Separable Equations

Michael Brodskiy
Professor: Meetal Shah

September 9, 2020

- Separable Differential − A differential equation that may be broken apart into a function of x and a function of y:

$$\frac{dy}{dx} = g(x)h(y)$$

- If this form is divided by $h(y)$ (where $h(y) = \frac{1}{p(y)}$:

$$p(y)\frac{dy}{dx} = g(x)$$

- If $y = \phi(x)$, then:

$$p(\phi(x))\phi'(x) = g(x)$$
$$\int p(\phi(x))\phi'(x)\,dx = \int g(x)\,dx$$
$$\frac{dy}{dx} = \phi'(x) \Rightarrow dy = \phi'(x)\,dx$$
$$\int p(\phi(x))\,dy = \int g(x)\,dx \Rightarrow H(y) = G(x) + c$$

- Often, it will be necessary to create an integral-defined function, where (x_o, y_o) is the initial condition:

$$y = y_o + \int_{x_o}^{x} f(t)\,dt$$

- One example of such a case would be where $\frac{dy}{dx} = e^{-x^2}$. It is not possible to differentiate this, and, therefore, one ends up with:

$$y = y_o + \int_{x_o}^{x} e^{-t^2}\,dt$$