

Fundamentals of Electromagnetics

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- There are two kinds of fields, electric (E) and magnetic (H)
- There are two kinds of fluxes, electric (D) and magnetic (B)

$$\boxed{D = \varepsilon E}$$

$$\boxed{B = \mu H}$$

– Where ε is the permittivity and μ is the permeability

- The electromagnetic force:

$$F = \underbrace{qE}_{\text{Electric}} + \underbrace{qv \times B}_{\text{Magnetic}}$$

– The magnetic component is referred to as Lorentz's Force

- Connection to gravitation

– The electric force from particle 1 to 2 may be written as:

$$F = \frac{q_1 q_2}{4\pi \varepsilon R^2} \hat{r}_{12}$$

– From particle 2 to 1, it may be written as

$$F = \frac{q_1 q_2}{4\pi \varepsilon R^2} \hat{r}_{21} = \frac{q_1 q_2}{4\pi \varepsilon R^2} (-\hat{r}_{12})$$

– This is known as the Coulomb force

– Combining this with force equation above, we see:

$$E_{q_1} = \frac{q_1}{4\pi \varepsilon R^2} \hat{r}_{12}$$

- Gravitation between two masses can be written as:

$$F_{1 \rightarrow 2} = \frac{m_1 m_2}{R^2} G \hat{r}_{12}$$

- Signals and Systems

- Systems take in some input (such as a signal), x , use a rule \mathcal{L} , and map it to an output, y
- Linear Systems — Obey the principle of superposition; that is, the output of a sum of inputs = the sum of outputs of the individual inputs

$$\mathcal{L}\Sigma = \Sigma\mathcal{L}$$

- * Linear systems can be split up into two further groups: time-variant and time-invariant (LTI)

- Fourier/Frequency Domain Representation

- F.T. of $x(t) = \tilde{x}(\omega)$
- F.T. of $y(t) = \tilde{y}(\omega)$
- This makes:

$$\tilde{y}(\omega) = H(\omega)\tilde{x}(\omega)$$

where $H(\omega)$ is the transfer function

- We can recall from circuits:

$$y(t) = V_0 \cos(\omega t + \phi)$$

- With Euler representation, we get:

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

$$\tilde{Y}(\omega) = \text{phasor} = V_0 e^{j\theta}$$

- By definition, the Fourier transform is:

$$\begin{aligned} f(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{F}(\omega) e^{j\omega t} d\omega, \text{ if signal } f(t) \in \text{real:} \\ &= \text{Re} \int_0^{\infty} d\omega e^{j\omega t} \tilde{A}(\omega) \\ &= \int_0^{\infty} d\omega \text{Re} (e^{j\omega t} \tilde{A}(\omega)) \end{aligned}$$

- This is called the analytic signal representation

- Waves

- A wave can be thought of as a propagating disturbance in a medium
- There is finite propagation time
- The most general way to describe a propagating, unidimensional wave traveling in the positive x direction can be written as follows:

$$f(x, t) = F\left(t - \frac{x}{c}\right)$$

* Special case: cosine

$$F(t) = A \cos(\omega t + \phi)$$

$$f(x, t) = A \cos\left(\omega t + \phi - \frac{x}{c}\right)^1$$

- The most general wave traveling in the $-x$ direction:

$$f(x, t) = F\left(t + \frac{x}{c}\right)$$

- We know $\omega = 2\pi f$
- This yields us $\frac{\omega}{c} = k$, or the wave number (of a wave)
- Thus,

$$\omega t - \frac{x}{c} = \omega \left(t - k \frac{x}{\omega}\right) = \omega t - kx$$

- Finally, we obtain:

$$f(x, t) = A \cos(\omega t + \phi - kx)$$

This is called the canonical form of a wave traveling in the positive x direction.

¹This wave is in the $+x$ direction