Types of Numbers

A course book exploring Numbers in GCSE Mathematics

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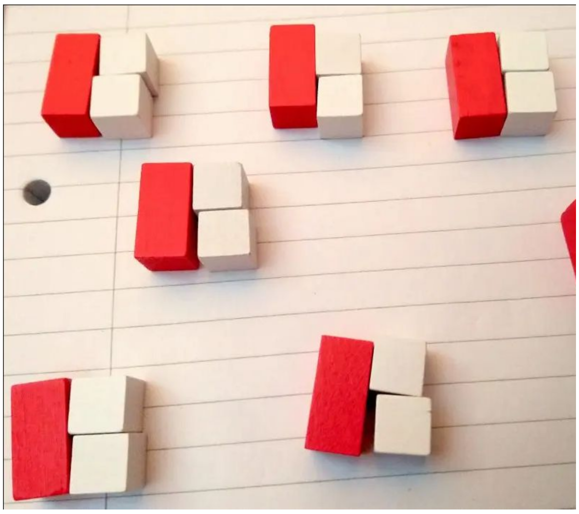

Introduction

Welcome to the course book on Numbers. This course book is aimed at the middle grades in the GCSE syllabus. It's also useful if you need some practice in these areas of Maths too. You won't need anything for this book, just a pen/pencil and yourself - there's plenty of room within for the exercises and, hopefully, your own notes and notations as we go through.

Have fun and enjoy yourself!

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Types of Numbers

There are a few different types of numbers you'll need to know for your GCSEs:

- Integers
- Fractions
- Decimals; including
  - Terminating
  - Recurring
  - Non-terminating, non-recurring
- Rational and Irrational Numbers

Exercise

Before we continue, what do you think these numbers are? If you don't know, can you give any example of each of them?

Integers

There are a range of numbers, and numbers can be grouped into different sets (something you may already be familiar with). One of these sets is called the Integers. Any number that can be written as a *whole number* is an integer. They can be negative, or positive and include the number zero. For instance, the numbers 7, -800, 58 and 1290000 are all integers.

If you can count it, it's probably an integer!

Fractions

What is a fraction?

A fraction is made up of two numbers; the **numerator** and the **denominator**. Separated by a line; called a **fraction** or **division bar**. That bar looks a bit familiar, doesn't it? Fractions are a form of ratio or a way of dividing something into parts.

A numer-whatnow and a demon-ator?

Numerator (*new-mer-ator*) and denominator (*de-nom-in-ator*). The number found below the division bar is the denominator. This tells us how many parts there are in whatever it is we're looking at. The numerator is found above the division bar, and lets us know how many of those parts we are dealing with.

Decimals

Money, lengths, weights, distances all use decimals. You're used to decimals and we learn about them in Primary School.

If a fraction will divide something into any number of parts, decimals only divide something into powers of 10.

What do I mean by that? Well… the prefix "dec-" comes from Roman and Greek and just means ten. Interesting side note, December also has the prefix "dec" and, yes, used to be the tenth month of the year, before the winter months January and February were introduced.

Depending on the number of decimal places a decimal has, depends on which power of ten we're dividing by.

Decimal places = Power of ten..?

Sure :)

If a decimal has only one decimal place (remember place values?) then it's a division of ten (ten to the power of 1). Three decimal places, it's a division of 1,000 (ten to the power of 3 - or ten cubed).

Recurring Decimals

Well, we're introduced to **terminating** decimals in primary school.
Put simply, terminating decimals stop. They have one, two, five, however many decimals places, but they all come to an end. 0.5, 0.97, 76.912 are all terminating decimals.

Recurring Decimals

These types of decimals have a *repeating* number or pattern after the decimal point.

So, something like $2.\overline{4646}$ would be recurring, or even $0.\overline{3}$. This is called the bar notation and the way we'll denote recurring decimals throughout this book.

We can use a dot (I personally like this one if I'm writing maths with pen and paper) - $2.\dot{3}$ or $2.\dot{2}\dot{4}$ which means the 24 repeats, ie 0.24242424…

It's important to note that "…" can also represent a non-recurring non-terminating decimal that just goes on an on and on and never stops - like π (3.14159265359…) or √2 (=1.414213…), so this is not considered "correct" mathematical notation.

Non-terminating, non-recurring Decimals

These are anything else. They are the ones that go on forever, with no pattern. The ellipsis (three dots "...") is used for these.
23.482…. , 1.76341 … are some examples of these types of decimals.

Rational and Irrational Numbers

An integer is what's known as a rational number - it is a number that can be written as a fraction (a ratio). For example, 7 can be written as $\frac{21}{3}$ or $\frac{49}{7}$.

Other fractions you have come across are also rational numbers; $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{-12}{1}$.

Terminating and recurring decimals are also rational numbers.

Irrational numbers however are, as many text books will put it "messy". These numbers cannot be written as fractions, and often will have non-ending and non-repeating decimals. π, and $\sqrt{2}$ are both examples of irrational numbers we'll come across on our GCSE journey.

Note: Not all square (or higher) roots are irrational. $\sqrt{2}$ for instance is, $\sqrt{4}$ however is not: $\sqrt{4} = \pm 2$

BODMAS

The Order of Operations - BODMAS (or BIDMAS or Americans use PODMAS/PIDMAS), they all mean the same thing.

Let's break it down:

B - Brackets

O/I - Orders/Indices (ie Powers and square roots)

D - Division

M - Multiplication

A - Addition

S - Subtraction

There's a secret here though, Division/Multiplication have the same importance as each other, as do Addition/Subtraction. This means that if you come across something with both subtraction and addition in it, then you should work it out left to right.

Let's have a look at an example.

Example

$(15 \div 3) - 7 + 8$

Following BODMAS, we start with the brackets, so $15 \div 3 = 5$.

So, now we have $5 - 7 + 8 = 6$

Or if we were to write it up, it would look like this:

$(15 \div 3) - 7 + 8$

$= 5 - 7 + 8$

$= -2 + 8$

$= 6$

Example

Without a calculator find the following: $\sqrt{(12-3) + 4 \times 2^2}$

That looks a little scary, huh? But we can do this....

Start with the brackets: $\sqrt{9 + 4 \times 2^2}$

Then deal with the power: $= \sqrt{9 + 4 \times 4}$

Then Multiply: $= \sqrt{9 + 16}$

Add: $= \sqrt{25}$

Then solve: $= 5$

So, $\sqrt{(12-3) + 4 \times 2^2} = 5$

Easy, right? Try one for yourself.

Exercise

Without a calculator find the following: $\sqrt{(2+3) \times 8 - 5}$

Prime Numbers

And we're on our final types of numbers - Prime Numbers!

It's been quite the fun journey so far, right? Don't worry, things get a little bit more interesting after this.

I'm sure we already know what a prime number is; "A number that is only divisible by itself and 1", correct? That's what you were probably taught. So, I'm going to change this slightly, a prime number has two factors; itself and 1. So, that should clear up that age-old question:

Is 1 a prime number?

No! 1 only has one factor - 1. There is a very interesting theorem that shows that 1 cannot be a prime number - and it was this theory that finally made us all agree on it. I won't get into it here, but if you're interested, I'll leave a few extra reading bits for you at the end of the section. The short of it is every number has a *unique* **prime factorization** and if you multiply anything by 1, you don't have a unique number, and it doesn't ever end.

For your GCSEs you'll need to be able to list all the primes to 100 (150 is best though, sometimes they'll sneak in a harder one on the Higher papers) and work out if a large number is prime - usually found in the proof sections of the exam.

Let's have a quick look at an example.

Example

Is the number 137 prime?

The most systematic way to approach this is to look at all the primes between 0 and $\frac{137+1}{2}$. So, $\frac{138}{2} = 69$. That's a lot of numbers! The primes between 0 and 69 are; 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61 and 67.

The long and short of it, is that yes, 137 is prime.

$137 \div 2 = 68.5$ obviously not divisible by 2

$137 \div 3 = 45.\overline{6}$

$137 \div 5 =$ obviously not divisible by 5
$137 \div 7 = 19.571...$
$137 \div 11 = 12.\overline{45}$
$137 \div 13 = 10.\overline{538461}$
$137 \div 17 = 8.05882...$
$137 \div 19 = 7.20105...$
$137 \div 23 = 5.956521...$
$137 \div 29 = 4.7541379...$
$137 \div 31 = 4.4193548...$
$197 \div 37 = 3.\overline{702}$
$137 \div 41 = 3.\overline{34146}$
$137 \div 43 = 3.1860465....$
$137 \div 47 = 2.9148936....$
$137 \div 53 = 2.58490566...$
$137 \div 59 = 2.32203389...$
$137 \div 61 = 2.245901639...$
$137 \div 67 = 2.04477611940...$

Before we continue, have a think about these following questions and write your thoughts below. Why do we just check the prime numbers (and not all the numbers)? Why did I suggest halving the number and just dealing with those primes? Are there any tricks you could use to make this just a little bit easier for yourself?
(think about what numbers would have to be multiplied by what to give you the last digit of the number, for example, $3 \times 7 = 21$, so any number with the last digit of 3 and/or 7 would give you a last digit of 1, try it and see for yourself).

Exercise

Is the number 73 prime? How can you be so sure?

You know some tricks to find if a number is divisible by certain numbers (eg all even numbers are divisible by 2, numbers divisible by 5 end in a 0 or 5), do you know any more? How about divisible by 3 or 9?

Quiz

1. The number -98 is an integer True/False

2. What kind of decimal is 0.5? Terminating/Recurring/Not a decimal

3. Is the number 93 Prime? Yes/No

4. Answer true or False to the following statements
    a. $\sqrt{2}$ is an integer
    b. All integers can be written as a fraction
    c. π is irrational
    d. $-0.\overline{14}$ is an integer
    e. $\sqrt{4}$ is irrational

5. For each of the following, give an example:
    a. A 3-digit negative integer
    b. Another 3- digit negative integer
    c. And another
    d. A recurring decimal
    e. A terminating decimal

6. Work out the following.
    $\sqrt{(4+3)^2 - 6 \times 4}$

7. Order these fractions from smallest to largest
    a. $\frac{4}{5}$, $\frac{2}{3}$, $\frac{7}{10}$
    b. $\frac{7}{5}$ $\frac{21}{20}$ $\frac{22}{15}$

Answers

BODMAS
Page 8

$\sqrt{15} = 3.872983346$ *(either would be acceptable)*

Prime Numbers
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Yes, 73 is prime.

Quiz
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1. True
2. Terminating
3. No
4.
    - False
    - True
    - True
    - False
    - False
5. *There are many, many different answers to this one - too many to list! Examples include*
   Negative integers include; -5, -400, -530, -1,234,500 etc
   Recurring decimals; $0.\overline{67},\ 8.\overline{2903}$
   (what's important here is the bar/dot)

6. 5
7.
    - $\frac{2}{3}, \frac{7}{10}, \frac{4}{5}$
    - $\frac{21}{20}, \frac{7}{5}, \frac{22}{15}$