Homework 7

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1. A hollow sphere of radius R centered at the origin is covered with a uniform surface charge σ and is rotating about the z-axis with angular frequency ω. A uniform external magnetic field is oriented in the y-direction: $\vec{B} = B_o\hat{\mathbf{y}}$
   (a) Find the total force on the sphere
   (b) Find the total torque on the sphere
   (c) Generalize the result of (b) to find the torque for a uniform magnetic field in an arbitrary direction, $\vec{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$
2. Calculate the magnetic field $\vec{B}(x, y)$ in the positive quadrant of the x-y plane due to a current coming on the y-axis from $y = +\infty$, turning 90° at the origin, and exiting along the x-axis to $x = +\infty$
3. A long cylindrical conductor has a uniform current density $\vec{J}$ oriented along the axis.
   (a) Find the strength of the magnetic field as a function of s (the perpendicular distance from the z-axis) for $s < a$.
   (b) Consider a charged particle with charge q and momentum p that passes through the cylinder in part (a) with an initial velocity parallel to the axis. As a function of the distance s of the particle from the axis, find the angle of deflection after passing through a short distance Δz. Considered as lens for such charged particles, what it the focal length of this segment of the conductor as a function of $\vec{J}, p, q, \Delta z$? (Such lenses are actually used in particle accelerators). Assume that $\Delta z << R$, where R is the radius of curvature of the particle in the magnetic field, so a small angle approximation is valid for the deflection
   (c) Find the current density $\vec{J}$ needed to focus particles of charge $e = 1.6 \cdot 10^{19}$[C] and momentum $p = 75\left[\frac{\text{GeV}}{c}\right]$ with a focal length of $f = 20$[m] for $\Delta z = 0.50$[m]
4. A double solenoid has two co-axial coils radii a and b,with n_a and n_b turns per unit length, and with currents I_a and I_b flowing in opposite directions. Find:

(a) the magnetic field for the region inside the first coil ($s < a$)

(b) the magnetic field for the region between he two coils ($a < s < b$)

(c) the magnetic field for the region inside outside both coils ($s > b$)

(d) What ratio of currents would be required to have $\vec{B} = 0$ for $s < a$?