

# Homework 10

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1. (a) Since we see that  $V_{GS} < V_{to}$ , the transistor is operating in the cutoff region. In the cutoff region, the drain current is  $I_D = 0$
- (b) Since  $V_{DS} \leq V_{GS} - V_{to}$  and  $V_{GS} \geq V_{to}$ , the transistor is operating in the linear (triode) region. This gives us the drain current as:

$$I_D = \left(\frac{W}{L}\right) \left(\frac{KP}{2}\right) [2(V_{GS} - V_{to})V_{DS} - V_{DS}^2] [1 + \lambda V_{DS}]$$

$$I_D = (10) (25 \cdot 10^{-6}) [2(-1)(5) - (5)^2] [1 + (0)(5)]$$

$$I_D = -8.75[\text{mA}]$$

- (c) Since  $V_{DS} \geq V_{GS} - V_{to}$  and  $V_{GS} \geq V_{to}$ , the transistor is operating in the saturation region. The drain current becomes:

$$I_D = KP \left(\frac{W}{L}\right) (V_G - V_S - V_t)^2$$

$$I_D = (50 \cdot 10^{-6}) (10) (3 - 1)^2$$

$$I_D = 2[\text{mA}]$$

- (d) Since  $V_{DS} \geq V_{GS} - V_{to}$  and  $V_{GS} \geq V_{to}$ , the transistor is operating in the saturation region. The drain current becomes:

$$I_D = KP \left(\frac{W}{L}\right) (V_G - V_S - V_t)^2$$

$$I_D = (50 \cdot 10^{-6}) (10) (5 - 1)^2$$

$$I_D = 8[\text{mA}]$$

2. First, we know that  $I_G = 0$  since the input impedance of the MOSFET is high. In this manner, we may write:

$$I_{DQ} = I_{SQ}$$

Using KVL, we may obtain:

$$V_{DD} = V_{GS} + I_{DQ}R_S$$

$$V_{GS} = 15 - 3000I_{DQ}$$

Assuming the MOSFET is operating in the saturated region, we may write:

$$I_{DQ} = K(V_{GS} - V_{to})^2$$

$$I_{DQ} = .25(15 - 3000I_{DQ} - 1)^2$$

$$I_{DQ} = .25(14 - 3000I_{DQ})^2$$

$$I_{DQ} = 2250I_{DQ}^2 - 21I_{DQ} + .049$$

$$0 = 2250I_{DQ}^2 - 22I_{DQ} + .049$$

Solving the equation, we obtain:

$$I_{DQ} = 4.889 \cdot 10^{-3} \pm 1.4572 \cdot 10^{-4}$$

$$\boxed{I_{DQ} = 6.3461, 3.4317[\text{mA}]}$$

We now check the voltage in both cases. Let us use the first value to find the gate-to-source voltage:

$$V_{GS1} = 15 - (3000)(I_{DQ})$$

$$V_{GS1} = 15 - (3)(6.3461)$$

$$\boxed{V_{GS1} = -4.0383[\text{V}]}$$

We may observe that, in this case, the transistor is off. Now, we use the second value:

$$V_{GS2} = 15 - (3000)(I_{DQ})$$

$$V_{GS2} = 15 - (3)(3.4317)$$

$$\boxed{V_{GS2} = 4.7049[\text{V}]}$$

We see that the transistor is on only for the second value. Thus, we proceed with the second drain current value. This gives us:

$$V_{DD} - I_{DQ}(R_D) - V_{DSQ} - I_{DQ}(R_S) + V_{DD} = 0$$

$$30 - I_{DQ}(R_D) - I_{DQ}(R_S) = V_{DSQ}$$

We can solve using our known values:

$$V_{DSQ} = 30 - 3.4317(4)$$

$$\boxed{V_{DSQ} = 16.273[\text{V}]}$$

We may observe that both  $V_{GS} > V_{to}$  and  $V_{DSQ} \geq V_{GS} - V_{to}$  are true, meaning that our saturation assumption was valid. As such, we have found our values for the given transistor.

3. First and foremost, we know that  $M_1$  is in saturation since  $V_{D1} > (V_{G1} - V_t)$ . As such, we may write:

$$I_{D1} = KP \left( \frac{W}{L} \right)_1 (V_{G1} - V_{S1} - V_t)^2$$

$$I_{D1} = 30 \cdot 10^{-6} (40) (5 - 3 - 1)^2$$

$$\boxed{I_{D1} = 1.2[\text{mA}]}$$

From here, since we know  $I_{D1} = I_{D2}$ , we may write:

$$I_{D1} = KP \left( \frac{W}{L} \right)_2 (V_{G2} - V_{S2} - V_t)^2$$

$$1.2 \cdot 10^{-3} = 30 \cdot 10^{-6} \left( \frac{W}{L} \right)_2 (3 - 0 - 1)^2$$

$$1.2 \cdot 10^{-3} = 120 \cdot 10^{-6} \left( \frac{W}{L} \right)_2$$

$$\boxed{\left( \frac{W}{L} \right)_2 = 10}$$