Undetermined Coefficients – Annihilator Approach

Michael Brodskiy
Professor: Meetal Shah

October 12, 2020

- An nth-order differential equation can be written in form (1), where $D^k y = d^k y/dx^k$

$$a_n D^n y + a_{n-1} D^{n-1} y + \cdots + a_1 Dy + a_0 y = g(x) \tag{1}$$

- Operators may be factored just like polynomial equations: (2)

$$(D^2 + 5D + 6)y = (D + 3)(D + 2)y \tag{2}$$

- The linear operator L is said to be an **annihilator** if it has the property shown in (3)

$$L(f(x)) = 0 \tag{3}$$

  - For example:
  - D annihilates $y = k$ because $Dk = 0$
  - D^2 annihilates $y = x$ because $D^2 x = 0$
  - D^3 annihilates $y = x^2$ because $D^3 x^2 = 0$

- Some common annihilators are listed:

$$\begin{gathered} D^n \text{ annihilates } x^{n-1} \\ (D-\alpha)^n \text{ annihilates } x^{n-1}e^{\alpha x} \\ [D^2 - 2\alpha D + (\alpha^2 + \beta^2)]^n \text{ annihilates } x^{n-1}e^{\alpha x}\cos\beta x,\ x^{n-1}e^{\alpha x}\sin\beta x \end{gathered} \tag{4}$$

- Once the annihilator is solved for, the differential equation becomes homogeneous and can be easily solved