

Signals, Systems, and Complex Numbers

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- What is a System?

- From electrical engineering perspective, a system is device or a group device or a set-up that takes an input signal x and manipulates it to generate a signal y
- Add another layer of abstraction over circuit (or other) models
- Can allow us to think about, analyze, and design circuits while paying attention only to what is important in a particular setting and ignoring lots of other details

- Complex Numbers

- The imaginary numbers consist of all numbers bi , where b is a real number and i is the imaginary unit, with the property that $i^2 = -1$
- The first four powers of i establish an important pattern and should be memorized:

$$\boxed{i^1 = i \quad i^2 = -1 \quad i^3 = -i \quad i^4 = 1}$$

- The complex numbers consist of all sums $a + bi$, where a and b are real numbers and i is the imaginary unit. The real part is a , and the imaginary part is bi
- Conjugates
 - * The conjugate of $a + bi$ is $a - bi$
 - * The conjugate of $a - bi$ is $a + bi$

- Systems

- Physical System
 - * Defining a system involves drawing a boundary around some part of the world (or conceptually, inside a computational device) so that quantities external to those boundaries may influence what happens inside the boundaries (the “inputs” to the system)
- Mathematical System

- LTI Systems
 - Linear System: Linear systems are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs
 - Time Invariant System: Time-invariant systems are systems where the output does not depend on when an input was applied. These properties make LTI systems easy to represent and understand graphically
- Fundamental Theorem of Algebra
 - A polynomial of form $ax^n + bx^{n-1} + \dots + c$ has n roots
- Complex Plane
 - Any complex number can be placed on a 2D coordinate imaginary plane
 - What would normal be the y axis becomes the imaginary axis, and the x axis acts as the real axis
- Properties of Complex Numbers
 - Instead of i , j is used for complex numbers in electrical engineering because i is used for current
 - Euler's Formula:

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

- The amplitude can be found using $c = \sqrt{a^2 + b^2}$, where

$$\begin{cases} a &= c \cos(\theta) \\ b &= c \sin(\theta) \end{cases}$$

- Important properties:

$$a + jb = ce^{j\theta}$$

$$\tan(\theta) = \frac{b}{a}$$

- Rectangular Form:

$$n = a + jb$$

- Polar Form:

$$n = ce^{j\theta}$$