Differential Equations – Exam Three

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\begin{aligned}
(e^t - e^{-t})^2 &= e^{2t} + e^{-2t} - 2 \\
\mathcal{L}\left\{e^{2t} + e^{-2t} - 2\right\} &= \frac{1}{s-2} + \frac{1}{s+2} - \frac{2}{s}
\end{aligned}
\tag{1}
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\mathcal{L}^{-1}\left\{\frac{4}{s} + \frac{6}{s^5} - \frac{1}{s+8}\right\} = 4 + \frac{1}{4}t^4 - e^{-8t}
\tag{2}
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\begin{aligned}
&\mathcal{L}^{-1}\left\{\frac{1}{s^2+s-20}\right\} \\
\frac{1}{s^2+s-20} &= \frac{1}{\left(s-\frac{1}{2}\right)^2 - \frac{81}{4}} \\
\mathcal{L}^{-1}\left\{\frac{1}{s^2+s-20}\right\} &= \frac{2}{9}e^{-\frac{1}{2}t} \cdot \sinh\left(\frac{9}{2}t\right)
\end{aligned}
\tag{3}
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\begin{aligned}
\mathcal{L}\left\{y'' + y = \sin(t)\right\} \to s^2F(s) - s + 1 + F(s) &= \frac{1}{s^2+1} \\
F(s) &= \frac{1}{\left(s^2+1\right)^2} + \frac{s}{s^2+1} - \frac{1}{s^2+1} \\
y(t) &= \frac{\sin(t) - t\cos(t)}{2} + \cos(t) - \sin(t)
\end{aligned}
\tag{4}
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\begin{aligned}
\mathcal{L}\left\{y' + y = \delta(t-1)\right\} \to sF(s) - 2 + F(s) &= e^{-s} \\
F(s) &= \frac{e^{-s}}{s+1} + \frac{2}{s+1} \\
y(t) &= 2e^{-t} + e^{-(t-1)}\mathcal{U}(t-1)
\end{aligned}
\tag{5}
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\begin{aligned}
\mathbf{X}' &= \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{X} \\
\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} &= (\lambda - 1)(\lambda - 4) \\
\lambda_n &= 1, 4 \\
\text{When } \lambda_n &= 1 \\
&\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \\
k_1 = 2,\ \ k_2 &= -1 \\
\text{When } \lambda_n &= 4 \\
&\begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix} \\
k_1 = 1,\ \ k_2 &= 1 \\
\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} &= c_1 \begin{pmatrix} 2 \\ -1 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}
\end{aligned}
\tag{6}
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\begin{aligned}
\text{Euler: } y''(c) \cdot \frac{h^2}{2!} \\
y'(x) = 2e^{2x} \\
y''(x) = 4e^{2x} \\
0 \leq c \leq .1 \\
.1^2 = \frac{1}{100} \\
\therefore \text{The bound is: } \frac{e^{\frac{1}{5}}}{50}
\end{aligned}
\tag{7}
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\begin{aligned}
f(x,y) = 4x - 2yk_1 = f(0,2) &= -4 \\
k_2 = f(.05, 1.8) &= -3.4 \\
k_3 = f(.05, 1.83) &= -3.46 \\
k_4 = f(.1, 1.654) &= -2.908 \\
RK4 = 2 + .1 \left(\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \right) \\
RK4 &= 1.6562
\end{aligned}
\tag{8}
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\begin{aligned}
\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} e^x \sin x \, dx \\
-e^x \cos(x) + \int e^x \cos(x) \, dx \\
\left. \frac{e^x(\sin(x) - \cos(x))}{2} \right|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\
\cos(x) = \sin(x) \text{ at } \frac{\pi}{4} \text{ and } \frac{5\pi}{4} \\
\therefore \text{This integral equals zero, so the functions are orthogonal}
\end{aligned}
\tag{9}
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\begin{aligned}
a_o = \frac{1}{\pi}\left(-\int_{-\pi}^{0} dx + 2\int_{0}^{\pi} dx\right) \\
\frac{1}{\pi}\left(-\pi + 2\pi\right) = 1 \\
\frac{a_o}{2} = \frac{1}{2} \\
a_n = \frac{1}{\pi}\left(-\int_{-\pi}^{0} \cos(nx)\, dx + 2\int_{0}^{\pi} \cos(nx)\, dx\right) \\
a_n = 0 \\
b_n = \frac{1}{\pi}\left(-\int_{-\pi}^{0} \sin(nx)\, dx + 2\int_{0}^{\pi} \sin(nx)\, dx\right) \\
b_n = \frac{3 + (-1)^n}{\pi} \\
f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left(\frac{3 + (-1)^n}{\pi}\right)\sin(nx)
\end{aligned}
\tag{10}
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\begin{aligned}
\mathcal{L}\left\{t^2 * te^t\right\} =& \mathcal{L}\left\{t^2\right\} \cdot \mathcal{L}\left\{te^t\right\} \\
\mathcal{L}\left\{t^2\right\} =& \frac{2}{s^3} \\
\mathcal{L}\left\{te^t\right\} =& \frac{1}{(s-1)^2} \\
\mathcal{L}\left\{t^2 * te^t\right\} =& \frac{2}{s^3(s-1)^2}
\end{aligned}
\tag{11}
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