Electric Fields in Matter

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- Hydrogen Atom
  - The charge density for a hydrogen atom would be:

    $$\rho(r) = \frac{q}{4\pi a^3} e^{-\frac{2r}{a}}$$

    when $r << a$:

    $$\rho(r) \approx \frac{q}{4\pi a^3}$$

  - Using Gauss's law, we can find the electric field:

    $$\vec{E}(4\pi r^2) = \frac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_o}$$
    $$\vec{E} = \frac{\rho r}{3\varepsilon_o}$$

  - From here, the force on the charges become:

    $$F = q\vec{E} = -\frac{q\rho r}{3\varepsilon_o}$$

- Molecule of Water
  - Since the hydrogens pull in different directions, we can describe the torque as:

    $$\vec{N} = \frac{d}{2}\sin(\theta)\vec{E}q + \frac{d}{2}\sin(\theta)\vec{E}q = d\sin(\theta)\vec{E}q$$
    $$\vec{N} = \vec{p} \times \vec{E}$$

- The net force can be described as:

$$F = q\vec{E}\left(\vec{r} + \frac{\vec{d}}{2}\right) - q\vec{E}\left(\vec{r} - \frac{\vec{d}}{2}\right)$$

$$F_x \approx q\left(E_x(\vec{r}) + \frac{\vec{d}}{2}\vec{\nabla}E_x\right) - q\left(\vec{E}(r) - \frac{\vec{d}}{2}\vec{\nabla}E_x\right)$$

$$F_x = q\vec{\nabla}E_x \cdot \vec{d} = \vec{p} \cdot \vec{\nabla}E_x$$

$$\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$$

- Polarized material

$$\vec{P} = \frac{\text{dipole moment}}{\text{volume}}$$

  - Units:

$$\frac{\mathrm{C}}{\mathrm{m}^2} \quad \text{like } \sigma$$

  - Potential due to a neutral polarized object

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o}\int \frac{\hat{\mathbf{R}} \cdot P(\vec{r}\prime)}{R^2}\, d\tau'$$

$$V(\vec{r}) = -\frac{1}{4\pi\varepsilon_o}\int \frac{1}{R'}\vec{\nabla}(\vec{r}\prime)\, d\tau'$$

- For a bound charge:

$$\sigma_b = \vec{P} \cdot \hat{\mathbf{n}} \qquad \text{bound surface charge}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P} \qquad \text{bound bulk charge}$$

- Note, in general:

$$\vec{E} \neq -\vec{\nabla}V_D$$

  - This is due to the fact that:

$$\vec{\nabla} \times \vec{D} \text{ is not always } 0$$

- Special Rule for a linear dielectric

$$\vec{P} = \chi_e \varepsilon_o \vec{E}$$

  - The "chi-sub-e" value is its electric susceptibility (which is the ability for something to become electrically polarized)
  - The permittivity is equal to:

$$\varepsilon = (1 + \chi_e)\varepsilon_o = \varepsilon_r \varepsilon_o$$

    * We can recognize ε_r as the dielectric constant
    * ε_o is the permittivity of free space
    * $\varepsilon_r = 1$ in a vacuum

$$\vec{E} = \frac{\vec{D}}{\varepsilon}$$

- Capacitor with Linear Dielectric

$$C = C_{vac}\varepsilon_r$$

  - The energy becomes:

$$W = \frac{1}{2}CV^2 = \frac{1}{2}\varepsilon_r C_{vac} V^2$$

  - The stored energy would be:

$$U = \frac{W}{Ad} = \frac{1}{2}\varepsilon_r \varepsilon_o \vec{E}^2 = \frac{1}{2}\vec{D}\vec{E}$$