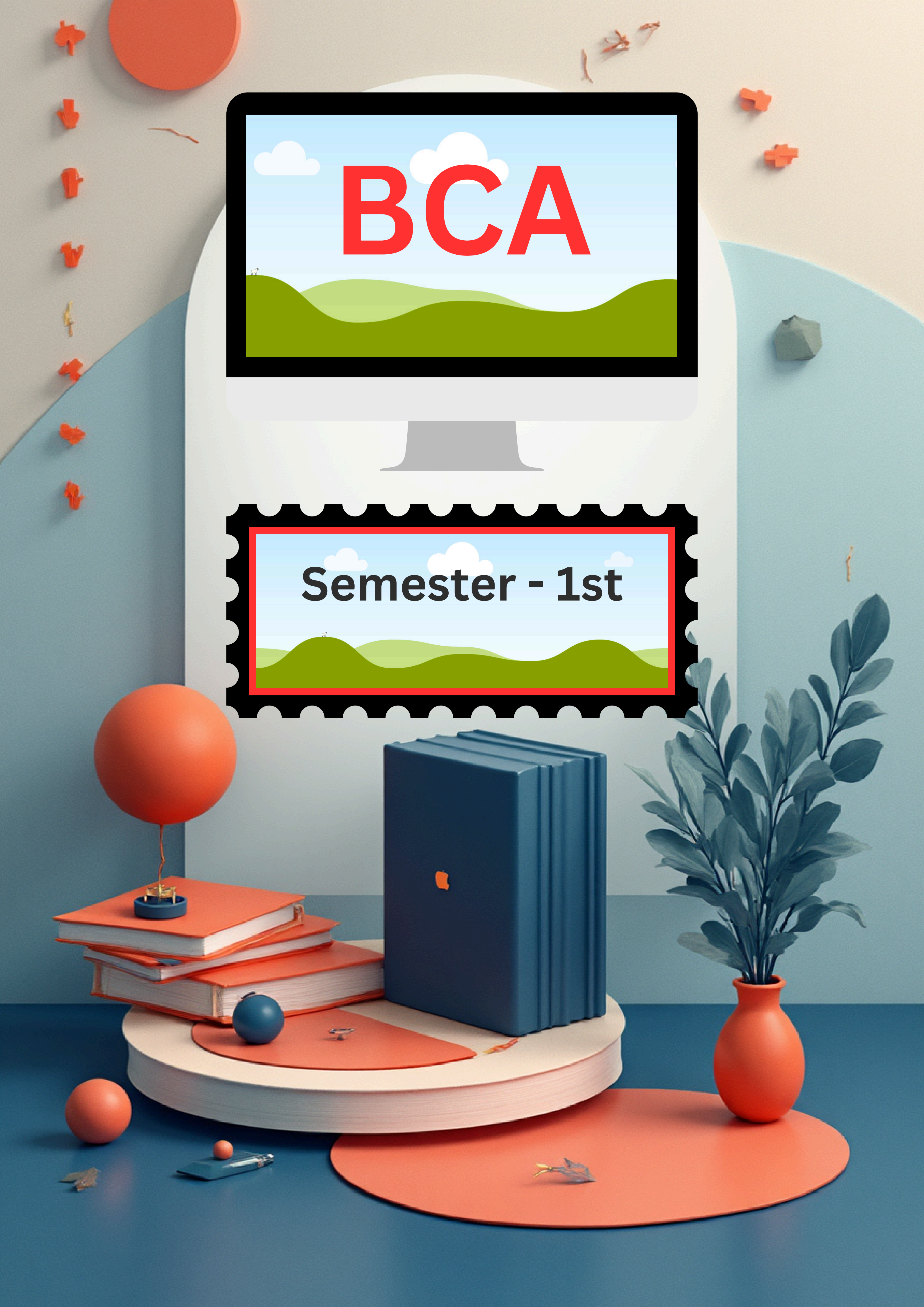
BCA
Semester - 1st

Mathematical Foundation

Notes - 1

Contents

Differential Calculus: Successive Differetiation, Leibnitz Theorem, Expansion of function of one variable in Taylor's and Meclaurin's infinite series, Maxima and minima of functions of one variable, Partial Derivatives, Euler's theorem, change of variables, Total differentiation, Taylor's series in two variables, Maxima and Minima of two variables.

Website - prepfolio.xyz

Differential Calculus: Successive Differentiation

1. Meaning of Successive Differentiation

- **Differentiation** means finding the **derivative** of a function.
- **Successive differentiation** means **repeatedly differentiating** a function more than once.

In other words, **if we take the derivative of a derivative**, and continue this process, it is called **successive differentiation.**

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2. First, Second, Third... Derivatives

Let a function be $y = f(x)$.

- The **first derivative** of y with respect to x is written as:

$$\frac{dy}{dx} \text{ or } f'(x)$$

  It gives the **rate of change** of y with respect to x.
- The **second derivative** is the derivative of the first derivative:

$$\frac{d^2y}{dx^2} \text{ or } f''(x)$$

  It gives the **rate of change of the rate of change**, i.e., how the slope itself is changing.
- The **third derivative** is:

$$\frac{d^3y}{dx^3} \text{ or } f'''(x)$$

- Similarly, the **nth derivative** is written as:

$$\frac{d^ny}{dx^n} \text{ or } f^{(n)}(x)$$

3. Example of Successive Differentiation

Let's take an example:

$$y = x^4$$

First derivative:

$$\frac{dy}{dx} = 4x^3$$

Second derivative:

$$\frac{d^2y}{dx^2} = 12x^2$$

Third derivative:

$$\frac{d^3y}{dx^3} = 24x$$

Fourth derivative:

$$\frac{d^4y}{dx^4} = 24$$

Fifth derivative:

$$\frac{d^5y}{dx^5} = 0$$

➡️ Notice that after a certain number of differentiations, the result becomes **zero** for polynomial functions.

4. Some Useful Standard Results

Function	1st Derivative	2nd Derivative	nth Derivative
x^n	nx^{n-1}	$n(n-1)x^{n-2}$	$n(n-1)(n-2)...(n-r+1)x^{n-r}$
e^{ax}	ae^{ax}	a^2e^{ax}	a^ne^{ax}
$\sin(ax)$	$a\cos(ax)$	$-a^2\sin(ax)$	$a^n\sin(ax+n\frac{\pi}{2})$
$\cos(ax)$	$-a\sin(ax)$	$-a^2\cos(ax)$	$a^n\cos(ax+n\frac{\pi}{2})$

5. Rules for Successive Differentiation

1. **Constant Rule:**

   Derivative of a constant is zero.

$$\frac{d}{dx}(C) = 0$$

2. **Sum Rule:**

   Derivative of a sum = sum of derivatives.

$$\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$$

3. **Product Rule:**

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

4. **Quotient Rule:**

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

5. **Chain Rule:**

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

These rules can be applied again and again for **successive differentiation.**

6. Important Notes

- **Higher order derivatives** (like 2nd, 3rd, etc.) are used in:
    - **Physics:** to find acceleration, jerk, etc.
    - **Mathematics:** to find **maxima, minima, and points of inflection.**
    - **Engineering:** to analyze motion, vibrations, and rates of change.
- **If nth derivative of a function becomes zero**, all higher derivatives will also be zero.

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7. Example for Practice

Find the **third derivative** of $y = \sin(x)$.

Solution:

1. $\frac{dy}{dx} = \cos(x)$
2. $\frac{d^2y}{dx^2} = -\sin(x)$
3. $\frac{d^3y}{dx^3} = -\cos(x)$

Hence,

$$\frac{d^3y}{dx^3} = -\cos(x)$$

Leibnitz's Theorem

1. Introduction

When we have a function that is the **product of two functions**, say

$$y = u \cdot v$$

we can find its **first derivative** using the **Product Rule**:

$$\frac{dy}{dx} = u'v + uv'$$

But what if we want to find the **second, third, or nth derivative** of $y = u \cdot v$?
👉 Then we use **Leibnitz's Theorem.**

It gives a **general formula** for finding the **nth derivative of a product of two functions.**

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2. Statement of Leibnitz's Theorem

If $y = u \cdot v$,
then the **nth derivative** of y with respect to x is given by:

$$\frac{d^n y}{dx^n} = \sum_{r=0}^{n} \binom{n}{r} \frac{d^{n-r} u}{dx^{n-r}} \cdot \frac{d^r v}{dx^r}$$

Or in simpler form:

$$y^{(n)} = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v'' + \ldots + uv^{(n)}$$

3. Explanation of Symbols

Symbol	Meaning
$y^{(n)}$	nth derivative of y
u, v	two functions of x
u', v'	first derivatives of u, v
$u^{(r)}, v^{(r)}$	r-th derivatives of u, v
$\binom{n}{r}$	"n choose r" = $\frac{n!}{r!(n-r)!}$

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4. Formula in Expanded Form

$$\frac{d^n(uv)}{dx^n} = u\frac{d^n v}{dx^n} + n\frac{du}{dx}\frac{d^{n-1}v}{dx^{n-1}} + \frac{n(n-1)}{2!}\frac{d^2u}{dx^2}\frac{d^{n-2}v}{dx^{n-2}} + \ldots + v\frac{d^n u}{dx^n}$$

So, each term has:

- a **derivative of** u multiplied by
- a **derivative of** v
  and multiplied by the **binomial coefficient** $\binom{n}{r}$.

5. Proof (Conceptual and Simple)

We know from the **binomial theorem** that:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

Similarly, in differentiation, we can treat "taking derivative" as an operation D.
Let $D = \frac{d}{dx}$.

Now, using the **product rule** in operator form:

$$D(uv) = uDv + Du \cdot v$$

If we apply the derivative **n times**,

$$D^n(uv) = (D_u + D_v)^n(u, v)$$

Expanding by binomial theorem gives:

$$D^n(uv) = \sum_{r=0}^{n} \binom{n}{r} D^{n-r}u \cdot D^r v$$

which is **Leibnitz's Theorem**.

6. Example 1

Find $\frac{d^2}{dx^2}(x^2e^x)$.

Here,

$u = x^2, v = e^x$

Using Leibnitz's Theorem:

$$\frac{d^2(uv)}{dx^2} = u''v + 2u'v' + uv''$$

Now,

$$u = x^2, \quad u' = 2x, \quad u'' = 2$$

$$v = e^x, \quad v' = e^x, \quad v'' = e^x$$

So,

$$\frac{d^2(uv)}{dx^2} = (2)e^x + 2(2x)e^x + (x^2)e^x$$

$$\boxed{\frac{d^2(uv)}{dx^2} = e^x(x^2 + 4x + 2)}$$

Expansion of a Function of One Variable in Taylor's and Maclaurin's Infinite Series

(From Differential Calculus – Successive Differentiation topic)

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1. Introduction

In calculus, sometimes we want to **find the approximate value of a function** near a particular point without directly calculating it.

For that, we use **Taylor's Series** or **Maclaurin's Series.**

These are used to **expand a function into an infinite series** involving powers of x.

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2. Taylor's Series Expansion

Definition:

If a function $f(x)$ is **infinitely differentiable** (i.e., has derivatives of all orders) near a point $x = a$,
then the function can be expressed as:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \ldots + \frac{(x-a)^n}{n!}f^{(n)}(a) + \ldots$$

This is called the **Taylor's Series Expansion** of $f(x)$ about $x = a$.

Formula (General Form):

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-a)^n}{n!} f^{(n)}(a)$$

Here:

- $f^{(0)}(a) = f(a)$
- $f^{(1)}(a) = f'(a)$
- $f^{(2)}(a) = f''(a)$, and so on.

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3. Steps to Find Taylor's Expansion

1. Find $f(a), f'(a), f''(a), f'''(a), \ldots$
2. Substitute these values into the formula

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \cdots$$

3. Simplify the series to get the final expansion.

4. Example (Taylor's Theorem)

Example 1: Expand $f(x) = e^x$ about $x = 1$.

We know:

$$f(x) = e^x \Rightarrow f'(x) = e^x, f''(x) = e^x, \text{ and so on.}$$

At $x = 1$,

$$f(1) = e, f'(1) = e, f''(1) = e, \ldots$$

Now,

$$f(x) = f(1) + (x-1)f'(1) + \frac{(x-1)^2}{2!}f''(1) + \frac{(x-1)^3}{3!}f'''(1) + \ldots$$

Substitute values:

$$f(x) = e[1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} + \ldots]$$

5. Maclaurin's Series Expansion

Definition:

If in Taylor's Series, we take $a = 0$,
then it becomes **Maclaurin's Series**.

So,

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$

This is called the **Maclaurin's Expansion** of $f(x)$.

Formula (General Form):

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} f^{(n)}(0)$$

6. Important Maclaurin's Series Expansions

These are **standard series** you should remember for exams:

Function	Maclaurin's Expansion
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
$\ln(1+x)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$, valid for (
$(1+x)^n$	$1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$
$\tan^{-1} x$	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$, valid for (

7. Example (Maclaurin's Theorem)

Example 1: Expand $f(x) = \sin x$ in powers of x.

We know:

$$f(x) = \sin x$$

$$f'(x) = \cos x, \quad f''(x) = -\sin x, \quad f'''(x) = -\cos x, \quad f^{(4)}(x) = \sin x$$

At $x = 0$:

$$f(0) = 0, \quad f'(0) = 1, \quad f''(0) = 0, \quad f'''(0) = -1$$

Now,

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$$

Substitute values:

$$f(x) = 0 + x(1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(-1) + \dots$$

So,

$$\boxed{\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots}$$

Maxima and Minima of Functions of One Variable

(From Differential Calculus — Successive Differentiation topic)

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1. Introduction

In mathematics, **maxima** and **minima** refer to the **highest and lowest points** of a function.

In simple terms:

- A **maximum** point is where the function reaches its **highest value** (a peak).
- A **minimum** point is where the function reaches its **lowest value** (a valley).

Together, they are called **extrema** (plural of *extreme*).

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2. Example (Real-Life Idea)

Think of a **hill and a valley:**

- The **top of the hill** → maximum point.
- The **bottom of the valley** → minimum point.

When you walk along the curve of a function, these are the **turning points** where the slope changes direction.

3. Mathematical Definition

Let $y = f(x)$ be a function of one variable.

- The function $f(x)$ is said to have a **maximum** at $x = a$
  if $f(a)$ is **greater than** all nearby values of $f(x)$.

$$f(a) > f(x) \text{ for all } x \text{ near } a$$

- The function $f(x)$ is said to have a **minimum** at $x = a$
  if $f(a)$ is **less than** all nearby values of $f(x)$.

$$f(a) < f(x) \text{ for all } x \text{ near } a$$

4. Conditions for Maxima and Minima

To find where a function has a **maximum or minimum**, we use **derivatives**.

(a) First Derivative Test

1. **Find the first derivative:**

$$f'(x) = \frac{dy}{dx}$$

   This tells us the **slope** of the curve.

2. **Set the derivative equal to zero:**

$$f'(x) = 0$$

   The points where $f'(x) = 0$ are called **critical points** or **stationary points**.

3. **Check the sign of $f'(x)$:**

Condition	Nature of Point
$f'(x)$ changes from **positive to negative**	**Maximum**
$f'(x)$ changes from **negative to positive**	**Minimum**
$f'(x)$ does **not change sign**	**Neither maximum nor minimum**

(b) Second Derivative Test

If $f'(x) = 0$ at $x = a$, then:

- If $f''(a) > 0$, $f(a)$ is a **minimum**.
- If $f''(a) < 0$, $f(a)$ is a **maximum**.
- If $f''(a) = 0$, the test is **inconclusive** (need higher derivatives).

---

5. Procedure to Find Maxima and Minima

1. Find the **first derivative** $f'(x)$.
2. Set $f'(x) = 0$ to find **critical points.**
3. Find the **second derivative** $f''(x)$.
4. Substitute each critical point into $f''(x)$ and use the rules:

Condition	Result
$f''(a) > 0$	Minimum
$f''(a) < 0$	Maximum
$f''(a) = 0$	Inconclusive

5. Substitute the values of x back into $f(x)$ to get the **maximum or minimum value.**

6. Example 1 (Maximum)

Find the maximum and minimum values of $f(x) = -x^2 + 4x + 1$.

Step 1:

$$f'(x) = -2x + 4$$

Step 2:
Set $f'(x) = 0$:

$$-2x + 4 = 0 \Rightarrow x = 2$$

Step 3:
Find second derivative:

$$f''(x) = -2$$

Since $f''(x) = -2 < 0$,

$$\text{Maximum at } x = 2$$

Step 4:
Find maximum value:

$$f(2) = -2^2 + 4(2) + 1 = -4 + 8 + 1 = 5$$

✅ **Maximum value = 5 at x = 2**

7. Example 2 (Minimum)

Find the minimum value of $f(x) = x^2 + 4x + 3$.

Step 1:

$$f'(x) = 2x + 4$$

Step 2:
Set $f'(x) = 0$:

$$2x + 4 = 0 \Rightarrow x = -2$$

Step 3:
Find $f''(x)$:

$$f''(x) = 2$$

Since $f''(x) = 2 > 0$,

$$\text{Minimum at } x = -2$$

Step 4:
Find minimum value:

$$f(-2) = (-2)^2 + 4(-2) + 3 = 4 - 8 + 3 = -1$$

✅ **Minimum value = -1 at x = -2**

8. Graphical Representation

```mathematica
For Maximum:           For Minimum:
     ▲                      ▼
    / \                    / \
   /   \                  /   \
--/-----\--           ----\-----/----
   ^                      ^
  Max                    Min
```

At **maximum**, the curve bends **downward (concave down)**.

At **minimum**, the curve bends **upward (concave up)**.

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9. Points of Inflection

Sometimes, $f''(x) = 0$, but the curve neither has a maximum nor a minimum.

This point is called a **point of inflection** — where the **concavity changes**.

Example:

$f(x) = x^3 \rightarrow$

$f'(x) = 3x^2, \; f''(x) = 6x$

At $x = 0$, $f'(x) = 0$, $f''(x) = 0$,

but $f(x) = x^3$ has **no maxima/minima**, only an **inflection point**.

🌟 1. Partial Derivatives

Definition:

When a function has **two or more variables**, its derivative with respect to **one variable**, while keeping the **other variables constant**, is called a **partial derivative**.

If $z = f(x, y)$, then:

$$\frac{\partial z}{\partial x} = \text{Partial derivative of } z \text{ w.r.t. } x$$

$$\frac{\partial z}{\partial y} = \text{Partial derivative of } z \text{ w.r.t. } y$$

Example:

Let $z = x^2y + 3xy^2$

Now,

$$\frac{\partial z}{\partial x} = 2xy + 3y^2$$

$$\frac{\partial z}{\partial y} = x^2 + 6xy$$

👉 Here, when we differentiate w.r.t. **x**, we treat **y as constant**, and vice versa.

Higher Order Partial Derivatives:

Just like in single-variable calculus, we can take second or higher derivatives.

$$\frac{\partial^2 z}{\partial x^2}, \quad \frac{\partial^2 z}{\partial y^2}, \quad \frac{\partial^2 z}{\partial x \partial y}$$

If the function is continuous, then:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

🌟 2. Euler's Theorem on Homogeneous Functions

Homogeneous Function:

A function $f(x, y)$ is **homogeneous of degree n** if:

$$f(tx, ty) = t^n f(x, y)$$

That means, if we multiply each variable by t, the function is multiplied by t^n.

Euler's Theorem:

If $f(x, y)$ is a **homogeneous function of degree n**, then:

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$$

Example:

Let $f(x, y) = x^2y + y^3$

Check degree:

- Each term has total degree 3 (since x^2y = 3 and y^3 = 3)
  ✅ So, f is homogeneous of degree 3.

Now compute partials:

$$\frac{\partial f}{\partial x} = 2xy$$

$$\frac{\partial f}{\partial y} = x^2 + 3y^2$$

Then:

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x(2xy) + y(x^2 + 3y^2) = 2x^2y + x^2y + 3y^3 = 3(x^2y + y^3) = 3f(x, y)$$

✅ Hence proved — Euler's theorem is verified.

🌟 3. Change of Variables

Meaning:

In many cases, it's easier to calculate derivatives when we **change variables** from x, y to **new variables** u, v.

For example:

$$u = x + y, \quad v = x - y$$

We can express a function $z = f(x, y)$ as $z = f(u, v)$.

Partial Derivatives using Chain Rule:

If $z = f(u, v)$ and u, v depend on x, y,
then

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

Example:

Let $z = u^2 + v^2$,

where $u = x + y$, $v = x - y$

Now:

$$\frac{\partial z}{\partial u} = 2u, \quad \frac{\partial z}{\partial v} = 2v$$

$$\frac{\partial u}{\partial x} = 1, \frac{\partial v}{\partial x} = 1$$

$$\frac{\partial u}{\partial y} = 1, \frac{\partial v}{\partial y} = -1$$

Then:

$$\frac{\partial z}{\partial x} = 2u(1) + 2v(1) = 2(u + v) = 2((x + y) + (x - y)) = 4x$$

$$\frac{\partial z}{\partial y} = 2u(1) + 2v(-1) = 2(u - v) = 2((x + y) - (x - y)) = 4y$$

✅ Hence, $\frac{\partial z}{\partial x} = 4x$ and $\frac{\partial z}{\partial y} = 4y$.

🌞 Total Differentiation, Taylor's Series in Two Variables, and Maxima & Minima of Two Variables

Explained in **simple, easy, and exam-oriented** language 👇

🧩 1. Total Differentiation

Definition:

If $z = f(x, y)$, where x and y are **independent variables**, then the **total derivative (or total differential)** of z represents how much z changes when **both x and y** change **simultaneously**.

Mathematically,

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Here:

- $\frac{\partial z}{\partial x}$ → rate of change of z w.r.t x
- $\frac{\partial z}{\partial y}$ → rate of change of z w.r.t y
- dx, dy → small changes in x and y

Example:

Let $z = x^2y + 3xy^2$

Now,

$$\frac{\partial z}{\partial x} = 2xy + 3y^2$$

$$\frac{\partial z}{\partial y} = x^2 + 6xy$$

So,

$$dz = (2xy + 3y^2)dx + (x^2 + 6xy)dy$$

✅ This expression shows how z changes due to small changes in both x and y.

If x and y depend on another variable (say t):

If $x = x(t)$ and $y = y(t)$, then

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

This is called the **Total Derivative** of z with respect to t.

📗 2. Taylor's Series in Two Variables

Definition:

Taylor's theorem helps to **expand a function** of **two variables** $f(x,y)$ near a point (a,b) in terms of its partial derivatives.

If $f(x,y)$ has continuous partial derivatives, then:

$$f(x,y) = f(a,b) + (x-a)f_x(a,b) + (y-b)f_y(a,b) + \frac{1}{2!}\left[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\right] + \cdots$$

where:

- f_x, f_y are first partial derivatives
- f_{xx}, f_{yy}, f_{xy} are second partial derivatives

Special Case: Maclaurin's Series (when a = 0, b = 0)

If the expansion is about the origin, then:

$$f(x,y) = f(0,0) + xf_x(0,0) + yf_y(0,0) + \frac{1}{2!}[x^2 f_{xx}(0,0) + 2xyf_{xy}(0,0) + y^2 f_{yy}(0,0)] + \cdots$$

Example:

Let $f(x,y) = e^{x+y}$

We know $f(0,0) = 1$

Partial derivatives:

$$f_x = e^{x+y}, \quad f_y = e^{x+y}$$

$$f_{xx} = e^{x+y}, \quad f_{yy} = e^{x+y}, \quad f_{xy} = e^{x+y}$$

At $(0,0)$, all derivatives = 1

So,

$$f(x,y) = 1 + (x+y) + \frac{1}{2!}(x+y)^2 + \frac{1}{3!}(x+y)^3 + \cdots$$

✅ Hence, $f(x,y) = e^{x+y}$ as expected.

3. Maxima and Minima of Two Variables

Definition:

For a function $z = f(x, y)$,

- **Maximum:** z is largest near a point.
- **Minimum:** z is smallest near a point.

These points are called **stationary points** or **critical points**.

Steps to Find Maxima/Minima:

1. **Find the first partial derivatives**

$$\frac{\partial f}{\partial x} = 0, \quad \frac{\partial f}{\partial y} = 0$$

   → Solve these equations to find critical points (a, b).

2. **Find the second partial derivatives:**

$$f_{xx}, \quad f_{yy}, \quad f_{xy}$$

3. **Compute**

$$D = f_{xx} f_{yy} - (f_{xy})^2$$

4. **Apply the following tests:**

Condition	Nature of Point
$D > 0$ and $f_{xx} > 0$	Minimum
$D > 0$ and $f_{xx} < 0$	Maximum
$D < 0$	Saddle point (neither max nor min)
$D = 0$	Test fails (inconclusive)

Example:

Let $f(x, y) = x^2 + y^2 - 4x - 6y + 13$

Step 1:

$$f_x = 2x - 4 = 0 \Rightarrow x = 2$$

$$f_y = 2y - 6 = 0 \Rightarrow y = 3$$

So, critical point = (2, 3)

Step 2:

$$f_{xx} = 2, \quad f_{yy} = 2, \quad f_{xy} = 0$$

$$D = (2)(2) - (0)^2 = 4 > 0$$

and $f_{xx} = 2 > 0$

✅ So, **Minimum point** at (2, 3)

Minimum value:

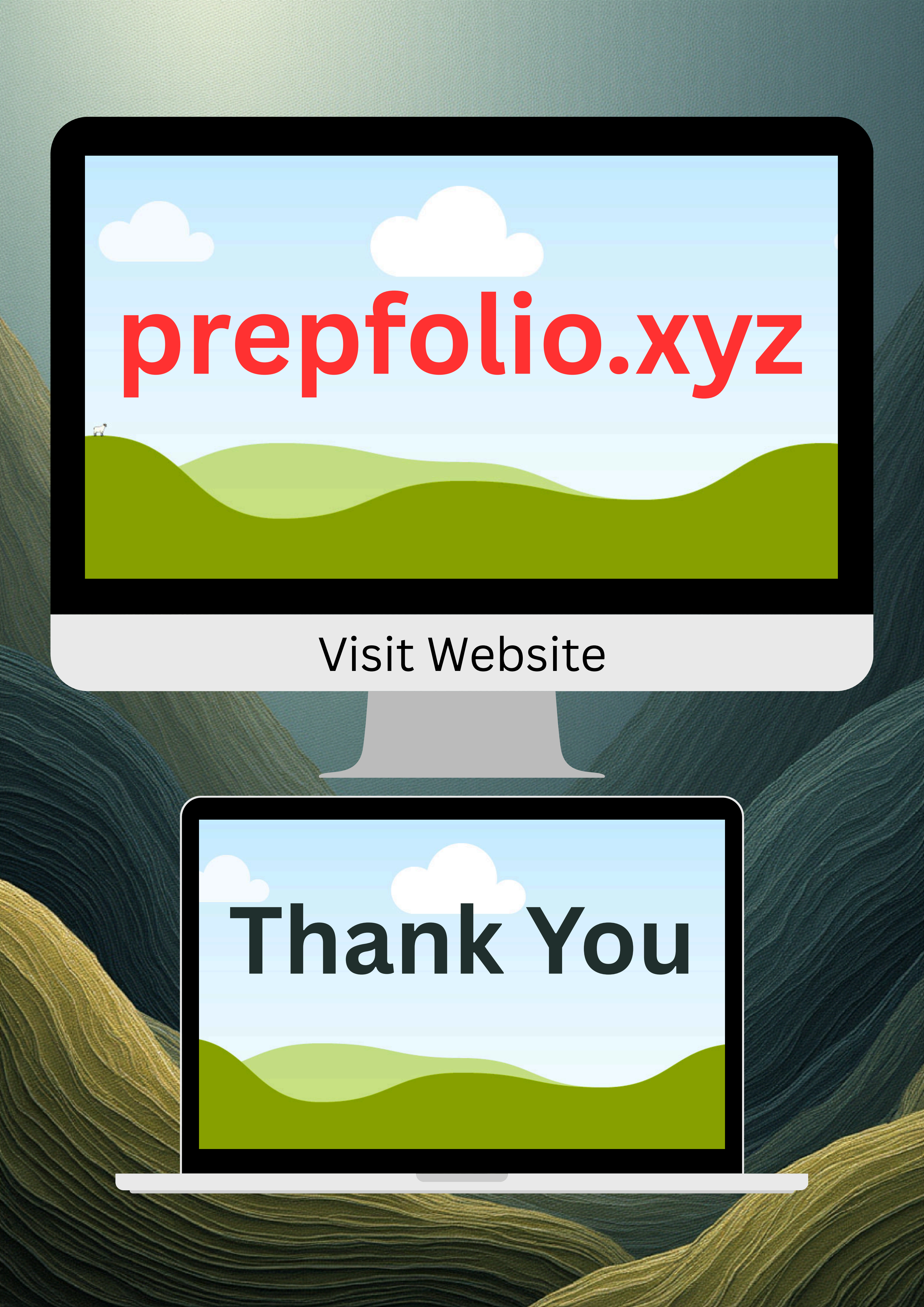
$$f(2, 3) = 2^2 + 3^2 - 4(2) - 6(3) + 13 = 4 + 9 - 8 - 18 + 13 = 0$$

✅ Hence, minimum value = 0 at (2, 3)

Practice Questions

1. Define successive differentiation and find the second and third derivatives of a given function with respect to xxx.
2. State and prove Leibnitz’s Theorem for the nthn^{th}nth derivative of the product of two functions.
3. Expand a function of one variable using Taylor’s Series and Maclaurin’s Series.
4. Find the Taylor’s Series expansion of exe^xex, sinx\sin xsinx, or cosx\cos xcosx up to the fourth term.
5. Define Maxima and Minima for a function of one variable. Find conditions for maximum and minimum values.
6. Explain Partial Derivatives with examples and find first and second order partial derivatives of a given function.
7. State and prove Euler’s Theorem on homogeneous functions. Verify it for a given example.
8. Explain the Change of Variables concept and derive the formulas for partial derivatives using the Chain Rule.
9. Define Total Differentiation and find the total derivative of a function z=f(x,y)z = f(x, y)z=f(x,y).
10. Explain how to find Maxima and Minima of functions of two variables and state the conditions for the same.

Check the answer in the Practice Questions section on our website.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