Homework 11

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November 26, 2024

1. (a) We may begin by saying that $V_{gs}(t)$ may be expressed as:

$$V_{gs}(t) = V_g(t) - V_s(t)$$

We may see that the source voltage is grounded, which lets us say:

$$V_{gs}(t) = V_g(t)$$

We may see that, taking the capacitor as a short, the voltage at G is simply the sum of the input voltage and the divided voltage from the 20[V] supply voltage. Thus, we may write:

$$V_g(t) = \frac{300k(20)}{300k + 1700k} + V_i$$
$$V_g(t) = 3 + \sin(2000\pi t)$$

Thus, we may say:

$$\boxed{V_{gs}(t) = 3 + \sin(2000\pi t)}$$

(b) Parts (b-d) are combined in the plot shown in (d)

(c) Parts (b-d) are combined in the plot shown in (d)

(d) We may generate the following plot:

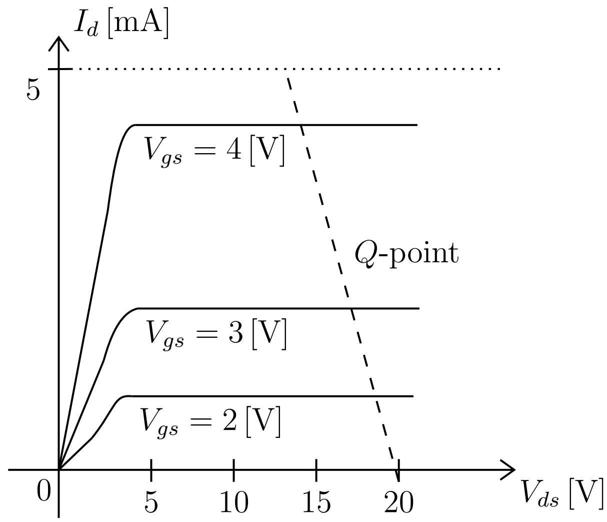

Figure 1: Plot for Parts (b)-(d)

We may observe, first and foremost, that the curve is zero for $V_{gs} = 1$[V], since, at this value, $V_{gs} = V_{to}$. We may observe via the load line in the plot that $V_{ds} \approx 16$[V] at the Q-point, and $V_{ds}^{max} \approx 20$[V] and $V_{ds}^{min} \approx 10$[V]

2. (a) We may observe that V_1 may be obtained using our equations:

$$I_D = \frac{\mu_p C_{ox} W}{2L}[V_{GS} - |V_t|]^2$$
$$8 = .5[V_{GS} - |-1|]^2$$
$$V_{GS}^2 - 2V_{GS} + 1 = 16$$
$$V_{GS}^2 - 2V_{GS} - 15 = 0$$

We see that the two solutions are $V_{GS} = 5, -3$[V]. We know the value is positive, so we write:

$$V_S - V_G = 5$$
$$V_G = V_S - 5$$
$$V_G = 10 - 5$$
$$\boxed{V_1 = V_G = 5[\text{V}]}$$

We may then proceed to calculate the voltage using the current source, which is equal to I_D, at V_2, which we may find as:

$$V_2 = -10 + 8(.5)$$
$$\boxed{V_2 = -6[\text{V}]}$$

(b) From the figure, we may observe:

$$V_3 = V_{G1}$$
$$V_S = 15[\text{V}]$$

We may use our formulas to write:

$$I_D = \frac{\mu_p C_{ox} W}{2L}[V_{SG1} - |V_t|]^2$$

Looking at the current source at the end, we see that $I_D = 8[\text{mA}]$. This lets us get:

$$8 = (.5)[V_{SG1} - |-1|]^2$$

This lets us get (simplifying $V_{SG1} \rightarrow V$):

$$V^2 - 2V + 1 = 16$$
$$V^2 - 2V - 15 = 0$$
$$(V-5)(V+3) = 0$$
$$V_{SG1} = 5, -3$$

We proceed with the positive value, which gives us:

$$5 = V_S - V_{G1}$$
$$V_s - 5 = V_{G1}$$
$$V_{G1} = 15 - 5$$

And finally:

$$\boxed{V_3 = V_{G1} = 10[\text{V}]}$$

We then use the same method to get $V_4 = V_{G2}$:

$$8 = (.5)[V_{SG2} - |-1|]^2$$

To simplify, take $V = V_{SG2}$:

$$V^2 - 2V + 1 = 16$$
$$V = 5, -3$$

Again, we proceed with the positive value to get:

$$V_S - V_{G2} = 5$$

$$V_{G2} = V_S - 5$$
$$V_{G2} = 10 - 5$$

This gets us:

$$\boxed{V_4 = V_{G2} = 5[\text{V}]}$$

3. (a) Taking the capacitors as open circuits, we end up with a circuit consisting solely of the 15[V] DC source, the NMOS, and the 10[MΩ], 5[MΩ], 7.5[kΩ], and 3[kΩ] resistors. This allows us to divide the voltage to find V_G:

$$V_G = (V_{DC})\frac{5}{10+5}$$
$$\boxed{V_G = 5[\text{V}]}$$

The drain current may be found as:

$$I_D = \frac{V_S}{R_S}$$

Alternatively, we may write:

$$I_D = \frac{V_G - V_{GS}}{R_S}$$

We can then equate this to our formula:

$$I_D = K(V_{GS} - V_t)^2$$

This gets us:

$$V_{GS}^2 - 2V_{GS} + 1 = \frac{5 - V_{GS}}{3}$$
$$V_{GS}^2 - \frac{5}{3}V_{GS} - \frac{2}{3} =$$
$$V_{GS} = \frac{(5/3) \pm \sqrt{(25/9) - (4)(1)(-2/3)}}{2}$$
$$V_{GS} = \frac{(5/3) \pm (7/3)}{2}$$
$$V_{GS} = 2, -\frac{1}{3}$$

We proceed with the positive value:

$$I_D = \frac{5-2}{3}[\text{mA}]$$

$$\boxed{I_D = 1[\text{mA}]}$$

We may find the transconductance gain to get:

$$g_m = \frac{2I_D}{V_{GS} - V_t}$$

$$g_m = \frac{2(.001)}{2-1}$$

$$\boxed{g_m = 2[\text{mS}]}$$

We can obtain:

$$r_{ds} = \frac{V_A}{I_D}$$

$$r_{ds} = \frac{100}{10^{-3}}$$

$$\boxed{r_{ds} = 100[\text{k}\Omega]}$$

(b) We may draw the small-signal equivalent circuit as:

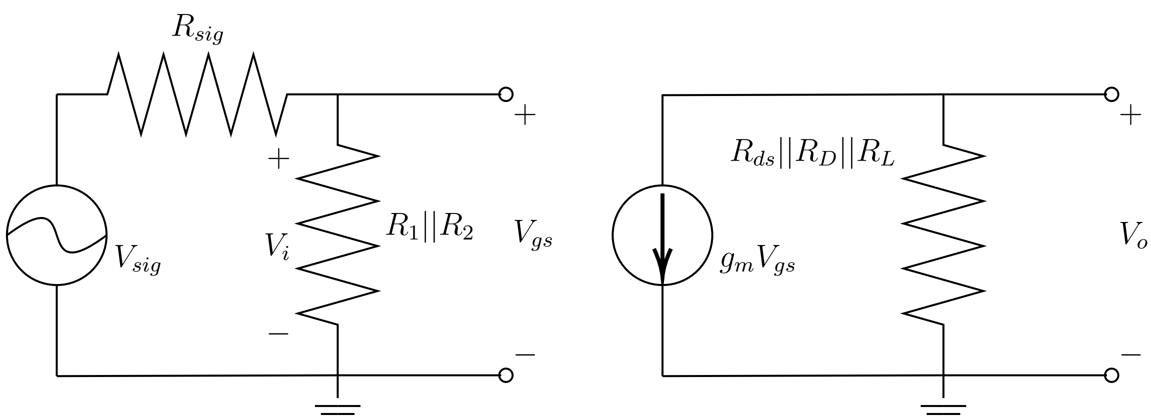

Figure 2: Small Signal Equivalent

From the circuit, we may conclude:

$$V_o = -g_m V_{gs}(r_{ds}||R_D||R_L)$$

Because no current flows through the gate, there is zero voltage drop, and we may conclude:

$$V_{gs} = V_i$$

Which gives us:

$$A_{vi} = \frac{V_o}{V_i}$$
$$A_{vi} = \frac{-g_m V_{gs}(r_{ds}||R_D||R_L)}{V_{gs}}$$
$$A_{vi} = -g_m(r_{ds}||R_D||R_L)$$

We substitute in known values to get:

$$A_{vi} = -.002(4.1096 \cdot 10^3)$$
$$\boxed{A_{vi} = -8.2192}$$

For A_{vs}, we may write:

$$A_{vs} = -g_m R'_L \left(\frac{R_G}{R_G + R_{sig}}\right)$$
$$A_{vs} = -g_m R'_L\,(.9709)$$
$$\boxed{A_{vs} = -7.9796}$$

(c) We may write:

$$Z_i = \frac{V_i}{I_i}$$
$$Z_i = \frac{V_i}{(V_i/R_G)}$$
$$\boxed{Z_i = 3.33[\mathrm{M}\Omega]}$$

(d) We short circuit the independent source and find the Thévenin equivalent to get:

$$Z_o = \frac{V_x}{I_x}$$
$$Z_o = R_{out}$$
$$\boxed{Z_o = (r_{ds}||R_D)}$$

Calculating from our known values, we get:

$$\boxed{Z_o = 6.976[\mathrm{k}\Omega]}$$

4. (a) We simulate the Bias Point to get:

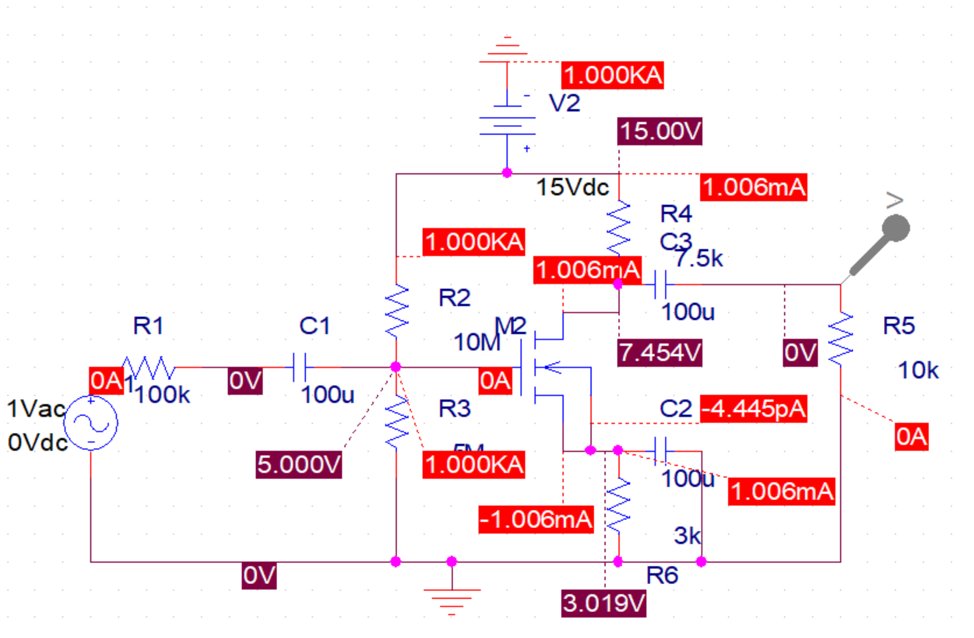

Figure 3: DC Operation

(b) Checking the operating point information, we get:

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****     OPERATING POINT INFORMATION      TEMPERATURE =    27.000 DEG C

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**** MOSFETS


NAME          M_M2
MODEL         Mbreakn
ID            1.01E-03
VGS           1.98E+00
VDS           4.44E+00
VBS           0.00E+00
VTH           1.00E+00
VDSAT         9.82E-01
Lin0/Sat1    -1.00E+00
if           -1.00E+00
ir           -1.00E+00
TAU          -1.00E+00
GM            2.05E-03
GDS           9.63E-06
GMB           0.00E+00
CBD           0.00E+00
CBS           0.00E+00
CGSOV         0.00E+00
CGDOV         0.00E+00
CGBOV         0.00E+00
CGS           0.00E+00
CGD           0.00E+00
CGB           0.00E+00
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Figure 4: DC Operating Point Information

We can calculate the r_{ds} value from this information using the GDS value to get:

$$r_{ds} = \frac{1}{9.63 \cdot 10^{-6}}$$

$$\boxed{r_{ds} = 103.84[\mathrm{k}\Omega]}$$

We may see that this is within 5% of the value obtained in (3).

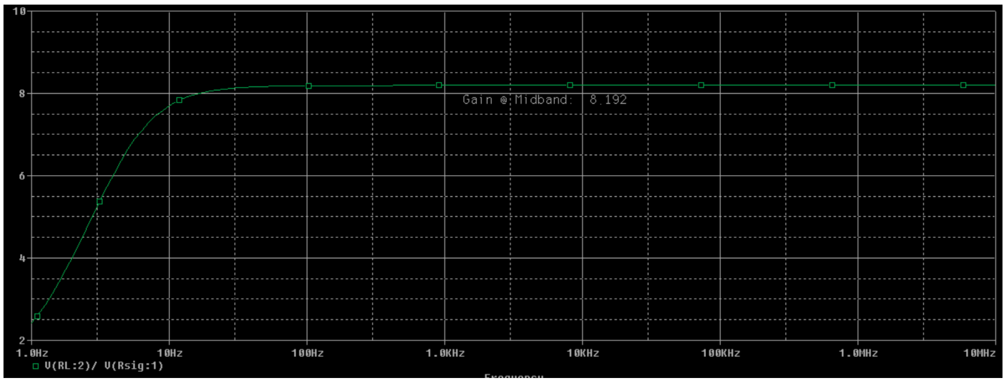

Figure 5: Gain Plot

(c) As expected, the gain is near the calculated result of $A_v = -8.2$

(d) Based on the transient simulation, we see:

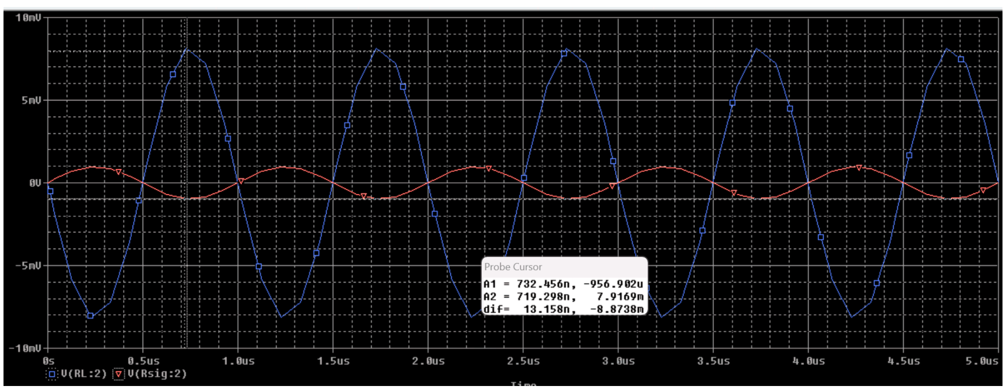

Figure 6: Transient Simulation

This agrees with our gain result.