Electrostatics, Boundary Conditions, Potentials, & More

Michael Brodskiy

Professor: E. Marengo Fuentes

October 2, 2023

- We begin with Maxwell's Equations:

$$\nabla \cdot D = P \quad \text{Gauss's Law}$$

  - Where D is the electric flux density

$$\nabla \times E = -\frac{\partial}{\partial t}B \quad \text{Faraday's Law}$$

$$\nabla \cdot B = 0 \quad \text{Magnetic Gauss's Law}$$

$$\nabla \times H = J + \frac{\partial}{\partial t}D \quad \text{Ampére's Law}$$

  - ρ is the charge density in coulombs per cubic meter, J is the current density, in ampéres per square meter

- In electrostatics, these laws mean:

$$\frac{\partial}{\partial t} = 0$$
$$\nabla \cdot D = \rho$$
$$\nabla \times E = 0$$

- In magnetostatics, these laws mean:

$$\frac{\partial}{\partial t} = 0$$
$$\nabla \cdot B = 0$$
$$\nabla \times H = J$$

- The emphasis in electrodynamics is:

$$\frac{\partial}{\partial t} \neq 0$$

- Combining equations, we get:

$$J = \rho v$$

where v is the velocity of the charge in meters per second

- Work
  - Moving a particle from point 1 to 2 would take work:

$$W = \int_1^2 -Eq\,dl$$

  - It can be expressed as a difference in potential energies:

$$V_2 - V_1 = \int_1^2 -Eq\,dl$$

  - Moving a charge from infinity:

$$V_2 - V_\infty = \int_\infty^2 -Eq\,dl$$

$$V = -\int E\,dl \longleftrightarrow E = -\nabla V$$

  - The Laplacian operator can be expressed as:

$$\nabla^2 = \nabla \cdot \nabla$$

$$\nabla^2(V) = -\frac{\rho}{\varepsilon}$$

- Source Point Analysis
  - The voltage with respect to a source point (r') and an observation point (r) can be expressed as:

$$V(r) = \int \frac{\rho(r')\,dr'}{4\pi\varepsilon|r - r'|}$$

- We can also learn:

$$\nabla^2 G(r, r') = \delta(r - r')$$

where

$$G(r, r') = -\frac{1}{4\pi\varepsilon|r - r'|}$$

- This is known as the Green's function or impulse response