Thevnin and Norton Equivalents

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- Circuit Equivalence
  - If there are two circuits, and the same voltage, v_a is applied to both, if $I_A = I_B$ then the circuits are considered equivalent
  - Same thing for same current applied and same voltage output
- Resistor Equivalence
  - Resistors can be connected in circuits like a delta connection
  - As well as a Y connection
  - A delta connection can be transformed into a Y connection and vice versa using transformation technique
  - This will often help in circuits
- Transformations can be established from one to the other, as shown in Figure 1, using the following

$$\boxed{R_1 = \frac{R_bR_c}{R_a+R_b+R_c}, \quad R_2 = \frac{R_cR_a}{R_a+R_b+R_c}, \quad R_3 = \frac{R_aR_b}{R_a+R_b+R_c}}$$

$$\boxed{R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}, \quad R_2 = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}, \quad R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}}$$

- A voltage source with a series resistor can be transformed into a current source with a resistor in parallel, as shown in Figure 2
- A parallel resistor with a voltage source can be removed (replaced by an open) for transformation, as shown in Figure 3
- A series resistor with a current source can be removed (replaced by a short) for transformation, as shown in Figure 4

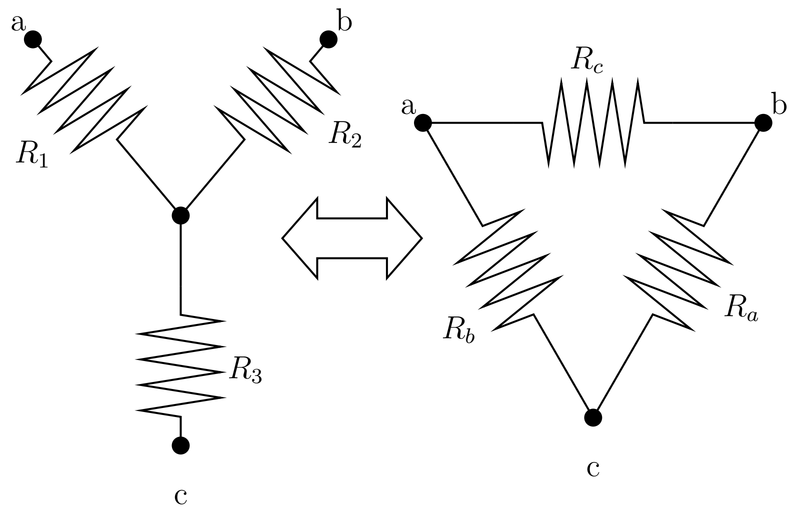

Figure 1: Delta and Y Circuit Configurations

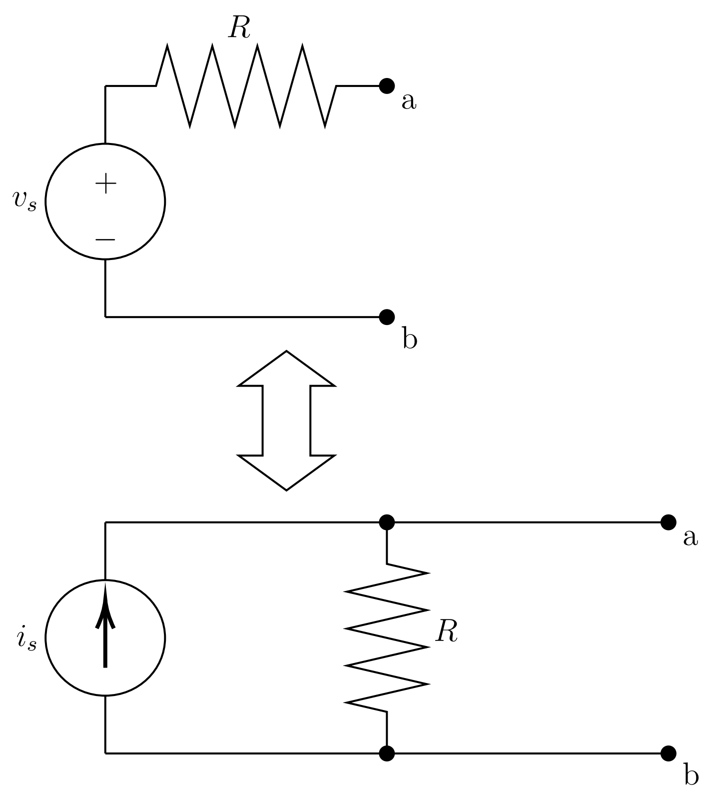

Figure 2: Converting Between Voltage and Current

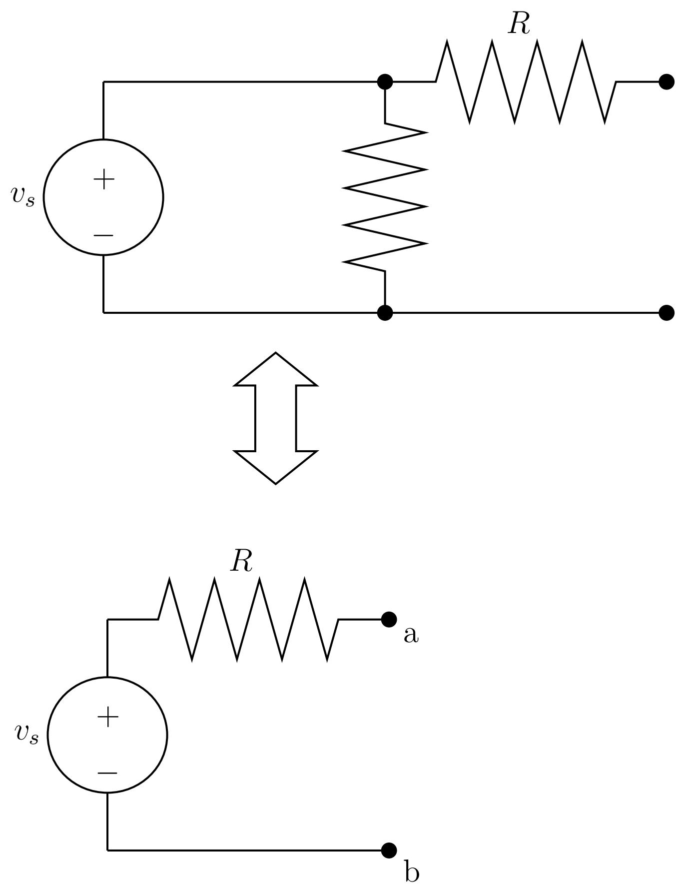

Figure 3: Converting Voltage Sources

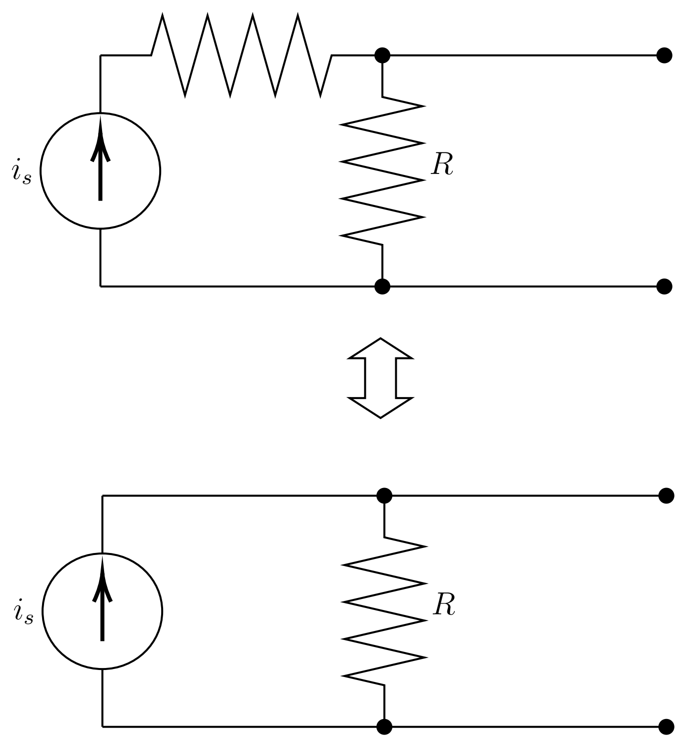

Figure 4: Converting Current Sources

- Some systems may be very complex, but we may be only interested in how it behaves at two terminals
  - Actual circuit can be converted into an equivalent circuit, called Thevnin equivalent
  - It essentially replaces the actual circuit with a voltage source with a series resistor
  - The voltage source is called the Thevnin voltage V_{TH}
  - The equivalent input resistance is called the Thevnin resistor R_{TH}
- Thevnin Equivalence is a two-step process:
  1. Analyze the circuit and find the open circuit voltage v_{oc}
  2. Analyze the circuit and find the short circuit current across the terminal i_{sc}

  - Some circuits only work properly over a certain range of loads (not with $R_L = 0$, for example)
  - For this type of circuit, to determine R_{TH} use an acceptable R_L and compute R_{TH} by using the voltage divider equation shown below

$$V_{RL} = V_{TH} \frac{R_L}{R_{TH} + R_L}$$

  - Note that only the process of determining R_{TH} is changed
- Norton Equivalent
  - Norton equivalent consists of a Norton current source in parallel with a Norton resistor
  - It's a dual of Thevnin equivalent circuit, and one can be derived from the other
  - It can be deduced from the source transform

$$\boxed{i_{nor} = \frac{V_{TH}}{R_{TH}} \qquad R_{TH} = R_{nor}}$$

- Determining R_{TH}
  - In a circuit with ideal voltage and/or current source[1]
  - Replace the voltage source with a short and a current source with an open
  - Find the equivalent resistance across a and b using resistor transform

[1]Note: This technique does not apply when there are dependent sources in the circuit