Homework 10

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November 13, 2024

1. (a) Since we see that $V_{GS} < V_{to}$, the transistor is operating in the cutoff region. In the cutoff region, the drain current is $\boxed{I_D = 0}$

   (b) Since $V_{DS} \leq V_{DS} - V_{to}$ and $V_{GS} \geq V_{to}$, the transistor is operating in the linear (triode) region. This gives us the drain current as:

$$I_D = \left(\frac{W}{L}\right)\left(\frac{KP}{2}\right)\left[2(V_{GS} - V_{to})V_{DS} - V_{DS}^2\right]\left[1 + \lambda V_{DS}\right]$$

$$I_D = (10)\left(25 \cdot 10^{-6}\right)\left[2(3-1)(1) - (1)^2\right]\left[1 + (0)(1)\right]$$

$$\boxed{I_D = .75[\text{mA}]}$$

   (c) Since $V_{DS} \geq V_{GS} - V_{to}$ and $V_{GS} \geq V_{to}$, the transistor is operating in the saturation region. The drain current becomes:

$$I_D = KP\left(\frac{W}{L}\right)(V_G - V_S - V_t)^2$$

$$I_D = \left(50 \cdot 10^{-6}\right)(10)(3-1)^2$$

$$\boxed{I_D = 2[\text{mA}]}$$

   (d) Since $V_{DS} \geq V_{GS} - V_{to}$ and $V_{GS} \geq V_{to}$, the transistor is operating in the saturation region. The drain current becomes:

$$I_D = KP\left(\frac{W}{L}\right)(V_G - V_S - V_t)^2$$

$$I_D = \left(50 \cdot 10^{-6}\right)(10)(5-1)^2$$

$$\boxed{I_D = 8[\text{mA}]}$$

2. First, we know that $I_G = 0$ since the input impedance of the MOSFET is high. In this manner, we may write:

$$I_{DQ} = I_{SQ}$$

Using KVL, we may obtain:

$$V_{DD} = V_{GS} + I_{DQ}R_S$$
$$V_{GS} = 15 - 3000I_{DQ}$$

Assuming the MOSFET is operating in the saturated region, we may write:

$$I_{DQ} = K(V_{GS} - V_{to})^2$$
$$I_{DQ} = .25(15 - 3000I_{DQ} - 1)^2$$
$$I_{DQ} = .25(14 - 3000I_{DQ})^2$$
$$I_{DQ} = 2250I_{DQ}^2 - 21I_{DQ} + .049$$
$$0 = 2250I_{DQ}^2 - 22I_{DQ} + .049$$

Solving the equation, we obtain:

$$I_{DQ} = 4.889 \cdot 10^{-3} \pm 1.4572 \cdot 10^{-4}$$
$$\boxed{I_{DQ} = 6.3461,\ 3.4317[\text{mA}]}$$

We now check the voltage in both cases. Let us use the first value to find the gate-to-source voltage:

$$V_{GS1} = 15 - (3000)(I_{DQ})$$
$$V_{GS1} = 15 - (3)(6.3461)$$
$$\boxed{V_{GS1} = -4.0383[\text{V}]}$$

We may observe that, in this case, the transistor is off. Now, we use the second value:

$$V_{GS2} = 15 - (3000)(I_{DQ})$$
$$V_{GS2} = 15 - (3)(3.4317)$$
$$\boxed{V_{GS2} = 4.7049[\text{V}]}$$

We see that the transistor is on only for the second value. Thus, we proceed with the second drain current value. This gives us:

$$V_{DD} - I_{DQ}(R_D) - V_{DSQ} - I_{DQ}(R_S) + V_{DD} = 0$$
$$30 - I_{DQ}(R_D) - I_{DQ}(R_S) = V_{DSQ}$$

We can solve using our known values:

$$V_{DSQ} = 30 - 3.4317(4)$$
$$\boxed{V_{DSQ} = 16.273[\text{V}]}$$

We may observe that both $V_{GS} > V_{to}$ and $V_{DSQ} \geq V_{GS} - V_{to}$ are true, meaning that our saturation assumption was valid. As such, we have found our values for the given transistor.

3. First and foremost, we know that M_1 is in saturation since $V_{D1} > (V_{G1} - V_t)$. As such, we may write:

$$I_{D1} = KP\left(\frac{W}{L}\right)_1 (V_{G1} - V_{S1} - V_t)^2$$
$$I_{D1} = 15 \cdot 10^{-6}\,(40)\,(5 - 3 - 1)^2$$
$$\boxed{I_{D1} = 0.6[\text{mA}]}$$

From here, since we know $I_{D1} = I_{D2}$, we may write:

$$I_{D1} = KP\left(\frac{W}{L}\right)_2 (V_{G2} - V_{S2} - V_t)^2$$
$$1.2 \cdot 10^{-3} = 30 \cdot 10^{-6}\left(\frac{W}{L}\right)_2 (3 - 0 - 1)^2$$
$$1.2 \cdot 10^{-3} = 120 \cdot 10^{-6}\left(\frac{W}{L}\right)_2$$
$$\boxed{\left(\frac{W}{L}\right)_2 = 10}$$