k}$ and $N > 1$. Then $W_N^{N/2}$ is equal to...

0.80 / 1 points

3. (Difficulty: ★) Which of the following signals (continuous- and discrete-time) are periodic signals?

Note that $t \in \mathbb{R}$ and $n \in \mathbb{Z}$.

1.33 / 2 points

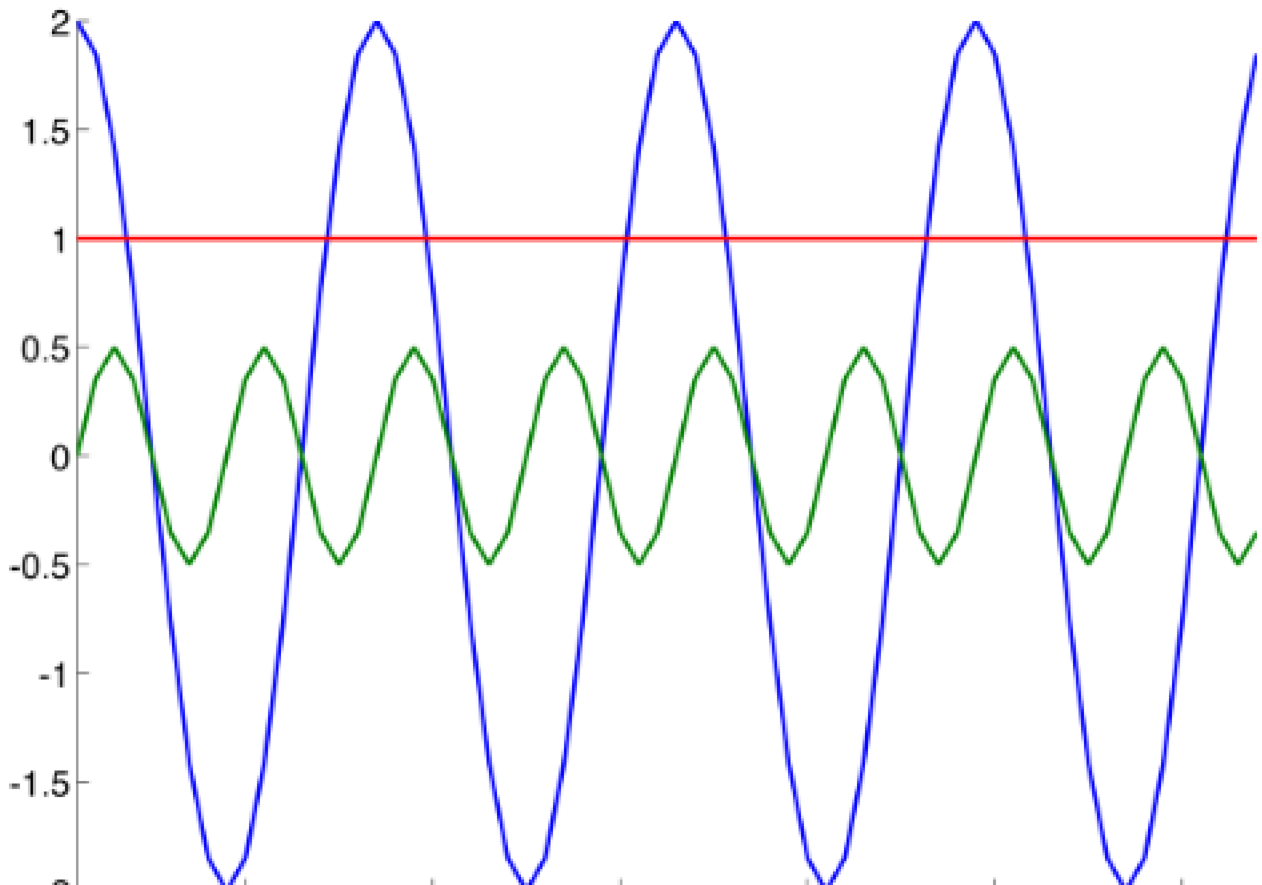
4. (Difficulty: ★ ★ ★) Choose the correct statements from the choices below.

1 / 1 points

5. (Difficulty: ★) Consider the Fourier basis $\{\mathbf{w}^k\}_{k=0,\ldots,N-1}$, where $\mathbf{w}^k[n] = e^{-j\frac{2\pi}{N}nk}$ for $0 \le n \le N-1$.

Select the correct statement below.

1 / 1 points

6. 

the DFT vectors.

0.75 / 1 points

7. (Difficulty: $\star\star\star$) Consider the length-N signal

$$x[n] = \cos\left(2\pi \tfrac{L}{M} n\right)$$

where M and L are integer parameter with $0 < L \leq N-1, 0 < M \leq N$.

Choose the correct statements among the choices below.

1 / 1 points

8. (Difficulty: $\star$) Consider an orthogonal basis $\{\phi_i\}_{i=0,\ldots,N-1}$ for $\mathbb{R}^N$. Select the statements that hold for any vector $\mathbf{x} \in \mathbb{R}^N$.

0.50 / 1 points

9. (Difficulty: $\star\star$) Pick the correct sentence(s) among the following ones regarding the DFT $\mathbf{X}$ of a signal $\mathbf{x}$ of length N, where N is odd.

Remember the following definitions for an arbitrary signal (asterisk denotes conjugation):

hermitian-symmetry: $x[0]$ real and $x[n] = x[N-n]^*$ for $n = 1, \ldots, N-1$.

hermitian-antisymmetry: $x[0] = 0$ and $x[n] = -x[N-n]^*$ for $n = 1, \ldots, N-1$.