

# Section 3

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# 1 Partial Antiderivatives

- Ex.  $\frac{\delta f}{\delta x} = 3x^2 - 5y^2 \Rightarrow f = x^3 - 5y^2x + g(y)$

$$\int_y^{y^2} 3x^2 - 5y^2 dx \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = y^6 - 5y^4 + 4y^3$$

$$\int_0^2 \int_y^{y^2} 3x^2 - 5y^2 dx dy \Rightarrow y^6 - 5y^4 - (y^3 - 5y^3) = \int_0^2 (y^6 - 5y^4 + 4y^3) dy =$$

$$\frac{1}{7}(2)^7 - (2)^5 + (2)^4 = \frac{128}{7} - 16 = \frac{16}{7}$$

- Ex.  $\int_1^3 \int_0^{\sin(x)} \frac{1+2y}{\sin(x)} dy dx =$

$$\frac{1}{\sin(x)} (\sin(x) + (\sin(x))^2) = \int_1^3 1 + \sin(x) dx = x - \cos(x) =$$

$$3 - \cos(3) - 1 + \cos(1) = 2 + \cos(1) - \cos(3)$$

- Ex.  $\int_0^2 \int_y^1 \int_z^{yz} 8xyz dx dz dy =$

$$yz((4(yz)^2 - 4z^2)) = \int_0^2 \int_y^1 4y^3 z^3 - 4yz^3 dz dy = (y^3 - y) - (y^7 - y^4) =$$

$$\int_0^2 -y^7 + y^4 + y^3 - y dy = -\frac{1}{8}(2)^7 + \frac{1}{5}(2)^5 + \frac{1}{4}(2)^4 - \frac{1}{2}(2)^2 = -16 + \frac{32}{5} + 4 - 2 = -\frac{38}{5}$$

- Ex.  $\frac{\delta f}{\delta x} = 3x^2 - 5y^2, \frac{\delta f}{\delta y} = -10xy + 8y^3, f = ?$

$$\int \frac{\delta f}{\delta x} dx = x^3 - 5xy^2 + g(y) = \frac{\delta f}{\delta y} = -10xy + g'(y) \Rightarrow g'(y) = 8y^3 \Rightarrow f(x, y) = x^3 - 5xy^2 + 2y^4 + c$$

# 2 Integration in $\mathbb{R}^2$

- Double Integral

$$- \iint_R f(x, y) dA$$

- Fubini's Theorem: Utilize iterated integration to calculate multiple-integration

$$- \iint_R f(x, y) dA \longrightarrow \int_a^b \int_c^d f(x, y) dy dx$$

- For Type I Regions:

$$- \int_a^b \int_{p(x)}^{q(x)} f(x, y) dy dx$$

- Occurs when  $y$  is bounded by functions of  $x$  and  $x$  is bounded by vertical lines ( $x = c$ )

- For Type II Regions:

$$- \int_c^d \int_{r(x)}^{s(x)} f(x, y) dx dy$$

- Occurs when  $x$  is bounded by functions of  $y$  and  $y$  is bounded by horizontal lines ( $y = c$ )

- A function can be Type I, Type II, both, or neither
- Regions can be broken down into parts to make calculations easier:

- Given  $R$  and two subregions,  $R'$  and  $R''$ , the integral becomes:

$$* \iint_R f dA = \iint_{R'} f dA + \iint_{R''} f dA$$

- Remark: If  $f(x, y) = 1$ , then:

$$- \iint_R dA = \text{Area of } R$$

### 3 Integration with Polar Coordinates

- Basic Properties  $\begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ x^2 + y^2 = r^2 \end{cases}$
- $dA = r dr d\theta$
- $\iint_R f(x, y) dA = \iint_R f(r \cos(\theta), r \sin(\theta)) r dr d\theta$

## 4 Integration in $\mathbb{R}^3$

- $\iiint_S f(x, y, z) \, dV$ , where  $S \rightarrow \begin{cases} p(x, y) \leq z \leq q(x, y) \\ u(x) \leq y \leq v(x) \\ a \leq x \leq b \end{cases}$
- This can be expressed as:  $\int_a^b \int_{u(x)}^{v(x)} \int_{p(x,y)}^{q(x,y)} f(x, y, z) \, dz \, dy \, dx$