Fourier Cosine and Sine Series

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- The definition of an even and odd function is defined in (1)

$$\begin{aligned} \text{Even if } f(-x) = f(x) \\ \text{Odd if } f(-x) = -f(x) \end{aligned} \tag{1}$$

- Some properties are:

1. The product of two even functions is even.
2. The product of two odd functions is even.
3. The product of an even function and an odd function is odd.
4. The sum (difference) of two even functions is even.
5. The sum (difference) of two odd functions is odd.
6. If f is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$
7. If f is odd, then $\int_{-a}^{a} f(x)\,dx = 0$

- The Fourier Series of an even function f defined on the interval $(-p, p)$ is the cosine series (2)

$$\begin{aligned} f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{p}x \\ a_0 = \frac{2}{p}\int_0^p f(x)\,dx \\ a_n = \frac{2}{p}\int_0^p f(x)\left(\cos\frac{n\pi}{p}x\right)dx \end{aligned} \tag{2}$$

- The Fourier Series of an odd function f defined on the interval $(-p, p)$ is the sine series (3)

$$\begin{aligned} f(x) &= \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{p} x \\ b_n &= \frac{2}{p} \int_0^p f(x) \left(\sin \frac{n\pi}{p} x \right) dx \end{aligned} \tag{3}$$

- Gibbs Phenomenon applies to points near discontinuities.
- Half Range Expansions – May be used to express f on interval $0 < x < L$ by using a "dummy" function setup.

  1. Even Reflection – Across y-axis, and then create a cosine series
  2. Odd Reflection – Across origin, and then create a sine series
  3. Periodic – Transform the function so it is periodic, then create a Fourier Series