The Dirac Delta Function

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- The Dirac Delta "Function" may be modeled by (1)

$$\delta(t-t_0)=\begin{cases}0, & 0\leq t<t_0-a\\ \frac{1}{2a}, & t_0-a\leq t<t_0+a\\ 0, & t\geq t_0+a\end{cases} \quad (1)$$

- This function could serve as a model for a big force exerted over little time (an impulse)
- The impulse (force over time), or the area under the graph of the force always equals 1. This means that, for a shorter period of time, the force is greater
- The function $\delta_a(t-t_0)$ is called the unit impulse, or the Dirac Delta Function. The limit that approximates this is defined as (2)

$$\delta(t-t_0)=\lim_{a\to 0}\delta_a(t-t_0) \quad (2)$$

- The Laplace transform is defined as (3)

$$\mathcal{L}\left\{\delta(t-t_0)\right\}=e^{-st_0} \quad (3)$$

- The integral of any function multiplied by the Dirac delta function is the function evaluated at the point t_0, or (4)

$$\int_a^b f(t)\delta(t-t_0)\,dt=f(t_0) \quad (4)$$