Exam 3

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1. (a) The average power density (assuming propagation is occurring in the $\hat{\mathbf{z}}$ direction) may be defined as:

$$S_{avg} = \frac{|\vec{E}|^2}{2|\eta|}\hat{\mathbf{z}}$$

We know that the incident wave occurs in air, and that, for air, $\eta = \sqrt{\mu_o/\varepsilon_o} = 376.819[\Omega]$. This gives us:

$$S_{avg} = \frac{|1750|^2}{376.819}$$

$$\boxed{S_{avg} = 8.127\left[\frac{\text{kW}}{\text{m}^2}\right]}$$

(b) First and foremost, we must check whether sea water is a good conductor. Given the parameters, we write:

$$\frac{\sigma}{\omega\varepsilon} = \frac{3.5}{2\pi(35\cdot 10^6)(50)(8.85\cdot 10^{-12})}$$

$$\frac{\sigma}{\omega\varepsilon} = 35.967$$

Since 35.967 >> 1, we know it is a good conductor. Thus, the wave impedance in the water may be found using:

$$\eta = (1+j)\frac{\alpha}{\sigma}$$

We can find the attenuation constant using:

$$\alpha = \sqrt{\pi f\mu\sigma}$$

$$\alpha = \sqrt{\pi(35\cdot 10^6)(4\pi\cdot 10^{-7})(3.5)}$$

$$\alpha \approx 22 \left[\frac{\text{Np}}{\text{m}}\right]$$

Furthermore, since this is a good conductor, we may write:

$$\beta \approx \alpha \approx 22 \left[\frac{\text{rad}}{\text{m}}\right]$$

Thus, the impedance becomes:

$$\eta = (1+j)\frac{22}{3.5}$$
$$\boxed{\eta = 6.286 + 6.286j}$$

(c) The reflection coefficient may be written as:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$
$$\Gamma = \frac{(6.286 + 6.286j) - 376.819}{(6.286 + 6.286j) + 376.819}$$
$$\boxed{\Gamma = -.9667 + .03227j}$$

(d) The transmission coefficient may be defined as:

$$\tau = 1 + \Gamma$$
$$\tau = 1 + (-.9667 + .03227j)$$
$$\boxed{\tau = .033\bar{3} + .03227j}$$

(e) We can find the average power density of the reflected wave using:

$$S_{avg,r} = |\Gamma|^2 S_{avg,i}$$

This gives:

$$S_{avg,r} = (.93555)S_{avg,i}$$
$$\boxed{S_{avg,r} = 7.603 \left[\frac{\text{kW}}{\text{m}^2}\right]}$$

(f) Since we know $.01[\mu\text{V}\,\text{m}^{-1}]$ is necessary, we may write:

$$|\tau|E_o e^{-\alpha z} = .01 \cdot 10^{-6}$$
$$(.0464)(1750)e^{-22z} = 10^{-8}$$
$$e^{-22z} = \frac{10^{-8}}{81.2}$$
$$-22z = \ln\left(1.2315 \cdot 10^{-10}\right)$$
$$\boxed{z = 1.03716[\text{m}]}$$

(g) We can use a formula similar to that from part (a):

$$S_{avg}(z) = \frac{|\vec{E}|^2}{2|\eta|} e^{-2\alpha z} \cos(\theta_\eta) \hat{\mathbf{z}}$$

This gives us:

$$S_{avg}(z) = \frac{|1750|^2}{2|8.89|} e^{-44z} \cos\left(\frac{\pi}{4}\right) \hat{\mathbf{z}}$$

$$S_{avg}(z) = 121795 e^{-44z} \hat{\mathbf{z}}$$

Now we plug in the value for the depth:

$$S_{avg}(1.03716) = 121795 e^{-44(1.03716)}$$

$$\boxed{S_{avg}(1.03716) = 1.847 \cdot 10^{-15} \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]}$$

2. (a) The wave number may be defined by the coefficients of the exponential:

$$k = \sqrt{2^2 + 3^2}$$

$$k = 3.61$$

We know that:

$$k = \frac{2\pi}{\lambda} \sqrt{\mu \varepsilon}$$

In air, this gives us:

$$\lambda = \frac{2\pi}{kc}$$

$$\lambda = \frac{2\pi}{(3.61)(3 \cdot 10^8)}$$

$$\boxed{\lambda = 5.8[\mathrm{nm}]}$$

(b) From part (a), we can find ω and f:

$$\omega = \frac{2\pi}{\lambda}$$

This then gives us:

$$\omega = \frac{2\pi}{5.8 \cdot 10^{-9}}$$

$$\boxed{\omega = 1.083 \left[\frac{\mathrm{rad}}{\mathrm{s}}\right]}$$

Or:

$$\boxed{f = 172.365\,[\text{MHz}]}$$

(c) The incidence angle can be found according to:

$$\theta_i = \cos^{-1}\left(\frac{2}{3}\right)$$

$$\boxed{\theta_i = 48.19^\circ}$$

(d) We can find the electric field via the formula:

$$E = \eta(\hat{\mathbf{z}} \times H)$$

This gives:

$$E = 376.819 e^{-j(2x+3z)} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & 0 & 1 \\ 12 & -14 & -8 \end{vmatrix}$$

$$E = 376.819 e^{-j(2x+3z)}(14\hat{\mathbf{x}} - 12\hat{\mathbf{y}})(10^{-3})$$

$$\boxed{E = (5.276\hat{\mathbf{x}} - 4.522\hat{\mathbf{y}}) e^{-j(2x+3z)} \left[\frac{\text{V}}{\text{m}}\right]}$$

(e) Since $\mu_1 = \mu_2$, we can find the perpendicular components as:

$$\Gamma_\perp = \frac{\cos(\theta_i) - \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2(\theta_i)}}{\cos(\theta_i) + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2(\theta_i)}}$$

This gives us:

$$\Gamma_\perp = \frac{\cos(48.19) - \sqrt{(3.5) - \sin^2(48.19)}}{\cos(48.19) + \sqrt{(3.5) - \sin^2(48.19)}}$$

$$\boxed{\Gamma_\perp = -.44}$$

According to $\tau_\perp = 1 + \Gamma_\perp$, we get:

$$\boxed{\tau_\perp = .56}$$

For the parallel reflection, we may write:

$$\Gamma_\parallel = \frac{-(\varepsilon_2/\varepsilon_1)\cos(\theta_i) + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2(\theta_i)}}{(\varepsilon_2/\varepsilon_1)\cos(\theta_i) + \sqrt{(\varepsilon_2/\varepsilon_1) - \sin^2(\theta_i)}}$$

This gives us:

$$\Gamma_{\parallel} = \frac{-(3.5)\cos(48.19) + \sqrt{(3.5) - \sin^2(48.19)}}{(3.5)\cos(48.19) + \sqrt{(3.5) - \sin^2(48.19)}}$$

$$\boxed{\Gamma_{\parallel} = -.1525}$$

We also know:

$$\tau_{\parallel} = (1 + \Gamma_{\parallel})\frac{\cos(\theta_i)}{\cos(\theta_t)}$$

From Snell's law, we may write:

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin(48.19)\right)$$

$$\theta_t = \sin^{-1}\left(\frac{1}{\sqrt{3.5}}\sin(48.19)\right)$$

$$\theta_t = 23.4789\,[^\circ]$$

Plugging this back in, we get:

$$\tau_{\parallel} = (1 + (-.1525))\frac{\cos(48.19)}{\cos(23.4789)}$$

$$\boxed{\tau_{\parallel} = .616}$$

(f) First, we find the parallel component of the reflected wave:

$$\Gamma_{\parallel} E_i = -.1525 E_i$$

This gives us:

$$E_{r,\parallel} = (-.80459\hat{\mathbf{x}} + .6896\hat{\mathbf{y}})e^{-j(2x+3z)}$$

Next, we find the perpendicular component:

$$\Gamma_{\perp} E_i = -.44 E_i$$

This gives us:

$$E_{r,\perp} = (-2.3214\hat{\mathbf{x}} + 1.9897\hat{\mathbf{y}})e^{-j(2x+3z)}$$

We then unite these to get the overall reflected wave (note, we keep the negative sign on the $\hat{\mathbf{x}}$ direction):

$$E_r = ((\sqrt{2.3214^2 + .80459^2})\hat{\mathbf{x}} + (\sqrt{.6896^2 + 1.9897^2})\hat{\mathbf{y}})e^{-j(2x+3z)}$$

$$\boxed{E_r = (-2.4569\hat{\mathbf{x}} + 2.1058\hat{\mathbf{y}})e^{-j(2x+3z)}\left[\frac{\mathrm{V}}{\mathrm{m}}\right]}$$

(g) First, we find the parallel component of the transmitted wave:

$$\tau_{\parallel} E_i = .616 E_i$$

This gives us:

$$E_{t,\parallel} = (3.25\hat{\mathbf{x}} - 2.79\hat{\mathbf{y}})e^{-j(2x+3z)}$$

Next, we find the perpendicular component:

$$\tau_{\perp} E_i = .56 E_i$$

This gives us:

$$E_{t,\perp} = (2.954\hat{\mathbf{x}} - 2.5323\hat{\mathbf{y}})e^{-j(2x+3z)}$$

We then unite these to get the overall transmitted wave (note, we keep the negative sign on the $\hat{\mathbf{y}}$ direction):

$$E_t = ((\sqrt{3.25^2 + 2.954^2})\hat{\mathbf{x}} + (\sqrt{2.79^2 + 2.5323^2})\hat{\mathbf{y}})e^{-j(2x+3z)}$$

$$\boxed{E_t = (4.392\hat{\mathbf{x}} - 3.768\hat{\mathbf{y}})e^{-j(2x+3z)} \left[\frac{\mathrm{V}}{\mathrm{m}}\right]}$$

(h) The average power density transmitted into the medium may be expressed as:

$$S_{avg} = \frac{|E_t|^2}{2|\eta_2|}$$

We first find the impedance of the medium:

$$\eta_2 = \sqrt{\frac{\mu_o}{3.5\varepsilon_o}}$$

$$\eta_2 = \sqrt{\frac{4\pi \cdot 10^{-7}}{3.5 \cdot 8.85 \cdot 10^{-12}}}$$

$$\eta_2 = 201.418[\Omega]$$

This gives us:

$$S_{avg} = \frac{(4.392)^2 + (3.768)^2}{2(201.418)}$$

$$\boxed{S_{avg} = .0831 \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]}$$