

Electric Fields in Matter

Michael Brodskiy

Professor: D. Wood

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- Hydrogen Atom

- The charge density for a hydrogen atom would be:

$$\rho(r) = \frac{q}{4\pi a^3} e^{-\frac{2r}{a}}$$

when $r \ll a$:

$$\rho(r) \approx \frac{q}{4\pi a^3}$$

- Using Gauss's law, we can find the electric field:

$$\vec{E}(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi r^3)}{\epsilon_o}$$
$$\vec{E} = \frac{\rho r}{3\epsilon_o}$$

- From here, the force on the charges become:

$$F = q\vec{E} = -\frac{q\rho r}{3\epsilon_o}$$

- Molecule of Water

- Since the hydrogens pull in different directions, we can describe the torque as:

$$\vec{N} = \frac{d}{2} \sin(\theta) \vec{E} q + \frac{d}{2} \sin(\theta) \vec{E} q = d \sin(\theta) \vec{E} q$$
$$\vec{N} = \vec{p} \times \vec{E}$$

- The net force can be described as:

$$F = q\vec{E}\left(\vec{r} + \frac{\vec{d}}{2}\right) - q\vec{E}\left(\vec{r} - \frac{\vec{d}}{2}\right)$$

$$F_x \approx q\left(E_x(\vec{r}) + \frac{\vec{d}}{2}\vec{\nabla}E_x\right) - q\left(E_x(\vec{r}) - \frac{\vec{d}}{2}\vec{\nabla}E_x\right)$$

$$F_x = q\vec{\nabla}E_x \cdot \vec{d} = \vec{p} \cdot \vec{\nabla}E_x$$

$$\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$$

- Polarized material

$$\vec{P} = \frac{\text{dipole moment}}{\text{volume}}$$

- Units:

$$\frac{\text{C}}{\text{m}^2} \quad \text{like } \sigma$$

- Potential due to a neutral polarized object

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_o} \int \frac{\hat{\mathbf{R}} \cdot \vec{P}(\vec{r}')}{R^2} d\tau'$$

$$V(\vec{r}) = -\frac{1}{4\pi\epsilon_o} \int \frac{1}{R'} \vec{\nabla}(\vec{r}') \cdot \vec{P}(\vec{r}') d\tau'$$

- For a bound charge:

$$\sigma_b = \vec{P} \cdot \hat{\mathbf{n}} \quad \text{bound surface charge}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P} \quad \text{bound bulk charge}$$

- Note, in general:

$$\vec{E} \neq -\vec{\nabla}V_D$$

- This is due to the fact that:

$$\vec{\nabla} \times \vec{D} \text{ is not always } 0$$

- Special Rule for a linear dielectric

$$\vec{P} = \chi_e \epsilon_o \vec{E}$$

- The “chi-sub-e” value is its electric susceptibility (which is the ability for something to become electrically polarized)
- The permittivity is equal to:

$$\varepsilon = (1 + \chi_e)\varepsilon_o = \varepsilon_r\varepsilon_o$$

- * We can recognize ε_r as the dielectric constant
- * ε_o is the permittivity of free space
- * $\varepsilon_r = 1$ in a vacuum

$$\vec{E} = \frac{\vec{D}}{\varepsilon}$$