Differential Equations – Exam One

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$$\frac{d^2R}{dt^2} = -\frac{k}{R^2} \tag{1}$$

The differential equation is NON-linear

The differential equation is an ODE

$$\int \frac{1}{y^{.5}}\,dy = \int x^{.5}\,dx$$

$$.5y^{.5} = \frac{2}{3}x^{1.5} + C \tag{2}$$

This function has a solution where: $x > 0, y > 0$

$$\frac{dx}{dt} = kxn - kx^2 \tag{3}$$

$$\begin{aligned} 10 + 3y - y^2 &= 0 \\ (5-y)(y+2) &= 0 \\ y &= 5, -2 \\ y = 5 \text{ is stable } y = -2 &\text{ is unstable} \end{aligned} \tag{4}$$

$$\begin{aligned} \int (2y-2)\,dy &= \int 3x^2 + 4x + 2\,dx \\ y^2 - 2y &= x^3 + 2x^2 + 2x + C \\ C &= 8 - 5 = 3 \\ y^2 - 2y &= x^3 + 2x^2 + 2x + 3 \\ \text{This function is defined in: } &(-\infty, \infty) \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{dx}{dy} - \frac{x}{y} &= 2y^2 \\ I = e^{-\int \frac{1}{y}} &= \frac{1}{y} \\ \int \left(\frac{x}{y}\right)' dx &= \int (2y)\ dy \\ \frac{x}{y} &= y^2 + C \\ C &= -\frac{124}{5} \\ \frac{x}{y} &= y^2 - \frac{124}{5} \\ \text{This function is defined in: } &y \neq 0 \end{aligned} \tag{6}$$

$$\begin{aligned} (xy + y^2 + y)\,dx + (x+2y)\,dy &= 0 \\ \text{The function is exact} & \\ \int (x+2y)\,dy &= xy + y^2 + h(x) \\ \cancel{y} + h'(x) &= xy + y^2 + \cancel{y} \\ h(x) &= \frac{x^2 y}{2} + xy^2 \\ xy + y^2 + xy^2 + \frac{x^2 y}{2} &= C \end{aligned} \tag{7}$$

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\begin{aligned}
(x^2+2y^2)\frac{dx}{dy} &= xy, x = vy \\
v + y\frac{dv}{dy} &= \frac{v}{v^2+2} \\
y\frac{dv}{dy} &= \frac{-v^3-v}{v^2+2} \\
\frac{v^2+2}{-v^3-v}\,dv &= \frac{1}{y}\,dy \\
-\int \frac{v}{v^2+1} + \frac{2}{v^3+v}\,dv &= \int \frac{1}{y} \\
\frac{1}{2}\ln(|(\frac{x}{y})^2+1|) - 2\ln(|(\frac{x}{y})|) &= \ln(|y|) + C \\
C &= \frac{1}{2}\ln(2) \\
\frac{1}{2}\ln(|(\frac{x}{y})^2+1|) - 2\ln(|(\frac{x}{y})|) &= \ln(|y|) + \frac{1}{2}\ln(2)
\end{aligned} \tag{8}
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\begin{aligned}
e(x,y) &= y + .2xy \\
e(1,1) &= 1.2 \\
e(1.1,1.2) &= 1.464 \\
e(1.2,1.464) &= 1.81536 \\
e(1.3,1.81536) &= 2.28735 \\
e(1.4,2.28735) &= 2.92781 \\
y(1.5) &\approx 2.92781
\end{aligned} \tag{9}
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\begin{aligned}
\frac{dx_1}{dt} &= \frac{x_2}{50} - \frac{3x_1}{50} \\
\frac{dx_2}{dt} &= \frac{3x_1}{50} - \frac{7x_2}{100} + \frac{x_3}{100} \\
\frac{dx_3}{dt} &= \frac{x_2}{20} - \frac{x_3}{20}
\end{aligned} \tag{10}
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\begin{aligned}
y' + \left(\frac{1+x}{x}\right)y &= \frac{\sin(2x)}{e^x \cdot x} \\
I &= e^{ln(|x|)+x} = xe^x \\
\int (xe^x y)'\,dy &= \int \sin(2x)\,dx \\
xe^x y &= \frac{-\cos(2x)}{2} \\
\text{The biggest interval is: } x &= (0,\infty)
\end{aligned} \tag{11}
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