Homework 5

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1. Four point charges are located a distance a from the origin on the y and z axes as shown. Find the approximate expression for the electric **potential** far from the charges. Use spherical coordinates and retain the only the **first** non-vanishing terms in the multipole expansion. [Hint: consider breaking down the distribution into a superposition of individual dipoles.]

2. Three point charges are located a distance a from the origin on the y and z axes as shown. Find the approximate expression for the electric **field** far from the charges. Use spherical coordinates and retain the **first two** non-vanishing terms in the multipole expansion.

3. For the charged spherical shell in Problem 4 of Assignment 4 (the one with $V(R,\theta) = V_o\sin^2(\theta)$ and $\sigma = \frac{V_o\varepsilon_o}{3R}(7 - 15\cos^2(\theta))$, find the monopole and dipole moments.

4. A thin rod on the z-axis goes from $z = -a$to $z = +a$ and carries a linear charge density of $\lambda(z)$. Find the leading term in the multipole expansion for:

   (a) $\lambda(z) = \lambda_o\cos\left(\frac{\pi z}{a}\right)$
   We can write the formula for the multipole expansion as:

$$V = \frac{1}{4\pi\varepsilon_o}\sum_{n=0}^{\infty}\frac{P_n(\cos(\theta))}{r^{n+1}}\int_{-a}^{a} z^n\lambda(z)\,dz$$

   For the $n = 0$ case, we can write:

$$V = \frac{1}{4\pi\varepsilon_o r}\int_{-a}^{a}\lambda(z)\,dz$$

   From this, we can see that the integral expression would evaluate to zero, meaning we have to try the next term. At $n = 1$, we get:

$$V = \frac{\cos(\theta)}{4\pi\varepsilon_o r^2}\int_{-a}^{a} z\lambda(z)\,dz$$

$$V = \frac{\lambda_o \cos(\theta)}{4\pi\varepsilon_o r^2} \int_{-a}^{a} z \cos\left(\frac{\pi z}{a}\right)\, dz$$

$$V = \frac{\lambda_o \cos(\theta)}{4\pi\varepsilon_o r^2} \left(\underbrace{\cancel{\frac{a}{\pi} z \sin\left(\frac{\pi z}{a}\right)}}_{0} + \frac{a^2}{\pi^2} \cos\left(\frac{\pi z}{a}\right) \right) \Bigg|_{-a}^{a}$$

$$V = \frac{\lambda_o \cos(\theta)}{4\pi\varepsilon_o r^2} \left(\frac{2a^2}{\pi^2}\right)$$

Thus, we see from the first non-zero term multipole expansion, we get:

$$\boxed{V(r,\theta) \approx \frac{2\lambda_o \cos(\theta) a^2}{4\pi^3 \varepsilon_o r^2}}$$

(b) $\lambda(z) = \lambda_1 \cos\left(\frac{\pi z}{2a}\right)$
where λ_o and λ_1 are constants.