Homework 5

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1. (a) We may begin by writing:

$$N_A = 10^{16}\left[\frac{1}{\text{cm}^3}\right] \quad \text{and} \quad N_D = 10^{18}\left[\frac{1}{\text{cm}^3}\right]$$

We know that this silicon ($N_A \neq N_D$) is n-type, so we may write the concentration of donors as:

$$\boxed{n \approx N_D = 10^{18}\left[\frac{1}{\text{cm}^3}\right]}$$

From here, we use the mass action law to write:

$$np = n_i^2$$

Where $n_i = 1.5 \cdot 10^{10}[\text{cm}^{-3}]$ for silicon at 300[K]. This gives us the hole concentration as:

$$p = \frac{(1.5 \cdot 10^{10})^2}{10^{18}}$$

$$\boxed{p = 225[\frac{1}{\text{cm}^3}]}$$

(b)

$$N_A = 10^{17}\left[\frac{1}{\text{cm}^3}\right] \quad \text{and} \quad N_D = 10^{17}\left[\frac{1}{\text{cm}^3}\right]$$

Given this, we know that this silicon is intrinsic like ($N_A = N_D$). This means that we may write:

$$n + N_A = p + N_D$$

$$n + 10^{15} = p + 10^{15}$$

$$n = p$$

Using the mass-action law, we may write:

$$n = n_i$$

Which gives us:

$$\boxed{n = p = 1.5 \cdot 10^{10} \left[\frac{1}{\text{cm}^3}\right]}$$

2. - V_{BE}

     We begin by using the transistor equation:

     $$I_e = I_{ES} e^{\frac{V_{BE}}{V_T}}$$

     This can be rearranged to get:

     $$V_{BE} = V_T \ln\left(\frac{I_E}{I_{ES}}\right)$$

     And now we enter known values:

     $$V_{BE} = .026 \ln\left(\frac{.01}{10^{-13}}\right)$$

     $$\boxed{V_{BE} = .6585[\text{V}]}$$

   - V_{BC}

     Since we are given $V_{CE} > .2[\text{V}]$, the BJT is active, and we can write:

     $$V_{BC} = V_{BE} - V_{CE}$$

     $$V_{BC} = .6585 - 10$$

     $$\boxed{V_{BC} = -9.3415[\text{V}]}$$

   - I_B

     We may use the value of β to find:

     $$I_B = (1 + \beta) I_E$$

     $$I_B = (1 + 100)^{-1}(.01)$$

     $$\boxed{I_B = 99[\mu\text{A}]}$$

- I_C
  We then know:

$$I_C = \beta I_B$$
$$I_C = 100(99 \cdot 10^{-6})$$
$$\boxed{I_C = 9.9[\text{mA}]}$$

- α
  Finally, we find α:

$$\alpha = \frac{\beta}{\beta + 1}$$
$$\alpha = \frac{100}{100 + 1}$$
$$\boxed{\alpha = .9901}$$

3. - V_1
     Since there is a voltage drop from the base (due to forward-bias) to the emitter of .7[V], we know:

$$\boxed{V_1 = -.7[\text{V}]}$$

   - V_2
     We may begin by finding the emitter current at transistor Q_1:

$$I_{EQ_1} = \frac{10 - .7}{4.7k}$$
$$I_{EQ_1} = 1.979[\text{mA}]$$

     Given the β value, we may find the collector current as:

$$I_{CQ_1} = \frac{100}{101} I_{EQ_1}$$
$$I_{CQ_1} = 1.9594[\text{mA}]$$

     We can then calculate V_2 based on KVL:

$$V_2 = 10 - (1.9594)(5.1k)$$
$$\boxed{V_2 = 7.0297[\text{mV}]}$$

- V_3

  Applying this voltage into a KVL equation for transistor Q_2, we may write:

$$I_{CQ_2} = \frac{10 - .0070297 - .7}{4.7k}$$
$$I_{CQ_2} = 1.9772[\text{mA}]$$

  We can thus get V_3 as:

$$V_3 = 10 - (1.9772)(4.7)$$
$$\boxed{V_3 = .707[\text{V}]}$$

- V_4

  We then find the emitter voltage based on the β:

$$I_{EQ_2} = \frac{101}{100}(1.9772)$$
$$I_{EQ_2} = 1.997[\text{mA}]$$

  Using KVL at the collector, we get:

$$V_4 = (3)(1.997) - 10$$
$$\boxed{V_4 = -4.0091[\text{V}]}$$

- V_5

  We may find V_5 using the voltage drop from a forward-biased diode:

$$V_5 = V_4 - .7$$
$$V_5 = -4.0091 - .7$$
$$\boxed{V_5 = -4.7091[\text{V}]}$$

- V_6

  Using KVL at the input of Q_3, we get:

$$I_{EQ_3} = \frac{10 - 4.0091 - .7}{1.3k}$$
$$I_{EQ_3} = 4.0699[\text{mA}]$$

  We then find the collector current:

$$I_{CQ_3} = \frac{\beta}{\beta + 1}(4.0699)$$
$$I_{CQ_3} = \frac{100}{101}(4.0699)$$

$$I_{CQ_3} = 4.0296[\text{mA}]$$

And, finally, we use KVL to get:

$$V_6 = 10 - (4.0296)(2)$$
$$\boxed{V_6 = 1.9408[\text{V}]}$$

We may demonstrate the values we found as:

$$\boxed{V \begin{cases} 1, & -.7 \\ 2, & 7.0297 \cdot 10^{-3} \\ 3, & .707 \\ 4, & -4.0091 \\ 5, & -4.7091 \\ 6, & 1.9408 \end{cases} [\text{V}]}$$

4. First, we find the thermal voltage at 180[°C]:

$$V_{T_2} = \frac{(1.38 \cdot 10^{-23})(273 + 180)}{1.6 \cdot 10^{-19}}$$
$$V_{T_2} = .039071[\text{V}]$$

From here, we may write our temperature difference equation to determine the initial V_{BE} voltage at 2[mA]:

$$V_{BE_o} = V_{BE_1} + .002(T_2 - T_1)$$
$$V_{BE_o} = -.7 + .002(150)$$
$$V_{BE_o} = -.4[\text{V}]$$

We then find the saturation current:

$$I_s = \frac{I_C}{e^{V_{BE_o}/V_{T_2}}}$$
$$I_s = \frac{.002}{e^{.4/.039071}}$$
$$I_s = 71.585[\text{nA}]$$

Finally, from this we may write:

$$V_{BE_2} = (.039071)\ln\left(\frac{.0001}{71.585 \cdot 10^{-9}}\right)$$
$$\boxed{V_{BE_2} = .283[\text{V}]}$$

5. (a)
   (b)
   (c)
   (d)