Homogeneous Linear Equations with Constant Coefficients

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October 5, 2020

- Given $ay' + by = 0$, we can solve the differential equation by using something close to a guess by using $y = e^{mx}$ (1)

$$\begin{aligned} ay' + by &= 0 \\ y = e^{mx},\ y' &= me^{mx} \\ e^{mx}(am + b) &= 0 \\ m &= \frac{-b}{a} \\ y &= c_1 e^{\frac{-b}{a}} \end{aligned} \tag{1}$$

- For the differential equation $ay'' + by' + cy = 0$, we may use the following method (2)

$$\begin{aligned} ay'' + by' + cy &= 0 \\ y = e^{mx},\ y' = me^{mx},\ y'' &= m^2 e^{mx} \\ e^{mx}(am^2 + bm + c) &= 0 \\ am^2 + bm + c &= 0 \end{aligned} \tag{2}$$

- If the quadratic obtained in (2) has no real (and therefore imaginary) solutions, Euler's formula is used (3)

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{3}$$

- Two important equations to know are (4)

$$\begin{aligned} &y'' + k^2 y = 0 \\ &y'' - k^2 y = 0 \\ &m^2 + k^2 = 0 \quad m^2 - k^2 = 0 \end{aligned} \tag{4}$$

- For higher order equations (5) a same solution could be applied (6)

$$a_n m^n + a_{n-1} m^{n-1} + \cdots + a_2 m^2 + a_1 m + a_0 = 0 \tag{5}$$

$$y = c_1 e_1^m x + c_2 e_2^m x + \cdots + c_n e_n^m x \tag{6}$$