

# Solution Curves Without a Solution

Michael Brodskiy

Professor: Meetal Shah

September 9, 2020

- The function  $f$  in the normal form is called the slope function or rate function

$$\frac{dy}{dx} = f(x, y)$$

- Lineal Element – An individual slope at a certain point
- Direction Field (Slope Field) – The collection of all lineal elements
- Autonomous First-Order Differential Equation:

$$\frac{dy}{dx} = f(y)$$

- Critical Points – Also called equilibrium or stationary points, they are points,  $c$ , which, when plugged into a function, yield:

$$f(c) = 0$$

- Equilibrium Solution – If  $c$  is a critical point, then  $y(x) = c$  is a constant solution of the autonomous differential equation. Equilibria are the only constant solutions
- One-Dimensional Phase Portrait – Simply called a phase portrait, it shows the intervals where a function is increasing and decreasing (essentially a sign chart). The line that the intervals are graphed on is named the phase line
- Attractors and Repellers – For a nonconstant solution,  $y(x)$ , there are basically three behaviors as it approaches some critical point,  $c$ .
  1. Attractor – When  $\lim_{x \rightarrow \pm\infty} y(x) = c$ . This means the critical point is asymptotically stable.
  2. Repeller – When both parts of a solution move away from a  $c$  point. These are unstable.
  3. Semi-stable – When both parts either move up or down.