First Order Time-Dependent Circuits

Michael Brodskiy
Professor: N. Sun

February 22, 2023

- Natural Response
  - An inductor or a capacitor can store energy when charge by current or a voltage source
  - If they are abruptly disconnected, they will lose energy through a resistor
  - The timing response of the circuit is called the natural response
- Time Constant
  - Notice the equation

$$\boxed{i(t) = I_0 e^{\frac{-Rt}{L}}}$$

  - The term $\frac{L}{R}$ is called the time constant of the circuit
  - The equation can be changed to:

$$\boxed{i(t) = I_0 e^{-\frac{t}{\tau}}}$$

  - Time constant is represented as τ and is the time it takes for the current to reduce to 37% of original value
  - When we say long time, we typically mean 5 or more time constants
  - Momentary events such as opening of the switch and circuit response that follows is called transient response
  - The response of the circuit after a long time (several time constants) is called the steady-state response
- Step Response of an RL Circuit

- Step response of a circuit is the timing response when you suddenly apply a voltage or current source across resistor
- Applying KVL, the differential equation comes out to be:

$$\boxed{-V_s + iR + L\frac{di}{dt} = 0}$$

- Solving this, we obtain:

$$\boxed{i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-\frac{R}{L}t}}$$

- RC Circuit
  - The differential equation for an RC Circuit is:

$$\boxed{-I_s + \frac{v}{R} + C\frac{dv}{dt} = 0}$$

  - Solving this equation, we get:

$$\boxed{I_s R + (V_0 - I_s R)e^{-\frac{t}{RC}}}$$