We have these T matrices which specify the orientation and location of one frame with respect to another frame. For example, ${}^0_1T$ shows the location of origin 1 and the orientation of the unit axes.

In Processing:

T1 = ${}^0_1T$, T2 = ${}^1_2T$, etc. Basically, Ti = ${}^{i-1}_{i}T$.

T1_2 = T1 * T2 = ${}^0_2T$. Basically, Ti_j = ${}^{i-1}_{j}T$.

These T matrices have the form:

$$\begin{bmatrix} r_{xx} & r_{xy} & r_{xz} & p_x \\ r_{yx} & r_{yy} & r_{yz} & p_y \\ r_{zx} & r_{zy} & r_{zz} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1 Jacobian

Given the position of the end effector $x = f(q)$, the velocity of the end effector can be expressed as follows:

$$\dot{x}_E = \frac{df}{dq}\frac{dq}{dt} = J\dot{q}$$

The velocity of the end effector can be split up into translational and rotational components, as can the Jacobian:

$$\dot{x}_E = \begin{bmatrix} v_E \\ \omega_E \end{bmatrix} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \dot{q}$$

1.1 Linear velocity

$$v_E = J_v\dot{q} = \sum_i v_E^i$$

Where v_E^i is the velocity caused by movement of joint i.

For us, we have:

$$\begin{bmatrix} v_{Ex} \\ v_{Ey} \\ v_{Ez} \end{bmatrix} = \begin{bmatrix} J_{v1,x} & J_{v2,x} & J_{v3,x} & J_{v4,x} & J_{v5,x} & J_{v6,x} \\ J_{v1,y} & J_{v2,y} & J_{v3,y} & J_{v4,y} & J_{v5,y} & J_{v6,y} \\ J_{v1,z} & J_{v2,z} & J_{v3,z} & J_{v4,z} & J_{v5,z} & J_{v6,z} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}$$

With our robot, we are dealing with revolute joints, so

$$v_E^i = \dot{q}_i\hat{z}_i \times r_{iE} \Rightarrow \begin{bmatrix} J_{vi,x} \\ J_{vi,y} \\ J_{vi,z} \end{bmatrix} = \hat{z}_i \times r_{iE}$$

where r_{iE} is the vector from joint i to the end effector, and $\hat{z}_i$ is the unit z vector the joint is rotating about.

The unit z_i vector can be obtained from the third column of the appropiate T matrix:

$$ {}^0_iT = \begin{bmatrix} . & . & \hat{z}_{ix} & . \\ . & . & \hat{z}_{iy} & . \\ . & . & \hat{z}_{iz} & . \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

This z vector would be expressed in the inertial frame. In processing, these vectors are z1 to z6.

Similarly, the r_{iE} can be obtained from the last column of the appropriate T matrix which expresses the origin of frame E with respect to frame i:

$$ {}^i_ET = \begin{bmatrix} . & . & . & r_{iEx} \\ . & . & . & r_{iEy} \\ . & . & . & r_{iEz} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

This will give ${}^ir_{iE}$, that is, the vector from joint i to E in frame i. Since we want the velocity in the inertial frame, we must translate this back.

$$ \begin{bmatrix} {}^0r_{iE} \\ 0 \end{bmatrix} = {}^0_i T \begin{bmatrix} {}^ir_{iE} \\ 0 \end{bmatrix} $$

The r_{iE} is padded with a 0 to ensure that only the rotation component of ${}^0_iT$ is used. In processing, these vectors are p1_e to p6_e.

With these vectors, we can get

$$ {}^0J_{vi} = {}^0\hat{z}_i \times {}^0r_{iE} $$

In processing these are jv1 to jv6.

Combining these, we get the overall J_v matrix, called JVMatrix in processing.

1.2 Angular velocity

$$ \omega_E = J_\omega \dot{q} = \sum_i \omega^i = \sum_i $$

Where ω^i is the angular velocity caused by movement of joint i.

Since our robot has revolute joints which rotate about the z axis of their frame,

$$ \omega^i_E = \hat{z}_i \dot{q}_i $$

Therefore in the inertial frame,

$$ {}^0\omega^i_E = {}^0\hat{z}_i \dot{q}_i $$

As in linear velocity, we can get ${}^0\hat{z}_i$ from the appropriate t matrix.

Hence

$$J_\omega = \begin{bmatrix} {}^0\hat{z}_1 & {}^0\hat{z}_2 & {}^0\hat{z}_3 & {}^0\hat{z}_4 & {}^0\hat{z}_5 & {}^0\hat{z}_6 \end{bmatrix}$$

In processing, this is the JOmegaMatrix.