Section 3

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Contents

1 Partial Antiderivatives

- *Ex.* $\dfrac{\delta f}{\delta x} = 3x^2 - 5y^2 \Rightarrow f = x^3 - 5y^2x + g(y)$

$$\int_y^{y^2} 3x^2 - 5y^2 \, dx \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = y^6 - 5y^4 + 4y^3$$

$$\int_0^2 \int_y^{y^2} 3x^2 - 5y^2 \, dx \, dy \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = \int_0^2 (y^6 - 5y^4 + 4y^3) \, dy =$$

$$\frac{1}{7}(2)^7 - (2)^5 + (2)^4 = \frac{128}{7} - 16 = \frac{16}{7}$$

- *Ex.* $\displaystyle\int_1^3 \int_0^{\sin(x)} \frac{1+2y}{\sin(x)} \, dy \, dx =$

$$\frac{1}{\sin(x)} \left(\sin(x) + (\sin(x))^2\right) = \int_1^3 1 + \sin(x) \, dx = x - \cos(x) =$$

$$3 - \cos(3) - 1 + \cos(1) = 2 + \cos(1) - \cos(3)$$

- *Ex.* $\displaystyle\int_0^2 \int_y^1 \int_z^{yz} 8xyz \, dx \, dz \, dy =$

$$yz((4(yz)^2 - 4z^2) = \int_0^2 \int_y^1 4y^3z^3 - 4yz^3 \, dz \, dy = (y^3 - y) - (y^7 - y^4) =$$

$$\int_0^2 -y^7 + y^4 + y^3 - y \, dy = -\frac{1}{8}(2)^7 + \frac{1}{5}(2)^5 + \frac{1}{4}(2)^4 - \frac{1}{2}(2)^2 = -16 + \frac{32}{5} + 4 - 2 = -\frac{38}{5}$$

- *Ex.* $\dfrac{\delta f}{\delta x} = 3x^2 - 5y^2, \ \dfrac{\delta f}{\delta y} = -10xy + 8y^3, \ f = ?$

$$\int \frac{\delta f}{\delta x} \, dx = x^3 - 5xy^2 + g(y) = \frac{\delta f}{\delta y} = -10xy + g'(y) \Rightarrow g'(y) = 8y^3 \Rightarrow f(x,y) = x^3 - 5xy^2 + 2y^4 + c$$

2 Integration in $\mathbb{R}^2$

- Double Integral
  - $\displaystyle\iint_R f(x,y) \, dA$
- Fubini's Theorem: Utilize iterated integration to calculate multiple-integration

- $\iint_R f(x,y)\,dA \longrightarrow \int_a^b \int_c^d f(x,y)\,dy\,dx$

- For Type I Regions:
  - $\int_a^b \int_{p(x)}^{q(x)} f(x,y)\,dy\,dx$
  - Occurs when y is bounded by functions of x and x is bounded by vertical lines $(x = c)$

- For Type II Regions:
  - $\int_c^d \int_{r(x)}^{s(x)} f(x,y)\,dx\,dy$
  - Occurs when x is bounded by functions of y and y is bounded by horizontal lines $(y = c)$

- A function can be Type I, Type II, both, or neither

- Regions can be broken down into parts to make calculations easier:
  - Given R and two subregions, R' and R'', the integral becomes:
    - $\iint_R f\,dA = \iint_{R'} f\,dA + \iint_{R''} f\,dA$

- Remark: If $f(x,y) = 1$, then:
  - $\iint_R dA = \text{Area of } R$