Differential Equations – Exam Two

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November 2, 2020

$$\begin{aligned} f_1'(x) &= e^{x+2} \\ f_2'(x) &= e^{x-3} \\ W &= \begin{vmatrix} e^{x+2} & e^{x-3} \\ e^{x+2} & e^{x-3} \end{vmatrix} \\ \left(e^{x+2}\right) e^{x-3} - \left(e^{x-3}\right) e^{x-2} &= 0 \end{aligned} \tag{1}$$

The Wronskian may be used to determine dependence.
In this case, it equals zero, signifying linear dependence

$$\begin{aligned} y_2(x) &= y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y_1(x))^2}\,dx \\ y_1(x) &= x^4 \\ P(x) &= \frac{-7}{x} \\ e^{\int \frac{7}{x}\,dx} &= x^7 \\ y_2(x) &= x^4 \left(\int \frac{x^7}{(x^4)^2}\,dx \right) \\ x^4 \left(\int \frac{1}{x}\,dx \right) &= x^4 \ln(x) \end{aligned} \tag{2}$$

$$\begin{aligned} y^{(4)} + y''' + y'' &= 0 \\ m^4 + m^3 + m^2 &= 0 \\ m^2(m^2 + m + 1) &= 0 \\ m &= 0, 0, -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i \\ y(x) = y_c(x) + y_p(x) = c_1 + c_2 x + e^{-\frac{1}{2}x} &\left(c_3 \sin \frac{\sqrt{3}}{2} x + c_4 \cos \frac{\sqrt{3}}{2} x \right) \end{aligned} \tag{3}$$

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\begin{aligned}
y'' + 2y' &= 2x + 5 - e^{-2x} \\
m^2 + 2m &= 0 \\
m(m+2) &= 0 \\
m &= 0, -2 \\
y_c(x) &= c_1 + c_2 e^{-2x} \\
y_p(x) &= (Ax^2 + Bx) - Cxe^{-2x} \\
y_p(x)' &= 2Ax + B - Ce^{-2x} + 2Cxe^{-2x} \\
y_p(x)'' &= 2A + 4Ce^{-2x} - 4Cxe^{-2x} \\
2A + 2Ce^{-2x} + 4Ax + 2B &= 2x + 5 - e^{-2x} \\
4A = 2 \rightarrow A &= \frac{1}{2} \\
2A + 2B = 5 \rightarrow B &= 2 \\
2C = -1 \rightarrow C &= -\frac{1}{2} \\
y_p(x) &= \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x} \\
y(x) &= y_p(x) + y_c(x) \\
y(x) &= c_1 + c_2 e^{-2x} + \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}
\end{aligned} \tag{4}
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\begin{gathered}
(2 - e^x)^2 = 4 - 4e^x + e^{2x} \\
\text{Annihilator: } D(D-1)(D-2)
\end{gathered} \tag{5}
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\begin{aligned}
y'' + 3y' + 2y &= \sin(e^x) \\
m^2 + 3m + 2 &= 0 \\
(m+1)(m+2) &= 0 \\
m &= -1, -2 \\
y_c(x) &= c_1 e^{-x} + c_2 e^{-2x} \\
y_1(x) = e^{-x}; y_2(x) &= e^{-2x} \\
f(x) &= \sin(e^x) \\
W &= \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} \\
&= -e^{-3x} \\
u_1(x) &= -\int \left(\frac{y_2(t) f(t)}{W(t)} \right) dx \\
-\int \left(\frac{e^{-2x} \sin(e^x)}{-e^{-3x}} \right) dx &= \int e^x \sin(e^x) \\
u = e^x \rightarrow &\int \sin(u)\, du \\
u_1(x) &= -\cos(e^x) \\
u_2(x) &= \int \left(\frac{y_1(t) f(t)}{W(t)} \right) dx \\
-\int \left(\frac{e^{-x} \sin(e^x)}{e^{-3x}} \right) dx &= -\int e^{2x} \sin(e^x) \\
u = e^x \rightarrow &-\int u \sin(u)\, du \\
u_2(x) &= e^x \cos(e^x) - \sin(e^x) \\
y(x) = c_1 e^{-x} + c_2 e^{-2x} + e^{-x}(-\cos(e^x)) &+ e^{-2x}(e^x \cos(e^x) - \sin(e^x))
\end{aligned} \tag{6}
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\begin{aligned}
(D+3)D^2 x - D(D+3)y &= 1 + 3t \\
(D+3)D^2 x + D^2(D+3)y &= 0 \\
(-D^3 - 4D^2 - 3D)y &= 0 \\
m^3 + 4m^2 + 3m &= 0 \\
m(m^2 + 4m + 3) &= 0 \\
m &= 0, -1, -3 \\
y(x) &= c_1 + c_2 e^{-x} + c_3 e^{-3x}
\end{aligned} \tag{7}
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\begin{aligned}
F &= kx \\
k &= 2 \\
8 &= m \cdot 32 \\
m &= \frac{1}{4} \\
x(0) = 0&; x'(0) = 5 \\
D^2x + 4\sqrt{2}Dx + 8x &= 0 \\
\lambda &= 2\sqrt{2} \\
\omega &= \sqrt{8} \\
\lambda^2 - \omega^2 &= 0 \\
x(t) &= e^{-2\sqrt{2}t}(c_1 + c_2 t) \\
c_1 &= 0 \\
x(t) &= c_2 t e^{-2\sqrt{2}t} \\
x'(t) &= c_2 e^{-2\sqrt{2}t} - 2\sqrt{2}c_2 t e^{-2\sqrt{2}t} \\
c_2 &= 5 \\
x(t) &= 5te^{-2\sqrt{2}t} \\
x'(t) &= 5e^{-2\sqrt{2}t} - 10\sqrt{2}te^{-2\sqrt{2}t} \\
5e^{-2\sqrt{2}t} - 10\sqrt{2}te^{-2\sqrt{2}t} &= 0 \\
5e^{-2\sqrt{2}t}(1 - 2\sqrt{2}t)& \\
t &= \frac{1}{2\sqrt{2}} \\
x(\frac{1}{2\sqrt{2}}) &= .65
\end{aligned}
\tag{8}
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\begin{aligned}
L &= \frac{\pi}{2} \\
\lambda_n &= \left(\frac{(2n-1)\pi}{2L}\right)^2 \\
\lambda_n &= \left(\frac{(2n-1)^2\cancel{\pi^2}}{\cancel{\pi^2}}\right) \\
\lambda_n &= (2n-1)^2 \\
y_n(x) &= \sin\left((2n-1)x\right)
\end{aligned}
\tag{9}
$$

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\begin{aligned}
y &= \sum_{n=0} c_n x^n \\
y' &= \sum_{n=1} n c_n x^{n-1} \\
y'' &= \sum_{n=2} n(n-1) c_n x^{n-2} \\
\sum_{n=2} n(n-1) c_n x^{n-2} &+ \sum_{n=0} c_n x^{n+2} \\
\sum_{k=0} [(k+2)(k+1)c_{k+2} &+ c_k] x^k \\
y_1(x) &= c_0 - \frac{c_0}{2}x^2 + \frac{c_0}{24}x^4 \dots \\
y_2(x) &= c_1 x - \frac{c_1}{6}x^3 + \frac{c_1}{120}x^5 \dots
\end{aligned}
\tag{10}
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\begin{aligned}
q_p(t) &= ? \\
D^2 q + 2Dq + 4q &= 50\cos t \\
m^2 + 2m + 4 &= 0 \\
m &= -1 \pm \sqrt{3}i \\
\text{Terms do not coincide with } y_p(x) \\
q_p(x) &= A\cos t + B\sin t \\
q_p(x)' &= -A\sin t + B\cos t \\
q_p(x)'' &= -A\cos t - B\sin t \\
(3A + 2B)\cos(t) + (3B - 2A)\sin(t) &= 50\cos(t) \\
(3A + 2B) &= 50 \\
(3B - 2A) &= 0 \\
A &= \frac{150}{13} \\
B &= \frac{100}{13} \\
q_p(t) &= \frac{150}{13}\cos t + \frac{100}{13}\sin t
\end{aligned}
\tag{11}
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