

Section 3

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1 Partial Antiderivatives

- Ex. $\frac{\delta f}{\delta x} = 3x^2 - 5y^2 \Rightarrow f = x^3 - 5y^2x + g(y)$

$$\int_y^{y^2} 3x^2 - 5y^2 dx \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = y^6 - 5y^4 + 4y^3$$

$$\int_0^2 \int_y^{y^2} 3x^2 - 5y^2 dx dy \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = \int_0^2 (y^6 - 5y^4 + 4y^3) dy =$$

$$\frac{1}{7}(2)^7 - (2)^5 + (2)^4 = \frac{128}{7} - 16 = \frac{16}{7}$$

- Ex. $\int_1^3 \int_0^{\sin(x)} \frac{1+2y}{\sin(x)} dy dx =$

$$\frac{1}{\sin(x)} (\sin(x) + (\sin(x))^2) = \int_1^3 1 + \sin(x) dx = x - \cos(x) =$$

$$3 - \cos(3) - 1 + \cos(1) = 2 + \cos(1) - \cos(3)$$

- Ex. $\int_0^2 \int_y^1 \int_z^{yz} 8xyz dx dz dy =$

$$yz((4(yz)^2 - 4z^2)) = \int_0^2 \int_y^1 4y^3 z^3 - 4yz^3 dz dy = (y^3 - y) - (y^7 - y^4) =$$

$$\int_0^2 -y^7 + y^4 + y^3 - y dy = -\frac{1}{8}(2)^7 + \frac{1}{5}(2)^5 + \frac{1}{4}(2)^4 - \frac{1}{2}(2)^2 = -16 + \frac{32}{5} + 4 - 2 = -\frac{38}{5}$$

- Ex. $\frac{\delta f}{\delta x} = 3x^2 - 5y^2, \frac{\delta f}{\delta y} = -10xy + 8y^3, f = ?$

$$\int \frac{\delta f}{\delta x} dx = x^3 - 5xy^2 + g(y) = \frac{\delta f}{\delta y} = -10xy + g'(y) \Rightarrow g'(y) = 8y^3 \Rightarrow f(x, y) = x^3 - 5xy^2 + 2y^4 + c$$