Differential Equations

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- Differential Equation – An equation containing the derivatives of one or more unknown functions (or dependent variables), with respect to one or more independent variables
- Ordinary Differential Equation – A differential equation that contains only ordinary derivatives of one or more unknown functions with respect to a single independent variable. Shortened to ODE
- Partial Differential Equation – A differential equation that contains partial derivatives of one or more unknown functions with respect to multiple independent variables. Shortened to PDE
- Ordinary derivatives will be written in two notations:
  1. Leibniz Notation $\rightarrow dy/dx\,,d^2y/dx^2\,,d^3y/dx^3$
  2. Prime Notation $\rightarrow y'\,,y''\,,y'''$
  3. (Not used often) Newton's Notation $\rightarrow \dot{y}\,,\ddot{y}\,,\dddot{y}$
- Partial derivatives may use Leibniz notation or:
  1. Subscript Notation $\rightarrow f_x\,,f_y\,,f_{xx}\,,f_{xy}\,,f_{yy}$
- The order of a differential equation (ODE or PDE) is determined by the highest derivative in the equation
- A first-order differential equation may be written in differential form:

$$M(x,y)\,dx+H(x,y)\,dy=0$$

- Normal Form – The result of finding an equation where the highest-order derivative is set equal to all else:

$$\frac{d^ny}{dx^n}=f(x,y,y',\ldots,y^{(n-1)})$$

For more common derivatives this looks as follows:

$$\frac{dy}{dx} = f(x,y) \quad \frac{d^2y}{dx^2} = f(x,y,y')$$

- An ordinary differential equation is said to be linear if F is linear in y, y',...,y^n. This means that the nth-order ODE is linear in this form:

$$a_n(x)y^n + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y' + a_0(x)y - g(x) = 0$$

- A nonlinear ordinary differential equation is simply one that is not linear
- Two important special cases of linear ODEs are as follows:

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \text{ and } a_2(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_0(x) = g(x)$$

- Any function ϕ, defined on an interval I, and possessing at least n derivatives that are continuous on I, which when substituted into an nth-order ODE reduces the equation to an identity, is said to be a solution of the equation on the interval
- The interval upon which a solution exists may be called many names: interval of definition, interval of existence, interval of validity, or domain of the solution
- Trivial Solution $-$ A solution of a differential equation that is identically zero on an interval I
- The graph of a solution ϕ of an ODE is called a solution curve. Because of ϕ's differentiability, it is continuous. This means that the domain of ϕ may have a domain bigger or smaller than that of the domain of the solution
- A relation $G(x,y) = 0$ is said to be an implicit solution of an ordinary differential equation on an interval I, provided that there exists at least one function ϕ that satisfies the relation as well as the differential equation on I
- A solution of $F(x,y,y') = 0$ containing a constant c is a set of solutions $G(x,y,c) = 0$ called a one-parameter family of solutions
- When solving an nth-order differential equation, we seek an n-parameter family of solutions
- A solution of a differential equation that is free of parameters is called a particular solution
- A solution that is not member of a family of solutions, and, therefore, cannot be obtained by specializing any of the parameters is called a singular solution