Undetermined Coefficients – Superposition Approach

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- Method of Undetermined Coefficients – A way of obtaining a particular solution to a nonhomogeneous equation.
- Ex. $y'' + 4y' - 2y = 2x^2 - 3x + 6$
  1. First solve the associated homogeneous equation.
  2. Next, Assume the particular solution, y_p, is a quadratic (as the DE equals a quadratic):
  $$y_p' = 2Ax + B,\, y_p'' = 2A$$
  $$2A + 8Ax + 4B - 2Ax^2 - 2Bx - 2C = 2x^2 - 3x + 6$$
  $$A = -1,\, B = -\frac{5}{2},\, C = -9$$
  $$\therefore y_p = -x^2 - \frac{5}{2}x - 9$$
  3. Then, find the complementary solution and add them together:
  $$y = y_c + y_p = c_1e^{-(2+\sqrt{6})x} + c_2e^{(-2+\sqrt{6})x} - x^2 - \frac{5}{2}x - 9$$
- The form of a prediction may be found using the following table:[1]
- If any y_p predictions are similar to terms in the complementary function, multiply by x^n, where n is the smallest integer which eliminates any correlation to the complementary function.

[1] Note: for functions like $8xe^{2x}$, it is necessary to create a second term, $(Ax + B)e^{2x} + e^{2x}$

$g(x)$	Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A \cos 4x + B \sin 4x$
6. $\cos 4x$	$A \cos 4x + B \sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. x^2e^{5x}	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x} \sin 4x$	$Ae^{3x} \cos 4x + Be^{3x} \sin 4x$
11. $5x^2 \sin 4x$	$(Ax^2 + Bx + C) \cos 4x + (Ex^2 + Fx + G) \sin 4x$
12. $xe^{3x} \cos 4x$	$(Ax + B)e^{3x} \cos 4x + (Cx + E)e^{3x} \sin 4x$

Figure 1: Table of Trials