

Magnetic Fields in Matter

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- Torque on a Dipole

$$\vec{N} = \vec{m} \times \vec{B}$$

- This is akin to the torque on an electric dipole:

$$\vec{N} = \vec{p} \times \vec{E}$$

- We can calculate the torque to be:

$$\vec{N} = m\vec{B} \sin(\theta)$$

- The energy can be defined as:

$$U = -\vec{m} \cdot \vec{B}$$

- The bound bulk and surface currents can be defined as:

$$\vec{J}_b = \vec{\nabla} \times \vec{M} \quad \text{and} \quad \vec{K}_b = \vec{M} \times \hat{\mathbf{n}}$$

- For a uniform magnetized sphere:

$$\vec{J}_b = 0 \quad \text{and} \quad \vec{K}_b = M \sin(\theta) \hat{\phi}$$

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$$\vec{A}(r) = \frac{\mu_o}{4\pi} \int \frac{\vec{J}_b}{R} d\tau + \frac{\mu_o}{4\pi} \int \frac{\vec{K}_b}{R} da$$

- Auxiliary Field (\vec{H})

$$\vec{H} = \frac{1}{\mu_o} \vec{B} - \vec{M} \Rightarrow \vec{B} = \mu(\vec{H} + \vec{M})$$

- From this formula, we can get:

$$\vec{\nabla} \times \vec{H} = \frac{\vec{\nabla} \times \vec{B}}{\mu_o} - \vec{\nabla} \times \vec{M}$$

- Which can become:

$$\vec{\nabla} \times \vec{H} = \vec{J} - \vec{J}_b = \vec{J}_F$$

- Linear Materials

$$\vec{M} = \chi_m \vec{H}$$

- This is not quite the same as the \vec{E} case, where $\vec{p} = \epsilon_o \chi_e \vec{E}$

$$\begin{aligned} \vec{B} &= \mu_o(\vec{H} + \vec{M}) = \mu_o(1 + \chi_m)\vec{H} \\ \mu &= \mu_o(1 + \chi_m) \end{aligned}$$

- Unlike dielectrics, χ_m could be either positive or negative