Homework 6

Michael Brodskiy
Professor: J. Adams

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1. (a) Per our formulas, we define capacitance as:

$$C_{eff} = \frac{\varepsilon_r \varepsilon_o A}{d}$$

We enter the given values to find:

$$C_{eff} = \frac{4.5 \cdot (8.85 \cdot 10^{-12}) \cdot 120(10^{-2})^2}{1.5 \cdot 10^{-3}}$$

This gives us:

$$\boxed{C_{eff} = .3186[\text{nF}]}$$

(b) The energy of a capacitor is given by:

$$E_C = \frac{1}{2}CV^2$$

We enter our given values and capacitance from (a) to get:

$$E_C = \frac{1}{2}(.3186 \cdot 10^{-9})(500)^2$$
$$\boxed{E_C = 3.9825 \cdot 10^{-5}[\text{J}]}$$

(c) We can calculate the maximum voltage as:

$$V = \frac{Ed}{\varepsilon_r}$$

Using our values, we get:

$$V = \frac{1}{4.5}(250 \cdot 10^5)(1.5 \cdot 10^{-3})$$
$$\boxed{V = 8.33\bar{3}[\text{kV}]}$$

(d) We know that the energy may also be expressed as:

$$E_C = \frac{Q^2}{2C}$$

Thus, we may calculate our charge quantity as:

$$Q = \sqrt{(2)(.3186 \cdot 10^{-9})(3.9825 \cdot 10^{-5})}$$
$$Q = 1.593 \cdot 10^{-7}[\mathrm{C}]$$

We can then calculate the changed capacitance, since the dielectric is removed, which would imply $\varepsilon_r = 1$:

$$C\prime = \frac{\varepsilon_o A}{d}$$

This gives us:

$$C\prime = 70.8[\mathrm{pF}]$$

We now recalculate the energy to see:

$$E_C = \frac{(1.593 \cdot 10^{-7})^2}{2(70.8 \cdot 10^{-12})}$$
$$\boxed{E_C = 1.7921 \cdot 10^{-4}[\mathrm{J}]}$$

We can see that the energy is increased by a factor of ε_r, or, in this case, 4.5 times.

2. (a) We may write the polarization as:

$$P = \chi_e \varepsilon_o E$$

Thus, we use our given values to write:

$$P = (4)(8.85 \cdot 10^{-12})(4 \cdot 10^5)$$

We solve to get:

$$\boxed{P = 1.416 \cdot 10^{-5} \left[\frac{\mathrm{C}}{\mathrm{m}^2}\right]}$$

(b)

(c)

3. (a) We can write each given case as:

$$\varepsilon_{r,1M} = 2.8 + \frac{4.5}{1+(.2)^2}$$
$$\varepsilon_{r,5M} = 2.8 + \frac{4.5}{1+(1)^2}$$
$$\varepsilon_{r,50M} = 2.8 + \frac{4.5}{1+(10)^2}$$

This gives us:

$$\boxed{\varepsilon_{r,1M} = 7.1269}$$
$$\boxed{\varepsilon_{r,5M} = 5.05}$$
$$\boxed{\varepsilon_{r,50M} = 2.8446}$$

(b) We may observe that permittivity decreases as frequency increases. Based on the capacitance formula, $C = \varepsilon A/d$, we know that capacitance is directly proportional to permittivity. Furthermore, we know that the frequency response of a capacitor is inversely proportional to frequency per $C = (j\omega Z)^{-1}$. As such, by extension we may conclude that permittivity is inversely proportional to frequency.

(c) By incorporating this dielectric into high-frequency RF applications, we would need to account for the decreasing dielectric relative permittivity with respect to frequency. Note that, one frequency is great enough, the relative permittivity remains fairly stable, just above 2.8.

(d)