# Steady-State Sinusoidal Analysis

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- The voltage for a sinusoid may be found using:

$$\boxed{v = V_m \cos(\omega t + \phi)}$$

- Where $V_m$ is the average voltage, $\omega$ is the angular frequency, and $\phi$ is the phase shift angle

- The two frequencies are:

$$f = \frac{1}{T}, \qquad \text{Frequency}$$

$$\omega = \frac{2\pi}{T}, \qquad \text{Angular Frequency}$$

- The mean value of a periodic signal is defined as:

$$V_m = \frac{1}{T} \int_0^T v\, dt$$

OR

$$\boxed{V_m = \int_0^T V_m \cos(\omega t + \phi)\, dt = 0}$$

- The root mean square (rms) may be calculated as follows:

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2\, dt}$$

OR

$$\boxed{V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \phi)\, dt} = \frac{V_m}{\sqrt{2}}}$$

- Solving for the differential equation of sinusoidal response, current becomes:

$$i = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-\frac{Rt}{L}} + \frac{V_m}{\sqrt{R^2 + \omega^2 L_2}} \cos(\omega t + \phi - \theta)$$

- The left term is the transient response, and the right term is the steady-state response
- The transient will go to 0, so the steady-state term is most important
- Frequency of the output signal is the same as the frequency of the input signal
- Amplitude and phase changes (new phase is $\phi - \theta$)
- The Phasor Transform
  - The phasor transform utilizes Euler's identity to rewrite the response
  - The frequency domain becomes

$$V = P\{V_m \cos(\omega t + \phi) i\} = V_m e^{j\phi} e^{j\omega t}$$

  - Where $V$ is the frequency domain
  - $V = 100\angle - 26°$ is the phasor representation of $100 \cos(\omega t - 25)$
  - For an inverse phasor transform, multiply by $e^{j\omega t}$. The exponential becomes the term inside of the cos. Convert.