1
point

1. (Difficulty: ★) Write the value for the inner product $\langle \mathbf{v}^{(0)}, \mathbf{v}^{(1)} \rangle$ where

$$\mathbf{v}^{(0)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} \text{ and } \mathbf{v}^{(1)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix},$$

0 0 0 0

1
point

2. (Difficulty: ★★) Consider the following vectors in $\mathbb{R}^4$

$$\mathbf{v}^{(0)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \mathbf{v}^{(1)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \text{ and } \mathbf{v}^{(2)} = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}.$$

You can verify that the vectors are mutually orthogonal and have unit norm.

How many different vectors $\mathbf{v}^{(3)}$ could we find such that $\{\mathbf{v}^{(0)}, \mathbf{v}^{(1)}, \mathbf{v}^{(2)}, \mathbf{v}^{(3)}\}$ is a full **orthogonal** basis in $\mathbb{R}^4$?

- ○ 0
- ○ 1
- ○ 2
- ○ 3
- ◉ >3

1
point

3. Let $\mathbf{y} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -1 \end{bmatrix}$,

what are the expansion coefficients of $\mathbf{y}$ in the basis $\{\mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ you found in the previous question?

Important: *Enter your answer as space separated floating point decimal numbers, e.g. the vector* $\mathbf{y}$ *would be entered as :*

2.0 1.0 0.0 -1.0

1 2 1 0

1
point

4. Which of the following sets form a basis of $\mathbb{R}^4$?

- [x] $\{\mathbf{y}, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$
- [ ] $\{\mathbf{y}, \mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_2\}$
- [x] $\{\mathbf{y}, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2, \mathbf{v}_3\}$
- [x] $\{\mathbf{y}, \mathbf{v}_0, \mathbf{v}_1, \mathbf{v}_3\}$

1
point

5. (Difficulty: ★) If we represent finite-length signals as vectors in Euclidean space, many operations on signals can be encoded as a matrix-vector multiplication. Consider for example a circular shift in $\mathbb{C}^3$: a delay by one (i.e. a right shift) transforms the signal $\mathbf{x} = [x_0\ x_1\ x_2]^T$ into $\mathbf{x}' = [x_2\ x_0\ x_1]^T$ and it can be described by the matrix

$$D = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

so that $\mathbf{x}' = D\mathbf{x}$.

Determine the matrix F that implements the one-step-difference operator in $\mathbb{C}^3$ i.e. the operator that transforms a signal $\mathbf{x}$ into $[(x_0 - x_2)\ (x_1 - x_0)\ (x_2 - x_1)]^T$.

Write the 9 integer matrix coefficients one after the other, row by row and separated by spaces.

1 0 -1 -1 1 0 1 -1 1

1
point

6. (Difficulty ★) Given the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

compute the matrix A^4 (i.e. the fourth power of A).

(Hint: there is a simple way to do that and, if you've solved the previous question, it should be obvious).

Write the 16 integer matrix coefficients one after the other, row by row and separated by spaces.

1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1

- [x] I, **Dafna Hirschfeld**, understand that submitting work that isn't my own may result in permanent failure of this course or deactivation of my Coursera account.

Learn more about Coursera's Honor Code

Submit Quiz

Homework for Module 3 Part 1

Quiz, 9 questions

1 point

1. (Difficulty: ★) Write out the phase of the complex numbers $a_1 = 1 - \mathrm{j}$ and $a_2 = -1 - \mathrm{j}$.

Express the phase in degrees and separate the two phases by a single white space. Each phase should be a number in the range $[-180, 180]$.

-45 -135

1 point

2. (Difficulty: ★) Let $W_N^k = e^{-\mathrm{j}\frac{2\pi}{N}k}$ and $N > 1$. Then $W_N^{N/2}$ is equal to...

- (selected) -1
- 1
- $-\mathrm{j}$
- $e^{-\mathrm{j}(2\pi/N)+N}$

1 point

3. (Difficulty: ★) Which of the following signals (continuous- and discrete-time) are periodic signals?

Note that $t \in \mathbb{R}$ and $n \in \mathbb{Z}$.

- [x] $x(t) = t - \text{floor}(t)$.
- [x] $x[n] = 1$.
- [x] $x[n] = \sin(n)$.
- [x] $x(t) = \cos(2\pi f_0 t + \phi)$ with $f_0 \in \mathbb{R}$.
- [ ] $x[n] = e^{-jf_0 n} + e^{+jf_0 n}$, where $f_0 = \sqrt{2}$.

2 points

4. (Difficulty: ★ ★ ★) Choose the correct statements from the choices below.

- [x] If we apply the DFT twice to a signal $x[n]$, we obtain the signal itself scaled by N, i.e. $Nx[n]$.
- [x] Consider the length-N signal $x[n] = (-1)^n$ with N even. Then $X[k] = 0$ for all k except $k = N/2$
- [ ] Consider the length-N signal $x[n] = \cos(\frac{2\pi}{N} Ln + \phi)$, where N is even and $L = N/2$. Then $X[k] = \begin{cases} \frac{N}{2} e^{j\phi} & \text{for } k = L \\ 0 & \text{otherwise} \end{cases}$.

1
point

5. (Difficulty: ★) Consider the Fourier basis $\{\mathbf{w}^k\}_{k=0,\ldots,N-1}$, where $\mathbf{w}^k[n] = e^{-j\frac{2\pi}{N}nk}$ for $0 \leq n \leq N-1$.

Select the correct statement below.

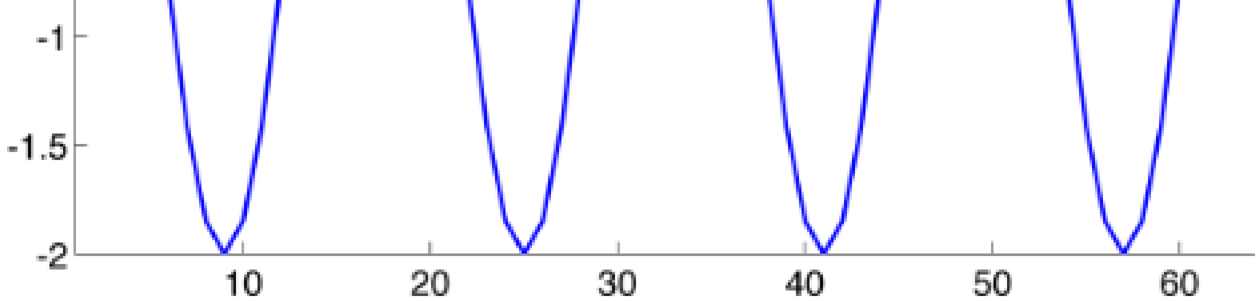
- [x] The elements of the basis are orthogonal:

$$\langle \mathbf{w}^i, \mathbf{w}^j \rangle = \begin{cases} N & \text{for } i = j \\ 0 & \text{otherwise.} \end{cases}$$

- [ ] The elements of the basis are orthonormal:

$$\langle \mathbf{w}^i, \mathbf{w}^j \rangle = \begin{cases} 1 & \text{for } i = j \\ 0 & \text{otherwise.} \end{cases}$$

- [ ] The orthogonality of the vectors depends on the length N of the elements of the basis.



(Difficulty: ★ ★) Consider the three sinusoids of length $N = 64$ as illustrated in the above figure; note that the signal values are shown from $n = 0$ to $n = 63$.

Call $y_1[n]$ the blue signal, $y_2[n]$ the green and $y_3[n]$ the red. Further, define $x[n] = y_1[n] + y_2[n] + y_3[n]$.

Choose the correct statements from the list below. Note that the capital letters indicate the DFT vectors.

- [ ] $\|x\|_2^2 = \|X\|_2^2 = 12800$
- [ ] $Y_2[k] = \begin{cases} 16j & \text{for } k = 8 \\ 16j & \text{for } k = 56 \\ 0 & \text{otherwise} \end{cases}$
- [ ] $Y_1[k] = \begin{cases} 2 & \text{for } k = 4, 60 \\ 0 & \text{otherwise} \end{cases}$
- [x] $Y_3[k] = \begin{cases} 64 & \text{for } k = 0 \\ 0 & \text{otherwise} \end{cases}$

1
point

7. (Difficulty: ★ ★ ★) Consider the length-N signal

$$x[n] = \cos\left(2\pi \frac{L}{M} n\right)$$

where M and L are integer parameter with $0 < L \leq N-1, 0 < M \leq N.$

Choose the correct statements among the choices below.

- [ ] Consider the circularly shifted signal $y[n] = x[(n-D) \mod N]$. In the Fourier domain, the two DFTs related by a modulation factor: $Y[k] = X[k]e^{-j2\pi k\frac{D}{N}}$.
- [ ] If $M = N$ and $2L < N$, the signal has exactly L periods for $0 \leq n < N$
- [ ] The DFT $X[k]$ has two elements different from zero if $N = M$ and $N \neq 2L$.
- [ ] In general, it will be easier to compute the norm of the signal $\|\mathbf{x}\|_2$ in the Fourier domain, using the Parseval's Identity.

1
point

8. (Difficulty: ★) Consider an orthogonal basis $\{\phi_i\}_{i=0,\ldots,N-1}$ for $\mathbb{R}^N$. Select the statements that hold for any vector $\mathbf{x} \in \mathbb{R}^N$.

- [ ] $\|\mathbf{x}\|_2^2 = \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2.$
- [x] $\|\mathbf{x}\|_2^2 = \frac{1}{P} \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2$ if and only if $\|\phi_i\|_2^2 = P \; \forall i.$
- [x] $\|\mathbf{x}\|_2^2 = \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2$ if and only if $\|\phi_i\|_2 = 1 \; \forall i.$
- [ ] $\|\mathbf{x}\|_2^2 = \frac{1}{P} \sum_{i=0}^{N-1} |\langle x, \phi_i \rangle|^2$

  if and only if $\|\phi_i\|_2 = P \; \forall i.$

Congratulations! You passed!

Next Item

1 / 1 points

1. (Difficulty: ★) Write out the phase of the complex numbers $a_1 = 1 - \mathrm{j}$ and $a_2 = -1 - \mathrm{j}$.

Express the phase in degrees and separate the two phases by a single white space. Each phase should be a number in the range $[-180, 180]$.

1 / 1 points

2. (Difficulty: ★) Let $W_N^k = e^{-\mathrm{j}\frac{2\pi}{N}k}$ and $N > 1$. Then $W_N^{N/2}$ is equal to...

0.80 / 1 points

3. (Difficulty: ★) Which of the following signals (continuous- and discrete-time) are periodic signals?

Note that $t \in \mathbb{R}$ and $n \in \mathbb{Z}$.

1.33 / 2 points

4. (Difficulty: ★ ★ ★) Choose the correct statements from the choices below.

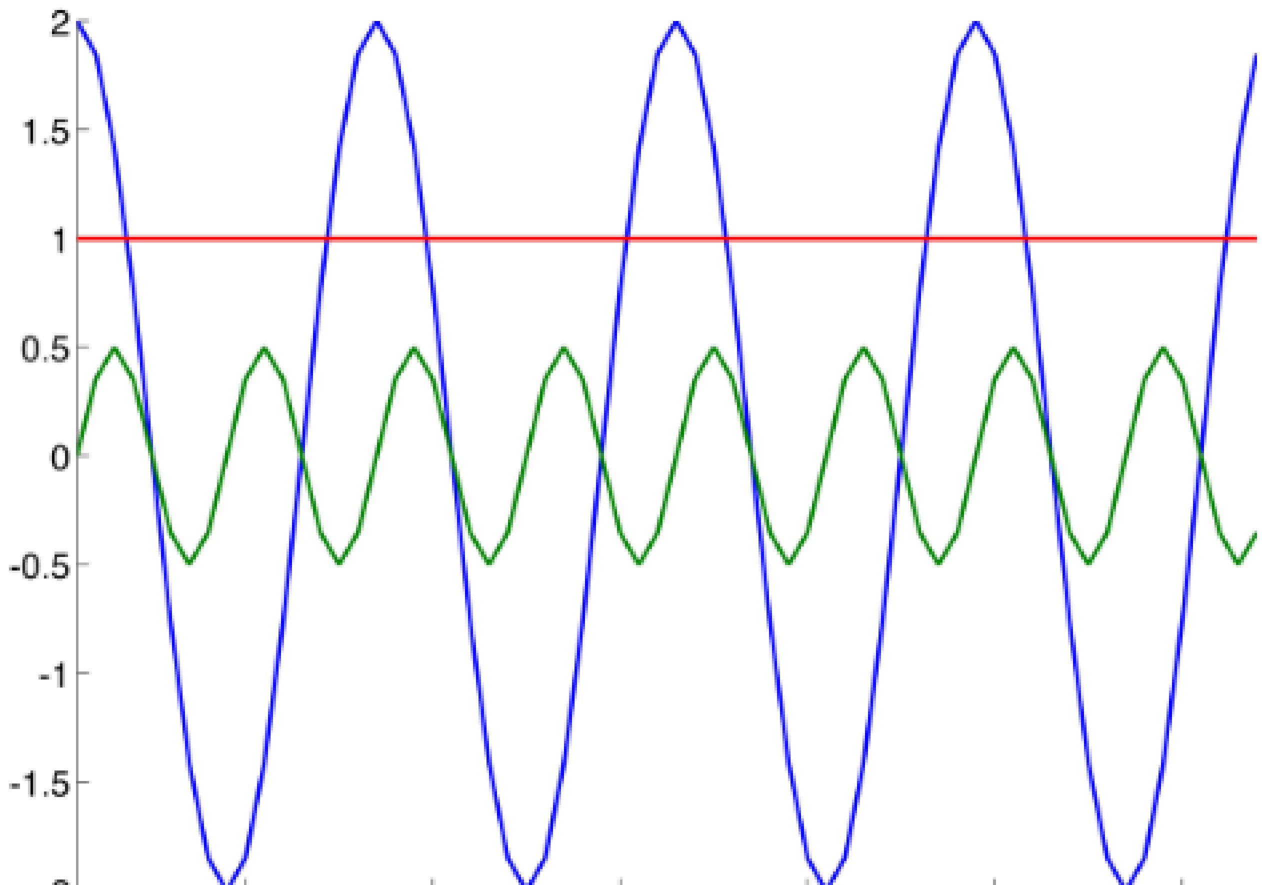
5. (Difficulty: ★) Consider the Fourier basis $\{\mathbf{w}^k\}_{k=0,\ldots,N-1}$, where $\mathbf{w}^k[n] = e^{-j\frac{2\pi}{N}nk}$ for $0 \leq n \leq N-1$.

1 / 1 points

Select the correct statement below.

6.

1 / 1 points



the DFT vectors.

0.75 / 1 points

7. (Difficulty: ★ ★ ★) Consider the length-N signal

$$x[n] = \cos\left(2\pi \frac{L}{M} n\right)$$

where M and L are integer parameter with $0 < L \leq N-1, 0 < M \leq N$.

Choose the correct statements among the choices below.

1 / 1 points

8. (Difficulty: ★) Consider an orthogonal basis $\{\phi_i\}_{i=0,\ldots,N-1}$ for $\mathbb{R}^N$. Select the statements that hold for any vector $\mathbf{x} \in \mathbb{R}^N$.

0.50 / 1 points

9. (Difficulty: ★ ★) Pick the correct sentence(s) among the following ones regarding the DFT $\mathbf{X}$ of a signal $\mathbf{x}$ of length N, where N is odd.

Remember the following definitions for an arbitrary signal (asterisk denotes conjugation):

hermitian-symmetry: $x[0]$ real and $x[n] = x[N-n]^*$ for $n = 1, \ldots, N-1$.

hermitian-antisymmetry: $x[0] = 0$ and $x[n] = -x[N-n]^*$ for $n = 1, \ldots, N-1$.