

Electrodynamics

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- Current

- Ohm's "Law"¹

- * Holds when there is some current density such that $\vec{J} = \sigma \vec{E}$, with σ as conductivity

- * The unit of conductivity is $[\frac{\text{A}}{\text{V m}}]$

- * The resistivity is the inverse of the conductivity, $\rho = \sigma^{-1}$, with units $[\Omega \text{ m}]$ ²

- The average velocity of a particle accelerated over an interval due to an electric field is:

$$v_{avg} = \sqrt{\frac{q\vec{E}d}{2m}}$$

- The current density can be defined as

$$\vec{J} = nq\vec{v}$$

- An electron's drift velocity may be defined as:

$$v_d = \frac{1}{2} \frac{q\vec{E}d}{mv}$$

- * As long as $v_d \ll v$

- Given a wire of length L and potential V_o , we can calculate:

$$\vec{E} = \frac{V_o}{L}$$

$$R = \frac{V}{I} = \frac{\vec{E}L}{\vec{J}A} = \frac{\rho L}{A}$$

¹Note: this is not a fundamental law

²Note: Ohms are equal to $\frac{\text{V}}{\text{A}}$

- Circuits and Power

- We know:

$$V = \frac{Q}{C}$$

$$W = \frac{1}{2}QV = \frac{Q^2}{2C}$$

- By conservation of charge, we can write:

$$P = \frac{dW}{dt} = \frac{1}{2C} \frac{d}{dt}(Q^2) = -IV$$

$$P = \frac{V^2}{R}$$

- We can also derive:

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$Q = Q_o e^{-\frac{t}{RC}}$$

- Electromotive Force (EMF)

- The EMF can be defined as:

$$\varepsilon = \int \vec{f} d\vec{l}$$

- Where:

$$\vec{f} = \frac{\vec{F}}{q}$$

- Magnetic flux can be defined as:

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

- The EMF can also be defined as:

$$\varepsilon = -\frac{d\Phi}{dt}$$

- Lenz's Law: Induced effect opposes the change

- There are several ways flux may be changed:

- * Loop is stationary, move B -field
- * Loop stationary, change strength of B -field

* Change relative direction of loop and \vec{B}

- Faraday's Law:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a}$$

– According to Stokes' Theorem, we may write:

$$\int_S (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = - \int_S \frac{d}{dt} (\vec{B} \cdot d\vec{a})$$

– Which gives us one of Maxwell's equations:

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

- We can define flux as:

$$\Phi_B(t) = BA \cos(\omega t)$$

– Which then gives us:

$$\varepsilon = -\frac{d}{dt}(\Phi_B(t)) = AB\omega \sin(\omega t)$$

– When the magnetic field is constant, but the area is changing, we can write:

$$\varepsilon = -B \frac{dA}{dt}$$

– Thus, for a moving loop, we can write:

$$\varepsilon = -Bwv$$

- Mutual Inductance

– Since \vec{B} is proportional to I (via Biot-Savart), we can also say that Φ will be proportional to the current I . Thus, we may write:

$$\Phi = MI$$

– Where M is known as the mutual inductance

– Likewise, we can define:

$$\varepsilon = -M \frac{dI}{dt}$$

– We can observe:

1. Φ is proportional to I
2. M depends only on the geometry
3. $M_{1,2} = M_{2,1} = M$

- Self Inductance

$$\Phi = LI$$

– L describes the self inductance