Electric Fields in Matter

Michael Brodskiy
Professor: D. Wood

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- Hydrogen Atom
  - The charge density for a hydrogen atom would be:
  $$\rho(r) = \frac{q}{4\pi a^3} e^{-\frac{2r}{a}}$$
  when $r << a$:
  $$\rho(r) \approx \frac{q}{4\pi a^3}$$
  - Using Gauss's law, we can find the electric field:
  $$\vec{E}(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_o}$$
  $$\vec{E} = \frac{\rho r}{3\varepsilon_o}$$
  - From here, the force on the charges become:
  $$F = q\vec{E} = -\frac{q\rho r}{3\varepsilon_o}$$
- Molecule of Water
  - Since the hydrogens pull in different directions, we can describe the torque as:
  $$\vec{N} = \frac{d}{2}\sin(\theta)\vec{E}q + \frac{d}{2}\sin(\theta)\vec{E}q = d\sin(\theta)\vec{E}q$$
  $$\vec{N} = \vec{p} \times \vec{E}$$

– The net force can be described as:

$$F = q\vec{E}\left(\vec{r} + \frac{\vec{d}}{2}\right) - q\vec{E}\left(\vec{r} - \frac{\vec{d}}{2}\right)$$

$$F_x \approx q\left(E_x(\vec{r}) + \frac{\vec{d}}{2}\vec{\nabla}E_x\right) - q\left(\vec{E}(r) - \frac{\vec{d}}{2}\vec{\nabla}E_x\right)$$

$$F_x = q\vec{\nabla}E_x \cdot \vec{d} = \vec{p} \cdot \vec{\nabla}E_x$$

$$\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$$

- Polarized material

$$\vec{P} = \frac{\text{dipole moment}}{\text{volume}}$$

  – Units:

$$\frac{\mathrm{C}}{\mathrm{m}^2} \quad \text{like } \sigma$$

  – Potential due to a neutral polarized object

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\hat{\mathbf{R}} \cdot P(\vec{r}\prime)}{R^2}\, d\tau'$$

$$V(\vec{r}) = -\frac{1}{4\pi\varepsilon_o}\int \frac{1}{R'}\vec{\nabla}(\vec{r}\prime)\, d\tau'$$