Section 2

Michael Brodskiy
Professor: A. Martsinkovsky

September 19, 2022

Contents

List of Figures

1 Partial Derivatives

- The slope of $f(x, y)$ depends on the direction in the xy-plane
  - The slope in the x-direction is called the partial derivative of f with respect to x
  - The slope in the y-direction is called the partial derivative of f with respect to y
  - Notation: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ or f_x, f_y
  - For Second Derivatives: $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial x \partial y}$ or f_{xx}, f_{xy}, f_{yx}, f_{yy}

If f, f_x, f_y, and f_{xy} are defined in a small disc around (x_o, y_o) and f_{yx} is continuous, then:

$$f_{xy} = f_{yx}$$

in that disc

- The gradient of f
  - Given $f(x_1, x_2, \ldots, x_n)$, the gradient of f, $\nabla f = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$
  - It can be computed at a point: $\nabla f(p) = \left\langle \frac{\partial f}{\partial x_1}(p), \frac{\partial f}{\partial x_2}(p), \ldots, \frac{\partial f}{\partial x_n}(p) \right\rangle$
  - $\nabla f \approx f'(x_o)\Delta x$
  - $\nabla f \approx \nabla f \cdot \Delta \overline{x}$