Lecture 7 — Conduction in Metals

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- Thermoelectrics
  - The thermoelectric effect is the direct conversion of temperature differences (ΔT) to electric voltage and vice versa
  - A thermoelectric device creates voltage when there is a different temperature on each side
  - Conversely, when a voltage is applied to it, it creates a temperature difference (ΔT)
  - At the atomic scale, an applied temperature gradient causes charge carries in the material to diffuse from the hot side to the cold side thus creating an electrical currentX-Ray Diffraction
- Average Velocity of Charge Carriers (v_{dx})

$$v_{dx} = \frac{1}{N}[v_{x1} + v_{x2} + \cdots + v_{xN}] = \frac{eE_x}{m_e}\overline{(t - t_i)}$$

  - Time-averaged velocity, acceleration over many collisions is zero
- Temperature Dependence of Resistivity
  - To determine the temperature dependence of σ, we must first consider the temperature dependence of τ, since this determines the resistivity
- Mean Free Path
  - Since τ is the time for one scattering process, then $\mu\tau$ is the length traversed before one scattering event (the mean free path):

$$l = \mu\tau$$

- Temperature Dependent Drift Mobility

  - The thermal vibrations of an atom can be considered as a simple harmonic oscillator (think mass on a spring)
  - The average kinetic energy of this system is:

$$E_k = \frac{1}{4}Ma^2\omega^2 \approx \frac{1}{2}kT$$

    * Where a is the amplitude of vibration, ω is the oscillation frequency, k is the Boltzmann constant, and T is the temperature, so $a^2 \propto T$

$$\tau \propto \frac{1}{\pi a^2} \propto \frac{1}{T} \quad \text{or} \quad \tau = \frac{C}{T}$$

$$\mu_d = \frac{eC}{m_e T}$$

- Matthiessen's Rule
  - The theory of conduction that only considers thermal vibration works only for pure metals, not for metallic alloys
  - States that there are two scattering contributions: thermal and impurity
  - We now have two τ: τ_T and τ_l
  - The net probability of scattering (the overall frequency of scattering) then becomes:

$$\frac{1}{\tau} = \frac{1}{\tau_T}\frac{1}{\tau_l}$$

  - The drift mobility, μ_d depends on the effective scattering time, so:

$$\frac{1}{\mu_d} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

    * Where μ_L is the lattice-scattering-limited drift mobility and μ_I is the impurity-scattering-limited drift mobility
  - The resistivity then becomes:

$$\rho = \frac{1}{en\mu_d} = \frac{1}{en\mu_L} + \frac{1}{en\mu_I}$$
$$\rho = \rho_T + \rho_R$$

  - Since we have $\rho_T = AT$, we know that the effective resistivity can be given by:

$$\rho = AT + B$$