Inverse Transforms and Transforms of Derivatives

Michael Brodskiy
Professor: Meetal Shah

November 18, 2020

- Some known values for inverse Laplace transforms, $\mathcal{L}^{-1}\{f(t)\}$, are shown:

$$\begin{gathered} 1 = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} \\ t^n = \mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} \qquad e^{at} = \mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} \\ \sin kt = \mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} \qquad \cos kt = \mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} \\ \sinh kt = \mathcal{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} \qquad \cosh kt = \mathcal{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} \end{gathered} \tag{1}$$

- If $f, f', \ldots, f^{n-1}$ are continuous on $[0, \infty)$, and are of exponential order and if $f^{(n)}(t)$ is piecewise continuous on $[0, \infty)$, then (2) where $F(s) = \mathcal{L}\{f(t)\}$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0) \tag{2}$$