Homework 2

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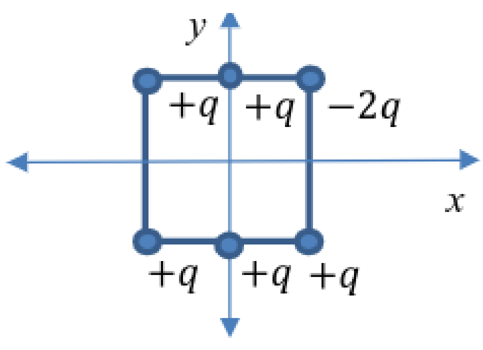
1. Six charges are arranged along the sides and corners of a square with sides of length L as shown. Calculate the magnitude and direction of the electric field at the origin. Use symmetry and superposition to make the calculation simple.



We know, by definition, that $\vec{F} = \vec{E}q$. Using the concepts we know about force, we know the following charges cancel out each other, as they are symmetric about the test charge at the origin:

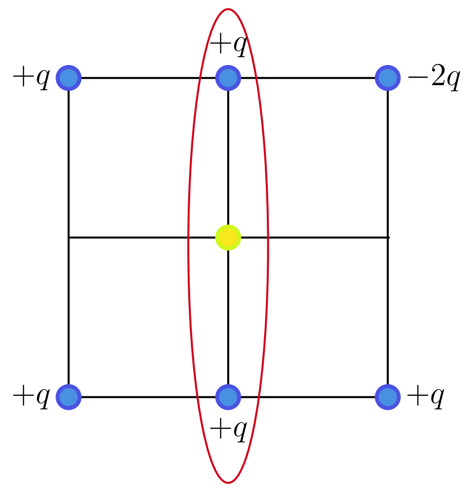


Figure 1: The Opposite Forces Negate Each Other

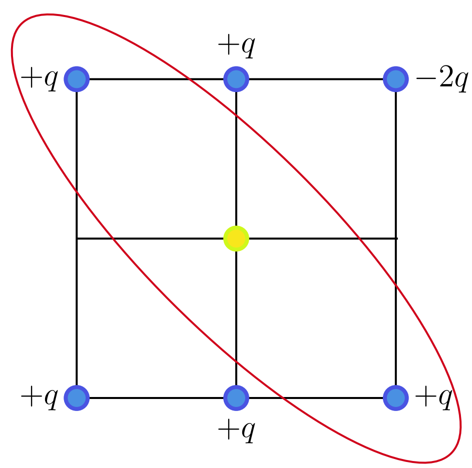


Figure 2: The Opposite Forces Negate Each Other

Thus, we need only consider the effects of these charges:

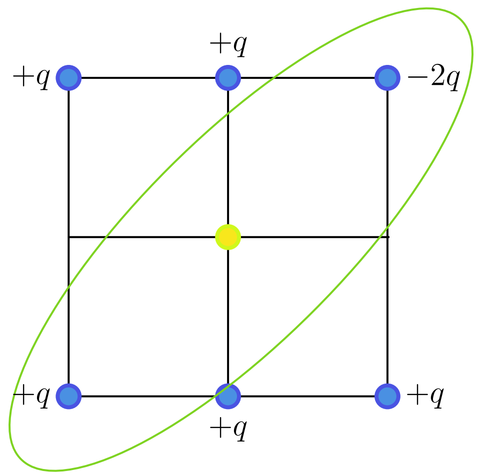


Figure 3: These Charges Remain Relevant

Using superposition, we know that the two charges can be summed, and they produce a force equal to $-3q$ at an angle of $\frac{\pi}{4}$ radians with respect to the x-axis. Decomposing this, we know the force can be expressed, with Q as the test charge, as:

$$F_Q = \frac{(Q)(-3q)}{4\pi\varepsilon_o R^2}\cos\left(\frac{\pi}{4}\right)\hat{\mathbf{x}} + \frac{(Q)(-3q)}{4\pi\varepsilon_o R^2}\sin\left(\frac{\pi}{4}\right)\hat{\mathbf{y}}$$

To find the electric field, we can simply divide by the test charge:

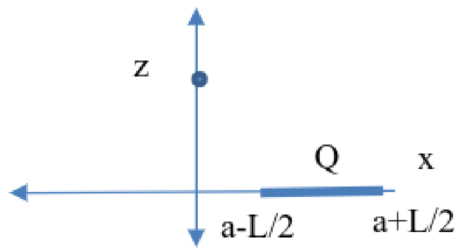
$$E_Q = -\frac{3q}{4\pi\varepsilon_o R^2}\cos\left(\frac{\pi}{4}\right)\hat{\mathbf{x}} - \frac{3q}{4\pi\varepsilon_o R^2}\sin\left(\frac{\pi}{4}\right)\hat{\mathbf{y}}$$

Additionally, we know that $R = \sqrt{\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2} = \sqrt{\frac{L^2}{2}} = \frac{L}{\sqrt{2}}$. This gives us:

$$\boxed{E_Q = -\frac{3\sqrt{2}q}{4\pi\varepsilon_o L^2}\hat{\mathbf{x}} - \frac{3\sqrt{2}q}{4\pi\varepsilon_o L^2}\hat{\mathbf{y}}}$$

2. A uniformly charged rod of length L and charge q is placed along the x-axis with its center at $x = a$. Find the x-component of the electric field at a point on the z axis. (Hint: use R as the variable of integration.) Check your expression in the following limit: $z = 0$ and $a >> L$.

3. Calculate the electric potential on the z-axis due to a uniformly charged annulus in the xy-plane centered at the origin with inner radius a and outer radius b. Then find the electric field from the gradient of the potential.



4. Consider an infinitely long uniformly-charged solid cylinder of radius a and charge per unit volume ρ surrounded by a coaxial cylindrical shell of radius b and charge per unit area of σ. Take the axis of the cylinders as the z-axis.

   (a) Calculate the electric field everywhere in space

   (b) Also calculate the potential as a function of the distance from the axis, taking the potential to be zero on the z-axis.

5. The electric field for two charged concentric spherical shells is given by

$$\begin{cases} 0, & r < a \\ \hat{\mathbf{r}} A_1/r^2, & a < r < b \\ \hat{\mathbf{r}} A_2/r^2, & r > b \end{cases}$$

   Where $A_1 = 5 \times 10^6 \left[\frac{\mathrm{N\,m^2}}{\mathrm{C}}\right]$, $A_2 = -3 \times 10^6 \left[\frac{\mathrm{N\,m^2}}{\mathrm{C}}\right]$, $a = .25[\mathrm{m}]$, and $b = .45[\mathrm{m}]$. Find the surface charge densities σ_a and σ_b on the two shells.