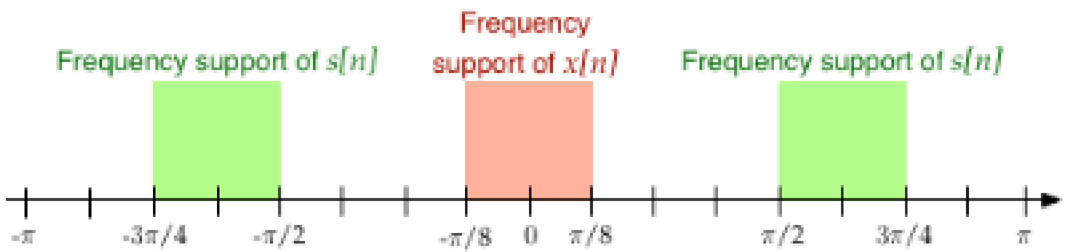
1 point

2. (Difficulty: ⋆) We will see in later lectures that communication systems must fulfill what is called the "bandwidth constraint", that is, the energy of the signals that they transmit must strictly fit into pre-defined frequency bands. In this problem we will look at the bandwidth constraint in the discrete-time domain.

The signal $x[n]$ is real-valued and its spectrum is nonzero only over the $[-\pi/8,\ \pi/8]$ interval. Due to the bandwidth constraint we need to "fit" the signal over the bands indicated in green in the following figure



To this aim, we need to design a processing block $\mathcal{H}$ in order to convert $x[n]$ into a sequence $s[n]$ satisfying the following requirements:

- The support of the DTFT of $s[n]$ must be limited to $[-3\pi/4,\ -\pi/2] \cup [\pi/2,\ 3\pi/4]$
- The sequence $s[n]$ must be real-valued ($x[n]$ is real-valued)

Which of the following input/output relationships for $\mathcal{H}$ meet both requirements? (check all correct answers) :

- [ ] $s[n] = e^{j\frac{5\pi}{8}n} \cdot x[n]$
- [ ] $s[n] = \sin(\frac{21\pi}{8}n) \cdot x[n]$
- [ ] $s[n] = \text{IDTFT}\{X(e^{j(\omega-5\pi/8)})\}$
- [x] $s[n] = \cos(\frac{5\pi}{8}n) \cdot x[n]$

1 point

3. (Difficulty: ⋆) Consider the length-L signal

$$x[n] = \begin{cases} 1 & 0 \le n \le M-1 \\ 0 & M \le n \le L-1 \end{cases},$$

Write out the closed-form analytical expression for its DFT coefficients $X[k]$.

Be careful with your typing since the regular-expression parser can be a bit picky. Check Coursera help to enter math expression. In particular, remember that in the Coursera platform the symbols are different:

- I (capital i) is used for the imaginary unit instead of j
- Euler's number is E instead of e
- you can also use the exponential function $\exp(\cdot)$
- π is defined as pi

The only other symbols you'll need for the answer are the case-sensitive variables k, M, L.

Finally, do not forget to validate your syntax by clicking "Preview" before submitting your answer.

For instance, the expression $e^{j(\pi L+3\pi)}/(k+M)$ should be entered as

E^(I*(pi*L + 3 * pi))/(k + M)

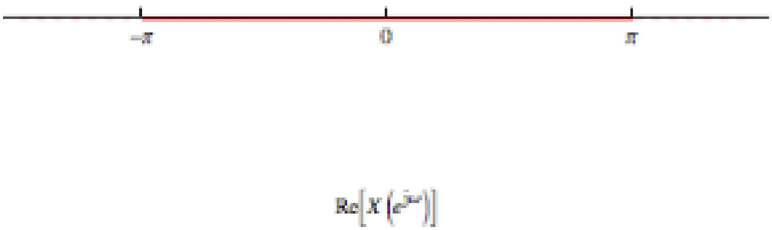
Preview

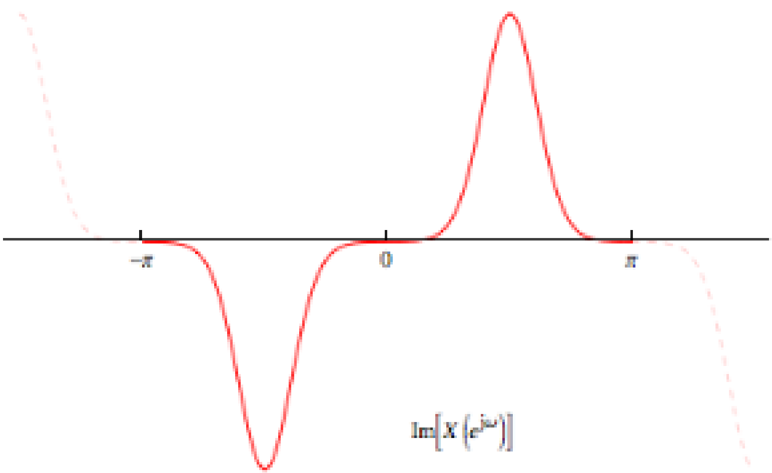
$$\frac{1-e^{-\frac{2\pi}{L}iMk}}{1-e^{-\frac{2\pi}{L}ik}}$$

(1-E^(-I*2*pi*k*M/L))/(1-E^(-I*2*pi*k/L))

1 point

4. The real and imaginary parts of $X(e^{j\omega})$ are:





After examining the plots, check all the correct statements below.

- [ ] $x[n]$ is Hermitian-symmetric $x[n] = x^*[-n]$.
- [x] $x[n]$ is 0-mean, i.e. $\sum_{n\in\mathbb{Z}} x[n] = 0$.
- [x] $x[n]$ is real valued.

1
point

5. (Difficulty: ⋆⋆) Consider a signal $x[n]$ and its DTFT $X(e^{j\omega})$. Assume $X(e^{j\omega})$ is differentiable. Compute the inverse DTFT of

$j\frac{d}{d\omega}X(e^{j\omega})$.

You should write your answer in terms of $x[n]$ and elementary functions and constants, for example $\frac{\pi}{2}x[n]$ would be written :

pi/2*x[n]

n*x[n]

1
point

6. (Difficulty: ⋆) Which property of the DTFT allows you to easily compute the inverse DTFT of $4X(e^{j\omega})/\pi - 2$ once you know $x[n]$? Just type the name of the property.

Linearity

1 point

7. (Difficulty: ⋆) Take a length-N signal $x[n]$ and its DFT $X[k]$, with $0 \leq n, k, \leq N-1$. Next, consider its periodized version $\tilde{x}[n] = x[n \bmod N]$ with its DFS $\tilde{X}[k]$ where now $n, k \in \mathbb{Z}$.

Which of the following statements are true?

- [x] $\tilde{X}[l] = X[l \bmod N]$, for all $l \in \mathbb{Z}$
- [x] $\tilde{X}[k + lN] = X[k]$, for all $l \in \mathbb{Z}$ and $k = 0, \ldots, N-1$.
- [ ] $\tilde{X}[-2] = X[2]$ for all $x[n]$ and $N > 2$

1 point

8. (Difficulty: ⋆) In the class, we learned how the modulation theorem can help us tune a musical instrument. Martin showed us an example with a bass but of course the same works with a classical guitar. Listen carefully to these two samples (with earphones, if possible); each audio clip is the recording of two notes played together:

- Audio clip A
- Audio Clip B

Select the correct options below.

- [ ] The notes are in tune in both audio clips
- [x] The notes are in tune in audio clip A and out of tune in audio clip B
- [ ] The notes are out of tune in both audio clips
- [ ] The notes are in tune in audio clip B and out of tune in audio clip A

1
point

9. (Difficulty: ★★) A ringback tone is the sound you hear in your landline telephone when the remote phone you are trying to call is ringing.

In most European countries, the ringback tone is a single sinusoid turned on and off periodically while in the USA, the ringbback tone is the sum of 2 sinusoids with relatively close frequencies turned on and off periodically.

Here are two audio clips:

[Sample A]

[Sample B]

Just by listening to the clips, you should be able to identify the US ringback tone. Explain in the box below what helped you identify the US tone. Use the wording and concepts that appear in the lecture slides. No credit, without a proper explanation, e.g., "I live there" is not an answer.

Sample b , it's possible to hear the beating

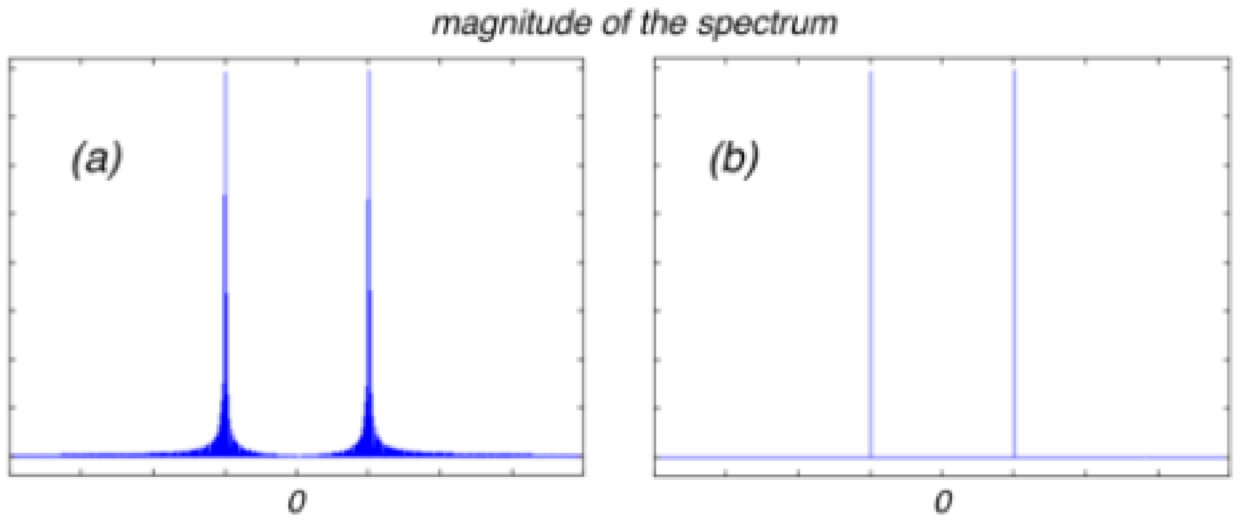
1

10. (Difficulty: ★) As explained in previous question, the European and US ringback tones are

1
point

10. (Difficulty: ⋆) As explained in previous question, the European and US ringback tones are composed of one or two sinusoids respectively, multiplied by a square wave switching between 0 and 1. This multiplication by a square wave periodically mutes the sinusoid(s).

Look at the following magnitude DTFT plots (each plot shows the interval $[-\pi\ \pi]$) :



Select the correct statement amongst the choices below.

- ○ Spectrum (a) corresponds to the European ringback tone.
- ○ Spectrum (b) corresponds to the US ringback tone.
- ● Spectrum (b) corresponds to the European ringback tone.
- ○ Spectrum (a) corresponds to the US ringback tone.

1 point

11. (Difficulty: ⋆) Consider the sequences

$$x_1[n] = \cos(2\pi n \cdot \sqrt{2}/30)$$

$$x_2[n] = \cos(2\pi n \cdot 1.41421356/30)$$

with $n \in \mathbb{Z}$

Select the correct statements below.

- [ ] There exists $N \in \mathbb{N}$ for which $x_1[n]$ has a DFS of size N
- [x] There exists $N \in \mathbb{N}$ for which $x_2[n]$ has a DFS of size N

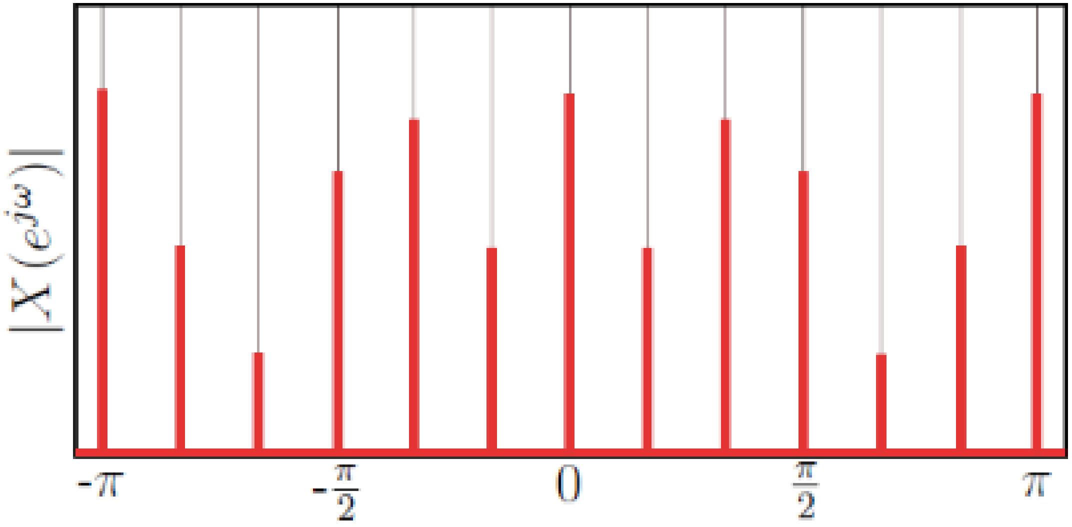
---

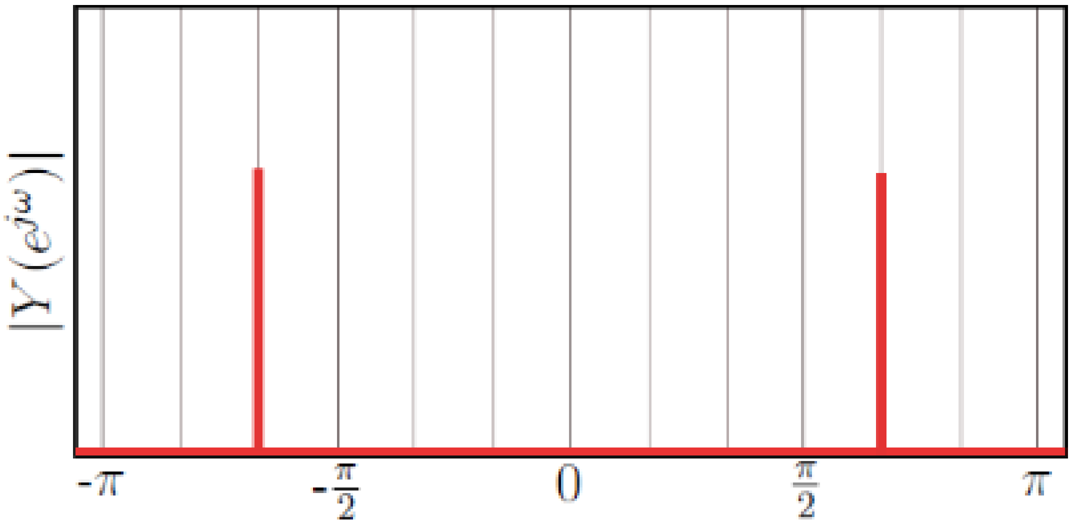
1 point

12. (Difficulty: ⋆⋆) Consider a signal $x[n]$; the only thing we know about the signal is that its DTFT is strictly bandlimited between $-\frac{\pi}{10}$ and $\frac{\pi}{10}$. We now modulate the signal to obtain $y[n] = x[n]\cos(\omega_c n)$.

Among the possibilities below, select all the values for ω_c that allow us to perfectly demodulate $y[n]$.

- [ ] $\frac{\pi}{20}$
- [x] $\frac{\pi}{3}$
- [ ] $\frac{11\pi}{12}$
- [x] $\frac{9\pi}{10}$





Both underlying signals $x[n]$ and $y[n]$ are periodic. Find their periods and write them below, separated by a space. Please write the smallest period, i.e. a 5-periodic signal is also obviously 15-periodic but we're interested in 5.

Enter the period of $x[n]$ and $y[n]$ with a unique white space in between.

6 3