Differential Equations – Final Exam

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$$\begin{aligned}
I &= e^{-\int dx} \\
I &= e^{-x} \\
e^{-x}y &= \int xe^{-x} \\
e^{-x}y &= -xe^{-x} - e^{-x} + c \\
y &= -x - 1 + ce^{x} \\
\text{The region is the whole } xy\text{-plane}
\end{aligned} \tag{1}$$

$$\begin{aligned}
\frac{dy}{dx} &= -2xy^2 \\
\frac{1}{y^2}\,dy &= -2x\,dx \\
\int \frac{1}{y^2}\,dy &= \int -2x\,dx \\
-\frac{1}{y} &= -x^2 + c \\
y &= \frac{1}{x^2 + c}
\end{aligned} \tag{2}$$

$$\begin{aligned}
L_1\frac{di_2}{dt} + (R_1 + R_2)i_2 + R_1 i_3 &= E(t) \\
L_2\frac{di_3}{dt} + (R_1 + R_3)i_3 + R_1 i_2 &= E(t)
\end{aligned} \tag{3}$$

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\begin{aligned}
\frac{d^2y}{d\theta^2} + y &= 0 \\
m^2 + 1 &= 0 \\
m &= \pm i \\
y &= e^{0\theta}\left(c_1 \sin\left(\theta\right) + c_2 \cos\left(\theta\right)\right) \\
y &= c_1 \sin\left(\theta\right) + c_2 \cos\left(\theta\right) \\
y\left(\frac{\pi}{3}\right) &= \frac{c_1\sqrt{3}}{2} + \frac{c_2}{2} = 0 \\
y'\left(\frac{\pi}{3}\right) &= \frac{c_1}{2} - \frac{c_2\sqrt{3}}{2} = 2 \\
c_1\sqrt{3} + c_2 &= 0 \\
c_1 - c_2\sqrt{3} &= 4 \\
c_1 = 1, \ \ c_2 &= -\sqrt{3} \\
y &= \sin(\theta) - \sqrt{3}\cos(\theta)
\end{aligned} \tag{4}
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\begin{aligned}
m^2 - 16 &= 0 \\
m &= \pm 4 \\
y &= c_1 e^{-4x} + c_2 e^{4x} + \ldots \\
y_p(x) &= (Ax + B)e^{4x} \\
y_p'(x) &= Ae^{4x} + 4Axe^{4x} + 4Be^{4x} \\
y_p''(x) &= 8Ae^{4x} + 16Axe^{4x} + 16Be^{4x} \\
8Ae^{4x} &= 2e^{4x} \\
A &= \frac{1}{4} \\
y &= c_1 e^{-4x} + c_2 e^{4x} + \frac{1}{4}xe^{4x}
\end{aligned} \tag{5}
$$

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\begin{aligned}
k &= 5 \\
m &= \frac{1}{4} \\
D^2x + 4Dx + 20x &= 0 \\
\lambda^2 - \omega^2 &= -16 \\
A = \sqrt{\omega^2 - \lambda^2} &= 4 \\
x(t) &= e^{-2t}\left(c_1 \cos\left(4t\right) + c_2 \sin\left(4t\right)\right) \\
c_1 &= \frac{1}{2} \\
x'(0) \longrightarrow -1 + 4c_2 &= 2 \\
c_2 &= \frac{3}{4} \\
x(t) &= e^{-2t}\left(\frac{1}{2}\cos\left(4t\right) + \frac{3}{4}\sin\left(4t\right)\right)
\end{aligned} \tag{6}
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\begin{aligned}
x^2 - 2x + 10 &= 0 \\
x &= 1 \pm 3i \\
x_1 = \sqrt{(1-0)^2 + (3-0)^2} &= \sqrt{10} \\
x_2 = \sqrt{(1-1)^2 + (3-0)^2} &= 3
\end{aligned} \tag{7}
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\begin{aligned}
f(t) &= 1 - 2\mathcal{U}\left(t-a\right) \\
\mathcal{L}\left\{y' + y = f(t)\right\} = sF(s) + F(s) &= \frac{1}{s} - \frac{2e^{-s}}{s} \\
F(s) &= \frac{1}{s(s+1)} - \frac{2e^{-s}}{s(s+1)} \\
\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\right\} &= -(e^{-t} - 1) \\
\mathcal{L}^{-1}\left\{-\frac{2e^{-s}}{s(s+1)}\right\} &= 2(e^{-(t-1)} - 1)\mathcal{U}(t-1) \\
f(t) &= -e^{-t} + 1 + (2e^{-(t-1)} - 2)\mathcal{U}(t-1)
\end{aligned} \tag{8}
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\begin{aligned}
|\mathbf{A} - \lambda\mathbf{I}| &= \begin{pmatrix} -6-\lambda & 2 \\ -3 & 1-\lambda \end{pmatrix} \\
\lambda^2 + 5\lambda &= 0 \\
\lambda_n &= -5, 0 \\
\text{When } \lambda_n &= -5 \\
&\begin{vmatrix} -1 & 2 \\ -3 & 6 \end{vmatrix} \\
k_1 = 2, \ \ k_2 &= 1 \\
\text{When } \lambda_n &= 0 \\
&\begin{vmatrix} -6 & 2 \\ -3 & 1 \end{vmatrix} \\
k_1 = 1, \ \ k_2 &= 3 \\
\mathbf{X} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-5t} + c_2 &\begin{pmatrix} 1 \\ 3 \end{pmatrix}
\end{aligned}
\tag{9}
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\begin{aligned}
y^{(5)}(c) \cdot \frac{h^5}{5!} \\
y^{(5)}(x) = 2^5 e^{2x} \\
0 \leq c \leq .1 \\
2^5 e^{\frac{1}{5}} \cdot \frac{.1^5}{120} = \frac{e^{\frac{1}{5}}}{375000}
\end{aligned}
\tag{10}
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\begin{aligned}
Odd \cdot Even = Odd \\
b_n = 2 \int_0^1 x|x| \sin{(n\pi x)} \ dx \\
b_n = 2 \cdot \frac{2x^2 \sin(n\pi x) + (2x - x^3)\cos(n\pi x)}{n^2\pi^2|x|}\bigg|_0^1 \\
b_n = \frac{2(-1)^n}{n^2\pi^2} \\
a_0 = 0 \\
a_n = 0 \\
y(x) = \sum_{n=1}^{\infty} \frac{2(-1)^n}{n^2\pi^2} \sin(n\pi x)
\end{aligned}
\tag{11}
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