Exam 2

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1. (a) We can tell from the given function that the direction of propagation is $-\hat{\mathbf{x}}$

   (b) The frequency is equal to the coefficient of t divided by 2π:

   $$\boxed{f = \frac{1}{2\pi}(5\pi \cdot 10^7) = 2.5 \cdot 10^7 \text{ [Hz]}}$$

   (c) The wavelength may be calculated using the following formula:

   $$\lambda = \frac{u_p}{f}$$

   First, we calculate the propagation speed:

   $$u_p = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{(36)(4\pi \cdot 10^{-7})(8.85 \cdot 10^{-12})}} = 5 \cdot 10^7 \left[\frac{\text{m}}{\text{s}}\right]$$

   And then the wavelength:

   $$\boxed{\lambda = \frac{5 \cdot 10^7}{2.5 \cdot 10^7} = 2[\text{m}]}$$

   (d) The wave number k can be defined in several ways, one of which is:

   $$k = \frac{2\pi}{\lambda}$$

   This gives us:

   $$\boxed{k = \frac{2\pi}{2} = \pi \left[\frac{\text{rad}}{\text{m}}\right]}$$

   (e) The medium is lossless because there is no imaginary part to the permittivity or permeability

(f) The wave impedance, η, may be defined as:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Thus we can find:

$$\boxed{\eta = \sqrt{\frac{(4\pi \cdot 10^{-7})(4)}{(8.85 \cdot 10^{-12})(9)}} = 251.213[\Omega]}$$

(g) As a phasor, we can express the electric field as:

$$\boxed{\tilde{E} = 10e^{j\pi x}\hat{\mathbf{y}} - 3e^{j\left(\pi x - \frac{\pi}{2}\right)}\hat{\mathbf{z}} \left[\frac{\mathrm{V}}{\mathrm{m}}\right]}$$

(h) The average power density can be found according to the Poynting vector:

$$S_{avg} = \hat{\mathbf{x}}\frac{|\tilde{E}|^2}{2\eta}$$

We can find the magnitude of $\tilde{E}$:

$$|\tilde{E}| = \sqrt{(10)^2 + (-3)^2} = 10.44$$

Using the formula, we get:

$$\boxed{S_{avg} = \hat{\mathbf{x}}\frac{10.44^2}{502.416} = .216939 \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]}$$

(i) Now, we can find the magnetic field by using:

$$\tilde{H} = \frac{1}{\eta}(\hat{\mathbf{x}} \times \tilde{E})$$

Substituting the values we know, we get:

$$\tilde{H} = \frac{1}{251.213}\left[\hat{\mathbf{x}} \times \left(10e^{j\pi x}\hat{\mathbf{y}} - 3e^{\pi x - \frac{\pi}{2}}\hat{\mathbf{z}}\right)\right]$$

$$\tilde{H} = \frac{1}{251.213}\left(10e^{j\pi x}\hat{\mathbf{z}} + 3e^{\pi x - \frac{\pi}{2}}\hat{\mathbf{y}}\right)$$

$$\boxed{\tilde{H} = .0398e^{j\pi x}\hat{\mathbf{z}} + .01194e^{j\left(\pi x - \frac{\pi}{2}\right)}\hat{\mathbf{y}} \left[\frac{\mathrm{A}}{\mathrm{m}}\right]}$$

(j) We can then convert the above to a time-domain function:

$$\boxed{H = .0398\cos\left(\left(5\pi \cdot 10^7\right)t + \pi x\right)\hat{\mathbf{z}} + .01194\sin\left(\left(5\pi \cdot 10^7\right)t + \pi x\right)\hat{\mathbf{y}} \left[\frac{\mathrm{A}}{\mathrm{m}}\right]}$$

2. We are given the equation:

$$H(0,t) = \hat{\mathbf{y}} 50 \sin(\omega t + 23^\circ) \left[\frac{\text{mA}}{\text{m}}\right]$$

(a) We know that, for a field equation, two terms go to zero when $z \to 0$. These are:

$$\text{The attenuation factor: } e^{-\alpha z}$$
$$\text{The phase value: } -\beta z$$

We first check whether this is a good conductor:

$$\frac{\sigma}{\omega} = \frac{3}{2\pi \cdot 10^4} = .000048$$
$$\varepsilon = (70)(8.85 \cdot 10^{-12}) = 6.195 \cdot 10^{-10}$$

Since the ratio of conductance to angular frequency is much greater than the permittivity, this is a good conductor. We can solve for these using the formulas:

$$\alpha = \sqrt{\pi f \mu \sigma}$$
$$\beta = \alpha$$

This gives us:

$$\alpha = \sqrt{\pi(10^4)(4\pi \cdot 10^{-7})(3)} = .344144, \left[\frac{\text{Np}}{\text{m}}\right]$$
$$\beta = .344144 \left[\frac{\text{rad}}{\text{m}}\right]$$

The phase shift may also be converted to radians:

$$23 \cdot \frac{2\pi}{360} = .401426$$

Thus, we can fully define the formula as:

$$H(z,t) = 50e^{-.344144z} \sin\Big(\big(2\pi \cdot 10^4\big)\, t - .344144z + .401426\Big)\hat{\mathbf{y}} \left[\frac{\text{mA}}{\text{m}}\right]$$

$$\boxed{H(z,t) = 50e^{-.344144z} \cos\Big(\big(2\pi \cdot 10^4\big)\, t - .344144z - 1.16937\Big)\hat{\mathbf{y}} \left[\frac{\text{mA}}{\text{m}}\right]}$$

(b) We know by definition, that:

$$E = \eta(\hat{\mathbf{z}} \times H)$$

Where:

$$\eta = (1+j)\frac{\alpha}{\sigma}$$

We can calculate the intrinsic impedance as:

$$\eta = (1+j)\frac{.344144}{3} = .114715 + .114715j$$

Multiplying $H(z,t)$ by η gives us:

$$E(z,t) = (.114715 + .114715j)50e^{-.344144z}\cos\big(\left(2\pi\cdot 10^4\right)t - .344144z - 1.16937\big)$$

$$E(z,t) = 8.11158e^{j\frac{\pi}{4}}e^{-.344144z}\cos\big(\left(2\pi\cdot 10^4\right)t - .344144z - 1.16937\big)$$

$$E(z,t) = 8.11158e^{-.344144z}\cos\big(\left(2\pi\cdot 10^4\right)t - .344144z - .383972\big)$$

$$\boxed{E(z,t) = 8.11158e^{-.344144z}\cos\big(\left(2\pi\cdot 10^4\right)t - .344144z - .383972\big)\hat{\mathbf{x}}\ \left[\frac{\text{mV}}{\text{m}}\right]}$$

(c) To find the average power density at depth z, we may use the Poynting vector. This can be defined as:

$$S_{avg} = \hat{\mathbf{z}}\frac{|\tilde{E}(0)|^2}{2|\eta|}e^{-2\alpha z}\cos(\theta)$$

We know the magnitude of η and $\tilde{E}$, as well as the angle and α, which gives us:

$$S_{avg} = \hat{\mathbf{z}}\frac{|.00811158|^2}{2|.16223|}e^{-2(.344144)z}\cos\left(\frac{\pi}{4}\right)$$

This gives:

$$\boxed{S_{avg} = .143e^{-.688288z}\hat{\mathbf{z}}\ \left[\frac{\text{mW}}{\text{m}^2}\right]}$$

(d) The power drop off can be found using:

$$10^{-\frac{20}{10}}\cdot 100 = 1\%$$

Which we can then use in:

$$.01 = e^{-.688z}$$

$$z = \frac{\ln(.01)}{-.688}$$

$$\boxed{z = 6.69356[\text{m}]}$$

3. First and foremost, we know $\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$ and $-\cos(x) = \cos(x - \pi)$. This will be important for the following problems.

   (a) We are given the equation:

$$\vec{E} = \hat{\mathbf{x}}5\sin(\omega t + z) - \hat{\mathbf{y}}5\cos(\omega t + z) \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

   This can be rewritten as:

$$\vec{E} = \hat{\mathbf{x}}5\cos\left(\omega t + z - \frac{\pi}{2}\right) + \hat{\mathbf{y}}5\cos\left(\omega t + z - \pi\right) \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

   We can see that E_y lags E_x by $\pi/2$, or $90°$. This, in tandem with the equal magnitudes define the above equation as <u>Right-Hand Polarized</u>.

   (b) We are given the equation:

$$\vec{E} = \hat{\mathbf{z}}\cos(\omega t + x) - \hat{\mathbf{y}}\sin(\omega t + x) \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

   This can be rewritten as:

$$\vec{E} = \hat{\mathbf{z}}\cos\left(\omega t + x\right) + \hat{\mathbf{y}}\cos\left(\omega t + z - \frac{3\pi}{2}\right) \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

$$\vec{E} = \hat{\mathbf{z}}\cos\left(\omega t + x\right) + \hat{\mathbf{y}}\cos\left(\omega t + z + \frac{\pi}{2}\right) \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

   We can see that E_y leads E_z by $\pi/2$, or $90°$. This, in tandem with the equal magnitudes define the above equation as <u>Right-Hand Polarized</u>.

   (c) We are given the equation:

$$\vec{H} = \hat{\mathbf{x}}5\cos(\omega t - z) + \hat{\mathbf{y}}5\sin(\omega t - z) \left[\frac{\mathrm{mA}}{\mathrm{m}}\right]$$

   This can be rewritten as:

$$\vec{H} = \hat{\mathbf{x}}5\cos(\omega t - z) + \hat{\mathbf{y}}5\cos\left(\omega t - z - \frac{\pi}{2}\right) \left[\frac{\mathrm{mA}}{\mathrm{m}}\right]$$

   We can see that H_y lags H_x by $\pi/2$, or $90°$. This, in tandem with the equal magnitudes define the above equation as <u>Right-Hand Polarized</u>.