Euler Methods and Error Analysis

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- The backbone of Euler's Method is formula (1)

$$y_{n+1} = y_n + hf(x_n, y_n) \tag{1}$$

- Round-off Error − This error occurs when an object doing the calculating has an error due to the finite number of digits it displays.
- Truncation Error − An error that occurs at each step, and, therefore, carries over to the next step, until the calculator reaches a (total) truncation error.
  - At a certain step, it is called <u>local</u> truncation error, while, over the whole scope, it is called global truncation error.
- When using Euler's method, the local truncation error can be found by using the formula for a Taylor's Series (2), where $\frac{h^2}{2!}$ is known as the local truncation error.

$$y(x_{n+1}) = y_n + hf(x_n, y_n) + y''(c)\frac{h^2}{2!} \tag{2}$$

- If $e(h)$ denotes the error in a numerical calculation depending on h then $e(h)$ is said to be of order h^n, denoted by $O(h^n)$, if there exists a constant C and a positive integer n such that $|e(h)| \leq Ch^n$ for h sufficiently small.
- The improved Euler's Method is defined as (3)

$$\begin{aligned} y_{n+1} = y_n + h\frac{f(x_n, y_n) + f(x_{n+1}, y^*_{n-1})}{2} \\ y^*_{n+1} = y_n + hf(x_n, y_n) \end{aligned} \tag{3}$$

- The improved Euler's Method is an example of a predictor-corrector method. The value of y^*_{n+1} given by (3) predicts a value of $y(x_n)$, whereas the value of y_{n+1} defined by formula (3) corrects this estimate.
- The local truncation error for the improved Euler's method is $O(h^3)$. The global truncation error, therefore, is $O(h^2)$