# Homework 3

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3. Per our formula, we know:

$$\mu_d = \frac{\sigma}{\eta e} = \frac{1}{\eta e \rho}$$

To find $\eta$, we use:

$$\eta = \frac{\rho N_A}{M_{at}}$$

Also, we need to find $\rho$, or the resistivity at $22\,[^\circ\text{C}]$. As such, we get:

$$\rho_{22} = \rho_o[1 + \alpha(T - T_o)]$$

Obtaining our values from a known value table, we get:

$$\rho_{22} = 2.44 \cdot 10^{-8}[1 + .003715(22 - 20)]$$
$$\rho_{22} = 2.458 \cdot 10^{-8}\,[\Omega\,\text{m}]$$

We return to the electron concentration to get:

$$\eta = \frac{2.458 \cdot 10^{-8} \cdot 6.022 \cdot 10^{23}}{196.67}$$
$$\eta = 590.96 \cdot 10^{26} \left[\frac{\text{electron}}{\text{m}^3}\right]$$

Finally, we calculate the drift mobility:

$$\mu_d = \frac{1}{590.96 \cdot 10^{26} \cdot 1.6 \cdot 10^{-19} \cdot 2.458 \cdot 10^{-8}}$$

$$\boxed{\mu_d = 4.3027 \cdot 10^{-3} \left[\frac{\mathrm{m}^2}{\mathrm{Vs}}\right]}$$

Furthermore, with the given velocity, we may find the mean free path as:

$$\lambda = \frac{\mu_d m_e \mu_V}{e}$$

This gives us:

$$\lambda = \frac{4.3027 \cdot 10^{-3} \cdot 9.1 \cdot 10^{-31} \cdot 1.4 \cdot 10^{6}}{1.6 \cdot 10^{-19}}$$

$$\boxed{\lambda = 3.426 \cdot 10^{-8}[\mathrm{m}]}$$

5. Let us begin by denoting the temperature coefficient resistivity as $\tau_\rho$. We can use the following formula:

$$\tau_\rho = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$

Alternatively, we may see that this is equivalent to:

$$\tau_\rho = \frac{1}{\rho_o}\left(\frac{d\rho}{dT}\right)\Big|_{T_o}$$

Thus, we may find the slope at the reference temperature and the resistivity at this temperature. Let us begin by looking at Fe (Iron) at 0[°C]. We may see:

$$\rho_0 \approx .12 \cdot 10^{-6}[\Omega\,\mathrm{m}]$$

For ease of visualization, let us take the reference point as $T_o = 400$[°C] this gives us:

$$\left(\frac{d\rho}{dT}\right)\Big|_{400[^\circ\mathrm{C}]} \approx \frac{(.37 - .12) \cdot 10^{-6}}{400 - 0}$$

$$\left(\frac{d\rho}{dT}\right)\Big|_{400[^\circ\mathrm{C}]} \approx 6.25 \cdot 10^{-10}[\Omega\,\mathrm{m}\,^\circ\mathrm{C}^{-1}]$$

We then calculate the coefficient as:

$$\tau_{\rho,\mathrm{Fe}\to 0[^\circ\mathrm{C}]} = \frac{1}{.12 \cdot 10^{-6}}\left(6.25 \cdot 10^{-10}\right)$$

$$\boxed{\tau_{\rho,\mathrm{Fe}\to 0[^\circ\mathrm{C}]} = 5.2083 \cdot 10^{-3}[^\circ\mathrm{C}^{-1}]}$$

Similarly, we can find Fe (Iron) at 500[°C]:

$$\rho_{500} \approx .57 \cdot 10^{-6} [\Omega\,\mathrm{m}]$$

We can then find the slope using a 900[°C] reference temperature:

$$\left(\frac{d\rho}{dT}\right)\Big|_{900[^\circ\mathrm{C}]} \approx \frac{(1.13 - .57) \cdot 10^{-6}}{900 - 500}$$
$$\left(\frac{d\rho}{dT}\right)\Big|_{900[^\circ\mathrm{C}]} \approx 1.4 \cdot 10^{-9} [\Omega\,\mathrm{m}\,^\circ\mathrm{C}^{-1}]$$

We then calculate the coefficient:

$$\tau_{\rho,\mathrm{Fe}\rightarrow 500[^\circ\mathrm{C}]} = \frac{1}{.57 \cdot 10^{-6}} \left(1.4 \cdot 10^{-9}\right)$$
$$\boxed{\tau_{\rho,\mathrm{Fe}\rightarrow 500[^\circ\mathrm{C}]} = 2.4561 \cdot 10^{-3} [^\circ\mathrm{C}^{-1}]}$$

We then move on to electrotechnical steel. We may get:

$$\rho_0 \approx .5 \cdot 10^{-6} [\Omega\,\mathrm{m}]$$

We then find the slope using a 400[°C] reference temperature:

$$\left(\frac{d\rho}{dT}\right)\Big|_{400[^\circ\mathrm{C}]} \approx \frac{(.69 - .5) \cdot 10^{-6}}{400 - 0}$$
$$\left(\frac{d\rho}{dT}\right)\Big|_{400[^\circ\mathrm{C}]} \approx 4.75\dot{1}0^{-10} [\Omega\,\mathrm{m}\,^\circ\mathrm{C}^{-1}]$$

Finally, we find:

$$\tau_{\rho,\mathrm{Fe}_4\%\,\mathrm{C}\rightarrow 0[^\circ\mathrm{C}]} = \frac{1}{.5 \cdot 10^{-6}} \left(4.75 \cdot 10^{-10}\right)$$
$$\boxed{\tau_{\rho,\mathrm{Fe}_4\%\rightarrow 0[^\circ\mathrm{C}]} = 9.5 \cdot 10^{-4} [^\circ\mathrm{C}^{-1}]}$$

Next, we find the coefficient for electrotechnical steel at 500[°C] using a 900[°C] reference point:

$$\rho_{500} \approx .72 \cdot 10^{-6} [\Omega\,\mathrm{m}]$$

$$\left(\frac{d\rho}{dT}\right)\Big|_{900[^\circ\mathrm{C}]} \approx \frac{(1.23 - .72) \cdot 10^{-6}}{900 - 500}$$
$$\left(\frac{d\rho}{dT}\right)\Big|_{900[^\circ\mathrm{C}]} \approx 1.275\dot{1}0^{-9} [\Omega\,\mathrm{m}\,^\circ\mathrm{C}^{-1}]$$

This gives us:

$$\tau_{\rho,\mathrm{Fe}_4\%\,\mathrm{C}\to 500[^\circ\mathrm{C}]} = \frac{1}{.72\cdot 10^{-6}}\left(1.275\cdot 10^{-9}\right)$$

$$\boxed{\tau_{\rho,\mathrm{Fe}_4\%\to 0[^\circ\mathrm{C}]} = 1.7708\cdot 10^{-3}[^\circ\mathrm{C}^{-1}]}$$

We may observe that, at higher temperatures, the coefficient generally increases for both forms of iron. Furthermore, we may observe that, since the difference between both irons decreases as temperature increases, the values of coefficients converge as well. It is important to note, however, that electrotechnical steel has a much more stable resistivity than regular iron.

9. Let us begin by simply recreating the table:

| | AgAu | AuAg | CuPd | AgPd | AuPd | PdPt | PtPd | CuNi |
|---|---|---|---|---|---|---|---|---|
| $X$ | 8.8%Au | 8.77%Ag | 6.2%Pd | 10.1%Pd | 8.88%Pd | 7.66%Pt | 7.1%Pd | 2.16%Ni |
| $\rho_o[\mathrm{n}\Omega\,\mathrm{m}]$ | 16.2 | 22.7 | 17 | 16.2 | 22.7 | 108 | 105.8 | 17 |
| $\rho_X[\mathrm{n}\Omega\,\mathrm{m}]$ | 44.2 | 54.1 | 70.8 | 59.8 | 54.1 | 188.2 | 146.8 | 50 |
| $C_{eff}$ | | | | | | | | |
| $X$ | 15.4%Au | 24.4%Ag | 13%Pd | 15.2%Pd | 17.1%Pd | 15.5%Pt | 13.8%Pd | 23.4%Ni |
| $\rho'_{X'}$ | | | | | | | | |
| $\rho'_{X'}$[Exp.] | 66.3 | 107.2 | 121.6 | 83.8 | 82.2 | 244 | 181 | 300 |

To solve, we can use the formula $\rho = \rho_o + C_{eff}X(1-X)$. We plug this into a solver (GNU Octave) to iterate over the given values, which allows us to obtain $C_{eff}$:

| | AgAu | AuAg | CuPd | AgPd | AuPd | PdPt | PtPd | CuNi |
|---|---|---|---|---|---|---|---|---|
| $X$ | 8.8%Au | 8.77%Ag | 6.2%Pd | 10.1%Pd | 8.88%Pd | 7.66%Pt | 7.1%Pd | 2.16%Ni |
| $\rho_o[\mathrm{n}\Omega\,\mathrm{m}]$ | 16.2 | 22.7 | 17 | 16.2 | 22.7 | 108 | 105.8 | 17 |
| $\rho_X[\mathrm{n}\Omega\,\mathrm{m}]$ | 44.2 | 54.1 | 70.8 | 59.8 | 54.1 | 188.2 | 146.8 | 50 |
| $C_{eff}$ | 348.88 | 392.46 | 925.1 | 480.18 | 388.06 | 1133.85 | 621.6 | 1561.51 |
| $X$ | 15.4%Au | 24.4%Ag | 13%Pd | 15.2%Pd | 17.1%Pd | 15.5%Pt | 13.8%Pd | 23.4%Ni |
| $\rho'_{X'}$ | | | | | | | | |
| $\rho'_{X'}$[Exp.] | 66.3 | 107.2 | 121.6 | 83.8 | 82.2 | 244 | 181 | 300 |

We then use these $C_{eff}$ values to calculate the $\rho'_{X'}$ values, which gives us:

| | AgAu | AuAg | CuPd | AgPd | AuPd | PdPt | PtPd | CuNi |
|---|---|---|---|---|---|---|---|---|
| $X$ | 8.8%Au | 8.77%Ag | 6.2%Pd | 10.1%Pd | 8.88%Pd | 7.66%Pt | 7.1%Pd | 2.16%Ni |
| $\rho_o[\mathrm{n}\Omega\,\mathrm{m}]$ | 16.2 | 22.7 | 17 | 16.2 | 22.7 | 108 | 105.8 | 17 |
| $\rho_X[\mathrm{n}\Omega\,\mathrm{m}]$ | 44.2 | 54.1 | 70.8 | 59.8 | 54.1 | 188.2 | 146.8 | 50 |
| $C_{eff}$ | 348.88 | 392.46 | 925.1 | 480.18 | 388.06 | 1133.85 | 621.6 | 1561.51 |
| $X$ | 15.4%Au | 24.4%Ag | 13%Pd | 15.2%Pd | 17.1%Pd | 15.5%Pt | 13.8%Pd | 23.4%Ni |
| $\rho'_{X'}$ | 61.654 | 95.094 | 121.629 | 78.093 | 77.712 | 256.506 | 179.743 | 296.891 |
| $\rho'_{X'}$[Exp.] | 66.3 | 107.2 | 121.6 | 83.8 | 82.2 | 244 | 181 | 300 |

We then use these $C_{eff}$ values to calculate the $\rho'_{X'}$ values, which gives us:

13. We may begin by writing out the formulas necessary to calculate the resistivity. First, we begin with the simple conductivity mixture rule:

$$\rho_{eff} = \rho_c \left( \frac{1 + .5X_d}{1 - X_d} \right)$$

Then, we write the Reynolds and Hough Rule:

$$\frac{\sigma_{eff} - \sigma_c}{\sigma_{eff} + 2\sigma_c} = X_d \left( \frac{\sigma_d - \sigma_c}{\sigma_d + 2\sigma_c} \right)$$

Note that $\sigma_d$ represents the conductivity of air, while $X_d$ refers to the pores in the brass. Using the given information, we write:

$$\rho_{eff} = 62 \cdot 10^{-9} \left( \frac{1 + .075}{1 - .15} \right)$$

$$\boxed{\rho_{eff} = 7.2941 \cdot 10^{-8} [\Omega\,\mathrm{m}] = 78.412 [\mathrm{n}\Omega\,\mathrm{m}]}$$

We then use the Reynolds and Hough Rule to write:

$$\frac{\sigma_{eff} - 1.6129}{\sigma_{eff} + 3.2258} = (.15) \left( \frac{-1.6129}{3.2258} \right)$$

We proceed to solve:

$$\frac{\sigma_{eff} - 1.6129}{\sigma_{eff} + 3.2258} = -.075$$

$$\sigma_{eff} - 1.6129 = (-.075)(\sigma_{eff} + 3.2258)$$

$$1.075\sigma_{eff} - 1.6129 = -.2419$$

$$\sigma_{eff} = \frac{1.6129 - .2419}{1.075}$$

$$\boxed{\sigma_{eff} = 1.2753 \cdot 10^{7} [\mathrm{m}^{-1}]}$$

We then take the inverse to find the resistivity:

$$\boxed{\rho_{eff} = 78.41 [\mathrm{n}\Omega\,\mathrm{m}]}$$