Magnetic Fields in Matter

Michael Brodskiy
Professor: D. Wood

November 1, 2023

- Torque on a Dipole

$$\vec{N} = \vec{m} \times \vec{B}$$

  - This is akin to the torque on an electric dipole:

$$\vec{N} = \vec{p} \times \vec{E}$$

  - We can calculate the torque to be:

$$\vec{N} = m\vec{B}\sin(\theta)$$

  - The energy can be defined as:

$$U = -\vec{m} \cdot \vec{B}$$

  - The bound bulk and surface currents can be defined as:

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \quad \text{and} \quad \vec{K}_b = \vec{M} \times \hat{\mathbf{n}}$$

  - For a uniform magnetized sphere:

$$\vec{J}_b = 0 \quad \text{and} \quad \vec{K}_b = M\sin(\theta)\hat{\phi}$$

  -

$$\vec{A}(r) = \frac{\mu_o}{4\pi}\int \frac{\vec{J}_b}{R}\,d\tau + \frac{\mu_o}{4\pi}\int \frac{\vec{K}_b}{R}\,da$$

- Auxiliary Field ($\vec{H}$)

$$\vec{H} = \frac{1}{\mu_o}\vec{B} - \vec{M} \Rightarrow \vec{B} = \mu(\vec{H} + \vec{M})$$

- From this formula, we can get:

$$\vec{\nabla} \times \vec{H} = \frac{\vec{\nabla} \times \vec{B}}{\mu_o} - \vec{\nabla} \times \vec{M}$$

- Which can become:

$$\vec{\nabla} \times \vec{H} = \vec{J} - \vec{J}_b = \vec{J}_F$$

- Linear Materials

$$\vec{M} = \chi_m \vec{H}$$

  - This is not quite the same as the $\vec{E}$ case, where $\vec{p} = \varepsilon_o \chi_e \vec{E}$

$$\vec{B} = \mu_o(\vec{H} + \vec{M}) = \mu_o(1 + \chi_m)\vec{H}$$
$$\mu = \mu_o(1 + \chi_m)$$

  - Unlike dielectrics, χ_m could be either positive or negative
  - Paramagnetism signifies $\chi_m > 0$, more exactly $10^{-6} \leq \chi_m \leq 10^{-1}$, which means $\vec{m}$ aligns with $\vec{B}$
  - Diamagnetic materials signify that $-10^{-9} \leq \chi_m \leq -10^{-4}$
  - Ferromagnetism signifies that the domains of magnetic dipoles align with an external magnetic field, which strengthens the field
    * Hysteresis causes the magnetic field to “lag behind” in a ferromagnetic, even when the inducing magnet/magnetic field is gone