Homework 6

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1. (a) Per our formulas, we define capacitance as:

$$C_{eff} = \frac{\varepsilon_r \varepsilon_o A}{d}$$

We enter the given values to find:

$$C_{eff} = \frac{4.5 \cdot (8.85 \cdot 10^{-12}) \cdot 120(10^{-2})^2}{1.5 \cdot 10^{-3}}$$

This gives us:

$$\boxed{C_{eff} = .3186[\text{nF}]}$$

(b) The energy of a capacitor is given by:

$$E_C = \frac{1}{2}CV^2$$

We enter our given values and capacitance from (a) to get:

$$E_C = \frac{1}{2}(.3186 \cdot 10^{-9})(500)^2$$
$$\boxed{E_C = 3.9825 \cdot 10^{-5}[\text{J}]}$$

(c) We can calculate the maximum voltage as:

$$V = Ed$$

Using our values, we get:

$$V = (250 \cdot 10^5)(1.5 \cdot 10^{-3})$$
$$\boxed{V = 37.5[\text{kV}]}$$

(d) We know that the energy may also be expressed as:

$$E_C = \frac{Q^2}{2C}$$

Thus, we may calculate our charge quantity as:

$$Q = \sqrt{(2)(.3186 \cdot 10^{-9})(3.9825 \cdot 10^{-5})}$$
$$Q = 1.593 \cdot 10^{-7}[\mathrm{C}]$$

We can then calculate the changed capacitance, since the dielectric is removed, which would imply $\varepsilon_r = 1$:

$$C\prime = \frac{\varepsilon_o A}{d}$$

This gives us:

$$C\prime = 70.8[\mathrm{pF}]$$

We now recalculate the energy to see:

$$E_C = \frac{(1.593 \cdot 10^{-7})^2}{2(70.8 \cdot 10^{-12})}$$
$$\boxed{E_C = 1.7921 \cdot 10^{-4}[\mathrm{J}]}$$

We can see that the energy is increased by a factor of ε_r, or, in this case, 4.5 times.

2. (a) We may write the polarization as:

$$P = \chi_e \varepsilon_o E$$

Thus, we use our given values to write:

$$P = (3)(8.85 \cdot 10^{-12})(4 \cdot 10^5)$$

We solve to get:

$$\boxed{P = 1.062 \cdot 10^{-5}\left[\frac{\mathrm{C}}{\mathrm{m}^2}\right]}$$

(b) Since the external electric field is uniform, the volume-bound charge density would be zero. This is confirmed by the relation:

$$-\nabla P = \rho_b$$

We may observe that, since the polarization is constant (again, due to the uniform magnetic field), the gradient, and, therefore, bound charge density, is zero.

(c) Per the formula from part (a), doubling the susceptibility would double the polarization; however, as stated in part (b), despite the increase in susceptibility, since the field is uniform, the volume-bound charge density remains 0

3. (a) We can write each given case as:

$$\varepsilon_{r,1M} = 2.8 + \frac{4.5}{1+(.2)^2}$$
$$\varepsilon_{r,5M} = 2.8 + \frac{4.5}{1+(1)^2}$$
$$\varepsilon_{r,50M} = 2.8 + \frac{4.5}{1+(10)^2}$$

This gives us:

$$\boxed{\varepsilon_{r,1M} = 7.1269}$$
$$\boxed{\varepsilon_{r,5M} = 5.05}$$
$$\boxed{\varepsilon_{r,50M} = 2.8446}$$

(b) We may observe that permittivity decreases as frequency increases; however, we must note that, as the frequency increases, the significance of the frequency-dependent term with respect to the permittivity decreases. That is, the permittivity stabilizes at higher frequencies (around 2.8 in this case). This falls in line with our expectations of dielectric dispersion, which states that dipolar polarization does not adjust fast enough to the rapid oscillations of the external field. As a result, the dipoles within the material do not respond to such changes in the field, which reduces the dielectric's charge storing capability, and, consequently, the permittivity.

(c) Incorporating this dielectric into a high-frequency RF circuit would mean we have to take the following into account:

i. Voltage Breakdown — The variation in permittivity with respect to frequency would mean a lower breakdown voltage at lower frequencies. Thus, we should account for the strictest (worst-case scenario) values of the breakdown voltage in case of a drop in frequency

ii. Shift in Capacitance — Due to the lowering of the permittivity at higher frequencies, we would expect the capacitance to drop at high frequencies as well, since one is directly proportional to the other

iii. Dielectric Losses — We must take into account the dissipation factor ($\tan(\delta)$) when designing the circuit, as even a small amount of heat dissipation at high frequencies may lead to overheating of the circuit

(d) We may calculate the energy loss per cycle as:

$$E = 2\pi \tan(\delta)$$

Thus, we get:

$$E = 2\pi(.02)$$

$$\boxed{E = .1257 = 12.57\%}$$

High losses in the dielectric, such as the one above, may result in significant energy dissipation (usually through heat) and degraded signal quality (Q factor). As such, we want to take into account factors that may lead to loss when designing circuits, one of which could be the loss factor of a dielectric.