Sinusoidal Instantaneous Power

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- Instantaneous power is given by

$$p = iv$$

- Where $v = V_m \cos(\omega t + \theta_v)$ and $i = I_m \cos(\omega t + \theta_i)$
- Changing the reference time for the sinusoidal, we can create:

$$\begin{cases} v = V_m \cos(\omega t + \theta_v - \theta_i) \\ i = I_m \cos(\omega t) \end{cases}$$

- This makes the instantaneous power:

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t)$$

$$p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \cos(2\omega t) - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin(2\omega t)$$

- Or, to simplify:

$$p = P + P \cos(2\omega t) - Q \sin(2\omega t)$$

- The average (real) power becomes $P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$
- The reactive power becomes $Q = \frac{V_m I_m}{w} \sin(2\omega t)$
- Units
  - Instantaneous power unit is V A (Volt-Ampere)
  - The average (real) power unit is watts
  - The reactive power units is V AR (Volt-Ampere Reactive)

- Using root-mean square

$$\begin{cases} P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \\ Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \end{cases}$$

$$\frac{Q}{P} = \tan(\theta_v - \theta_i)$$

- Next, we use $|S| = \sqrt{P^2 + Q^2}$ and $S = P + jQ$ to define a phasor transform

$$S = V_{rms} I_{rms} \angle(\theta_v - \theta_i)$$

- In Euler notation:

$$\frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)}$$