Homework 5

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1. (a) We may begin by writing:

$$N_A = 10^{16}\left[\frac{1}{\text{cm}^3}\right] \quad \text{and} \quad N_D = 10^{18}\left[\frac{1}{\text{cm}^3}\right]$$

We know that this silicon ($N_A \neq N_D$) is n-type, so we may write the concentration of donors as:

$$\boxed{n \approx N_D = 10^{18}\left[\frac{1}{\text{cm}^3}\right]}$$

From here, we use the mass action law to write:

$$np = n_i^2$$

Where $n_i = 1.5 \cdot 10^{10}[\text{cm}^{-3}]$ for silicon at 300[K]. This gives us the hole concentration as:

$$p = \frac{(1.5 \cdot 10^{10})^2}{10^{18}}$$

$$\boxed{p = 225[\frac{1}{\text{cm}^3}]}$$

(b)

$$N_A = 10^{17}\left[\frac{1}{\text{cm}^3}\right] \quad \text{and} \quad N_D = 10^{17}\left[\frac{1}{\text{cm}^3}\right]$$

Given this, we know that this silicon is intrinsic like ($N_A = N_D$). This means that we may write:

$$n + N_A = p + N_D$$

$$n + 10^{15} = p + 10^{15}$$

$$n = p$$

Using the mass-action law, we may write:

$$n = n_i$$

Which gives us:

$$\boxed{n = p = 1.5 \cdot 10^{10} \left[\frac{1}{\text{cm}^3} \right]}$$

2. • V_{BE}

   We begin by using the transistor equation:

$$I_e = I_{ES} e^{\frac{V_{BE}}{V_T}}$$

   This can be rearranged to get:

$$V_{BE} = V_T \ln \left(\frac{I_E}{I_{ES}} \right)$$

   And now we enter known values:

$$V_{BE} = .026 \ln \left(\frac{.01}{10^{-13}} \right)$$

$$\boxed{V_{BE} = .6585[\text{V}]}$$

   • V_{BC}

   Since we are given $V_{CE} > .2[\text{V}]$, the BJT is active, and we can write:

$$V_{BC} = V_{BE} - V_{CE}$$

$$V_{BC} = .6585 - 10$$

$$\boxed{V_{BC} = -9.3415[\text{V}]}$$

   • I_B

   We may use the value of β to find:

$$I_B = (1 + \beta) I_E$$

$$I_B = (1 + 100)^{-1}(.01)$$

$$\boxed{I_B = 99[\mu\text{A}]}$$

- I_C

  We then know:

$$I_C = \beta I_B$$
$$I_C = 100(99 \cdot 10^{-6})$$
$$\boxed{I_C = 9.9[\text{mA}]}$$

- α

  Finally, we find α:

$$\alpha = \frac{\beta}{\beta + 1}$$
$$\alpha = \frac{100}{100 + 1}$$
$$\boxed{\alpha = .9901}$$

3. - V_1

     Since there is a voltage drop from the base (due to forward-bias) to the emitter of .7[V], we know:

$$\boxed{V_1 = -.7[\text{V}]}$$

   - V_2

     We may begin by finding the emitter current at transistor Q_1:

$$I_{EQ_1} = \frac{10 - .7}{4.7k}$$
$$I_{EQ_1} = 1.979[\text{mA}]$$

     Given the β value, we may find the collector current as:

$$I_{CQ_1} = \frac{100}{101} I_{EQ_1}$$
$$I_{CQ_1} = 1.9594[\text{mA}]$$

     We can then calculate V_2 based on KVL:

$$V_2 = 10 - (1.9594)(5.1k)$$
$$\boxed{V_2 = 7.0297[\text{mV}]}$$

- V_3

  Applying this voltage into a KVL equation for transistor Q_2, we may write:

$$I_{CQ_2} = \frac{10 - .0070297 - .7}{4.7k}$$
$$I_{CQ_2} = 1.9772[\text{mA}]$$

  We can thus get V_3 as:

$$V_3 = 10 - (1.9772)(4.7)$$
$$\boxed{V_3 = .707[\text{V}]}$$

- V_4

  We then find the emitter voltage based on the β:

$$I_{EQ_2} = \frac{101}{100}(1.9772)$$
$$I_{EQ_2} = 1.997[\text{mA}]$$

  Using KVL at the collector, we get:

$$V_4 = (3)(1.997) - 10$$
$$\boxed{V_4 = -4.0091[\text{V}]}$$

- V_5

  We may find V_5 using the voltage drop from a forward-biased diode:

$$V_5 = V_4 - .7$$
$$V_5 = -4.0091 - .7$$
$$\boxed{V_5 = -4.7091[\text{V}]}$$

- V_6

  Using KVL at the input of Q_3, we get:

$$I_{EQ_3} = \frac{10 - 4.0091 - .7}{1.3k}$$
$$I_{EQ_3} = 4.0699[\text{mA}]$$

  We then find the collector current:

$$I_{CQ_3} = \frac{\beta}{\beta + 1}(4.0699)$$
$$I_{CQ_3} = \frac{100}{101}(4.0699)$$

$$I_{CQ_3} = 4.0296[\text{mA}]$$

And, finally, we use KVL to get:

$$V_6 = 10 - (4.0296)(2)$$
$$\boxed{V_6 = 1.9408[\text{V}]}$$

We may demonstrate the values we found as:

$$\boxed{V \begin{cases} 1, & -.7 \\ 2, & 7.0297 \cdot 10^{-3} \\ 3, & .707 \\ 4, & -4.0091 \\ 5, & -4.7091 \\ 6, & 1.9408 \end{cases} [\text{V}]}$$

4. First, we find the thermal voltage at 180[°C]:

$$V_{T_2} = \frac{(1.38 \cdot 10^{-23})\,(273 + 180)}{1.6 \cdot 10^{-19}}$$
$$V_{T_2} = .039071[\text{V}]$$

From here, we may write our temperature difference equation to determine the initial V_{BE} voltage at 2[mA]:

$$V_{BE_o} = V_{BE_1} + .002(T_2 - T_1)$$
$$V_{BE_o} = -.7 + .002(150)$$
$$V_{BE_o} = -.4[\text{V}]$$

We then find the saturation current:

$$I_s = \frac{I_C}{e^{V_{BE_o}/V_{T_2}}}$$
$$I_s = \frac{.002}{e^{.4/.039071}}$$
$$I_s = 71.585[\text{nA}]$$

Finally, from this we may write:

$$V_{BE_2} = (.039071)\ln\left(\frac{.0001}{71.585 \cdot 10^{-9}}\right)$$
$$\boxed{V_{BE_2} = .283[\text{V}]}$$

5. (a) From running the simulation, we obtain the following result:

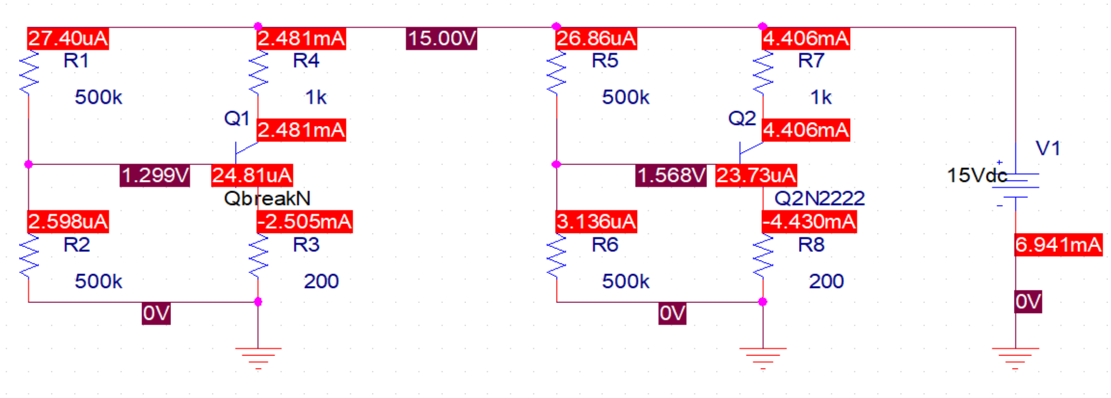

Figure 1: Simulation Results for Part (a)

From this, we may see that $I_B = 23.73[\mu A]$ and $I_C = 4.406[mA]$. Thus, we may obtain:

$$\beta_{Q2} = \frac{4.406}{.02373}$$

$$\boxed{\beta_{Q2} = 185.67}$$

(b) Once again, we simulate to obtain new values:

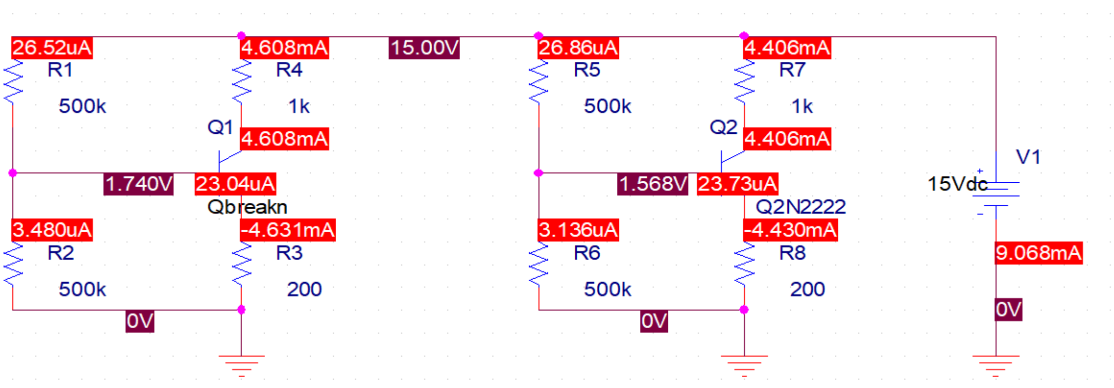

Figure 2: Simulation Results for Part (b)

Now, we get $I_C = 4.608[mA]$ and $I_B = 23.04[\mu A]$, which gives us:

$$\beta_{QB} = \frac{4.608}{.02304}$$

$$\boxed{\beta_{QB} = 200}$$

This is the expected value, since we hard coded β.

(c) We may find the value of BF in the table below:

	Qbreakn	Q2N2222
	NPN	NPN
IS	100.000000E-18	14.340000E-15
BF	200	255.9
NF	1	1
VAF		74.03
IKF		.2847
ISE		14.340000E-15
NE		1.307
BR	1	6.092
NR	1	1
RB		10
RC		1
CJE		22.010000E-12
MJE		.377
CJC		7.306000E-12
MJC		.3416
TF		411.100000E-12
XTF		3
VTF		1.7
ITF		.6
TR		46.910000E-09
XTB		1.5
CN	2.42	2.42
D	.87	.87

Figure 3: Model Parameters

From this, we see that the expected BF value for the Q2N2222 is 255.9. Furthermore, we can confirm that BF for the other transistor is 200.

**** BIPOLAR JUNCTION TRANSISTORS

NAME	Q_Q1	Q_Q2
MODEL	Qbreakn	Q2N2222
IB	2.30E-05	2.37E-05
IC	4.61E-03	4.41E-03
VBE	8.14E-01	6.82E-01
VBC	-8.65E+00	-9.03E+00
VCE	9.47E+00	9.71E+00
BETADC	2.00E+02	1.86E+02
GM	1.78E-01	1.68E-01
RPI	1.12E+03	1.19E+03
RX	0.00E+00	1.00E+01
RO	1.00E+12	1.88E+04
CBE	0.00E+00	1.06E-10
CBC	0.00E+00	3.04E-12
CJS	0.00E+00	0.00E+00
BETAAC	2.00E+02	1.99E+02
CBX/CBX2	0.00E+00	0.00E+00
FT/FT2	2.84E+18	2.44E+08

Figure 4: Operating Point Parameters

Highlighted in the above figure, we see that BETADC is about 186 and BETAAC is about 199.

(d) The following data sheet was obtained online:

ELECTRICAL CHARACTERISTICS

TC=25°C unless otherwise noted

Symbol	Ratings	Test Condition(s)		Min	Typ	Mx	Unit
$V_{CE(SAT)}$	Collector-Emitter saturation Voltage (*)	I_C=150 mA, I_B=15 mA	2N2221-2N2222	-	-	0.4	V
			2N2221A-2N2222A	-	-	0.3	
		I_C=500 mA, I_B=50 mA	2N2221-2N2222	-	-	1.6	
			2N2221A-2N2222A	-	-	1	
$V_{BE(SAT)}$	Base-Emitter saturation Voltage (*)	I_C=150 mA, I_B=15 mA	2N2221-2N2222	-	-	1.3	V
			2N2221A-2N2222A	0.6	-	1.2	
		I_C=500 mA, I_B=50 mA	2N2221-2N2222	-	-	2.6	
			2N2221A-2N2222A	-	-	2	
f_T	Transition frequency	I_C=20 mA, V_{CE}=20 V f= 100MHz	2N2221-2N22218A 2N2222	250	-	-	MHz
			2N2222A	300	-	-	
h_{fe}	Small signal current gain	I_C=1 mA, V_{CE}=10 V f= 1kHz	2N2221A	30	-	150	-
			2N2222A	50	-	300	
		I_C=10 mA, V_{CE}=10 V f= 1kHz	2N2221A	50	-	300	
			2N2222A	75	-	375	
t_d	Delay time	I_C=150 mA, I_B=15 mA $-V_{BB}$=0.5 V, V_{CC}=30 V	2N2221A	-	-	10	ns
			2N2222A				
t_r	Rise time	I_C=150 mA, I_B=15 mA $-V_{BB}$=0.5 V, V_{CC}=30 V	2N2221A	-	-	25	ns
			2N2222A				
t_s	Storage time	I_C=150 mA, V_{CC}=30 V I_{B1} = $-I_{B2}$=15 mA	2N2221A	-	-	225	ns
			2N2222A				
t_f	Fall time	I_C=150 mA, V_{CC}=30 V I_{B1} = $-I_{B2}$=15 mA	2N2221A	-	-	60	ns
			2N2222A				
r_b,C_C	Feedback time constant	I_C=20 mA, V_{CE}=20 V f= 31.8MHz	2N2221A	-	-	150	ps
			2N2212A				

(*) Pulse conditions : tp < 300 μs, δ =2%

Figure 5: Q2N2222 Sample Data Sheet

From the highlighted row, we may see that, for $I_C = 1$[mA] and $V_{CE} = 10$[V], the current gain is between 50 and 300. Given that in our simulation I_C is just over 4 times as big, and V_{CE} is about 13.5[V], we can estimate that the value will be on the higher end of this range. As observed, β was 185.67, which is within the range specified by the data sheet.