

# Potentials and Fields

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- The energy density of a field may be written as:

$$U = \frac{1}{2} \left( \epsilon_o \vec{E}^2 + \frac{1}{\mu_o} \vec{B}^2 \right)$$

- The Poynting Vector may be defined as:

$$\vec{S} = \frac{1}{\mu_o} (\vec{E} \times \vec{B})$$

- For some volume  $\tau$ , we can write the total energy in it using:

$$\mathcal{U}_{tot} = \int_V U d\tau$$

- The Poynting Vector represents the flow of energy out of the surface

$$\vec{\nabla} \cdot \vec{S} = \frac{1}{\mu_o} \vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \frac{1}{\mu_o} (\vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E} \cdot (\vec{\nabla} \times \vec{B}))$$

$$\vec{\nabla} \cdot \vec{S} = -\frac{\partial}{\partial t} \left( \frac{\epsilon_o \vec{E}^2}{2} + \frac{\vec{B}^2}{2\mu_o} \right)$$

- This can be simplified to attain:

$$\vec{\nabla} \cdot \vec{S} + \frac{\partial U}{\partial t} = 0$$

- A similar equation can be seen in conservation of charge:

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

- Thus, we have obtained a conservation of energy formula