Homework 4

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1. (a) We can write the maximum as:

$$E_g = \frac{hc}{\lambda}$$

Using our known information, we can express this as:

$$E_g = \frac{(4.136 \cdot 10^{-15})(3 \cdot 10^8)}{600 \cdot 10^{-9}}$$

Using a solver, we obtain:

$$\boxed{E_g = 2.068[\text{eV}] = 3.31 \cdot 10^{-19}[\text{J}]}$$

(b) Given the cross-sectional area (A) and light intensity (I), we know:

$$P = IA$$

Which then gives us:

$$P = (5 \cdot 10^{-2})(2)$$
$$\boxed{P = .1[\text{mW}]}$$

We can then calculate the number of pairs as:

$$N = \frac{P}{E_{ph}}$$

This gives us:

$$N = \frac{.1 \cdot 10^{-3}}{3.31 \cdot 10^{-19}}$$
$$N = 3.0211 \cdot 10^{14} \left[\frac{\text{photons}}{\text{s}}\right]$$

Since each photon creates one pair, we get:

$$\boxed{E = 3.0211 \cdot 10^{14} \left[\frac{\text{electrons}}{\text{s}}\right]}$$

(c) From the same formula as part (a), we may rearrange to get:

$$\lambda = \frac{hc}{E_g}$$

We then plug in our known values to get:

$$\lambda = \frac{(4.136 \cdot 10^{-15})(3 \cdot 10^{8})}{1.42}$$

This gives us:

$$\boxed{\lambda = 873.8[\text{nm}]}$$

(d) Since light is visible from roughly 380 to 700 nanometers, this is not in the visible range (it is in the infrared range)

(e) We can find the energy gap of the given material to be $E_g = 1.1[\text{eV}]$. Thus, we may calculate the wavelength as:

$$\lambda = \frac{(4.136 \cdot 10^{-15})(3 \cdot 10^{8})}{1.1}$$

This gives us:

$$\lambda_{\text{GaAs}} = 1.128[\mu\text{m}]$$

Since the cutoff wavelength of the given material is higher than that of the aforementioned set up, we may conclude that this detector <u>will be able to detect</u> the 873.8[nm] wavelength.

2. From the table, we may obtain the following important values:

$$E_g = .66[\text{eV}]$$
$$m_h^* = .40 m_e$$
$$m_e^* = .56 m_e$$

Then, we may use our formula for the density of states:

$$N_C = 2\left[\frac{2\pi m_e^* k_B T}{h^2}\right]^{\frac{3}{2}}$$

$$N_V = 2\left[\frac{2\pi m_h^* k_B T}{h^2}\right]^{\frac{3}{2}}$$

We may thus write:

$$N_C = 2\left[\frac{2\pi \cdot .56 \cdot 9.1 \cdot 10^{-31} \cdot 1.38 \cdot 10^{-23} \cdot 300}{(6.626 \cdot 10^{-34})^2}\right]^{\frac{3}{2}}$$

$$N_V = 2\left[\frac{2\pi \cdot .4 \cdot 9.1 \cdot 10^{-31} \cdot 1.38 \cdot 10^{-23} \cdot 300}{(6.626 \cdot 10^{-34})^2}\right]^{\frac{3}{2}}$$

We calculate to get:

$$\boxed{N_C = 1.0493 \cdot 10^{25} \left[\mathrm{m}^{-3}\right]}$$

$$\boxed{N_V = 6.3343 \cdot 10^{24} \left[\mathrm{m}^{-3}\right]}$$

We then use the expression for the intrinsic carrier concentration:

$$n_i = \sqrt{N_C N_V} e^{-\frac{E_g}{2k_B T}}$$

Using our values:

$$n_i = \sqrt{(1.0493 \cdot 10^{25})(6.3343 \cdot 10^{24})} e^{-\frac{.66}{2(8.6 \cdot 10^{-5})(300)}}$$

This gives us:

$$\boxed{n_i = 2.2718 \cdot 10^{19} [\mathrm{m}^{-3}]}$$

Using the values from the table, we may get:

$$n_i = \sqrt{(1.04 \cdot 10^{25})(6 \cdot 10^{24})} e^{-\frac{.66}{2(8.6 \cdot 10^{-5})(300)}}$$

$$\boxed{n_i = 2.2012 \cdot 10^{19} [\mathrm{m}^{-3}]}$$

Finally, we can use this to calculate the intrinsic resistivity. We know that:

$$\rho = \frac{1}{\sigma}$$

And that:

$$\sigma = e n_1 (\mu_c + \mu_h)$$

We then get:

$$\rho = \frac{1}{(1.6 \cdot 10^{-19})(2.2012 \cdot 10^{19})(.39 + .19)}$$

$$\rho = (2.0427)^{-1}$$

$$\boxed{\rho = .4895[\Omega\,\mathrm{m}]}$$

Note that, using our calculated intrinsic carrier concentration we obtain a slightly different value:

$$\rho = \frac{1}{(1.6 \cdot 10^{-19})(2.2718 \cdot 10^{19})(.39 + .19)}$$

$$\rho = (2.1082)^{-1}$$

$$\boxed{\rho = .4743[\Omega\,\mathrm{m}]}$$

3. We can express the formula for Fermi level as:

$$\Delta E = \frac{k_B T}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

We expand ΔE to:

$$\Delta E = E_f - E_i$$

With E_f representing the Fermi level and E_i representing the intrinsic (middle of the bandgap) Fermi level. As such, we write:

$$E_f - E_i = \frac{k_B T}{2} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

We can write m_h^* and m_e^* from the table to write the ratios for each material:

$$\mathrm{Ge} \rightarrow \frac{.4}{.56} = .7143$$

$$\mathrm{Si} \rightarrow \frac{.6}{1.08} = .5556$$

$$\mathrm{GaAs} \rightarrow \frac{.5}{.067} = 7.4627$$

We then apply this to the formula:

$$\mathrm{Ge} \rightarrow \frac{(8.617 \cdot 10^{-5} \cdot 300)}{2} \ln(.7143)$$

$$\mathrm{Si} \rightarrow \frac{(8.617 \cdot 10^{-5} \cdot 300)}{2} \ln(.5556)$$

$$\text{GaAs} \to \frac{(8.617 \cdot 10^{-5} \cdot 300)}{2} \ln(7.4627)$$

This gives us:

$$\Delta E_{\text{Ge}} = -4.3488 \cdot 10^{-3}[\text{eV}]$$
$$\Delta E_{\text{Si}} = -7.5964 \cdot 10^{-3}[\text{eV}]$$
$$\Delta E_{\text{GaAs}} = .025979[\text{eV}]$$

Given that we may rewrite with the intrinsic energy band gap, we may observe that Germanium and Silicon are below the intrinsic band gap by -4.3488 and -7.5964 milli-electron-volts, respectively, and Gallium-Arsenide .025979 electron-volts above the intrinsic band gap. We can calculate the precise Fermi level as:

$$E_{f,\text{Ge}} = .33 - 4.3488 \cdot 10^{-3}[\text{eV}]$$
$$E_{f,\text{Si}} = .55 - 7.5964 \cdot 10^{-3}[\text{eV}]$$
$$E_{f,\text{GaAs}} = .71 + .025979[\text{eV}]$$

Finally, we find:

$$\boxed{E_{f,\text{Ge}} = .3257[\text{eV}]}$$
$$\boxed{E_{f,\text{Si}} = .5424[\text{eV}]}$$
$$\boxed{E_{f,\text{GaAs}} = .736[\text{eV}]}$$

4. First and foremost, we are given:

$$N_D = 10^{15}[\text{cm}^{-3}]$$

From the tables from the problems above, we may obtain:

$$\mu_e = 1350 \left[\frac{\text{cm}^2}{\text{Vs}}\right]$$

We can apply our formula for conductivity to get:

$$\sigma = eN_D\mu_e$$

This gives us:

$$\sigma = (1.6 \cdot 10^{-19})(1350)(10^{15})$$
$$\boxed{\sigma = .216[\Omega^{-1}\,\text{cm}] = 21.6[\Omega^{-1}\,\text{m}]}$$

We then find:

$$\rho = \frac{1}{\sigma}$$

$$\rho = \frac{1}{21.6}$$

$$\boxed{\rho = .046296[\Omega\,\mathrm{m}]}$$