Fourier Series

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- Periodic Signals
  - Sinusoidal signals are periodic
  - A periodic function is a function that repeats itself after T seconds
- Fourier Series
  - The Fourier series representation of a periodic signal is:

$$\boxed{f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)}$$

  - The coefficients may be determined in the following manner:

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_k = \frac{2}{T}\int_0^T f(t)\cos(k\omega_0 t)\,dt$$

$$b_k = \frac{2}{T}\int_0^T f(t)\sin(k\omega_0 t)\,dt$$

    * Even though the indicated limits of integration are from 0 to T, the expressions are equally valid if the lower limit is changed to t_0 and the upper limit to $(t_0 + T)$ for any value of t_0; in some cases, the evaluation is easier to perform by integrating from $-\frac{T}{2}$ to $\frac{T}{2}$

∗ For an even function ($f(t) = f(-t)$):

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\,dt$$

$$a_k = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(k\omega_0 t)\,dt$$

$$b_k = 0$$

∗ For an odd function ($f(t) = -f(-t)$):

$$a_0 = 0$$

$$a_k = 0$$

$$b_k = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(k\omega_0 t)\,dt$$

- Some common Fourier transforms are:

Table 13-2: Fourier series expressions for a select set of periodic waveforms.

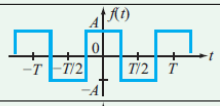
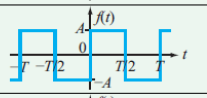
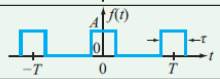
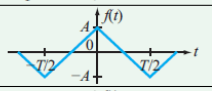
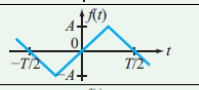
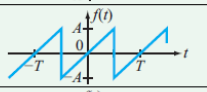
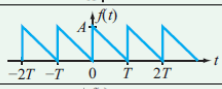
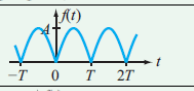
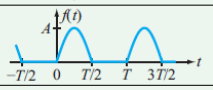
Waveform	Fourier Series
1. Square Wave	 $f(t) = \sum_{n=1}^{\infty} \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{2n\pi t}{T}\right)$
2. Time-Shifted Square Wave	 $f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{4A}{n\pi}\sin\left(\frac{2n\pi t}{T}\right)$
3. Pulse Train	 $f(t) = \frac{A\tau}{T} + \sum_{n=1}^{\infty} \frac{2A}{n\pi}\sin\left(\frac{n\pi\tau}{T}\right)\cos\left(\frac{2n\pi t}{T}\right)$
4. Triangular Wave	 $f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{8A}{n^2\pi^2}\cos\left(\frac{2n\pi t}{T}\right)$
5. Shifted Triangular Wave	 $f(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{8A}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{2n\pi t}{T}\right)$
6. Sawtooth	 $f(t) = \sum_{n=1}^{\infty} (-1)^{n+1}\frac{2A}{n\pi}\sin\left(\frac{2n\pi t}{T}\right)$
7. Backward Sawtooth	 $f(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{A}{n\pi}\sin\left(\frac{2n\pi t}{T}\right)$
8. Full-Wave Rectified Sinusoid	 $f(t) = \frac{2A}{\pi} + \sum_{n=1}^{\infty} \frac{4A}{\pi(1-4n^2)}\cos\left(\frac{2n\pi t}{T}\right)$
9. Half-Wave Rectified Sinusoid	 $f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\left(\frac{2\pi t}{T}\right) + \sum_{\substack{n=2 \\ n=\text{even}}}^{\infty} \frac{2A}{\pi(1-n^2)}\cos\left(\frac{2n\pi t}{T}\right)$

Figure 1: Common Fourier Transform Table