Modeling with Systems of First-Order DEs

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- Say we are given two populations which interact, $x(t)$ and $y(t)$. The two differential equations (1) can be used to model population growth. A **linear system** would be of form (2), where c_i could depend on t. Any model of another form is said to be nonlinear.

$$\frac{dx}{dt}\frac{dy}{dt} \tag{1}$$

$$\begin{aligned} g_1(t,x,y) &= c_1x + c_2y + f_1(t) \\ g_2(t,x,y) &= c_3x + c_4y + f_2(t) \end{aligned} \tag{2}$$

- Given different elements with radioactive decay, where y is gaining atoms from decay of x and itself decaying, it depends on x and y the differential equations (3)

$$\begin{aligned} \frac{dx}{dt} &= -\lambda_1 x \\ \frac{dy}{dt} &= \lambda_1 x - \lambda_2 y \\ \frac{dz}{dt} &= \lambda_2 y \end{aligned} \tag{3}$$

- Say $x(t)$ and $y(t)$ are fox and rabbit populations, respectively. The model for the fox population, without rabbits, may be found using equation 4 (4). If there are rabbits in the system, there could be a better model, equation 5 (5). The rabbit population without any foxes present may be like (7). The rabbit population with foxes in the system would be (8). This is known as the **Lotka-Volterra predator-prey model**.

$$\frac{dx}{dt} = -ax \tag{4}$$

$$\frac{dx}{dt} = -ax + bxy \tag{5}$$

$$\frac{dy}{dt} = dy \tag{6}$$

$$\frac{dy}{dt} = dy - cxy \tag{7}$$

- Say there are two species which are competing for some kind of condition. If the two species are in isolated environments, their population rates are modeled by (8) and (9), respectively. If the two are present in the same system, their population rates may be modeled by (10) and (11), respectively. If we assume that these species compete proportionally to each other, then the best model is (12) and (13). Also, recalling back to Calculus II (or BC), a logistic curve is modeled by (14) and (15). Combining what we know, the logistic curve and predator-prey model, we get the **competition model**, (16) and (17).

$$\frac{dx}{dt} = ax \tag{8}$$

$$\frac{dy}{dt} = cy \tag{9}$$

$$\frac{dx}{dt} = ax - by \tag{10}$$

$$\frac{dy}{dt} = cy - dx \tag{11}$$

$$\frac{dx}{dt} = a_1x - b_1xy \tag{12}$$

$$\frac{dy}{dt} = c_1y + d_1xy \tag{13}$$

$$\frac{dx}{dt} = ax - bx^2 \tag{14}$$

$$\frac{dy}{dt} = cy - dy^2 \tag{15}$$

$$\begin{aligned} \frac{dx}{dt} &= a_1x - b_1x^2 - c_1xy \\ & x(a_1 - b_1x - c_1y) \end{aligned} \tag{16}$$

$$\begin{aligned} \frac{dy}{dt} &= a_2y - b_2y^2 - c_2xy \\ &= y(a_2 - b_2y - c_2x) \end{aligned} \tag{17}$$