Solutions by Substitutions

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- Often the first step in solving a differential equation consists of transforming it into another differential equation by means of a **substitution**

- **Homogeneous Function** – A function f that possesses the property $f(tx, ty) = t^{\alpha} f(x, y)$ for some real number α

$$M(x, y)\, dx + N(x, y)\, dy = 0$$

- Is homogeneous if both coefficient functions M and N are homogeneous functions of the same degree.

- Either substitution, $y = ux$ or $x = vy$, will reduce a homogeneous equation to a separable first-order differential equation.

- **Bernoulli's Equation** – An equation which fits the following form, where n is any real number:

$$\frac{dy}{dx} + P(x)y = f(x)y^n$$

- To solve differential equations with the given form, substitute:

$$u = y^{1-n}$$

- **Reduction to Separation of Variables** – A differential equation of the following form may be reduced to separation of variables when substituted with $u = Ax + By + C, B \neq 0$

$$\frac{dy}{dx} = f(Ax + By + C)$$