Electrostatics

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- In this section, we focus on electrostatics
  - Not doing (for now):
    * Magnetic field
    * Forces on moving charges
    * Finite of propagation
- Coulomb's Law
  - Given a source charge, q, and a test charge, Q, with $\vec{R}$ as the difference between their positions ($\vec{r} - \vec{r'}$), we can generate Coulomb's Law:

$$F = \frac{qQ\hat{\mathbf{R}}}{4\pi\varepsilon_o R^2}$$

    * ε_o is known as the permittivity of free space
    * $\varepsilon_o = 8.85 \cdot 10^{-12} \left[\frac{\text{C}}{\text{N m}^2}\right]$
  - A Coulomb is defined as an Ampère per second
- Superposition
  - A force per charge (q) can be calculated and then summed to find the total force on a test charge (Q)

$$\vec{F} = \sum_n \frac{Q}{4\pi\varepsilon_o}\frac{q_n\hat{\mathbf{R}}_n}{R_n^2}$$

$$\vec{F} = Q\vec{E} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_n \frac{q_n\hat{\mathbf{R}}_n}{R_n^2}$$

- Continuous

$$q_n \to dq = \rho(\vec{r\prime})\, d\tau$$
$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\int \frac{1}{R^2}\hat{\mathbf{R}}\, dq = \frac{1}{4\pi\varepsilon_o}\int \frac{\rho(\vec{r\prime})\, d\tau'}{R^2}\hat{\mathbf{R}}$$

- For various shapes:
  - Volume: $dq = \rho\, d\tau$
  - Line: $dq = \lambda\, dl$
  - Surface: $dq = da$

- Electric Potential — V (volts)

$$\vec{E} = -\vec{\nabla}V \Longleftrightarrow V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

  - Note: V is a scalar function
  - For a charge q and some reference radius, r:

$$V(r) - V_{ref} = -\int_{r_{ref}}^{r} \vec{E}\cdot d\vec{r\prime}$$

  - This yields

$$V(r) = \frac{q}{4\pi\varepsilon_o r}$$

  - With multiple charges:

$$V(r) = \frac{1}{4\pi\varepsilon_o}\sum_n \frac{q_n}{R_n}$$