Electrodynamics

Michael Brodskiy
Professor: D. Wood

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- Current
  - Ohm's "Law"[1]
    - Holds when there is some current density such that $\vec{J} = \sigma\vec{E}$, with σ as conductivity
    - The unit of conductivity is $\left[\frac{\mathrm{A}}{\mathrm{V\,m}}\right]$
    - The resistivity is the inverse of the conductivity, $\rho = \sigma^{-1}$, with units $[\Omega\,\mathrm{m}]$[2]
  - The average velocity of a particle accelerated over an interval due to an electric field is:

$$v_{avg} = \sqrt{\frac{q\vec{E}d}{2m}}$$

  - The current density can be defined as

$$\vec{J} = nq\vec{v}$$

  - An electron's drift velocity may be defined as:

$$v_d = \frac{1}{2}\frac{q\vec{E}d}{mv}$$

    - As long as $v_d << v$
  - Given a wire of length L and potential V_o, we can calculate:

$$\vec{E} = \frac{V_o}{L}$$

$$R = \frac{V}{I} = \frac{\vec{E}L}{\vec{J}A} = \frac{\rho L}{A}$$

[1] Note: this is not a fundamental law
[2] Note: Ohms are equal to $\frac{\mathrm{V}}{\mathrm{A}}$

- Circuits and Power
  - We know:

$$V = \frac{Q}{C}$$
$$W = \frac{1}{2}QV = \frac{Q^2}{2C}$$

  - By conservation of charge, we can write:

$$P = \frac{dW}{dt} = \frac{1}{2C}\frac{d}{dt}(Q^2) = -IV$$
$$P = \frac{V^2}{R}$$

  - We can also derive:

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$
$$Q = Q_o e^{-\frac{t}{RC}}$$

- Electromotive Force (EMF)
  - The EMF can be defined as:

$$\varepsilon = \int \vec{f} d\vec{l}$$

  - Where:

$$\vec{f} = \frac{\vec{F}}{q}$$

  - Magnetic flux can be defined as:

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

  - The EMF can also be defined as:

$$\varepsilon = -\frac{d\Phi}{dt}$$

  - Lenz's Law: Induced effect opposes the change
  - There are several ways flux may be changed:
    * Loop is stationary, move B-field
    * Loop stationary, change strength of B-field

    * Change relative direction of loop and $\vec{B}$

- Faraday's Law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a}$$

  - According to Stokes' Theorem, we may write:

$$\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = -\int_S \frac{d}{dt} (\vec{B} \cdot d\vec{a})$$

  - Which gives us one of Maxwell's equations:

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

- We can define flux as:

$$\Phi_B(t) = BA\cos(\omega t)$$

  - Which then gives us:

$$\varepsilon = -\frac{d}{dt}(\Phi_B(t)) = AB\omega \sin(\omega t)$$

  - When the magnetic field is constant, but the area is changing, we can write:

$$\varepsilon = -B\frac{dA}{dt}$$

  - Thus, for a moving loop, we can write:

$$\varepsilon = -Bwv$$

- Mutual Inductance
  - Since $\vec{B}$ is proportional to I (via Biot-Savart), we can also say that Φ will be proportional to the current I. Thus, we may write:

$$\Phi = MI$$

  - Where M is known as the mutual inductance
  - Likewise, we can define:

$$\varepsilon = -M\frac{dI}{dt}$$

- We can observe:
  1. Φ is proportional to I
  2. M depends only on the geometry
  3. $M_{1,2} = M_{2,1} = M$

- Self Inductance

$$\Phi = LI$$

  - L describes the self inductance

- Work

$$P = \varepsilon I$$

  - From the above, we may write for an inductor:

$$\int \frac{dW}{dt} = -L \int \frac{dI}{dt} I$$
$$W = \frac{1}{2} LI^2$$

  - The energy may be written as:

$$U = \frac{1}{2}(\mu_o n^2 I^2)(Al)$$
$$U = \frac{1}{2\mu_o} B^2 (Al)$$

  - For energy density we may simply write:

$$\mathcal{U}_M = \frac{1}{2\mu_o} B^2$$

  - Given that $\tau = \frac{L}{R}$, we may write:

$$I = I_o e^{-\frac{t}{\tau}}$$