Homework C

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Abstract

The purpose of this document is to analyze how the voltage, and, subsequently, the electric field and force between a parallel plate capacitor may be plotted in the GNU Octave program. We assume a charge-discharge cycle for the capacitor, as indicated in the attached circuit schematic. Various oscillation periods will be tested.

Keywords: parallel plate capacitor, electric field, GNU Octave, charge-discharge, cycle

Contents

1 Preliminary Assumptions and Calculations

We may use the following schematic to design the capacitor testing circuit:

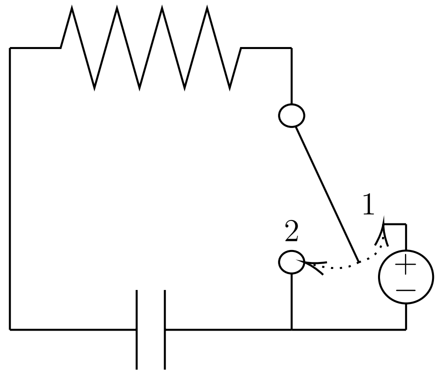

Figure 1: Circuit Schematic

The switch flips between 1 and 2 based on a periodic cycle defined by a time interval T. Let us assume that a standard 1[µF] capacitor is used, along with a 100[Ω] resistor. Furthermore, suppose that the battery has a voltage $V = 5$[V], and that, at time $t = 0$, the switch has been in position 1 for a long time, and is moved to 2. We may begin by assuming that the initial voltage is entirely across the capacitor, such that:

$$V_o = 5$$

According to discharging capacitors, we may write:

$$V = 5e^{-\frac{t}{RC}}$$

for the stage in which the capacitor is losing charge. Let us start with a case in which the capacitor discharges to 25% of its initial voltage value, and then the switch flips until fully charged (this repeats every T). This occurs when:

$$\ln(.01) = -\frac{T}{RC}$$

$$T = -(100)(1 \cdot 10^{-6}) \ln(.1)$$

$$T = .139[\text{ms}]$$

Finally, let us assume that the plates are separated by 1[mm]. Let us now proceed to the computational portion.

2 Computational Analysis

2.1 $T \to 25\%$ of Initial Voltage Discharge Time

Using the scenario laid out in the preliminary assumptions, we may begin plotting. Let us choose a time step size of $t = 10[\mu s]$ and 100 iterations:

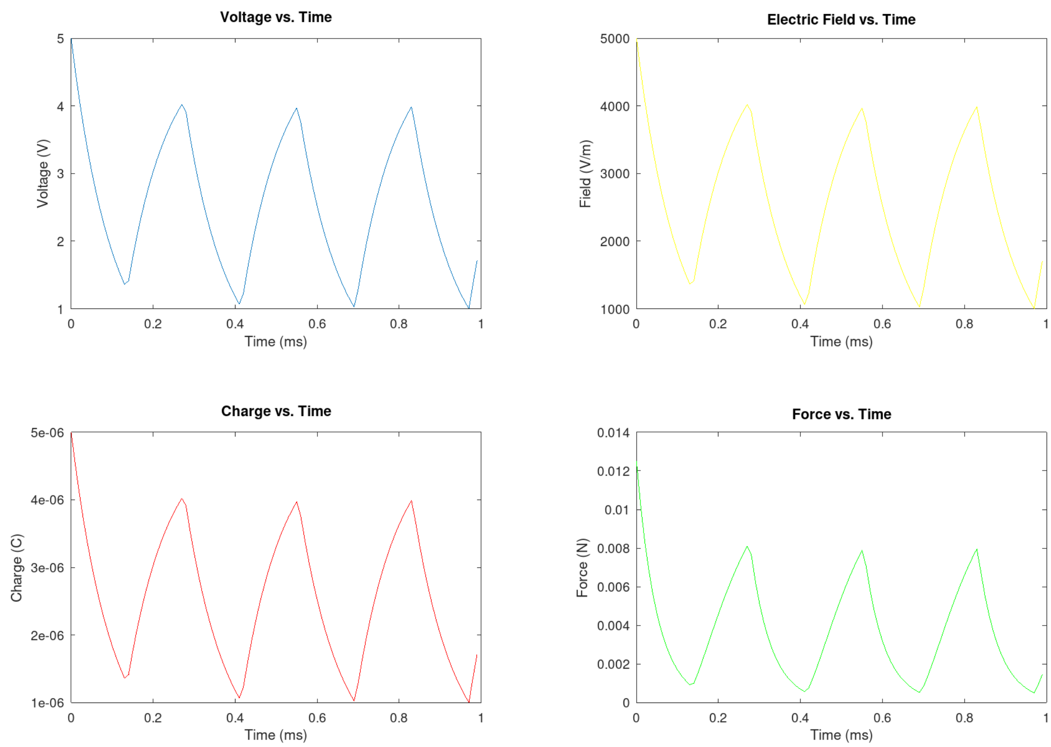

Figure 2: Period (T) is Time to Discharge to 25% Initial Voltage

As expected, we can see the capacitor discharging in the long run, as it is unable to reach its initial value on recharge. Note that we do see the initial discharge fall to 25% of the initial value ($5 \to 1.25[V]$).

2.2 $T = 20[\mu s]$ (Small Period)

Let us repeat the analysis with the same parameters specified in Section 2.1, but with a set period of $20[\mu s]$. This yields:

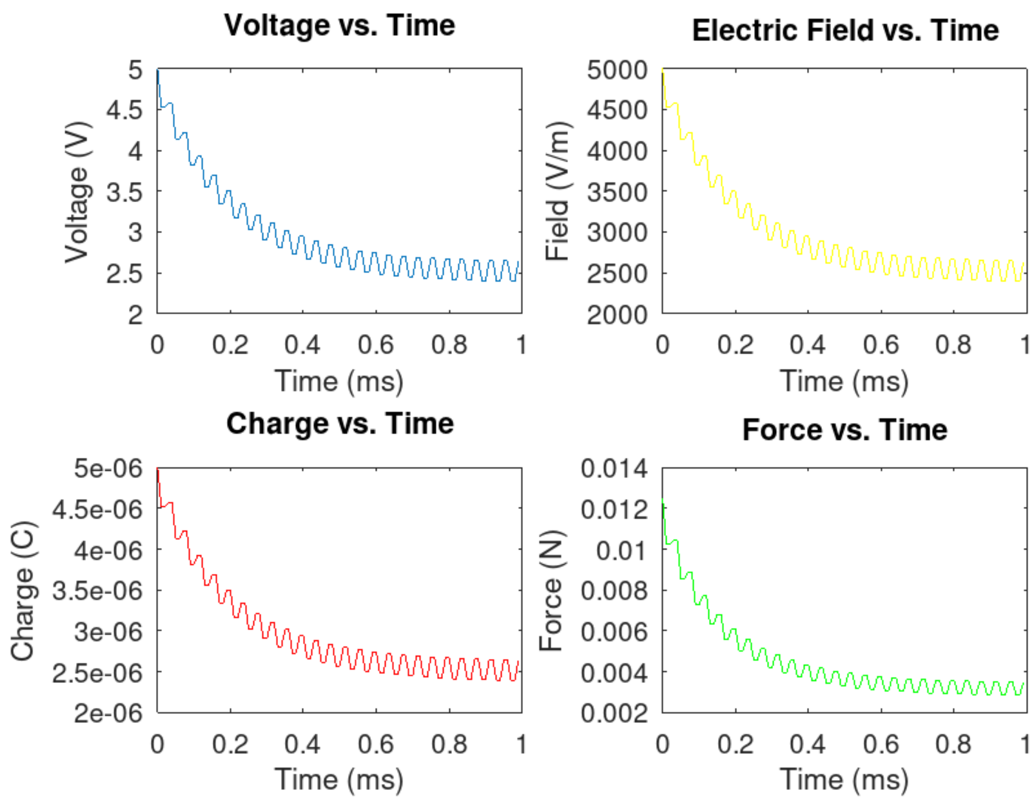

Figure 3: Period of 20[µs]

We can see that, as compared to the initial period setting, this setting has oscillations occur much more frequently, as expected.

2.3 Arbitrary T

We can also test a case in which the oscillations occur randomly. That is, each interval for charge-discharge is randomly timed. We will set reasonable time boundaries so that the random time interval rests in the range $[2 \cdot 10^{-5}, 9.2 \cdot 10^{-4}]$. The following was generated:

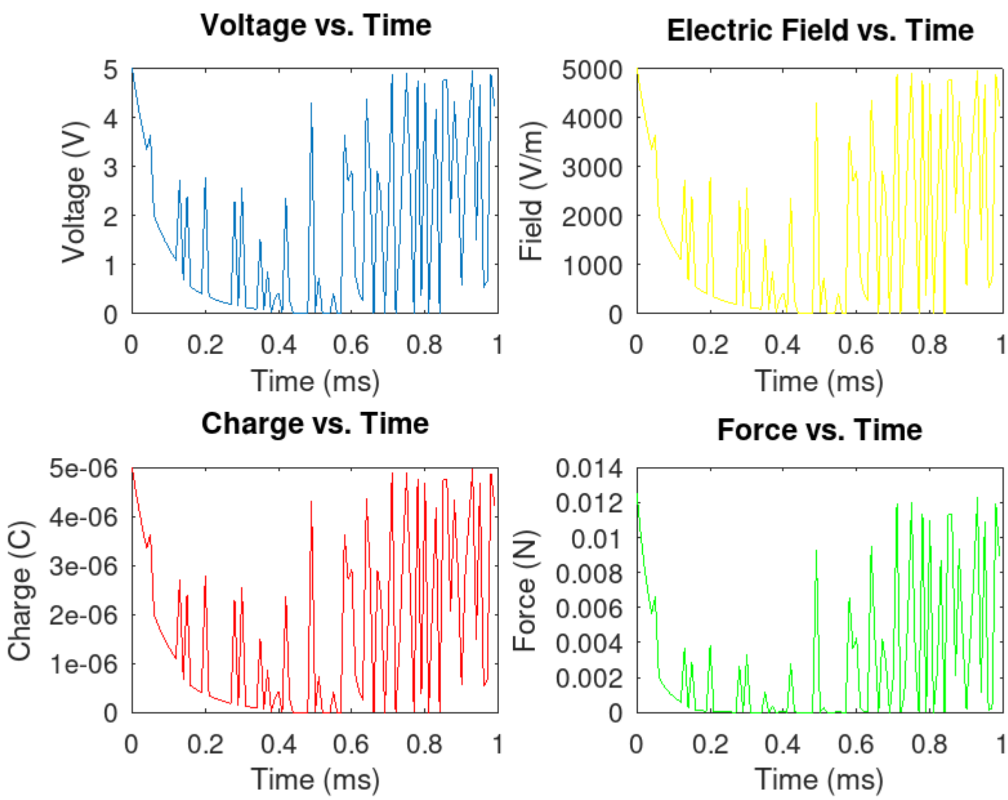

Figure 4: Random Period Value

2.4 Edge Case Check

We can confirm the accuracy of this projection by testing edge cases. Let us first see the capacitor discharge to 100% of its initial value:

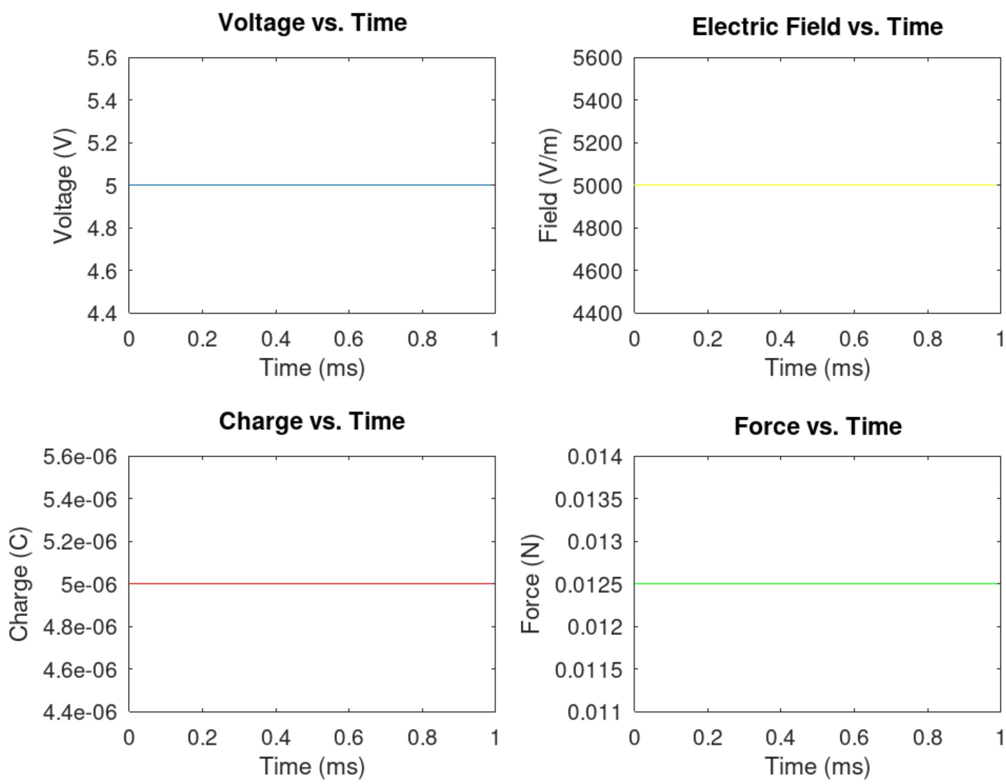

Figure 5: Discharge to 100% (No Discharge)

As expected, the switch never flips to position 2, which does not allow the capacitor to discharge. As expected, the voltage remains at its initial value. For the second edge case, we can see what happens when the capacitor discharges to 0% of its initial value (no subsequent charge). This gives us:

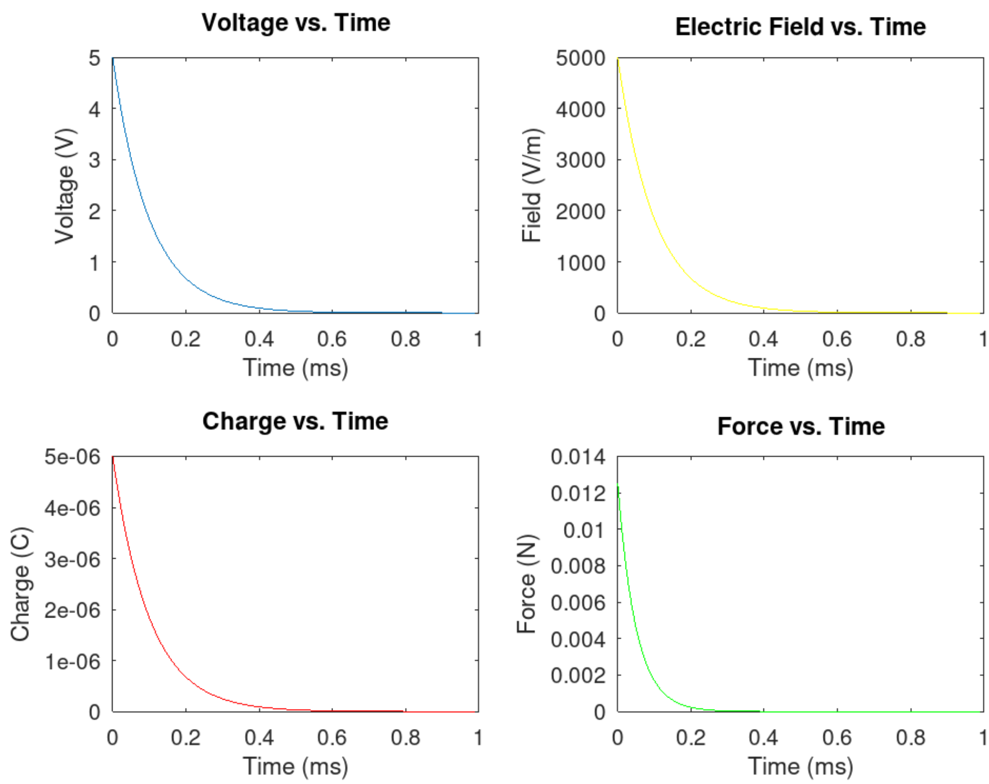

Figure 6: Discharge to 0% (No Charge After Discharge)

As expected, we can see that the capacitor simply discharges.

3 Conclusion

As such, we are successfully able to model oscillating capacitors with the attached code (see Appendix).

4 Appendix

Listing 1: GNU Octave Code

```
capacitance = 1e-06; % Set capacitance value (farads)
resistance = 100; % Set resistance value (ohms)
Vo = 5; % Set battery voltage value (volts)
Vprev = Vo; % "Previous-step" value tracker
V1 = Vo; % Swap variable used for switch position 2 (discharging)
V2 = V1; % Swap variable used for switch position 1 (charging)
iteration = 1; % Used to keep track of iterations
dt = 1e-05; % Set the time-step value for calculations
voltages = [Vo]; % Stores voltage values for later plotting
times = [0]; % Stores time values for later plotting
separation = .001; % Plate separation
dischargePercent = .25; % Used to set the period as dependent on initial charge percent

%T = -resistance * capacitance * log(dischargePercent); % Define period of switch oscillation
%T = 20e-06; % Small period case

time = dt; % Begin the time based on dt

while iteration < 100 % Define iteration quantity
  if (mod(((time - mod(time,T)) / T), 2) == 0) % Define when to flip switch (every time t exceeds period T)
    % This block is discharging
    Vprev = V1 * exp(-(time - (time - mod(time,T)))/(resistance * capacitance)); % Calculate instantaneous V
    V2 = Vprev; % Swap variable when flipping switch
  else
    % This block is charging
    Vprev = 5 - (5 - V2)*exp(-(time - (time - mod(time,T)))/(resistance * capacitance)); % Calculate instantaneous V
    V1 = Vprev; % Swap variable when flipping switch
  end
  voltages = [voltages, Vprev]; % Add values to voltage matrix
  times = [times, time]; % Add time increments
  iteration = iteration + 1; % Increase iteration count
  time = iteration * dt; % Take another time step
  T = 9e-04 * rand() + 20e-06
endwhile

% Plot mechanics included below
```

```
subplot(2,2,1);
plot(times * 1000, voltages);
title("Voltage vs. Time");
xlabel("Time (ms)");
ylabel("Voltage (V)");

subplot(2,2,2);
plot(times * 1000, voltages / separation, 'y');
title("Electric Field vs. Time");
xlabel("Time (ms)");
ylabel("Field (V/m)");

subplot(2,2,3);
plot(times * 1000, voltages * capacitance, 'r');
title("Charge vs. Time");
xlabel("Time (ms)");
ylabel('Charge (C)');

subplot(2,2,4);
plot(times * 1000, ((voltages .* voltages * capacitance) / (2 * separation)), 'g');
title("Force vs. Time");
xlabel("Time (ms)");
ylabel('Force (N)');
```