Homework 1.2

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1.
- Magnitude: $\sqrt{3^2+4^2} = 5$ feet per second
- Direction: $\frac{1}{5}\langle 3,4\rangle = \langle \frac{3}{5}, \frac{4}{5}\rangle$

3.
- Magnitude: $\sqrt{(-6)^2+(1)^2+(6)^2} = \sqrt{73}$ meters per second
- Direction: $\frac{1}{\sqrt{73}}\langle -6,1,6\rangle = \langle \frac{-6}{\sqrt{73}}, \frac{1}{\sqrt{73}}, \frac{6}{\sqrt{73}}\rangle$

5.
- Magnitude: $\sqrt{(1)^2+(-1)^2+(1)^2+(-1)^2} = 2$
- Direction: $\frac{1}{2}\langle 1,-1,1,-1\rangle = \langle \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\rangle$

7.
- Magnitude: $\sqrt{(2)^2+(-3)^2+(1)^2} = \sqrt{14}$
- Direction: $\frac{1}{14}(2\mathbf{i}-3\mathbf{j}+\mathbf{k}) = \frac{2}{14}\mathbf{i} - \frac{3}{14}\mathbf{j} + \frac{1}{14}\mathbf{k}$

9. See figure below:

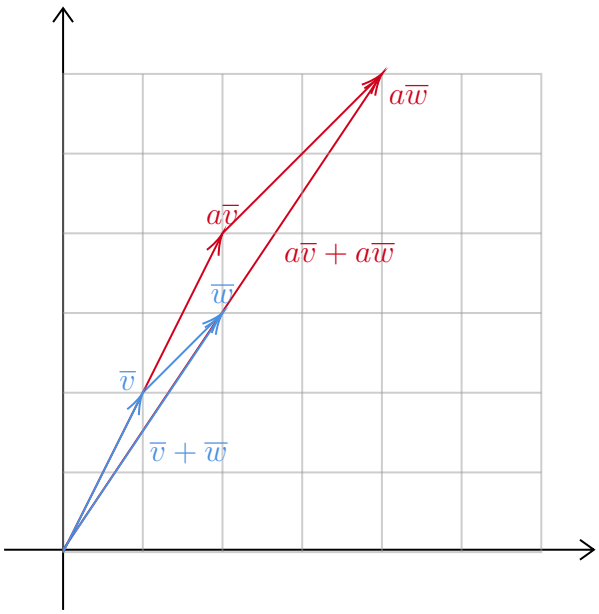

Figure 1: $\overline{v} = \langle 1,2\rangle$, $\overline{w} = \langle 1,1\rangle$, $a = 2$

10. See figure below:

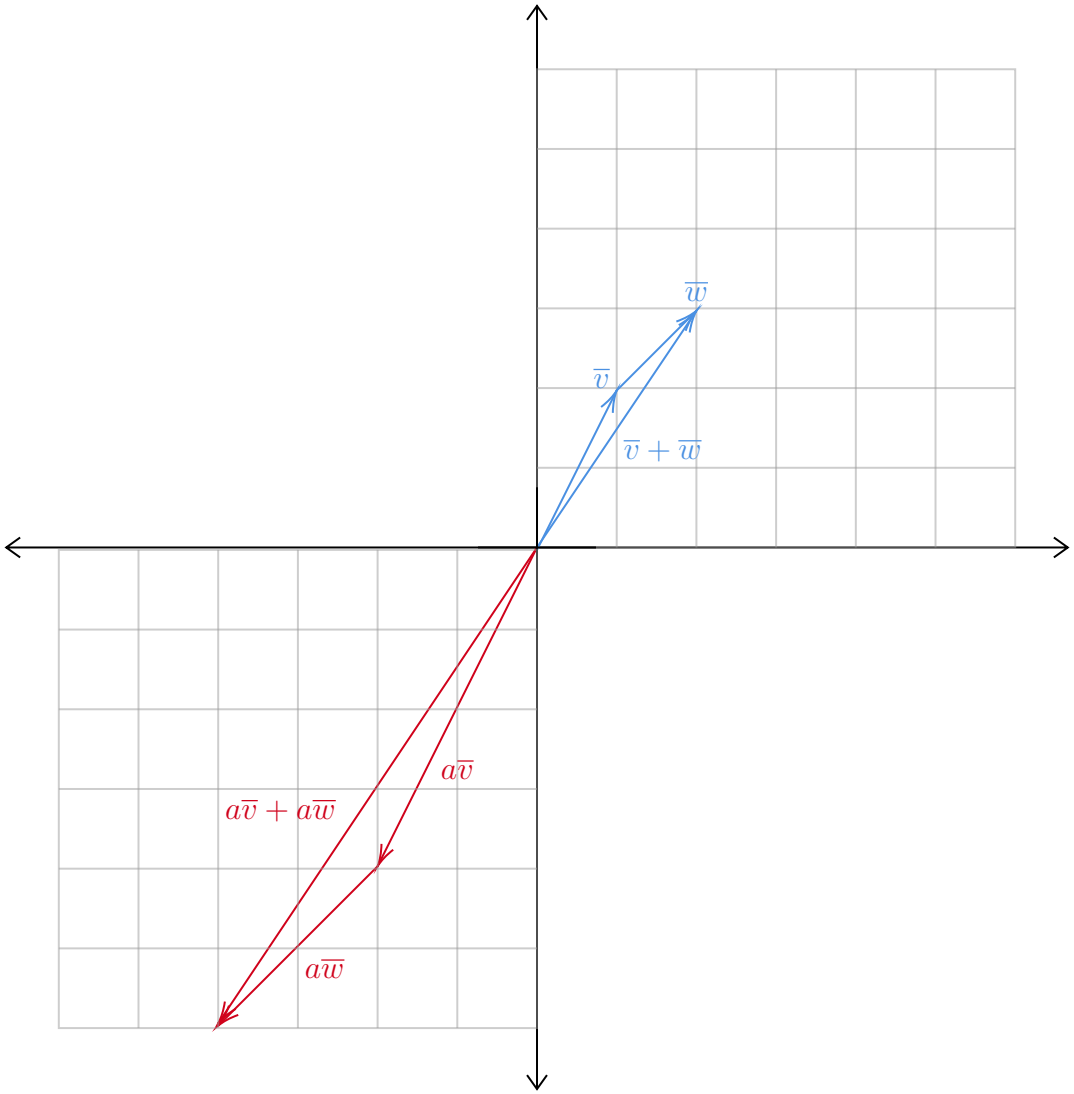

Figure 2: $\overline{v} = \langle 1, 2\rangle$, $\overline{w} = \langle 1, 1\rangle$, $a = -2$

13. $1\left(-\frac{1}{2}\right)+2(-1) = -2.5 \Rightarrow \left(\frac{-2.5}{\sqrt{5}}\right)\left(\frac{\sqrt{5}}{2}\right) = -1 \Rightarrow \cos^{-1}(-1) = 0$, thus the angle between them is zero. Because one of the vectors is negative and one is positive, they are in opposite directions.

14. $3(-6) + 4(-7) = -46 \Rightarrow \frac{-46}{(5)(\sqrt{85})} \neq \pm 1$, so they are not parallel

15. $1(2) + (-2)(-4) + 3(5) = 25 \Rightarrow \frac{25}{(\sqrt{14})(\sqrt{45})} \neq \pm 1$, so they are not parallel

16. The second vector is a scaled, positive multiple of the first one ($3\overline{v}_1 = \overline{v}_2$), so they are parallel and in the same direction

19. $a = \frac{\sum \overline{F}}{m} = \frac{1}{2}\left(\langle 0, 4\rangle\right) = \langle 0, 2\rangle$

20. $a = \frac{1}{2}\left(\langle -1, 10, 7\rangle\right) = \langle -.5, 5, 3.5\rangle$

21. $a = \frac{1}{2}\left(\mathbf{i} + \mathbf{j} + \mathbf{k}\right) = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$

23. $\langle 6, -9\rangle \Rightarrow \sqrt{36 + 81} = \sqrt{117}$ feet per second

24. $\langle 7, 4, -2\rangle \Rightarrow \sqrt{49 + 16 + 4} = \sqrt{69}$ feet per second

27. $\overline{d} = b - a = \langle -3, -5\rangle$

- Magnitude: $\sqrt{9 + 25} = \sqrt{34}$
- Direction: $\frac{1}{\sqrt{34}}\langle -3, -5\rangle = \langle -\frac{3}{\sqrt{34}}, -\frac{5}{\sqrt{34}}\rangle$

29. $\overline{d} = \langle -1, -4, -3\rangle$

- Magnitude: $\sqrt{1 + 16 + 9} = \sqrt{26}$
- Direction: $\frac{1}{\sqrt{26}}\langle -1, -4, -3\rangle = \langle -\frac{1}{\sqrt{26}}, -\frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}}\rangle$

33. $\overline{u} = \frac{\langle 3,4\rangle}{5} = \langle \frac{3}{5}, \frac{4}{5}\rangle$

- Magnitude of 3: $3\langle \frac{3}{5}, \frac{4}{5}\rangle = \langle \frac{9}{5}, \frac{12}{5}\rangle$
- Magnitude of 7: $7\langle \frac{3}{5}, \frac{4}{5}\rangle = \langle \frac{21}{5}, \frac{28}{5}\rangle$

36. $\overline{u} = \frac{\langle 2,-1,3\rangle}{\sqrt{1+4+9}} = \langle \frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$

- Magnitude of 3: $3\langle \frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle = \langle \frac{6}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, \frac{9}{\sqrt{14}}\rangle$
- Magnitude of 7: $7\langle \frac{2}{\sqrt{14}}, -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle = \langle \frac{14}{\sqrt{14}}, -\frac{7}{\sqrt{14}}, \frac{21}{\sqrt{14}}\rangle$

41. It becomes -15

42. It is 13

43. $\overline{v} = \langle \sqrt{3}, \sqrt{3}\rangle$

45. $\overline{a}_i = \frac{30\mathbf{j}}{3} = 10\mathbf{j} \Rightarrow \overline{g} = -9.81\overline{j} \Rightarrow \sum \overline{a} = 10\mathbf{j} - 9.81\mathbf{j} = .19\mathbf{j}$, so it has an upward acceleration of .19 $\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]$, but the direction of movement can not be determined

46. $F_g = \frac{Gm_1m_2}{|\overline{r}|^2} \cdot \frac{\overline{r}}{|\overline{r}|} = \frac{6.674(10)^{-11}(3)(5)}{26} \cdot \frac{\langle 1,3,4\rangle}{\sqrt{26}} = -G\langle \frac{15}{26\sqrt{26}}, \frac{45}{26\sqrt{26}}, \frac{60}{26\sqrt{26}}\rangle$