Lecture 3

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- Frequency Dependence (Impedance)
  - Capacitor
    $$Z_c = \frac{1}{j\omega C}$$
  - Inductors
    $$Z_L = j\omega L = SL$$
  - Note, for capacitors impedance decreases with frequency, while it increases with frequency for inductors
- DC Coupling
  - Amplifier stages are directly connected together
  - High-frequency gain decreases ("rolls off") due to unwanted ("parasitic") capacitances and inductances
- AC Coupling
  - Input-coupling capacitors are sometimes referred to as DC-blocking Capacitors
  - Improved isolation between stages because the capacitors "block" DC current/voltages ($Z_c = 1/j\omega C \rightarrow$ infinite impedance at $\omega = 0$)
  - Impacts the low-frequency response
- Impact of Parasitics (Stray Inductances/Capacitances)
  - Stray inductances/capacitances (often called "parasitics") result from non-ideal properties of materials:
    * Integrated circuits, chip packages, printed circuit boards, cables, . . .

- High-frequency gain reduction from:
  * Capacitors in parallel with the signal path
  * Inductors in series with the signal path
- Computer-based simulations are used for complex models and circuits

- Half-Power Bandwidth
  - $P_o = (AV_{\text{inRMS}})^2/R_L \to P_o = P_{max}/2$ when$A = A_{max}/\sqrt{2}$
  - By convention, the frequencies f_H and f_L at which $P_o = P_{max}/2$ are referred to as half-power frequencies or -3db frequencies
    * Note: $20\log\left(A_{max}/\sqrt{2}\right) = 20\log(A_{max}) - 20\log\left(\sqrt{2}\right) = A_{max(\text{dB})} - 3.01\text{dB}$
  - Amplifier bandwidth: $B = f_H - f_L$

- Complex Gain, Frequency Response
  - Complex transfer function $T(j\omega)$
    * $s = j\omega = j(2\pi f) \to T(s) = {}^{V_o(s)/V_i(s)}$
    * Frequency-dependent gain and phase
    * $|T|\angle\phi = R + jX$, where $|T| = (R^2 + X^2)^{1/2}$, $\phi = \tan^{-1}\left(\frac{X}{R}\right)$

- First-Order Low-Pass Filter
  - $V_o(s) = V_i(s)\frac{Z_c}{Z_c+Z_r}$, where $Z_r = R$, $Z_c = \frac{1}{sC} = \frac{1}{j\omega C}$
  - $T(j\omega) = \dfrac{V_o(j\omega)}{V_i(j\omega)} = \dfrac{1/(j\omega C)}{(1/j\omega C) + R} = \dfrac{1}{1 + j\omega RC}$
  - Let $\omega_o = (1/RC) = (1/\tau)$ and $K = 1$
    * $\tau = RC$ is the only time constant of this circuit with a single pole formed by the resistor and capacitor
    * $T(j\omega) = \dfrac{K}{[1 + j\frac{\omega}{\omega_o}]}$

- Transfer Function Normalization (First-Order LPF Case)
  - Typically, $K \neq 1$
  - When normalizing a magnitude response, plot: $|T(j\omega)/K|$
    * $20\log(|T(j\omega)/K|) = 20\log(1) = 0[\text{dB}]$ becomes max gain
  - Low-pass filter characteristics:
    * For $\omega << \omega_o$: $|T(j\omega)/K| \approx 1$ (0[dB])
    * For $\omega >> \omega_o$: $|T(j\omega)/K| \approx \dfrac{\omega_o}{\omega} \to$ high-frequency roll-off

    * Slope is -20[dB]/decade (or -6[dB]/octave)

- Bode Plot of the Low-Pass Filter
  - Attenuates high-frequency signal components
  - "Corner frequency" $\leftrightarrow$ -3[dB] frequency is the "cutoff frequency"
    * Often labeled $f_c(\omega_c)$, f_{3dB}, (ω_{3dB}), $f_o(\omega_o)$, or $f_B(\omega_B)$
    * In the LPF case, the corner frequency is often called "bandwidth of the filter"

- First-Order High Pass Filter

$$T(j\omega) = \frac{1}{1 - j(\omega_o/\omega)}$$

  - Where $\omega_o = 1/(RC) = (1/\tau)$, with $\tau = RC$ as the time constant
  - In general:

$$T(j\omega) = \frac{K}{1 - j(\omega_o/\omega)}$$

    * As $\omega \to 0$, $T(j\omega \to)$ (low frequency rejection)
    * As $\omega \to \infty$, $T(j\omega \to K)$ (high frequency transmission)

- Bode Plot of the High-Pass Filter
  - Attenuates low-frequency components
  - "Corner frequency" $\leftrightarrow$ -3[dB] frequency is the "cutoff frequency"
    * Often labeled $f_c(\omega_c)$, f_{3dB}, (ω_{3dB})
    * $f_{\mathrm{3dB}} \neq$ bandwidth

- Bandpass (Mid-Band) Filter
  - Attenuates signal components outside of bandwidth
  - Bandwidth: $B = f_{c(LP)} - f_{c(HP)} = f_{\mathrm{High3DB}} - f_{\mathrm{Low3dB}}$

- Ideal Operational Amplifiers (Op-Amps)
  - Infinite open-loop differential gain $A_{dOL} = V_o/(V_+ - V_-)$
  - Infinite input impedance ($R_i = \infty$, $i_{in+} = i_{in-} = 0$)
  - Zero output impedance ($R_o = 0$)
  - Zero common-mode gain (CMRR=$A_{dOL}/A_{cm} = \infty$)
  - Infinite bandwidth (no high or low frequency gain roll-off)

- The Summing-Point Constraint
  - Only applies when the op-amp is used in negative feedback, which is often the case
  - Assuming the ideal op-amp (in particular: $A_{dOL} = \infty$), the feedback action forces $V_+ - V_- = 0$ (a virtual short-circuit between the terminals)
    - No current flow into the input terminals