

Laplace Transform Definition:

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) dt$$

Unit Step Definition:

$$\mathcal{U}(t-a) = \begin{cases} 0, & 0 \leq t < a \\ 1, & t \geq a \end{cases}$$

$\underline{f(t)}$	$\underline{F(s)}$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$t^{-\frac{1}{2}}$	$\sqrt{\frac{\pi}{s}}$
$t^{\frac{1}{2}}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
$\sin(kt)$	$\frac{k}{s^2 + k^2}$
$\cos(kt)$	$\frac{s}{s^2 + k^2}$
$\sin^2(kt)$	$\frac{2k^2}{s(s^2 + 4k^2)}$
$\cos^2(kt)$	$\frac{s^2 + 2k^2}{s(s^2 + 4k^2)}$
e^{at}	$\frac{1}{s-a}$
$\sinh(kt)$	$\frac{k}{s^2 - k^2}$
$\cosh(kt)$	$\frac{s}{s^2 - k^2}$
$\sinh^2(kt)$	$\frac{2k^2}{s(s^2 - 4k^2)}$
$\cosh^2(kt)$	$\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$
$t \sin(kt)$	$\frac{2ks}{(s^2 + k^2)^2}$
$t \cos(kt)$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$
$\sin(kt) + kt \cos(kt)$	$\frac{2ks^2}{(s^2 + k^2)^2}$
$\sin(kt) - kt \cos(kt)$	$\frac{2k^3}{(s^2 + k^2)^2}$
$\delta(t-t_0)$	e^{-st_0}

$\underline{f(t)}$	$\underline{F(s)}$
$t \sinh(kt)$	$\frac{2ks}{(s^2 - k^2)^2}$
$t \cosh(kt)$	$\frac{s^2 + k^2}{(s^2 - k^2)^2}$
$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
$1 - \cos(kt)$	$\frac{k^2}{s(s^2 + k^2)}$
$kt - \sin(kt)$	$\frac{k^3}{s^2(s^2 + k^2)}$
$\frac{a \sin(bt) - b \sin(at)}{ab(a^2 - b^2)}$	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$
$\frac{\cos(bt) - \cos(at)}{a^2 - b^2}$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$
$\sin(kt) \sinh(kt)$	$\frac{2k^2 s}{s^4 + 4k^4}$
$\sin(kt) \cosh(kt)$	$\frac{k(s^2 + 2k^2)}{s^4 + 4k^4}$
$\cos(kt) \sinh(kt)$	$\frac{k(s^2 - 2k^2)}{s^4 + 4k^4}$
$\cos(kt) \cosh(kt)$	$\frac{s^3}{s^4 + 4k^4}$
$\frac{e^{bt} - e^{at}}{t}$	$\ln\left(\frac{s-a}{s-b}\right)$
$e^{at} f(t)$	$F(s-a)$
$\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$
$f * g = \int_0^t f(\tau)g(t-\tau) d\tau$	$F(s)G(s)$
$g(t)\mathcal{U}(t-a)$	$e^{-as}\mathcal{L}\{g(t+a)\}$

Laplace Derivative Formulas:

$$t^n f(t) = (-1)^n \frac{d^n}{ds^n} [F(s)]$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0)$$

$$|\mathbf{A} - \lambda_n \mathbf{I}| \mathbf{K} = 0$$

If a k value is not explicitly zero,
assume it is equal to one

$$k_1 = f(x, y)$$

$$k_2 = f\left(x + \frac{h}{2}, y + \frac{h \cdot k_1}{2}\right)$$

$$k_3 = f\left(x + \frac{h}{2}, y + \frac{h \cdot k_2}{2}\right)$$

$$k_4 = f(x + h, y + h \cdot k_3)$$

$$RK4 = y + h \left(\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \right)$$

For truncation errors:

$$O(h^n) = y^{(n)}(c) \frac{h^n}{n!}$$

$O(h^4)$ for global truncation on RK4

$O(h^5)$ for local truncation on RK4

Orthogonal if:

$$(\phi_m, \phi_n) = \int_a^b \phi_m(x) \phi_n(x) dx = 0$$

$$\|\phi_n(x)\|^2 = \int_a^b \phi_n^2(x) dx$$

$$\|\phi_n(x)\| = \sqrt{\int_a^b \phi_n^2(x) dx}$$

$$c_n = \frac{\int_a^b f(x) w(x) \phi_n(x) dx}{\|\phi_n(x)\|^2}$$

$$\|\phi_n(x)\|^2 = \int_a^b w(x) \phi_n^2(x) dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right) \right)$$

$$a_0 = \frac{1}{p} \int_{-p}^p f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^p f(x) \cos\left(\frac{n\pi}{p}x\right) dx$$

$$b_n = \frac{1}{p} \int_{-p}^p f(x) \sin\left(\frac{n\pi}{p}x\right) dx$$

$$\int_a^b f(t) \delta(t - t_0) dt = f(t_0)$$

$$\delta(t - t_0) = \begin{cases} 0, & 0 \leq t < t_0 - a \\ \frac{1}{2a}, & t_0 - a \leq t < t_0 + a \\ 0, & t \geq t_0 + a \end{cases}$$