

$$\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos(\theta) = \vec{v}_1 \vec{w}_1 + \dots \vec{v}_n \vec{w}_n$$

$$\text{Proj}_{\vec{v}}(\vec{F}) = \left(\frac{\vec{F} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

$$\text{Eq } \perp \vec{n} \rightarrow \vec{n} \cdot \langle x - x_o, y - y_o, \dots \rangle = 0$$

$$|\vec{v} \times \vec{w}| = |\vec{v}||\vec{w}| \sin(\theta)$$

$$\text{Parametrization: } \vec{r}(t) = \vec{r}_o + t\vec{v}$$

$$\text{Or: } \vec{r}(t) = \vec{r}(p) + t\vec{r}_u(p) + s\vec{r}_v(p)$$

$$\text{Point to point: } \vec{r}(t) = (1-t)\vec{p} + t\vec{q}, 0 \xrightarrow{t} 1$$

$$L_f(x, y) \approx f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \quad \frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \dots = \nabla f \cdot \frac{d\vec{x}}{dt}$$

$$D_{\vec{u}}f(p) = \nabla f(p) \cdot \vec{u}$$

$$\text{Lagrange Multipliers: } \nabla \bar{f} = \lambda \nabla g$$

$$\text{Max. magnitude: } \frac{\nabla f(p)}{|\nabla f(p)|}$$

$$\text{Min. magnitude: } -\frac{\nabla f(p)}{|\nabla f(p)|}$$

$$\text{Max/Min Rate of Change: } \pm |\nabla f(p)|$$

$$D = f_{xx}(p)f_{yy}(p) - f_{xy}^2(p)$$

$$\text{Degenerate if } D = 0$$

$$\text{If } D > 0, \text{ and } f_{xx}(p) > 0, \text{ min}$$

$$\text{If } D > 0, \text{ and } f_{xx}(p) < 0, \text{ max}$$

$$\text{If } D < 0, \text{ saddle point}$$

$$\text{Surface Area: } \iint_D |\vec{r}_u \times \vec{r}_v| du dv$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) dt$$

$$V = \iiint_S dV$$

$$= \iiint_S r dr d\theta dz \begin{cases} x = r \cos(\theta) \\ y = r \sin(\theta) \\ z = z \\ x^2 + y^2 = r^2 \end{cases}$$

$$= \iiint_S \rho^2 \sin(\phi) d\rho d\phi d\theta \begin{cases} x = \rho \sin(\phi) \cos(\theta) \\ y = \rho \sin(\phi) \sin(\theta) \\ z = \rho \cos(\phi) \\ x^2 + y^2 + z^2 = \rho^2 \end{cases}$$

$$\text{Gradient: } \nabla f$$

$$\text{Divergence: } \nabla \cdot f$$

$$\text{Curl: } \nabla \times f$$

$$\text{If conservative: } \int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$$

$$\text{Where } \vec{F} = \nabla f$$

$$\text{Cons. if: } \text{curl}(\vec{F}) = 0$$

$$\int_C P dx + Q dy + R dz = \int_C (P \cdot x'(t) + Q \cdot y'(t) + R \cdot z'(t)) dt$$

$$\text{Green's Theorem:}$$

$$\int_{\partial R} \vec{F} d\vec{r} = \iint_R Q_x - P_y dA$$

$$\text{Flux: } \iint_{D(u,v)} V(\vec{r}(u, v)) \cdot (\vec{r}_u \times \vec{r}_v) du dv$$

$$\text{Divergence Theorem:}$$

$$\iint_{\partial E} (\vec{F} \cdot \vec{n}) dS = \iiint_E \text{div}(\vec{F}) dV$$

$$\text{Stokes' Theorem:}$$

$$\int_{\partial M} \vec{F} d\vec{r} = \iint_M (\text{curl}(\vec{F}) \cdot \vec{n}) dS$$