

Section 3

Michael Brodskiy

Professor: A. Martsinkovsky

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Contents

1	Partial Antiderivatives	3
2	Integration in \mathbb{R}^2	3
3	Integration with Polar Coordinates	4
4	Integration in \mathbb{R}^3	5
5	Volume	5
6	Cylindrical and Spherical Coordinates	5
7	Density and Mass	5

1 Partial Antiderivatives

- Ex. $\frac{\delta f}{\delta x} = 3x^2 - 5y^2 \Rightarrow f = x^3 - 5y^2x + g(y)$

$$\int_y^{y^2} 3x^2 - 5y^2 dx \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = y^6 - 5y^4 + 4y^3$$

$$\int_0^2 \int_y^{y^2} 3x^2 - 5y^2 dx dy \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = \int_0^2 (y^6 - 5y^4 + 4y^3) dy =$$

$$\frac{1}{7}(2)^7 - (2)^5 + (2)^4 = \frac{128}{7} - 16 = \frac{16}{7}$$

- Ex. $\int_1^3 \int_0^{\sin(x)} \frac{1+2y}{\sin(x)} dy dx =$

$$\frac{1}{\sin(x)} (\sin(x) + (\sin(x))^2) = \int_1^3 1 + \sin(x) dx = x - \cos(x) =$$

$$3 - \cos(3) - 1 + \cos(1) = 2 + \cos(1) - \cos(3)$$

- Ex. $\int_0^2 \int_y^1 \int_z^{yz} 8xyz dx dz dy =$

$$yz((4(yz)^2 - 4z^2)) = \int_0^2 \int_y^1 4y^3 z^3 - 4yz^3 dz dy = (y^3 - y) - (y^7 - y^4) =$$

$$\int_0^2 -y^7 + y^4 + y^3 - y dy = -\frac{1}{8}(2)^7 + \frac{1}{5}(2)^5 + \frac{1}{4}(2)^4 - \frac{1}{2}(2)^2 = -16 + \frac{32}{5} + 4 - 2 = -\frac{38}{5}$$

- Ex. $\frac{\delta f}{\delta x} = 3x^2 - 5y^2, \frac{\delta f}{\delta y} = -10xy + 8y^3, f = ?$

$$\int \frac{\delta f}{\delta x} dx = x^3 - 5xy^2 + g(y) = \frac{\delta f}{\delta y} = -10xy + g'(y) \Rightarrow g'(y) = 8y^3 \Rightarrow f(x, y) = x^3 - 5xy^2 + 2y^4 + c$$

2 Integration in \mathbb{R}^2

- Double Integral

$$- \iint_R f(x, y) dA$$

- Fubini's Theorem: Utilize iterated integration to calculate multiple-integration

$$- \iint_R f(x, y) dA \longrightarrow \int_a^b \int_c^d f(x, y) dy dx$$

- For Type I Regions:

$$- \int_a^b \int_{p(x)}^{q(x)} f(x, y) dy dx$$

- Occurs when y is bounded by functions of x and x is bounded by vertical lines ($x = c$)

- For Type II Regions:

$$- \int_c^d \int_{r(x)}^{s(x)} f(x, y) dx dy$$

- Occurs when x is bounded by functions of y and y is bounded by horizontal lines ($y = c$)

- A function can be Type I, Type II, both, or neither
- Regions can be broken down into parts to make calculations easier:

- Given R and two subregions, R' and R'' , the integral becomes:

$$* \iint_R f dA = \iint_{R'} f dA + \iint_{R''} f dA$$

- Remark: If $f(x, y) = 1$, then:

$$- \iint_R dA = \text{Area of } R$$

3 Integration with Polar Coordinates

- Basic Properties $\begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ x^2 + y^2 = r^2 \end{cases}$
- $dA = r dr d\theta$
- $\iint_R f(x, y) dA = \iint_R f(r \cos(\theta), r \sin(\theta)) r dr d\theta$

4 Integration in \mathbb{R}^3

- $\iiint_S f(x, y, z) dV$, where $S \rightarrow \begin{cases} p(x, y) \leq z \leq q(x, y) \\ u(x) \leq y \leq v(x) \\ a \leq x \leq b \end{cases}$
- This can be expressed as: $\int_a^b \int_{u(x)}^{v(x)} \int_{p(x,y)}^{q(x,y)} f(x, y, z) dz dy dx$

5 Volume

- $\iiint_S f(x, y, z) dV \Rightarrow f(x, y, z) = 1 \Rightarrow \iiint_S dV$ gives the volumes of solid S

6 Cylindrical and Spherical Coordinates

- For cylindrical coordinates, refer to polar:

$$- \begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ z = z \end{cases}, \text{ and } dV = r dr d\theta dz$$

$$- 0 \leq r < \infty, 0 \leq \theta \leq 2\pi, \text{ and } -\infty < z < \infty$$

- Spherical coordinates use variables ρ , ϕ , and θ , where ρ is the distance to the origin, ϕ is the vertical deviation (with respect to the positive z -axis), and θ is the horizontal deviation (or polar angle)

$$- \begin{cases} x = \rho \sin(\phi) \cos(\theta) \\ y = \rho \sin(\phi) \sin(\theta) \\ z = \rho \cos(\phi) \end{cases}, \text{ and } dV = \rho^2 \sin(\phi) d\rho d\phi d\theta$$

$$- \text{It is important to remember the property } x^2 + y^2 + z^2 = \rho^2$$

$$- 0 \leq \rho < \infty, 0 \leq \theta \leq 2\pi, \text{ and } 0 \leq \phi < \pi$$

7 Density and Mass

$$\bullet \text{ Mass} = \begin{cases} \int_a^b \delta(x) dx \\ \iint_R \delta(x, y) dA \\ \iiint_S \delta(x, y, z) dV \end{cases}$$