Linear Equations

Michael Brodskiy
Professor: Meetal Shah

September 14, 2020

- A first-order linear differential equation takes the form:

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

- The standard form (where $P(x) = \frac{a_0(x)}{a_1(x)}$ and $f(x) = g(x)$) is as follows:

$$\frac{dy}{dx} + P(x)y = f(x)$$

- The solution of such differential equations may be obtained by using the product rule:

$$\frac{d}{dx}[\mu(x)y] = \mu\frac{dy}{dx} + \frac{d\mu}{dx}y = \mu\frac{dy}{dx} + \mu Py$$

This is proven true when:

$$\frac{d\mu}{dx} = \mu P, \text{ or } \mu(x) = c_2 e^{\int P(x)\,dx}$$

- This value is known as the integrating factor, I
- Solving a Linear First-Order Equation
  1. First, rearrange the equation into standard form
  2. Find the integrating factor by locating $P(x)$
  3. Multiply both sides by the integrating factor (and add constant c).
  4. Then, solve for y
- Values of x where $a_1(x) = 0$ are known as singular points ($a_1(x)$ is the coefficient of $\frac{dy}{dx}$)

- A transient term is one that approaches zero as x approaches infinity (ex. e^{-x})
- When either $P(x)$ or $f(x)$ is a piecewise-defined function, the equation is said to be a piecewise-linear differential equation
- Two important functions defined by integrals exist. These are the error function and the complimentary error function, respectively:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \qquad erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2}\, dt$$

- These two functions are related by an identity:

$$erf(x) + erfc(x) = 1$$