Wave Reflection and Transmission

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- Given a phasor wave $\tilde{E}$ and a perpendicular propagation vector $\hat{k}$, we may write:

$$\tilde{E} = E_o\hat{u}e^{-jk\hat{k}}$$
$$\hat{u}\cdot\hat{k} = 0$$
$$r = (x, y, z)$$

- The propagation constant may be defined as:

$$c_{prop} = \frac{\omega}{c}$$

where

$$c = \frac{c_o}{\sqrt{\varepsilon_r\mu_r}}$$

- We will apply transmission line theory to normal incidence and oblique incidence
  - The reflection coefficient, Γ, will be defined relative to the electric field
  - There is also a transmission coefficient (τ)
  - We can write our formulas as:

$$\tilde{E}_t = E_o\tau\hat{\mathbf{x}}e^{-jk_z z}$$
$$\tilde{H}_t = \frac{E_o\tau}{\eta_2}\hat{\mathbf{y}}e^{-jk_z z}$$

  - We can then obtain the reflections

$$\tilde{E}_r = -\hat{\mathbf{x}}E_o\Gamma e^{jk_1 z}$$
$$\tilde{H}_r = -\hat{\mathbf{y}}\frac{E_o\Gamma}{\eta_1}e^{jk_1 z}$$

- We can also get the formula

$$1 + \Gamma = \tau$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

- The Poynting Vector can be redefined as

$$S_{av} = \frac{\hat{\mathbf{z}}|E_o|^2}{2\eta_1}\left(1 - |\Gamma|^2\right)$$

- Oblique Incidence
  - Snell's Laws apply
  - The index of refraction is:

$$c = \frac{c_o}{n}$$
$$n = \sqrt{\varepsilon_r}$$
$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t)$$