Digital Signal Processing - week5 quiz solutions

June 15, 2018

1 Question 1

Which of the following are linear systems:

1.1

1. Envelope detection (via squaring), i.e. $y[n] = |[n]|^2 * h[n]$ where h[n] is the impulse response of a lowpass filter such as the moving average filter.
2. This is obviously not liner since squaring is not linear

1.2

1. AM radio modulation, i.e. multiply a signal x[n] by a cosine at the carrier frequency : $y[n] = x[n]\cos(2\pi\omega_c n)$
2. It is linear: $(ax_1[n] + bx_2[n])\cos(2\pi\omega_c n) = ax_1[n]\cos(2\pi\omega_c n) + bx_2[n]\cos(2\pi\omega_c n) = ay_1[n] + by_2[n]$

1.3

1. Second derivative, i.e. $y(t) = \frac{d^2}{dt^2}x(t)$
2. It is easy to see that derivative operation is linear

1.4

1. Clipping, i.e. enforce a maximum signal amplitude MM,e.g:

$$y[n] = \begin{cases} x[n] & \text{if } x[n] \leq M \\ M & \text{otherwise} \end{cases}$$

2. take $x_1[n] = x_2[n] = M+1$ then $x_3[n] = x_1[n]+x_2[n] = 2M+2$ but $y_1[n] = y_2[n] = y_3[n] = M$ So $y_3[n] \neq y_1[n] + y_2[n]$

2 Question 5

2.1 Q

Consider the filter $h[n] = \delta[n] - \delta[n-1]$, and the input $x[n] = \begin{cases} n & \text{if } n \leq 0 \\ 0 & \text{otherwise} \end{cases}$

and the output $y[n] = x[n] * h[n]$.
Compute $y[-1], y[0], y[1], y[2]$

2.2 A

$y[n] = x[n] * h[n] = x[n] * (\delta[n] - \delta[n-1]) = x[n] * \delta[n] - x[n] * \delta[n-1] = x[n] - x[n-1]$. Therefore:

1. $y[-1] = x[-1] - x[-2] = 0$
2. $y[0] = x[0] - x[-1] = 0$
3. $y[1] = x[1] - x[0] = 1$
4. $y[2] = x[2] - x[1] = 1$

3 Question 6

3.1 Q

Which of the following filters are BIBO-stable? Assume $N \in \mathbb{N}$ and $0 < \omega < \pi$

3.2 A

1. Any filter $h[n]$ with finite support and bounded coefficients. Is BIBO-stable - it is absolutly summable
2. the moving avarage. Is BIBO stable - if the input is bounded so is the output.
3. The following smoothing filter: $h[n] = \sum_{k=0}^{\infty} \frac{1}{k+1}\delta[n-k]$. is NOT BIBO-stable - this is the harmonic series which is not absolutly summble.
4. The ideal low pass filter with a cutoff frequency ω:

$$H(e^{j\omega}) = \begin{cases} 1 & \text{if } |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$

   is NOT BIBO stable since $sinc$ is not absolutly summble.

4 Question 7

4.1 Q

See the figure for question 7.

4.2 A

We need to find the impules response $h[n]$ from the given input and output.

$y[n] = x[n] * h[n] = \Sigma x[k]h[n-k] = x[0]h[n] + x[1]h[n-1] = 1h[n] + (-1)h[n-1] = h[n] - h[n-1]$

1. for $n \leq -2, n \geq 2$ we have $y[n] = h[n] - h[n-1] = 0$ so $h[n] = h[n-1]$
2. for $n = -1$: $y[-1] = h[-1] - h[-2] = 1 \rightarrow h[-1] = h[-2] + 1$
3. for $n = 0$: $y[0] = h[0] - h[-1] = -1 \rightarrow h[0] = h[-1] - 1 = h[-2] + 1 - 1 = h[-2]$
4. for $n = 1$: $y[1] = h[1] - h[0] = 1 \rightarrow h[1] = h[0] + 1 = h[-2] + 1$

define $A := h[-2]$ so $h[-1] = A + 1$, $h[0] = A$, $h[1] = A + 1$.

So $h[n]$ looks: $...A, (A)_{-2}, (A+1)_{-1}, (A)_0, (A+1)_1, (A+1)_2, ...$

Now we have $x_1[n] = ..0, 0, -2_{-1}, 1_0, -2_1, 3_2, 0, 0, ..$ so:

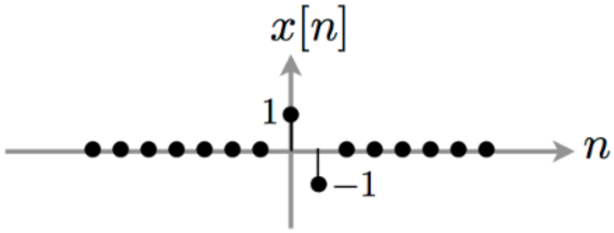
1.

$$y_1[-2] = (x_1[n]*h[n])(-2) = \Sigma x_1[k]h[-2-k] = x_1[-1]h[-1] + x_1[0]h[-2] + x_1[1]h[-3] + x_1[2]h[-4]$$

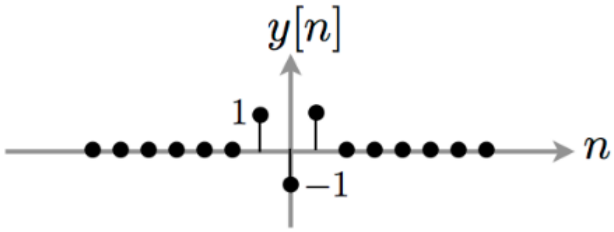
$$= -2(A+1) + (1A) + (-2A) + (3A) = -2$$

I
int

7. (Difficulty: ⋆⋆) Consider an LTI system $\mathcal{H}$. When the input to $\mathcal{H}$ is the following signal



then the output is



Assume now the input to $\mathcal{H}$ is the following signal

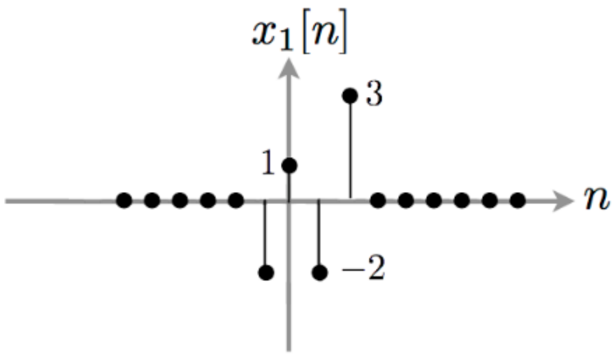
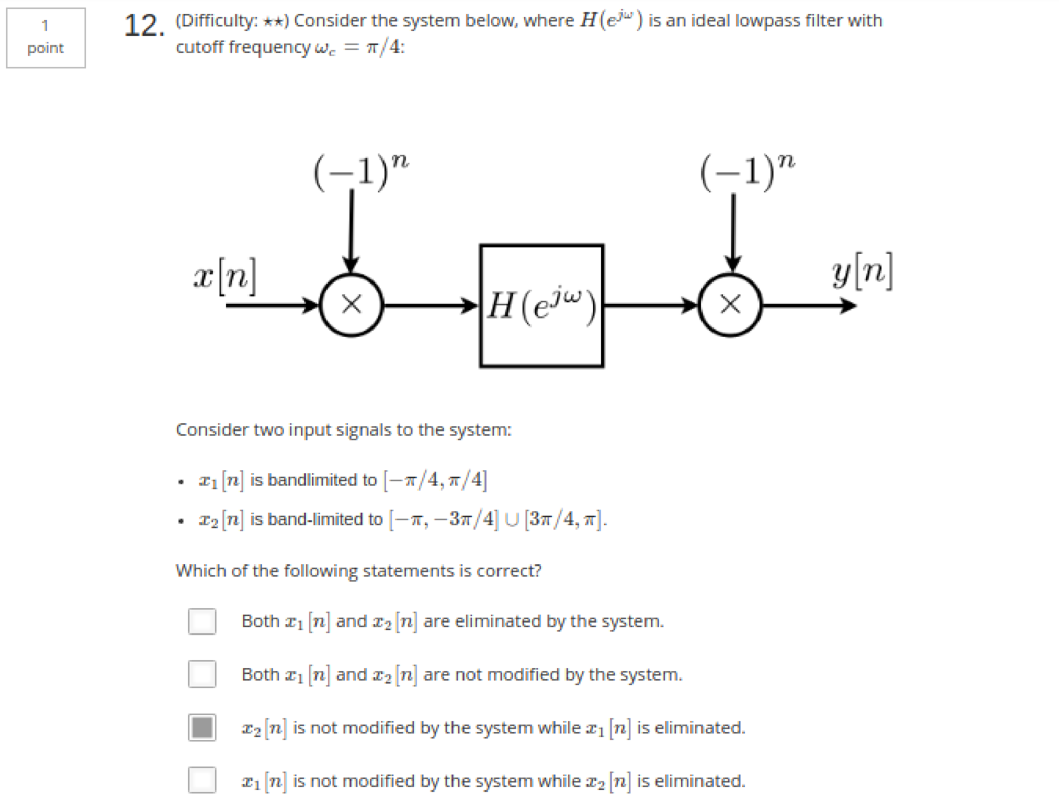


Figure 1: Question 7



1 point

12. (Difficulty: ★★) Consider the system below, where $H(e^{j\omega})$ is an ideal lowpass filter with cutoff frequency $\omega_c = \pi/4$:

Consider two input signals to the system:

- $x_1[n]$ is bandlimited to $[-\pi/4, \pi/4]$
- $x_2[n]$ is band-limited to $[-\pi, -3\pi/4] \cup [3\pi/4, \pi]$.

Which of the following statements is correct?

- [ ] Both $x_1[n]$ and $x_2[n]$ are eliminated by the system.
- [ ] Both $x_1[n]$ and $x_2[n]$ are not modified by the system.
- [x] $x_2[n]$ is not modified by the system while $x_1[n]$ is eliminated.
- [ ] $x_1[n]$ is not modified by the system while $x_2[n]$ is eliminated.

Figure 2: Question 12

2.

$$y_1[-1] = (x_1[n]*h[n])(-1) = \Sigma x_1[k]h[-1-k] = x_1[-1]h[0]+x_1[0]h[-1]+x_1[1]h[-2]+x_1[2]h[-3]$$

$$= (-2A) + 1(A+1) + (-2A) + 3A = 1$$

5 Question 12

5.1 Q

See the figure for question 12.

5.2 A

The way to solve it is to note that $(-1)^n = \cos(\pi n)$ so multiply by $(-1)^n$ is equivalent to modultation with $\omega_0 = \pi$.

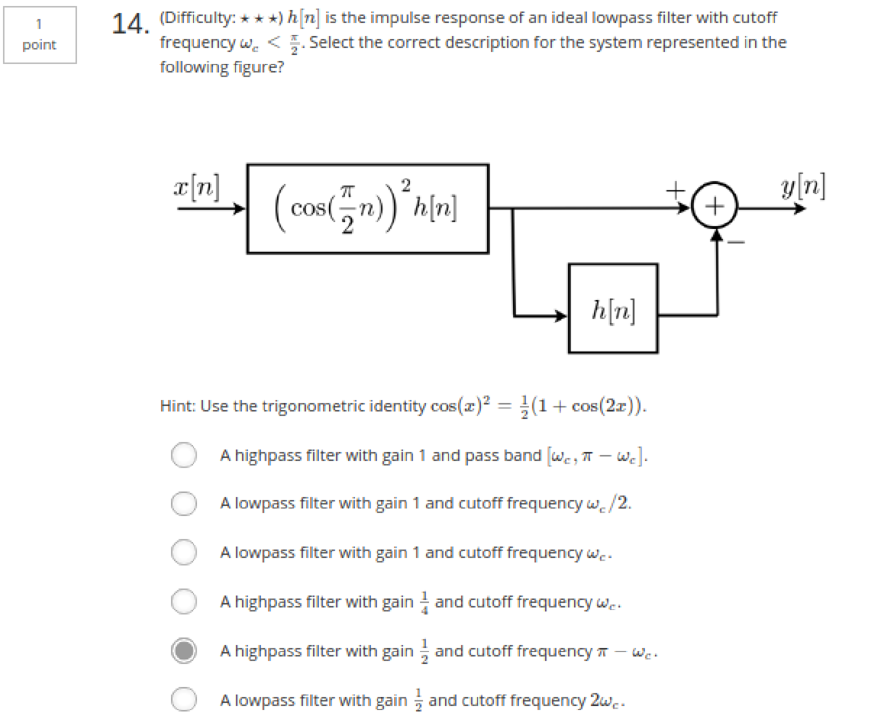
From week 4 we know $DTFT\{x[n]cos(\omega_0 n)\} = \frac{1}{2}(X(e^{j(\omega-\omega_0)}) + X(e^{j(\omega+\omega_0)}))$.

Also, one should remember the 2π preiodicity, from there it is easy to note that the system eliminates x_1 and does not modify x_2

6 Question 14

6.1 Q

See the figure for question 14.



1 point

14. (Difficulty: ⋆ ⋆ ⋆) $h[n]$ is the impulse response of an ideal lowpass filter with cutoff frequency $\omega_c < \frac{\pi}{2}$. Select the correct description for the system represented in the following figure?

Hint: Use the trigonometric identity $\cos(x)^2 = \frac{1}{2}(1+\cos(2x))$.

- A highpass filter with gain 1 and pass band $[\omega_c, \pi - \omega_c]$.
- A lowpass filter with gain 1 and cutoff frequency $\omega_c/2$.
- A lowpass filter with gain 1 and cutoff frequency ω_c.
- A highpass filter with gain $\frac{1}{4}$ and cutoff frequency ω_c.
- A highpass filter with gain $\frac{1}{2}$ and cutoff frequency $\pi - \omega_c$.
- A lowpass filter with gain $\frac{1}{2}$ and cutoff frequency $2\omega_c$.

Figure 3: Question 14

6.2 A

The thing to note here is that the $\cos(\frac{\pi}{2}n)^2$ and the $h[n]$ are not convulated but multiplyed, so it relates to modulation of the $h[n]$ filter.

From the hint we have $\cos(\frac{\pi}{2}n)^2 = \frac{1}{2}(1+\cos(\pi n))$ and we have from modulation:

$$DTFT\{\frac{1}{2}(1+\cos(\pi n))h[n]\} = \frac{1}{2}DTFT\{h[n]\} + \frac{1}{4}(H(e^{j(\omega-\pi)}) + H(e^{j(\omega+\pi)}))$$
$$= \frac{1}{2}H(e^{j\omega}) + \frac{1}{4}(H(e^{j(\omega-\pi)}) + H(e^{j(\omega+\pi)}))$$

So the system in the frequency domain is:

$$X(e^{j\omega})(\frac{1}{2}H(e^{j\omega}) + \frac{1}{4}(H(e^{j(\omega-\pi)}) + H(e^{j(\omega+\pi)}))(1 - H(e^{j\omega}))$$

First we multiply with a filter that take the low frequencies and the high frequencies, so we eliminate the bandpass frequencies. Then we multiply with a highpass filter, so overall the final system is a highpass filter. Note that because of perudicity, the high frequencies is a sum of the modulation from $0 \to \pi$ and from $\pi \leftarrow 2\pi$ and since it is multiplied by $\frac{1}{4}$ the overall gain is $\frac{1}{2}$.

7 Question 15

7.1 Q

See the figure for question 15.

7.2 A

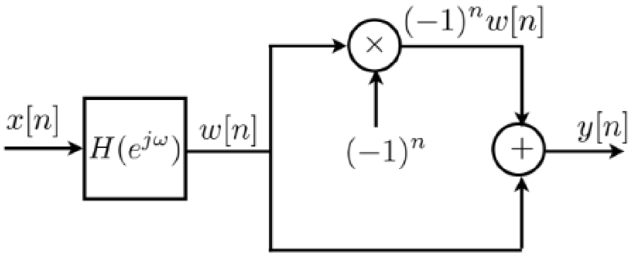
Since $x[n] = \delta[n]$ then $y[n]$ is the system itself. Note again that multiplying by $(-1)^n$ is a modulation with $\cos(\pi n)$ so the system is the sum of the filter $h[n]$ and it's modulation with $\cos(\pi n)$

Homework for Module 4 Part 1

Quiz, 15 questions

1 point

15. (Difficulty: ⋆ ⋆ ⋆) Consider the following system, where $H(e^{j\omega})$ is a half-band filter, i.e. an ideal lowpass with cutoff frequency $\omega_c = \pi/2$:



Assume the input to the system is $x[n] = \delta[n]$. Compute

$\sum_{n=-\infty}^{\infty} y[n]$

- Hint: Perform the derivations in the frequency domain.

Enter answer here

Figure 4: Question 15

but $h[n]$ is a lowpass filter with cutoff freq $\frac{\pi}{2}$ so the system in the frequency sapce (call it $S(e^{j\omega})$) is $S(e^{j\omega}) = 1$ for each ω (the sum of H and it's modulation covers the whole range).

Now, note that $\Sigma y[n] = (y[k] * 1)[0]$. So in the frequency domain it is $S(e^{j\omega})DTFT\{1\}$. Also $DTFT\{1\}$ is an impulse train with period 2π and amplitude 2π.

So we have:

$$\Sigma y[n] = (y[k]{*}1)[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} S(e^{j\omega})DTFT\{1\}e^{j\omega 0}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} DTFT\{1\}d\omega = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\omega)d\omega = 1$$