Fundamentals of Electromagnetics

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- There are two kinds of fields, electric (E) and magnetic (H)
- There are two kinds of fluxes, electric (D) and magnetic (B)

$$\boxed{D = \varepsilon E}$$

$$\boxed{B = \mu H}$$

  - Where ε is the permittivity and μ is the permeability

- The electromagnetic force:

$$F = \underbrace{qE}_{\text{Electric}} + \underbrace{qv \times B}_{\text{Magnetic}}$$

  - The magnetic component is referred to as Lorentz's Force

- Connection to gravitation
  - The electric force from particle 1 to 2 may be written as:

$$F = \frac{q_1 q_2}{4\pi\varepsilon R^2}\hat{r}_{12}$$

  - From particle 2 to 1, it may be written as

$$F = \frac{q_1 q_2}{4\pi\varepsilon R^2}\hat{r}_{21} = \frac{q_1 q_2}{4\pi\varepsilon R^2}(-\hat{r}_{12})$$

  - This is known as the Coulomb force
  - Combining this with force equation above, we see:

$$E_{q_1} = \frac{q_1}{4\pi\varepsilon R^2}\hat{r}_{12}$$

- Gravitation between two masses can be written as:

$$F_{1\to 2} = \frac{m_1 m_2}{R^2} G\hat{r}_{12}$$

- Signals and Systems
  - Systems take in some input (such as a signal), x, use a rule $\mathcal{L}$, and map it to an output, y
  - Linear Systems — Obey the principle of superposition; that is, the output of a sum of inputs = the sum of outputs of the individual inputs

$$\mathcal{L}\Sigma = \Sigma\mathcal{L}$$

    * Linear systems can be split up into two further groups: time-variant and time-invariant (LTI)

- Fourier/Frequency Domain Representation
  - F.T. of $x(t) = \tilde{x}(\omega)$
  - F.T. of $y(t) = \tilde{y}(\omega)$
  - This makes:

$$\tilde{y}(\omega) = H(\omega)\tilde{x}(\omega)$$

    where $H(\omega)$ is the transfer function
  - We can recall from circuits:

$$y(t) = V_0 \cos(\omega t + \phi)$$

  - With Euler representation, we get:

$$e^{j\theta) = \cos(\theta) + j\sin(\theta)}$$

$$\tilde{Y}(\omega) = \text{ phasor } = V_0 e^{j\theta}$$

  - By definition, the Fourier transform is:

$$\begin{aligned} f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{F}(\omega)e^{j\omega t}\,d\omega, &\text{ if signal } f(t) \in \text{real:} \\ = Re\int_0^{\infty} d\omega\, e^{j\omega t}\tilde{A}(\omega) \\ = \int_0^{\infty} d\omega\, Re\left(e^{j\omega t}\tilde{A}(\omega)\right) \end{aligned}$$

  - This is called the analytic signal representation

- Waves
  - A wave can be thought of as a propagating disturbance in a medium
  - There is finite propagation time
  - The most general way to describe a propagating, unidimensional wave traveling in the positive x direction can be written as follows:

$$f(x,t) = F\left(t - \frac{x}{c}\right)$$

    * Special case: cosine

$$F(t) = A\cos(\omega t + \phi)$$
$$f(x,t) = A\cos\left(\omega t + \phi - \frac{x}{c}\right)$$ [1]

  - The most general wave traveling in the $-x$ direction:

$$f(x,t) = F\left(t + \frac{x}{c}\right)$$

  - We know $\omega = 2\pi f$
  - This yields us $\frac{\omega}{c} = k$, or the wave number (of a wave)
  - Thus,

$$\omega t - \frac{x}{c} = \omega\left(t - k\frac{x}{\omega}\right) = \omega t - kx$$

  - Finally, we obtain:

$$f(x,t) = A\cos(\omega t + \phi - kx)$$

    This is called the canonical form of a wave traveling in the positive x direction.

- Traveling Waves
  - Wave maintains shape, no distortion
  - Medium is nondispersive (properties do not depend on frequency)
  - For a sinusoidal, T is the period; for a delayed sinusoid, $T \cong$ pulse length
  - Given a pulse defined by $f(t - d/c)$:
  - If $d/c << T$, the delayed version $\approx$ original
    * Limited delay, same signal at input and at output, we can ignore the finite propagation speed of wave, circuit theory applies
  - If d/c is NOT $<< T$

[1] This wave is in the $+x$ direction

    * We need electromagnetics, circuit theory does NOT apply
  - If $d << \lambda$ (where λ is wavelength), circuit theory applies
  - $f(t - x/c) = y(t) \rightarrow f(t) = A\cos(\omega t + \phi) \rightarrow y(t) = A\cos(\omega t + \phi - kx)$
    * $k = \frac{2\pi}{\lambda}$ is the wave number
    * $\lambda = \frac{c}{f}$
    * Propagates in $+x$
    * A is the amplitude, ϕ is the phase, ω is the frequency, f is the frequency in hertz, $2\pi f$ is the frequenzy in rad/s, and λ is the wavelength
  - An exponential term adds attenuation: $f(t - x/c) = f(t)e^{-\alpha x}$
    * α is the attentuation factor
    * $y(x,t) = Ae^{-\alpha x}$

* $y(x,t) = Ae^{\alpha x}$
* Phasor: $\tilde{y}(x) = \underbrace{\left(Ae^{-\alpha x}\right)}_{\text{attenuation}} \underbrace{\left(e^{-jkx}\right)}_{\text{phase form}} \left(e^{j\phi}\right)$