Runge-Kutta Methods

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- Much like in probability, the sum of the weights, $w_1 + w_2 + \cdots + w_m = 1$. The order of the Runge-Kutta Method is given by m, or the amount of weights given. Therefore, a first-order Runge-Kutta Method is Euler's Method.
- To find the fourth-order Runge-Kutta procedure, one must use formula (1)

$$\begin{aligned} y_{n+1} &= y_n + h(w_1k_1 + w_2k_2 + w_3k_3 + w_4k_4) \\ &\text{Where} \\ k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \alpha_1 h, y_n + \beta_1 hk_1) \\ k_3 &= f(x_n + \alpha_2 h, y_n + \beta_2 hk_1 + \beta_3 hk_2) \\ k_4 &= f(x_n + \alpha_3 h, y_n + \beta_4 hk_1 + \beta_5 hk_2 + \beta_6 hk_3) \end{aligned} \tag{1}$$

- For our purposes, we go with the formula (2)

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\ k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1) \\ k_3 &= f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2) \\ k_4 &= f(x_n + h, y_n + hk_3) \end{aligned} \tag{2}$$

- (2) is known as the **RK4 method**
- Because RK4 is of order 4, the local truncation error is $O(h^5)$ and the global truncation error is $O(h^4)$
- More accurate methods, called adaptive methods do exist. One such method is the RKF45 method, or the Runge-Kutta-Fehlberg method.