Notes 1

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- What is a vector?
  - A magnitude and a direction? (not all vectors in the real world can be added, so not entirely true)
  - For our course, vectors exist in vector spaces ($\mathbb{R}^2$, $\mathbb{R}^3$, ..., $\mathbb{R}^n$)
  - $\overline{v} = \langle v_1, v_2, \dots, v_n \rangle$
  - $\mathbb{R}^1$ represents scalars, while $\mathbb{R}^2$, $\mathbb{R}^3$, ..., $\mathbb{R}^n$ are vectors
- Properties of Vectors
  - Can be added
    * $\overline{v} = \langle v_1, v_2, \dots, v_n \rangle + \overline{w} = \langle w_1, w_2, \dots, w_n \rangle = \langle v_1 + w_1, v_2 + w_2, \dots, v_n + w_n \rangle$
    * If forming a parallelogram from the vectors, the diagonal is the sum, $\overline{v} + \overline{w}$, of two vectors
  - Can be scaled (scalar multiplication)
    * $2\overline{v} = \langle 2v_1, 2v_2, \dots, 2v_n \rangle$
    * Magnitude is multiplied by the factor
  - Can find magnitude (length)
    * $|\overline{v}| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$
    * *Ex.* $\overline{v} = \langle 2, -3 \rangle \Rightarrow |\overline{v}| = \sqrt{(2)^2 + (-3)^2)} = \sqrt{13}$
  - A vector divided by its own magnitude becomes a vector of magnitude 1 (unit vector)
    * $|\frac{\overline{v}}{|\overline{v}|}| = 1$
    * Unit vectors are dimensionless (no units)
    * A vector that is by itself of length 1 is not a unit vector
    * A unit vector is simply a direction (all unit vectors from a given point form a circle)

- Any non-zero vector is the product of its magnitude and its direction
    - $\overline{v} = |\overline{v}| \cdot \frac{\overline{v}}{|\overline{v}|}$

- Linear Combinations
  - $\overline{v}_1,\ \overline{v}_2,\ \ldots,\ \overline{v}_s$
  - A linear combination of $\overline{v}_i$ is any sum of the form $r_1\overline{v}_1 + r_2\overline{v}_2 + \cdots + r_n\overline{v}_n$, where r_i are scalars

- Basis Vectors
  - $\mathbb{R}^n$ standard basis vectors: $\overline{e}_1,\ \overline{e}_2,\ \ldots,\ \overline{e}_n \Rightarrow \begin{cases} \overline{e}_1 = \langle 1, 0, \ldots, 0 \rangle \\ \overline{e}_2 = \langle 0, 1, \ldots, 0 \rangle \\ \quad\vdots \\ \overline{e}_n = \langle 0, 0, \ldots, 1 \rangle \end{cases}$
  - Any vector is a linear combination of the standard basis vectors
  - $\overline{w} = \langle w_1,\ w_2,\ \ldots,\ w_n \rangle = w_1\overline{e}_1 + w_2\overline{e}_2 + \cdots + w_n\overline{e}_n$
  - *Ex.* $\overline{v} = \langle 2, -3 \rangle = 2\langle 1, 0 \rangle + -3\langle 0, 1 \rangle$

- Dot Product
  - The dot product of two vectors is always a scalar
  - Geometric Definition: $\overline{v} \cdot \overline{w} = |\overline{v}||\overline{w}|\cos(\theta)$, where θ is the angle between $\overline{v}$ and $\overline{w}$
    - $\overline{v} \cdot \overline{w} = 0$ when $\theta = \frac{\pi}{2}$
    - $\overline{v} \cdot \overline{w} > 0$ when θ is acute
    - $\overline{v} \cdot \overline{w} < 0$ when θ is obtuse