$$W(f_1, f_2, f_n) = \begin{vmatrix} f_1 & f_2 & f_n \\ f_1' & f_2' & f_n' \\ \vdots & \vdots & \vdots \\ f_1^{n-1} & f_2^{n-1} & f_n^{n-1} \end{vmatrix} \quad (1)$$

If $W(f) \neq 0$, f is linearly independent

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y_1(x))^2}\,dx \quad (2)$$

Where $y_1(x)$ is a known function
And $P(x)$ is the coefficient of y'

$$\begin{aligned} \alpha \pm \beta i &\Rightarrow e^{\alpha x}\left(c_1 \cos(\beta x) + c_2 \sin(\beta x)\right) \\ \beta i &\Rightarrow c_1 \cos(\beta x) + c_2 \sin(\beta x) \\ \alpha, \beta &\Rightarrow c_1 e^{\alpha x} + c_2 e^{\beta x} \\ \alpha &\Rightarrow c_1 e^{\alpha x} + c_2 x e^{\alpha x} \end{aligned} \quad (3)$$

$$\begin{gathered} D^n;\ x^{n-1} \\ (D-\alpha)^n;\ x^{n-1} e^{\alpha x} \\ [D^2 - 2\alpha D + \alpha^2 + \beta^2]^n;\ x^{n-1} e^{\alpha x} \cos \beta x \end{gathered} \quad (4)$$

$$\begin{aligned} u_1' &= \frac{W_1}{W} = -\frac{y_2 f(x)}{W} \\ u_2' &= \frac{W_2}{W} = -\frac{y_1 f(x)}{W} \\ W &= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \\ W_1 &= \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} \\ W_2 &= \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} \end{aligned} \quad (5)$$

$$\begin{aligned} u_1(x) &= -\int_{x_0}^{x} \frac{y_2(t) f(t)}{W(t)}\,dt \\ u_2(x) &= \int_{x_0}^{x} \frac{y_1(t) f(t)}{W(t)}\,dt \\ y_p(x) &= u_1(x) y_1(x) + u_2(x) y_2(x) \end{aligned} \quad (6)$$

$y_1(x)$ is complementary function 1
$y_2(x)$ is complementary function 2
$f(x)$ is the right-side function of x

$$\begin{aligned} \lambda^2 - \omega^2 &> 0 \\ m_{1,2} &= -\lambda \pm \sqrt{\lambda^2 - \omega^2} \\ x(t) &= e^{-\lambda t}\left(c_1 e^{\sqrt{\lambda^2-\omega^2}t} + c_2 e^{-\sqrt{\lambda^2-\omega^2}}\right) \end{aligned} \quad (7)$$

$$\begin{aligned} \lambda^2 - \omega^2 &= 0 \\ m_{1,2} &= 0 \\ x(t) &= e^{-\lambda t}(c_1 + c_2 t) \end{aligned} \quad (8)$$

$$\begin{aligned} \lambda^2 - \omega^2 &< 0 \\ m_{1,2} &= -\lambda \pm \sqrt{\omega^2 - \lambda^2}\, i \\ A &= \sqrt{\omega^2 - \lambda^2} \\ x(t) &= e^{-\lambda t}\left(c_1 \cos At + c_2 \sin At\right) \end{aligned} \quad (9)$$

Spring Parallel: $k_eff = k_1 + k_2$

Spring Series: $k_eff = \dfrac{k_1 k_2}{k_1 + k_2}$

$$\begin{gathered} \frac{d^2x}{dt^2} + \omega^2 x = 0 \\ x(t) = c_1 \cos \omega t + c_2 \sin \omega t \\ A \sin(\omega t + \phi) \end{gathered} \quad (10)$$

$$\begin{aligned}
\text{Inductor (henries): } & L\frac{di}{dt} \\
\text{Resistor (ohms): } & iR \\
\text{Capacitor (farads): } & q\frac{1}{C} \\
L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + q\frac{1}{C} &= E(t) \\
R \neq 0 \text{ then } & q_c(t) \text{ is transient} \\
E(t) \text{ is } \cos, \sin, c \to & \; q_p(t) \text{ steady-state}
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\text{Overdamped if:} \\
&R^2 - \frac{4L}{C} > 0 \\
&\text{Critically Damped if:} \\
&R^2 - \frac{4L}{C} = 0 \\
&\text{Underdamped if:} \\
&R^2 - \frac{4L}{C} < 0
\end{aligned} \tag{12}$$

$$\begin{aligned}
\text{Embedded: } & y = 0,\ y' = 0 \\
\text{Free: } & y'' = 0,\ y''' = 0 \\
\text{Simply Supported: } & y = 0,\ y'' = 0 \\
\text{Horizontal Column: } & EI\frac{d^4y}{dx^4} = \omega(x) \\
\text{Vertical Column: } & EI\frac{d^2y}{dx^2} + Py = 0 \\
\text{Euler Load: } & \frac{\pi^2 EI}{L}
\end{aligned} \tag{13}$$

$$\begin{aligned}
&y(0) = 0,\ y(L) = 0: \\
&\lambda_n = \frac{\pi^2 n^2}{L^2} \\
&y_n(x) = \sin\left(\frac{n\pi}{L}x\right) \\
&y(0) = 0,\ y'(L) = 0 \\
&\lambda_n = \frac{(2n-1)^2\pi^2}{4L^2} \\
&y_n(x) = \sin\left(\frac{(2n-1)\pi x}{2L}\right)
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\text{For a twirling rope:} \\
&T\frac{d^2y}{dx^2} + \rho\omega^2 y = 0 \\
&\text{Where } T \text{ is the tension force} \\
&\rho \text{ is the density per unit length} \\
&\text{and } \omega \text{ is the angular velocity}
\end{aligned} \tag{15}$$

$$\begin{aligned}
y &= \sum_{n=0} c_n x^n \\
y' &= \sum_{n=1} n c_n x^{n-1} \\
y'' &= \sum_{n=2} n(n-1) c_n x^{n-2}
\end{aligned} \tag{16}$$

$$\begin{aligned}
x^3 - x + 1 &\to y_p(x) = Ax^3 + Bx^2 + Cx + E \\
\sin 4x &\to y_p(x) = A\cos 4x + B\sin 4x \\
\cos 4x &\to y_p(x) = A\cos 4x + B\sin 4x \\
x^2 e^{5x} &\to y_p(x) = (Ax^2 + Bx + C)e^{5x} \\
5x^2 \sin 4x &\to y_p(x) = (Ax^2 + Bx + C)\cos 4x + (Ex^2 + Fx + G)\sin 4x \\
xe^{3x}\cos 4x &\to y_p(x) = (Ax + B)e^{3x}\cos 4x + (Cx + E)e^{3x}\sin 4x
\end{aligned} \tag{17}$$