# Magnetostatics

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October 18, 2023

- Magnetostatics:

$$\vec{F}_{lorentz} = q\vec{v} \times \vec{B}$$

  - This is "static" in the sense of steady flow of magnetic field
  - The units of $\vec{B}$ are Teslas [T]
  - In the simple case where $\vec{B} = B_o\hat{\mathbf{z}}$ and $v_z = 0$, the force could be described as:

$$|\vec{F}| = qvB$$

  - With inward direction. We can then write:

$$qvB = \frac{mv^2}{R}$$
$$qB = \frac{mv}{R}$$
$$qB = \frac{p}{R}$$
$$R = \frac{p}{qB}$$

  - The frequency of rotation may be written:

$$\omega = \frac{qB}{m}$$

  - Total force may be written as:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = q(\vec{E} + \vec{v} \times \vec{B})$$

    * Special Case: $\vec{F} = 0$

$$\vec{E} + \vec{v} \times \vec{B} = 0$$

* Given $\vec{E} = E_o\hat{\mathbf{z}}$ and $\vec{B} = B_o\hat{\mathbf{x}}$, we would get:

$$E_o\hat{\mathbf{z}} + v_z B_o\hat{\mathbf{y}} - v_y B_o\hat{\mathbf{z}} = 0$$

* From this, we get $v_z = 0$ and $v_y = \frac{E_o}{B_o}$
* This can be used to construct a velocity selector
* In the event that $\vec{E} \parallel \vec{B}$, an expand or contracting helix about the fields as a pole would be constructed

- We now go back to a general case: $\vec{E} \perp \vec{B}$, $\vec{F}_{net} \neq 0$, $\vec{E} = E\hat{\mathbf{z}}$, and $\vec{B} = B\hat{\mathbf{x}}$

$$\frac{1}{m}\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

* The solutions for this would look as follows:

$$y(t) = c_1\cos(\omega t) + c_2\sin(\omega t) + \frac{E}{B}t + c_3$$
$$z(t) = c_2\cos(\omega t) + c_1\sin(\omega t) + c_4$$
$$\omega = \frac{qB}{m}$$

* This shape is known as a cycloid
* Special case: starting from rest at (0,0); this would give us:

$$c_1 = 0 \qquad c_2 = -\frac{E}{\omega B} \qquad c_3 = -c_1 \qquad c_4 = -c_2$$
$$y(t) = -\frac{E}{\omega B}\sin(\omega t) + \frac{E}{B}t = R(\omega t - \sin(\omega t))$$
$$z(t) = R(1 - \cos(\omega t))$$
$$\vec{v}_{cent} = \left(\frac{\vec{E} \times \vec{B}}{B^2}\right)$$

* Work:

$$dW = \vec{F} \cdot d\vec{l} = (q\vec{v} \times \vec{B}) \cdot \vec{v}\,dt = q\,dt(\vec{v} \times \vec{B}) \cdot \vec{v} = 0$$

* Critical note: magnetic fields <u>do no work</u>
* Magnetic fields may induce electric fields to do work, but <u>do not do work themselves</u>

- Continuous Systems
  - Given a wire in space, with a bit of charge, $dq$, moving with velocity $\vec{v}$ shaped in rectangle, and placed in a magnetic field $\vec{B} = \frac{A}{z}\hat{\mathbf{x}}$ :

$$dq\,\vec{v} = I\,d\vec{l}$$
$$d\vec{F} = I\,d\vec{l} \times \vec{B}$$
$$\vec{F}_{mag} = \int I\,d\vec{l} \times \vec{B}$$

– Solving would give us:

$$F_{tot} = -IL\left(\frac{A}{a}\right)\hat{\mathbf{z}} + \hat{\mathbf{y}}\int_a^b \frac{A}{z}\,dz + \hat{\mathbf{z}}IL\left(\frac{A}{b}\right) - \hat{\mathbf{y}}I\int_a^b \frac{A}{z}\,dz$$
$$= -\hat{\mathbf{z}}ILA\left(\frac{1}{a} - \frac{1}{b}\right)$$

– There are several types of densities:

$$\text{Linear:}\quad I\,d\vec{l}$$
$$\text{Surface:}\quad \vec{K}\,da$$
$$\text{Bulk:}\quad \vec{J}\,d\tau$$