Potentials and Fields

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- The energy density of a field may be written as:

$$U = \frac{1}{2}\left(\varepsilon_o \vec{E}^2 + \frac{1}{\mu_o}\vec{B}^2\right)$$

- The Poynting Vector may be defined as:

$$\vec{S} = \frac{1}{\mu_o}(\vec{E} \times \vec{B})$$

- For some volume τ, we can write the total energy in it using:

$$\mathcal{U}_{tot} = \int_V U\, d\tau$$

- The Poynting Vector represents the flow of energy out of the surface

$$\vec{\nabla} \cdot \vec{S} = \frac{1}{\mu_o}\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = \frac{1}{\mu_o}\left(\vec{B} \cdot (\vec{\nabla} \times \vec{E}) - \vec{E}(\vec{\nabla} \times \vec{B})\right)$$

$$\vec{\nabla} \cdot \vec{S} = -\frac{\partial}{\partial t}\left(\frac{\varepsilon_o \vec{E}^2}{2} + \frac{\vec{B}^2}{2\mu_o}\right)$$

  - This can be simplified to attain:

$$\vec{\nabla} \cdot \vec{S} + \frac{\partial U}{\partial t} = 0$$

  - A similar equation can be seen in conservation of charge:

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

  - Thus, we have obtained a conservation of energy formula

- The Maxwell Stress Tensor ($\overleftrightarrow{T}$)

$$d\vec{F} = \overleftrightarrow{T} \cdot d\vec{a}$$

  - T_{ij} is the force per area in the j-direction of an area oriented in the i-direction

$$\oint_S \overleftrightarrow{T} \cdot d\vec{a} = \vec{F}_{ext}$$

$$\oint_V (\vec{\nabla} \cdot \overleftrightarrow{T}) \cdot d\tau = \vec{F}_{ext}$$