# Definition of the Laplace Transform


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- Derivatives and integrals follow the linearity property, or, for constants $\alpha$ and $\beta$:

$$\begin{gathered}\frac{d}{dx}[\alpha f(x) + \beta g(x)] = \alpha f'(x) + \beta g'(x) \\ \int [\alpha f(x) + \beta g(x)]\,dx = \alpha \int f(x)\,dx + \beta \int g(x)\,dx\end{gathered} \tag{1}$$

- Integral transforms are done as such:

$$\int_0^\infty K(s,t)f(t)\,dt = \lim_{b\to\infty} \int_0^b K(s,t)f(t)\,dt \tag{2}$$

- The Laplace Transform is defined as, where $K(s,t)$ is the kernel of the transform:

$$\mathcal{L} = \int_0^\infty e^{-st} f(t)\,dt \tag{3}$$

- Then, for two different functions, the Laplace transform would be (4), which means it is a linear transform

$$\mathcal{L}\{\alpha f(x) + \beta g(x)\} = \alpha \int e^{-st} f(x)\,dx + \beta \int e^{-st} g(x)\,dx \tag{4}$$

- Laplace Transforms for common functions:

$$\begin{gathered}\mathcal{L}\{1\} = \frac{1}{s} \\ \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \\ \mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2} \qquad \mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2} \\ \mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2} \qquad \mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}\end{gathered} \tag{5}$$

- For a Laplace Transform to exist, the function presented must be piecewise continuous and of exponential order

- A function $f$ is said to be of exponential order if there exist constants, $c$, $M > 0$, and $T > 0$ such that $|f(t)| \leq Me^{ct}$ for all $t > T$

- If $f$ is piecewise continuous on $[0, \infty)$ and of exponential order, then $\mathcal{L}\{f(t)\}$ exists for $s > c$

- If $f$ is piecewise continuous on $[0, \infty)$ and of exponential order and $F(s) = \mathcal{L}\{f(t)\}$, then $\lim_{s\to\infty} F(s) = 0$