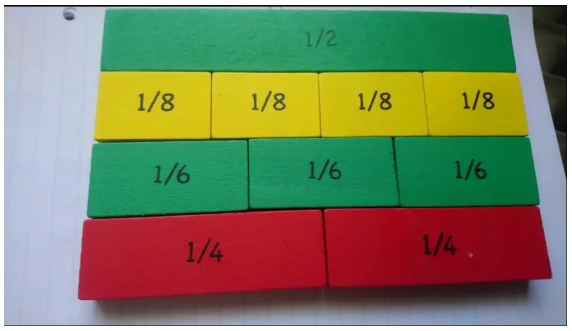
Fractions

A course book exploring fractions in GCSE Mathematics

Verity H Hawthorn-Thwaites
Verity's Maths Place.



Introduction

Welcome to the course book on . This course book is aimed at the middle grades in the GCSE syllabus. It's also useful if you need some practice in these areas of Maths too. You won't need anything for this book, just a pen/pencil and yourself - there's plenty of room within for the exercises and, hopefully, your own notes and notations as we go through.

Have fun and enjoy yourself!

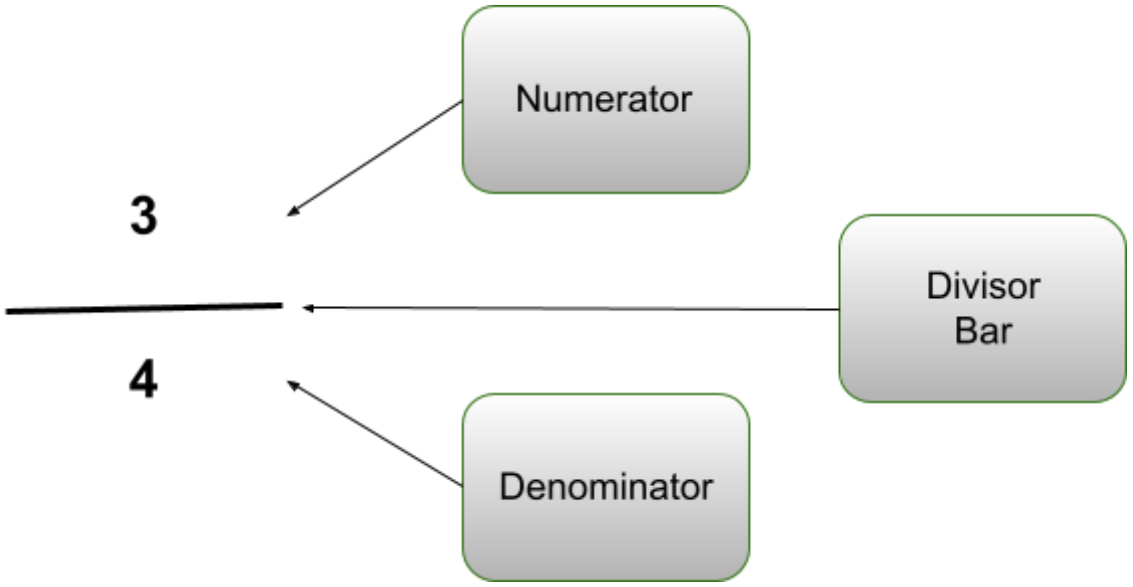
Contents

What is a fraction?

If you were to be asked this question: "what is a fraction and why do we want to use them?" What would you answer with? Use the space here to answer if you wish to do so.

A fraction is part of a number. Why would we want to use them? How else would you share 2 cookies between you and three friends?

There are three parts to a fraction; the **numerator** (top number), the **denominator** and the **divisor bar.** If you have been through the *"Types of Numbers"* course book, we've already been through these parts and you'll know that a fraction is just a way of dividing numbers.



Break down of the parts of a fraction

Cancelling Down

Cancelling down is fairly straightforward, and something you will have met already in KS3 and earlier. It uses factors of both the numerator and the denominator, or rather common factors of both. In exam questions, they'll generally ask you to "give your answer in its simplest form" - this is just a very wordy way of saying "cancel down".

Let's look at an example

Example

Cancel down the following fraction ...

$$\frac{18}{33}$$

Both 18 and 33 share a factor of 3. $18 \div 3 = 6$ and $33 \div 3 = 11$, giving us a fraction of

$$\frac{6}{11}$$

6 and 11 are what are known as co-prime (that is, they don't share any common factors [other than 1]), and so we end here.

Hints:

- Start with 2 - if both numbers are even, that's generally the easiest place to start from
- Use some tricks to quickly work it out, for instance, we can see fairly quickly that the digit sum of both 18 and 33 are factors of 3 (9 and 6, respectively).
- Use your times tables - if you recognise the two numbers are in the same times table, then that's a good place to start also.

Exercise

Cancel down the following (without a calculator!)

$\frac{12}{48}$ $\frac{45}{50}$

$\frac{9}{63}$ $\frac{20}{60}$

$\frac{55}{80}$ $\frac{45}{90}$

$\frac{25}{75}$ $\frac{6}{120}$

Equivalent Fractions

This seems like a good place to talk about equivalent fractions. All these fractions are equivalent. That is, although the numbers are different, they all share the same value. For example:

$\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$

Can you make any equivalent fractions with the following unit fractions? Unit fractions are any fractions with a numerator of 1.

$\frac{1}{2}$

$\frac{1}{5}$

$\frac{1}{3}$

We continue on our journey to meet mixed numbers. Now, you may remember these from your earlier school days. They have a whole part and a fractional part.

$2\frac{1}{2}$is a mixed number. And during this lesson, we're going to see how we can make them "top heavy" and turn top heavy fractions into mixed numbers.

Top Heavy, Improper Fractions

Improper fractions are heavier on the top - that is the numerator is larger than the denominator. $\frac{22}{15}$would be an improper fraction.

Improper to mixed numbers

How do we turn it into a mixed number? Quite simple. Remember what the bar is called? A division bar. So all we need to do is divide 22 by 15. $22 \div 15 = 1$with seven remaining. The remainder becomes the numerator to the fractional part and so, $\frac{22}{15} = 1\frac{7}{15}$. Remember, we need to keep the denominator the same, but you can always cancel down if required afterwards.

Exercise

Try these:

$\frac{15}{10}$ $\frac{25}{15}$

$\frac{6}{5}$ $\frac{10}{3}$

$\frac{15}{9}$ $\frac{17}{8}$

$\frac{40}{18}$ $\frac{32}{15}$

Mixed Numbers to Improper Fractions

What we want to do is change the whole part into a fraction. Can you think of the best way to do this? Think about how many parts there are in a single whole part.

There is a really nice little trick to this. Firstly, you keep the denominator the same. So then you multiply the whole number by the denominator and then add the numerator. Or, times the whole part by the bottom and add the top. This number becomes your numerator, and then you can cancel down/simplify as required.

Example

Turn the mixed number $5\ \frac{4}{10}$ into an improper fraction.

We keep the denominator the same: $\frac{\quad}{10}$

Multiply the whole part by the denominator and add the top, to give us the new numerator:

$5 \times 10 + 4 = 54$so, our improper fraction is: $\frac{54}{10}$and then cancel down to $\frac{27}{5}$

Common Denominators

You've probably met common denominators before - so this will be a really quick recap more than anything. We'll need common denominators for most work using fractions, but mostly when we're adding and subtracting.

How do we find common denominators? We look at the factors of both and find a common multiple. Take the following: $\frac{3}{20} + \frac{7}{10}$ this one seems easy, both have a common multiple of 20.

So we would rewrite this as $\frac{3}{20} + \frac{14}{20}$. ($\frac{7}{10}$ and $\frac{14}{20}$ are **equivalent fractions**).

I'm not interested in the answer for the moment, more how it works.

We would also use common denominators for ordering fractions.

If we take a look at the following $\frac{7}{15}, \frac{13}{30}, \frac{1}{5}, \frac{3}{10}$ we may notice that all the denominators share a multiple of 30 (15×2, 30×2, 5×6, 10×3). Remember for these ones to always write the fractions *you're given* in order, not the equivalent ones you've found.

Exercise

1. Find a common denominator for the following groups of fractions
   a. $\frac{5}{7}, \frac{3}{14}, \frac{15}{28}$
   b. $\frac{1}{3}, \frac{3}{5}$
   c. $\frac{2}{5}, \frac{3}{10}, \frac{17}{25}$
   d. $\frac{4}{9}, \frac{4}{5}$
2. Order the following fractions, smallest to largest
   a. $\frac{1}{3}, \frac{2}{5}, \frac{9}{10}, \frac{1}{2}$
   b. $\frac{3}{4}, \frac{19}{20}, \frac{3}{10}$

Adding and Subtracting Fractions

The most important rule is to make your denominators the same. And remember: If you multiply the bottom number by something, you must multiply the top by the same thing. Also, to make it easier on yourself, change all mixed numbers into improper fractions. Although, it can be done without doing so.

Example

Calculate (without a calculator!)
$\frac{7}{15} + 2\frac{2}{5}$

$= \frac{7}{15} + \frac{12}{5}$ *Change the fraction into an improper fraction:* $2 \times 5 + 2$

$= \frac{7}{15} + \frac{36}{15}$ *Multiply both top and bottom of* $\frac{12}{5}$ *by 3 to make the denominator of both to 15*

$= \frac{43}{15}$ *Add*

$= 2\frac{13}{15}$ *Turn back into a mixed number*

And done!

Exercise

Calculate (without a calculator!)

a. $\frac{5}{6} + \frac{7}{12}$
b. $\frac{3}{5} + 3\frac{1}{2}$
c. $2\frac{1}{3} - \frac{5}{7}$
d. $3\frac{1}{2} - 1\frac{2}{5}$
e. $2\frac{1}{3} + 1\frac{1}{2}$
f. $3\frac{7}{10} - 1\frac{3}{5}$
g. $\frac{4}{5} - \frac{8}{9}$
h. If $a = 4$ and $b = -3$, find the value of $\frac{1}{a} - \frac{1}{b}$

i. Look at shapes A, B, and C below

A B C

$\frac{6}{7}$ of shape A is shaded and $\frac{2}{5}$ of shape B is shaded. How much of shape C is shaded?

Multiplying and Dividing Fractions

Multiplying and Dividing fractions doesn't involve equivalent fractions! We just leave it as is. Multiplying is the easiest, just multiply the numerators together and multiply the denominators together.

Dividing is just as easy with a little KCF. Keep Change Flip. We **keep** the first fraction, **change** the division sign into a multiplication sign, **flip** the second fraction upside down.

Example

Find the value of

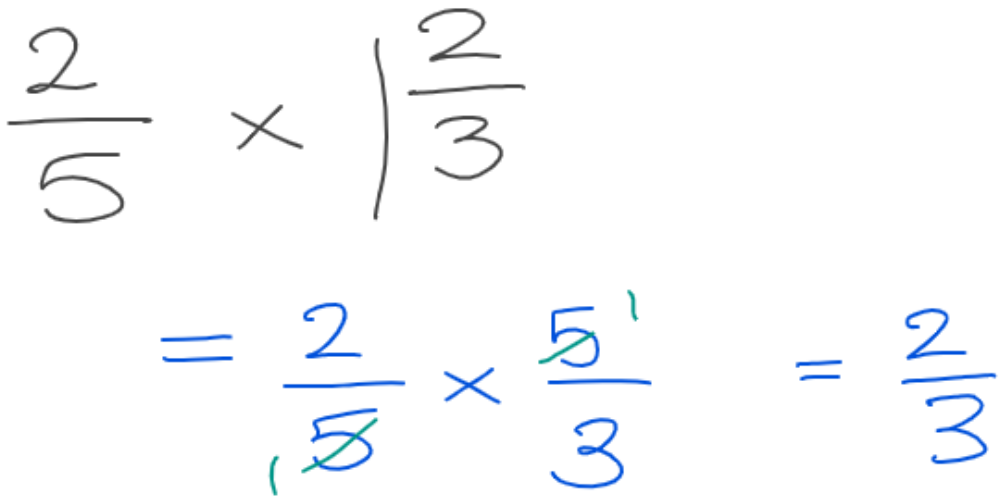
$\frac{2}{5} \times 1\frac{2}{3}$

$= \frac{2}{5} \times \frac{5}{3}$ *Turn the fraction into an improper*

$= \frac{2 \times 5}{5 \times 3}$ *Multiply numerators together and the denominators together*

$= \frac{10}{15} = \frac{2}{3}$ *Cancel down and simplify*

One of the properties of fractions I like is that we can *cross cancel* to make the multiplication easier for ourselves:



Example

Calculate the value of:

$\frac{3}{7} \div \frac{9}{14}$

$= \frac{3}{7} \times \frac{14}{9}$ ***Keep** the first, **change** the division to a multiplication, **flip** the second one upside down*

$= \frac{1}{1} \times \frac{2}{3}$ *Cross cancel*

$= \frac{2}{3}$

Exercise

Work out the following

1. $\frac{5}{7} \times \frac{2}{5}$
2. $\frac{8}{9} \times \frac{5}{8}$
3. $\frac{3}{5} \times \frac{10}{13}$
4. $\frac{8}{11} \div \frac{16}{33}$
5. $2\frac{1}{5} \div 1\frac{4}{3}$
6. $1\frac{3}{4} \div \frac{7}{8}$

Fractions in Fractions

What happens if we wanted to work something like the following out?

$$\frac{1}{\frac{1}{2}}$$

That is, 1 over a half?

Before we look at it, use the space below to mark your thoughts.

If you remember, the line in a fraction is called the *divisor bar*. Does that give you an extra hint?

$$\frac{1}{1/2} = 1 \div \frac{1}{2}$$

$$= \frac{1}{1} \times \frac{2}{1} = 2.$$

As you can see, we take the fraction in the denominator, and divide the top number by this fraction.

Let's have a look at the following question:

Example

If $a = \frac{3}{4}$ and $b = 2\frac{1}{2}$, find the value of $\frac{1}{a} + \frac{1}{b}$

$a = \frac{3}{4}$ $b = 2\frac{1}{2}$ $\frac{1}{a} + \frac{1}{b}$

$$\frac{1}{3/4} + \frac{1}{2\,1/2} = \frac{4}{3} + \frac{2}{5}$$

$$= \frac{20+6}{15} = \frac{26}{15} = 1\frac{11}{15}$$

$\frac{1}{3/4} = 1 \div \frac{3}{4} = \frac{1}{1} \times \frac{4}{3} = \frac{4}{3}$

Where did $\frac{2}{5}$ come from?

What do you notice about any fractions written as "one over a fraction"?

Exercise

If $a = \frac{2}{3}$ and $b = 2\frac{1}{4}$, find the value of $\frac{1}{a} + \frac{1}{b}$

If $a = 1\frac{3}{4}$ and $b = \frac{-4}{5}$, find the value of $\frac{1}{a} - \frac{1}{b}$

Answers

Cancelling Down
Page 8

I. $\frac{12}{48} = \frac{1}{4}$

II. $\frac{45}{50} = \frac{45}{50}$

III. $\frac{9}{63} = \frac{1}{7}$

IV. $\frac{20}{60} = \frac{1}{3}$

V. $\frac{55}{80} = \frac{11}{16}$

VI. $\frac{45}{90} = \frac{1}{2}$

VII. $\frac{25}{75} = \frac{1}{3}$

VIII. $\frac{6}{120} = \frac{1}{20}$

Top Heavy to Mixed Numbers
Page 8

$\frac{15}{10} = 1\frac{1}{2}$

$\frac{25}{15} = 1\frac{2}{3}$

$\frac{6}{5} = 1\frac{1}{5}$

$\frac{10}{3} = 3\frac{1}{3}$

$\frac{15}{9} = 1\frac{2}{3}$

$\frac{17}{8} = 2\frac{1}{8}$

$\frac{40}{18} = 2\frac{2}{9}$

$\frac{32}{15} = 2\frac{2}{15}$

Adding and subtraction
Page 10

a. $1\frac{5}{12}$
b. $4\frac{1}{10}$
c. $\frac{14}{15}$
d. $2\frac{1}{10}$
e. $3\frac{5}{6}$
f. $2\frac{1}{10}$
g. $-\frac{4}{45}$

Fractions in Fractions
Page 15

a. $1\frac{17}{18}$

b. $1\frac{23}{26}$

h. $\frac{7}{12}$ i. $\frac{19}{35}$	