Solutions About Ordinary Points

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- A point $x = x_0$ is said to be an ordinary point of the differential of the differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$ if both coefficients $P(x)$ and $Q(x)$ in the standard form $y'' + P(x)y' + Q(x)y = 0$ are analytic (can be expressed as a series) at x_0. A point that is not an ordinary point is said to be a singular point of the DE. (1)

$$P(x) = \frac{a_1(x)}{a_2(x)} \text{ and } Q(x) = \frac{a_0(x)}{a_2(x)} \tag{1}$$

- A point is ordinary as long as $a_2(x) \neq 0$ at x_0[1]

- If $x = x_0$ is an ordinary point of the differential equation, we can always find two linearly independent solutions in the form of a power series centered at x_0, that is (2). A power series solution converges at least on some interval defined by $|x - x_0| < R$, where R is the distance from x_0 to the closest singular point.

$$y = \sum_{n=0}^{\infty} c_n (x - x_0)^n \tag{2}$$

- A solution about the ordinary point x_0 is a solution of form (2)

[1]NOTE: singular points do not have to real. For example, in $(x^2 + 1)y'' + xy' - y = 0$, $x^2 + 1$ has roots ± 1