

Electromagnetic Waves

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- Maxwell's Equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_o}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J} + \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t}$$

- The second equation indicates no magnetic monopoles exist
- Where $J_d = \oint \vec{B} \cdot d\vec{l} = \mu_o I$
- Electric fields diverge with strength proportional to charge
- Magnetic fields do not diverge
- Curling electric field opposes change in magnetic field

$$\vec{F} = q(\vec{E} + \vec{v}\vec{B})$$

- Describes the Lorentz force, which indicates how charges move in fields
- A key term, the displacement current (\vec{J}_d), was added by Maxwell

$$\vec{J}_d = \epsilon_o \frac{\partial \vec{E}}{\partial t} = \frac{\partial \vec{D}}{\partial t}$$

$$I_d = \int \vec{J}_d d\vec{a}$$

- Within a capacitor, the displacement current may be defined by:

$$I_d = \epsilon_o \frac{d\vec{E}}{dt} A$$

- In matter, the equivalent to \vec{J}_d is polarization current

$$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{J}_p = -\frac{\partial \rho_b}{\partial t}$$

- Thus, we can rewrite some of the equations:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_f}{\epsilon_o} + \frac{-1}{\epsilon_o}(\vec{\nabla} \cdot \vec{P}) \rightarrow \vec{\nabla} \cdot \vec{D} = \rho_f$$

$$\vec{\nabla} \times \vec{B} = \mu_o \vec{J}_f + \mu_o \epsilon_o \frac{\partial \vec{E}}{\partial t} + \mu_o \frac{\partial \vec{P}}{\partial t} + \mu_o (\vec{\nabla} \times \vec{m}) \rightarrow \vec{\nabla} \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

- Electromagnetic Waves

$$\vec{\nabla}(\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}) = -\mu_o \epsilon_o \frac{\partial}{\partial t} \left(\frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{\nabla}^2 \vec{E} = \mu_o \epsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}$$

- Applying our knowledge of the wave equation, we may write:

$$v = \frac{1}{\sqrt{\mu_o \epsilon_o}} = c$$

- We obtain the same result:

$$\vec{\nabla}^2 \vec{E} = c^2 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{\nabla}^2 \vec{B} = c^2 \frac{\partial^2 \vec{B}}{\partial t^2}$$

- Magnetic and electric fields propagate at the speed of light, perpendicular to each other, to create light
- The direction of the wave is $\vec{E} \times \vec{B}$