Magnetostatics

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- From the prior lecture, we know:

$$\begin{cases} \nabla \cdot D = \rho \\ \nabla \times E = -\frac{\partial}{\partial t} B \end{cases}$$

- From statics, we know:

$$\begin{cases} \frac{\partial}{\partial t} = 0 \\ \nabla \times E = 0 \end{cases}$$

- We also know Poisson's equation:

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

- In this chapter, we learn:

$$\nabla \cdot B = 0$$
$$\nabla \times H = J + \frac{\partial}{\partial t} D$$

  - Where σ is conductivity, we find:

$$J = \sigma E$$

  - We can see the B is divergence-less
  - Furthermore, since we know:

$$\nabla \times (\text{curl}) = 0$$

    we can say:

$$B = \nabla A$$

since $B = \mu H$:

$$\mu H = \nabla \times A$$

this means:

$$\nabla \times \nabla \times A = \mu J$$

and finally, since we know $\nabla \times \nabla \times = -\nabla^2 + \nabla(\nabla \cdot)$

$$-\nabla^2 A + \nabla(\nabla A) = \mu J$$

since A is auxiliary, we can say $\nabla \cdot A = 0$:

$$\nabla^2 A = \mu J$$

Since we know the solution to Poisson's equation, we can apply it here as well, which gives us:

$$A(r) = \frac{\mu}{4\pi} \int_{\forall} \frac{J(r')}{|r - r'|} \, dr'$$

- From Faraday's Law, we can rewrite:

$$\nabla \times E = -\frac{\partial}{\partial t} \mu H$$

- From equations of linear systems, we can obtain:

$$\nabla \times \tilde{E}(r) = -j\omega\mu\tilde{H}(r)$$
$$\nabla \times \tilde{H}(r) = j\omega\varepsilon\tilde{E}(r)$$

  - The magnetic field phasor would then be:

$$\tilde{H}(r) = \frac{\nabla \times \tilde{E}}{-j\omega\mu}$$

  - The electric field phasor would then be:

$$\nabla \times \tilde{H} = j\omega\varepsilon\tilde{E}$$