Homework 4

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October 12, 2023

1. Consider an infinite grounded conducting plane bent at a $90°$ angle between the yz and xz planes as shown, with a charge placed at $x = 4a$, $y = a$. Use appropriate image charge(s) to find an expression for the potential $V(x, y, z)$ in the region $x > 0$, $y > 0$.

2. The boundary at $x = 0$ consists of two metal strips: one, from $y = 0$ to $y = a/2$ is held at a constant potential $+V_0$ and the other, from $y = 2/a$ to $y = a$ is held at a constant potential of V_0 . Solve for the potential $V(x, y, z)$ inside the slot. Feel free to use the relevant results from Example 3.3 or from lecture as a starting point.

3. Consider a long (semi-infinite) rectangular conducting pipe oriented V_0 parallel to the z-axis, with dimensions $a \times b$ in the xy-plane. The pipe itself is grounded, and the rectangle at the closed end is at a constant potential V_0 . Find an expression for the potential everywhere inside the pipe (for $z > 0$).

4. Consider an empty spherical shell of charge of radius R where the potential on the surface is given by $V(R, \theta) = V_o \sin^2(\theta)$.

   Hint: Express $\sin^2(\theta)$ as a polynomial function of $\cos(\theta)$.

   (a) Find $V(r, \theta)$ inside the shell.
   (b) Find $\vec{E}(R, \theta)$ just inside the shell.
   (c) Find $V(r, \theta)$ out of the shell.
   (d) Find $\vec{E}(R, \theta)$ just outside the shell.
   (e) Find $\sigma(R, \theta)$ on the shell. [answer: $\sigma = \frac{V_o \varepsilon_o}{3R}(7 - 15\cos^2(\theta))$]

5. An empty spherical shell of radius R has potential V_0 on the upper hemisphere and V_0 on the lower hemisphere

   (a) Calculate the first two non-zero terms of the expression for the potential outside of the sphere to obtain an approximate expression for $V(r, \theta)$ in this region.
   (b) From this approximate expression, compute the value of $V(R, \theta)$ (on the surface of the shell) for $\theta = 0, \theta = \pi/4$, and $\theta = 3\pi/4$ compare the results with the exact values at those locations