Homework 5

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9. a. 1. To find the conductivity, we may use the formula:

$$\sigma = ne\mu_e$$

Where n represents the electron concentration, e represents the charge of an electron, and μ_e is the electron mobility. Given that the mobility of electrons in silicon is:

$$\mu_{e,27[^\circ\text{C}]} = 8 \cdot 10^2 \left[\frac{\text{cm}^2}{\text{Vs}}\right]$$

We may substitute to get:

$$\sigma = \left(1 \cdot 10^{17}\right)\left(1.6 \cdot 10^{-19}\right)(800)$$

This gives us:

$$\boxed{\sigma_{27[^\circ\text{C}]} = 12.8 \left[\frac{\text{S}}{\text{cm}}\right]}$$

2. In intrinsic silicon, we know that:

$$n_i = N_c e^{-\frac{E_c - E_{fi}}{k_B T}}$$

In n-type doped silicon, we take $n \to N_d$ to get:

$$N_d = N_c e^{-\frac{E_c - E_{fn}}{k_B T}}$$

We divide the doped equation by the original to find a ratio:

$$\frac{N_d}{n_i} = e^{\frac{E_{fn} - E_{fi}}{k_B T}}$$

We can simplify to get:

$$\ln\left(\frac{N_d}{n_i}\right) = \frac{E_{fn} - E_{fi}}{k_B T}$$

We can take the difference in Fermi energies as:

$$\Delta E_f = E_{fn} - E_{fi}$$

Using this, we get:

$$\Delta E_f = k_B T \ln\left(\frac{N_d}{n_i}\right)$$

We can then substitute our known values to find:

$$\Delta E_f = \left(8.617 \cdot 10^{-5}\right)(300)\ln\left(\frac{10^{17}}{1.45 \cdot 10^{10}}\right)$$

$$\boxed{\Delta E_f = .407[\text{eV}]}$$

We see that the Fermi energy of the n-type doped silicon is .407[eV] higher than intrinsic silicon

3. At $T = 127[^\circ\text{C}] = 400[\text{K}]$, we may find the mobility to be:

$$\mu_{e,127[^\circ\text{C}]} = 4.5 \cdot 10^2 \left[\frac{\text{cm}^2}{\text{Vs}}\right]$$

This gives us:

$$\sigma = \left(10^{17}\right)\left(1.6 \cdot 10^{-19}\right)(450)$$

And finally:

$$\boxed{\sigma_{127[^\circ\text{C}]} = 7.2\left[\frac{\text{S}}{\text{cm}}\right]}$$

b. 1. Similar to part (a), we may write:

$$\sigma = ne\mu_e$$

We note the net effect of doping as:

$$n = N_d - N - a$$
$$n = 10^{17} - 9 \cdot 10^{16}$$
$$n = 1 \cdot 10^{16}\left[\text{cm}^{-3}\right]$$

Thus, we see that this is still an n-type semiconductor. We may note, however, that electron scattering occurs, and, thus, the drift mobility becomes:

$$\mu_{e,27[^\circ\text{C}]} = 7 \cdot 10^2 \left[\frac{\text{cm}^2}{\text{Vs}}\right]$$

We then plug in our values to get:

$$\sigma = (10^{16})(1.6 \cdot 10^{-19})(700)$$

Now we get:

$$\boxed{\sigma_{n,p,27[°\mathrm{C}]} = 1.12 \left[\frac{\mathrm{S}}{\mathrm{cm}}\right]}$$

2. Using the same formula from part (a), except now with $n = N_d - N_a$, we may get:

$$\Delta E_f = k_B T \ln\left(\frac{N_d - N_a}{n_i}\right)$$

This gives us:

$$\Delta E_f = \left(8.617 \cdot 10^{-5}\right)(300) \ln\left(\frac{10^{16}}{1.45 \cdot 10^{10}}\right)$$

$$\boxed{\Delta E_f = 3.475[\mathrm{eV}]}$$

We may notice that the Fermi energy difference has shrunk.

6. a No, doping will not always increase the conductivity. For example, take the intrinsic state with $n = p$. Let us then take a p-type material (say, Boron), and use it to slightly dope. This will result in $p > n$, which would mean more holes with lower mobility (as opposed to electrons with higher mobility), and, as such, the conductivity would decrease.

   b We may begin by applying the mass-action law:

$$np = n_i^2$$

We isolate n to write:

$$n = \frac{n_i^2}{p}$$

We substitute this into the given conductivity equation to get:

$$\sigma = \frac{e n_i^2 \mu_e}{p} + pe\mu_h$$

We want to find the minimum p value, so we proceed to differentiate:

$$\frac{\partial \sigma}{\partial p} = -\frac{e n_i^2 \mu_e}{p^2} + e\mu_h$$

We want to find points at which the sign changes, so we set the partial equal to 0 to get:

$$-\frac{en_i^2\mu_e}{p^2} + e\mu_h = 0$$

We can then solve for p:

$$\frac{en_i^2\mu_e}{p^2} = e\mu_h$$

$$\frac{en_i^2\mu_e}{e\mu_h} = p^2$$

$$p = \pm\sqrt{\frac{en_i^2\mu_e}{e\mu_h}}$$

Of course, p can not physically be negative, so we get:

$$\boxed{p = n_i\sqrt{\frac{\mu_e}{\mu_h}}}$$

From here, we can substitute this into our conductivity equation to get:

$$\sigma = \frac{en_i\mu_e}{\sqrt{\frac{\mu_e}{\mu_h}}} + n_i\sqrt{\frac{\mu_e}{\mu_h}}e\mu_h$$

This gives us a simplified form as:

$$\sigma_{min} = e\left[n_i\sqrt{\mu_e\mu_h} + n_i\sqrt{\mu_e\mu_h}\right]$$

$$\boxed{\sigma_{min} = 2en_i\sqrt{\mu_e\mu_h}}$$

c From Table 5.1, we may get:

$$\mu_e = 1.35 \cdot 10^3 \left[\frac{\text{cm}^2}{\text{Vs}}\right]$$

$$\mu_h = 4.5 \cdot 10^2 \left[\frac{\text{cm}^2}{\text{Vs}}\right]$$

Using the equations obtained in (b) we get:

$$p_m = (1.45 \cdot 10^{10})\sqrt{\frac{1.35}{.45}}$$

$$\boxed{p_m = 2.512 \cdot 10^{10}\left[\text{cm}^{-3}\right]}$$

We then take our conductivity equation as:

$$\sigma_{min} = 2(1.6 \cdot 10^{-19})(1.45 \cdot 10^{10})\sqrt{(1.35 \cdot 10^3)(.45 \cdot 10^3)}$$

$$\boxed{\sigma_{min} = 3.617 \left[\frac{\mathrm{S}}{\mathrm{cm}}\right]}$$

We know the intrinsic value of p, so we take the ratio to get:

$$\frac{p_m}{n_i} = \frac{2.512}{1.45}$$
$$\frac{p_m}{n_i} = 1.7324$$

We can then find the intrinsic conductivity as:

$$\sigma_i = \left(1.6 \cdot 10^{-19}\right)\left(1.45 \cdot 10^{10}\right)[1.35 + .45] \cdot 10^3$$
$$\sigma_i = 4.176 \cdot 10^{-6}[\frac{\mathrm{S}}{\mathrm{cm}}]$$

We then take the ratio to write:

$$\frac{\sigma_{min}}{\sigma_i} = \frac{3.617}{4.176}$$
$$\frac{\sigma_{min}}{\sigma_i} = .8661$$

We may see that p-type doping to the quantity calculated above results in a 73.24% increase in hole concentration, with a 13.39% decrease in conductivity.

7. We may begin by writing:

$$\sigma = pq\mu_h$$

We then convert this to resistivity:

$$\rho = [pq\mu_h]^{-1}$$

We can substitute known values (including $p \to N_{dopant}$) to get:

$$\rho = (1.6 \cdot 10^{-19})^{-1} N_{dopant}^{-1} \left[\frac{1 + 3.745 \cdot 10^{-18} N_{dopant}}{461.3 + 2.0335 \cdot 10^{-18} N_{dopant}}\right]$$

We take $\rho \to .1$ and then solve:

$$.1 = (1.6 \cdot 10^{-19})^{-1} N_{dopant}^{-1} \left[\frac{1 + 3.745 \cdot 10^{-18} N_{dopant}}{461.3 + 2.0335 \cdot 10^{-16} N_{dopant}}\right]$$
$$1.6 \cdot 10^{-20} N_{dopant} = \left[\frac{1 + 3.745 \cdot 10^{-18} N_{dopant}}{461.3 + 2.0335 \cdot 10^{-16} N_{dopant}}\right]$$
$$7.3808 \cdot 10^{-18} N_{dopant} + 3.2536 \cdot 10^{-36} N_{dopant}^2 = 1 + 3.745 \cdot 10^{-18} N_{dopant}$$

$$3.2536 \cdot 10^{-36} N_{dopant}^2 + 3.6358 \cdot 10^{-18} N_{dopant} - 1 = 0$$

We can put this in quadratic form:

$$N_{dopant}^2 + 1.175 \cdot 10^{18} N_{dopant} - 3.0735 \cdot 10^{35} = 0$$

Solving this gives us:

$$N_{dopant} = 2.2028535 \cdot 10^{17}, -1.3952853 \cdot 10^{18}$$

Since we know the quantity must be positive, we conclude:

$$\boxed{N_{dopant} = 2.2028535 \cdot 10^{17} [\mathrm{cm}^{-3}]}$$

12. a Per the problem, we are given:

$$N_D = 10^{17} [\mathrm{cm}^{-3}]$$

We use this in the given formulas to get:

$$\mu_e = 88 + \frac{1252}{1 + 6.984 \cdot 10^{-18} (10^{17})}$$

This gives us:

$$\mu_e = 825.16 \left[\frac{\mathrm{cm}^2}{\mathrm{Vs}} \right]$$

We then calculate:

$$\sigma = ne\mu_e$$
$$\sigma = (10^{17})(1.6 \cdot 10^{-19})(825.16)$$
$$\boxed{\sigma = 13.203 \left[\frac{\mathrm{S}}{\mathrm{cm}} \right]}$$

b Similar to part (1), we write:

$$N_D = 10^{17} [\mathrm{cm}^{-3}]$$
$$\mu_h = 54.3 + \frac{407}{1 + 3.745 \cdot 10^{-18} (10^{17})}$$

This gives us:

$$\mu_h = 350.41 \left[\frac{\mathrm{cm}^2}{\mathrm{Vs}} \right]$$

From here, we may write:

$$\sigma = pq\mu_h$$

We take $p \to N_A$ to get:

$$13.203 = N_A(1.6 \cdot 10^{-19})(350.41)$$

This gives us:

$$N_A = \frac{13.203}{350.41(1.6 \cdot 10^{-19})}$$

$$\boxed{N_A = 2.3548 \cdot 10^{17}[\text{cm}^{-3}]}$$