lista 2 zad 1

za pomocą wyznacznika określić osobliwość i uwarunkowanie (złe lub dobre)

a)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix} = \begin{Bmatrix} w_2 - 2w_1 \\ w_3 - 3w_1 \end{Bmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -4 \end{pmatrix}$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -2 & -4 \end{vmatrix} = (-1)^2 \cdot 1 \cdot \begin{vmatrix} -1 & -2 \\ -2 & -4 \end{vmatrix} = (-1)(-4) - (-2)(-2) = 0$$

$(-2)(-2)$ $(-1)(-4)$

2 1

$\det(A) = 0$ osobliwa i źle uwarunkowana

Macierz osobliwa, to taka której wyznacznik jest równy 0

b)

$$\begin{array}{ccc|cc} 2{,}11 & -0{,}80 & 1{,}72 & 2{,}11 & -0{,}80 \\ -1{,}84 & 3{,}03 & 1{,}29 & -1{,}84 & 3{,}03 \\ -1{,}54 & 5{,}25 & 4{,}30 & -1{,}54 & 5{,}25 \end{array} =$$

$$2{,}11 \cdot 3{,}03 \cdot 4{,}30 + (-0{,}80) \cdot 1{,}29 \cdot (-1{,}54) + 1{,}72 \cdot (-1{,}84) \cdot (5{,}25$$
$$- (-1{,}54)(3{,}03)(1{,}72) - (5{,}25)(1{,}29)(-0{,}80) - (4{,}30)(-1{,}84)(-0{,}80) =$$

$$= 27{,}49119 + 1{,}58929 - 16{,}6152 + (8{,}072526) +$$
$$+ 5{,}418 - 6{,}3296 \approx 19{,}63$$

nieosobliwa dobrze uwarunkowana

c) $\det A = \begin{vmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} \underset{L}{\overset{w_1+2w_2}{=\!=\!\rightarrow}} \begin{vmatrix} 0 & 3 & -2 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{vmatrix} =$

$= (-1)^3 \cdot (-1) \cdot \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = (6) - (2) = 4$

nieosobliwa

d) $\det A = \begin{vmatrix} 4 & 3 & -1 \\ 7 & -2 & 3 \\ 5 & -18 & 13 \end{vmatrix} \underset{w_3+13w_1}{\overset{w_2+3w_1}{=\!=}} \begin{array}{c} \quad \quad L \\ \begin{vmatrix} 4 & 3 & -1 \\ 19 & 7 & 0 \\ 57 & 21 & 0 \end{vmatrix} \end{array} =$

$= (-1)^{1+3} (-1) \begin{vmatrix} 19 & 7 \\ 57 & 21 \end{vmatrix} = -1 \left(19 \cdot 21 - 7 \cdot 57 \right) =$

$= -1 (399 - 399) = 0$

dolne uwarunkowana

zad 2

a)

$$L=\begin{pmatrix}1&0&0\\1&1&0\\1&5/3&1\end{pmatrix}\qquad U=\begin{pmatrix}1&2&4\\0&3&21\\0&0&0\end{pmatrix}$$

$$LU=\begin{pmatrix}1&0&0\\1&1&0\\1&\frac{5}{3}&1\end{pmatrix}\cdot\begin{pmatrix}1&2&4\\0&3&21\\0&0&0\end{pmatrix}=\begin{pmatrix}1&2&4\\1&5&25\\1&7&39\end{pmatrix}=A$$

$$\det(A)=\begin{vmatrix}1&2&4\\1&5&25\\1&7&39\end{vmatrix}\underset{w_3-w_1}{\overset{w_2-w_1}{=}}\ L\to\begin{vmatrix}1&2&4\\0&3&21\\0&5&35\end{vmatrix}=$$

$$(-1)^2(1)\begin{vmatrix}3&21\\5&35\end{vmatrix}=105-105=0$$

b)

$$\begin{array}{ccc|ccc} & & & 2&-1&1\\ & & & 0&1&-3\\ & & & 0&0&1\\ \hline 2&0&0&4&-2&2\\ -1&1&0&-2&2&-4\\ 1&-3&1&2&-4&11\end{array}=A$$

$$\det(A)=\begin{vmatrix}4&-2&2\\-2&2&-4\\2&-4&11\end{vmatrix}\underset{w_3+w_2}{\overset{w_1+2w_2}{=}}$$

$$=\ L\to\begin{vmatrix}0&2&-6\\-2&2&-4\\0&-2&7\end{vmatrix}=(-1)^3(-2)\begin{vmatrix}2&-6\\-2&7\end{vmatrix}=$$

$$2(14-12)=4$$

zad 3

$$\vec{x}=U^{-1}\left(L^{-1}\vec{b}\right)$$

$$L^{-1}:\ \left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\3/2&1&0&0&1&0\\1/2&11/13&1&0&0&1\end{array}\right]\underset{w_3-\frac{1}{2}w_1}{\overset{w_2-\frac{3}{2}w_1}{=}}\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&-\frac{3}{2}&1&0\\0&\frac{11}{13}&1&-\frac{1}{2}&0&1\end{array}\right]\overset{w_3-\frac{11}{13}w_2}{=}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 & \frac{20}{26} & -\frac{11}{13} & 1 \end{array}\right] \quad L^{-1}$$

$$U^{-1} = \left[\begin{array}{ccc|ccc} 2 & -3 & -1 & 1 & 0 & 0 \\ 0 & 13/2 & -7/2 & 0 & 1 & 0 \\ 0 & 0 & 32/13 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} k_2 + \frac{3}{2} k_1 \\ = \\ k_3 + \frac{1}{2} k_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & \frac{13}{2} & -\frac{7}{2} & 0 & 1 & 0 \\ 0 & 0 & \frac{32}{13} & 0 & 0 & 1 \end{array}\right] \begin{array}{l} k_3 + \frac{7}{13} k_2 \\ = \\ \frac{1}{2} w_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 0 & \frac{13}{2} & 0 & 0 & 1 & \frac{7}{13} \\ 0 & 0 & \frac{32}{13} & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \frac{2}{13} w_2 \\ = \\ \frac{13}{32} w_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 0 & 1 & 0 & 0 & \frac{2}{13} & \frac{14}{169} \\ 0 & 0 & 1 & 0 & 0 & \frac{13}{32} \end{array}\right] \quad u^{-1}$$

$$\vec{x} = u^{-1} \left(L^{-1} \vec{b} \right) = \begin{pmatrix} \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 0 & 2/13 & 14/169 \\ 0 & 0 & 13/32 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ \frac{20}{26} & -\frac{11}{13} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} =$$

$$\begin{pmatrix} \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ 0 & 2/13 & 14/169 \\ 0 & 0 & 13/32 \end{pmatrix} \begin{pmatrix} 1 \\ -\frac{5}{2} \\ 4 \end{pmatrix} = \begin{pmatrix} -\frac{3}{8} \\ -\frac{9}{169} \\ \frac{13}{8} \end{pmatrix}$$