Homework 7

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1. We may begin by finding the DC equivalent by changing all capacitors to open circuits. This gets us:

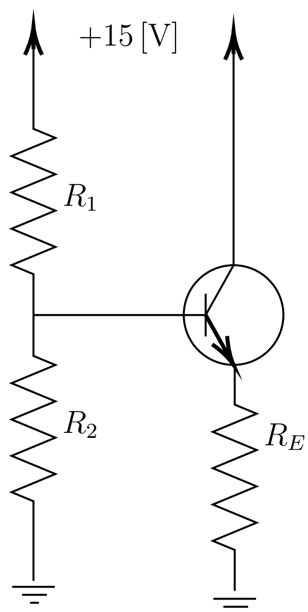

Figure 1: DC Equivalent Circuit

We can find a Thévenin equivalent for the R_1-R_2 voltage as:

$$V_{th} = 15\left[\frac{10k}{10k+10k}\right]$$
$$V_{th} = 7.5[\text{V}]$$

And then the equivalent resistance:

$$R_{th} = \frac{10^2}{20}$$
$$R_{th} = 5[\mathrm{k}\Omega]$$

This allows us to draw the circuit as:

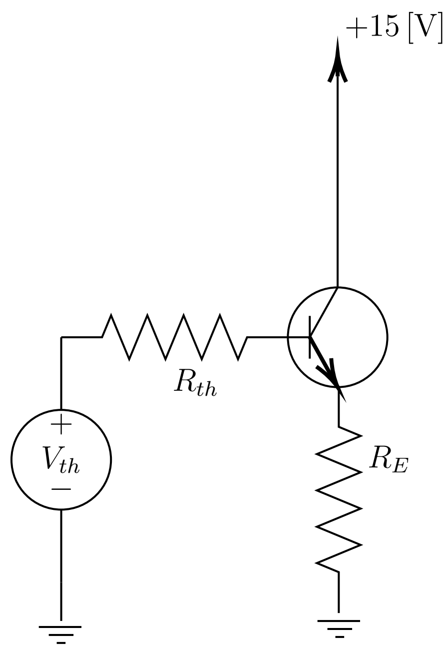

Figure 2: DC Equivalent Simplified

We use KVL at the base-emitter loop to getL

$$-V_{th} + I_B R_{th} + V_{BE} + I_E R_E = 0$$

Since we know $I_E = (1+\beta)I_B$, we may write:

$$-V_{th} + I_B R_{th} + V_{BE} + (1+\beta)I_B R_E = 0$$

Rearranging, we get:

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + (1+\beta)R_E}$$

We plug in our known values to get:

$$I_B = \frac{7.5 - .7}{5k + (1+100)1k}$$

$$I_B = 64.151[\mu\text{A}]$$

From this, we may find:

$$I_{CQ} = \beta I_B$$
$$I_{CQ} = (100)(64.151 \cdot 10^{-6})$$
$$\boxed{I_{CQ} = 6.4151[\text{mA}]}$$

We can find r_π by using the equation:

$$r_\pi = \frac{\beta}{g_m}$$

Where $g_m = \dfrac{I_{CQ}}{V_T}$:

$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$
$$r_\pi = \frac{100(.026)}{.0064151}$$
$$\boxed{r_\pi = 405.29[\Omega]}$$

We can then perform a small-signal analysis by finding r_e:

$$r_e = \frac{\beta V_T}{(1+\beta)I_{CQ}}$$
$$r_e = 4.0128[\Omega]$$

Using this, we can use our small-signal analysis model to find the gain. First, we get:

$$V_o = V_i \left[\frac{R_E R_L}{(R_E + R_L)r_e + R_E R_L}\right]$$

Thus, we see that the voltage gain is:

$$A_v = \left[\frac{R_E R_L}{(R_E + R_L)r_e + R_E R_L}\right]$$

We plug in our known values to find:

$$A_v = \left[\frac{(1000)(500)}{(1500)(4.0128) + (500000)}\right]$$
$$\boxed{A_v = .9881}$$

Furthermore, we can find the open-loop voltage gain when $R_L \to \infty$:

$$A_{voc} = \left[\frac{R_E}{r_e + R_E}\right]$$

$$A_{voc} = \left[\frac{1000}{4.0128 + 1000}\right]$$

$$\boxed{A_{voc} = .996}$$

Now, we can work to find the input impedance. First and foremost, we know:

$$i_e = V_i \left[\frac{R_E R_L}{(R_E + R_L)r_e + R_E R_L}\right]$$

Using KCL for the input current node, we may observe:

$$i_i + \frac{\beta}{\beta + 1} i_e = V_i \left[\frac{R_1 + R_2}{R_1 R_2}\right] + i_e$$

We can rearrange this to find:

$$\frac{V_i}{i_i} = \left[\frac{R_1 + R_2}{R_1 R_2} + \frac{R_E + R_L}{(1 + \beta)[(R_E + R_L)r_e + R_E R_L]}\right]^{-1}$$

$$Z_i = \left(2.2935 \cdot 10^{-4}\right)^{-1}$$

$$\boxed{Z_i = 4360.2[\Omega]}$$

The current gain may be expressed as the ratio of input (Z_i) and output (R_L) impedances:

$$A_i = A_v \frac{Z_i}{R_L}$$

$$\boxed{A_i = 8.6854}$$

From here, we know the power gain is:

$$G = A_i A_v$$

$$G = (.996)(8.6854)$$

$$\boxed{G = 8.6507}$$

Finally, we can find the output resistant by computing several parallel resistances:

$$Z_o = R_E || \left[\frac{R_B || R_S + r_\pi}{\beta + 1}\right]$$

We plug in known values:

$$Z_o = 1k || \left[\frac{833 + 405.29}{101} \right]$$

$$Z_o = 1k || 12.264$$

$$\boxed{Z_o = 12.115[\Omega]}$$

2. We begin by investigating $V_{hum} \to 0$, which shows that the circuit is the same for both figures:

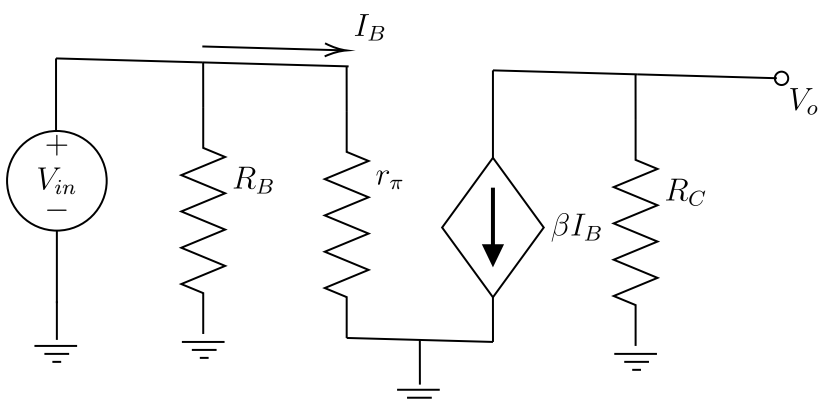

Figure 3: Equivalent Circuit is the Same for $V_{hum} \to 0$

From this, we may observe that:

$$\boxed{A_v \approx -\frac{\beta R_C}{r_\pi}}$$

We can then investigate the case when $V_{hum} \neq 0$ and $V_{in} \to 0$. For the first figure, we see:

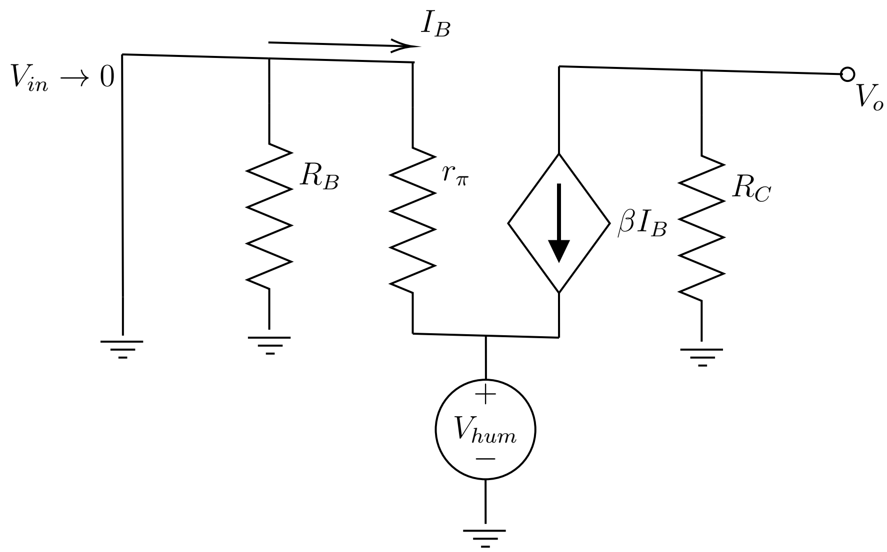

Figure 4: Equivalent Circuit for Figure (a) as $V_{in} \to 0$

We may conclude that the hum gain can be expressed as:

$$\boxed{A_{(a)} = \frac{\beta R_C}{r_\pi}}$$

Finally, we may draw the equivalent circuit for the second figure to get:

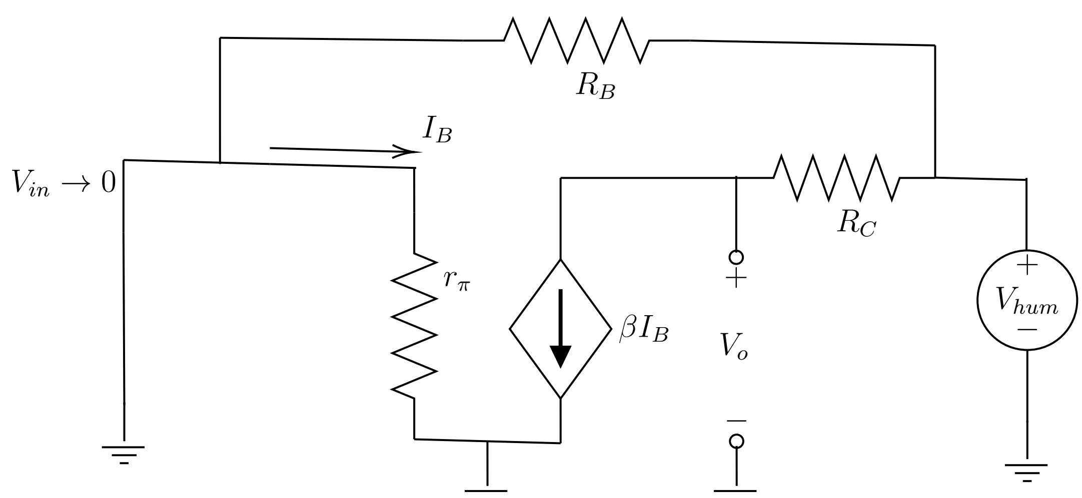

Figure 5: Equivalent Circuit for Figure (b) as $V_{in} \to 0$

First, note that $V_o = V_{hum}$ since the base current, $I_B = 0$, which causes the current-controlled current source to become null as well. As such, we may write that, for the second circuit, there is no gain, as the output is equivalent to the input, and thus:

$$\boxed{A_{(b)} = 1}$$

We can calculate the gain for the provided values by first finding r_π:

$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

We can perform analyses through the loops to write:

$$I_{BQ} = \frac{V_{CC} - .7}{R_B} \quad \text{and} \quad I_{CQ} = \beta I_{BQ}$$

This gives us:

$$I_{BQ} = \frac{15 - .7}{1M}$$
$$I_{BQ} = 14.3[\text{µA}]$$

Consequently, we find:

$$I_{CQ} = 1.43[\text{mA}]$$

This gives us:

$$r_\pi = \frac{100(.026)}{1.43 \cdot 10^{-3}}$$
$$r_\pi = 1.818[\text{k}\Omega]$$

We can then find:

$$A_{(a)} = \frac{(100)(4700)}{1818}$$
$$\boxed{A_{(a)} = 258.8}$$

We know that when $V_{hum} \to 0$, the gain becomes the negative of the above. As such, we find the following values:

$$\boxed{\begin{cases} A_{v(a)} &= -258.8 \\ A_{hum(a)} &= 258.8 \\ A_{v(b)} &= -258.8 \\ A_{hum(b)} &= 1 \end{cases}}$$

3. We begin by constructing the circuit:

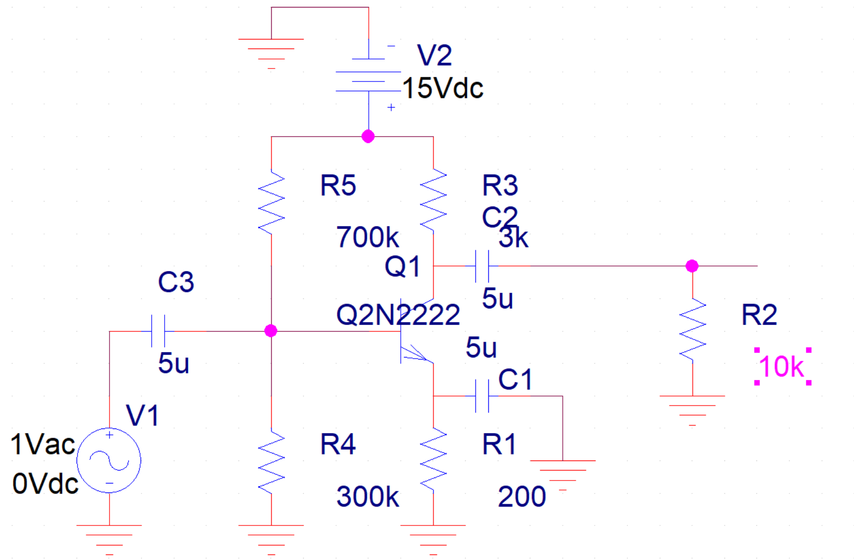

Figure 6: Circuit Schematic

(a) From here, we can run a DC bias point simulation to obtain:

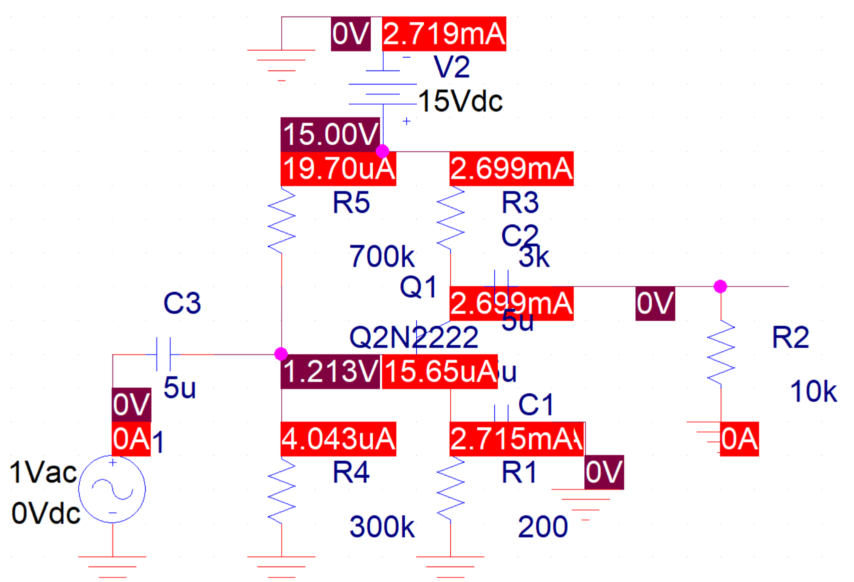

Figure 7: Bias Point Simulation Results

From the above, we may observe that the voltage at the collector may be written as:

$$V_{CE} = 15 - (3)(2.699)$$

$$V_{CE} = 6.903[\mathrm{V}]$$

The base voltage may be observed to be 1.213[V]. Therefore, because $V_{CE} > V_{BE}$, the BJT is operating in its active mode. Checking the output file, we may find:

```
**** BIPOLAR JUNCTION TRANSISTORS


NAME         Q_Q1
MODEL        Q2N2222
IB           1.57E-05
IC           2.70E-03
VBE          6.70E-01
VBC         -5.69E+00
VCE          6.36E+00
BETADC       1.72E+02
GM           1.03E-01
RPI          1.81E+03
RX           1.00E+01
RO           2.95E+04
CBE          7.96E-11
CBC          3.51E-12
CJS          0.00E+00
BETAAC       1.87E+02
CBX/CBX2     0.00E+00
FT/FT2       1.98E+08
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Figure 8: Operating Point Information

(b) Running an AC Simulation, we obtain the following:

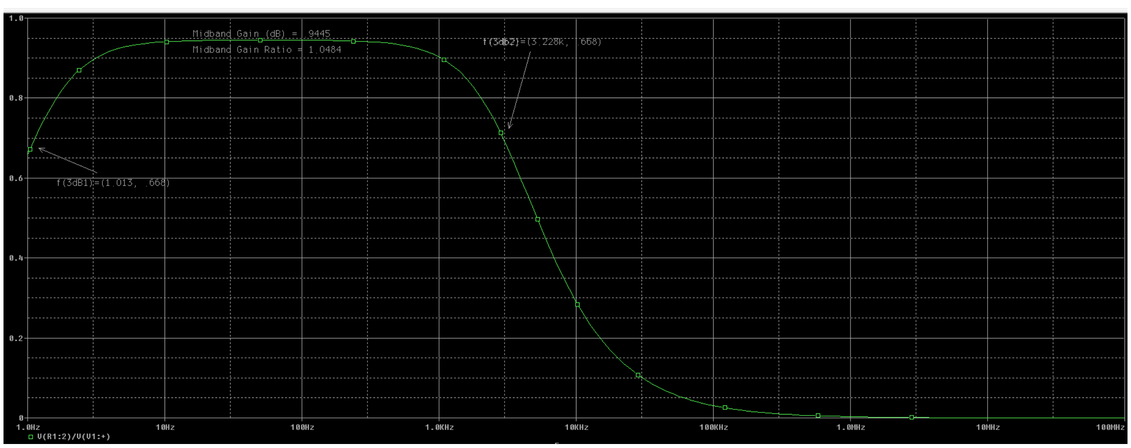

Figure 9: AC Simulation, 1[Hz] – 100[MHz], 20 Points per Decade

Note that critical values are labeled in the figure above.

(c) Running a transient simulation produces the following:

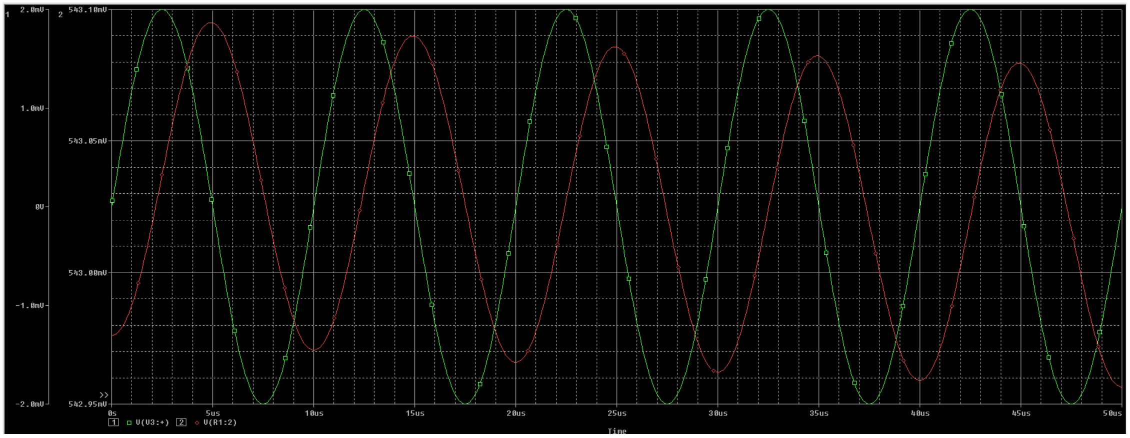

Figure 10: Transient Simulation, Sinusoidal Input (100[kHz], 2[mV]), .1[µs] Step

Note that, in the above figure, the green sinusoid corresponds to the input and is shown with respect to vertical axis 1 (on the left). The red sinusoid represents the output voltage, and corresponds to the second vertical axis, on the right. Based on the graph, we observe that the input has a peak amplitude of 2[mV], as expected, while the output has a peak amplitude of 543[mV]. Thus, we can calculate the gain as:

$$A_v = \frac{543}{2}$$

$$\boxed{A_v = 271.5}$$

(d) We now repeat the transient simulation; however, the input voltage now has a peak amplitude of 20[mV]. This produces:

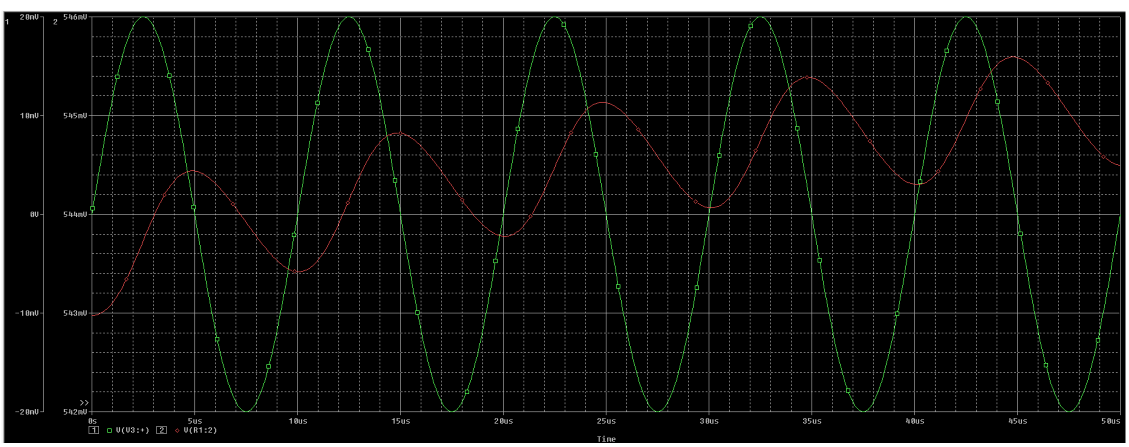

Figure 11: Transient Simulation, Sinusoidal Input (100[kHz], 20[mV]), .1[µs] Step

We may observe that, indeed, the output voltage becomes distorted. This is because the BJT becomes saturated due to the increase in input amplitude.

(e) Finally, we run both an AC and transient simulation on the circuit with C_3 removed. This produces:

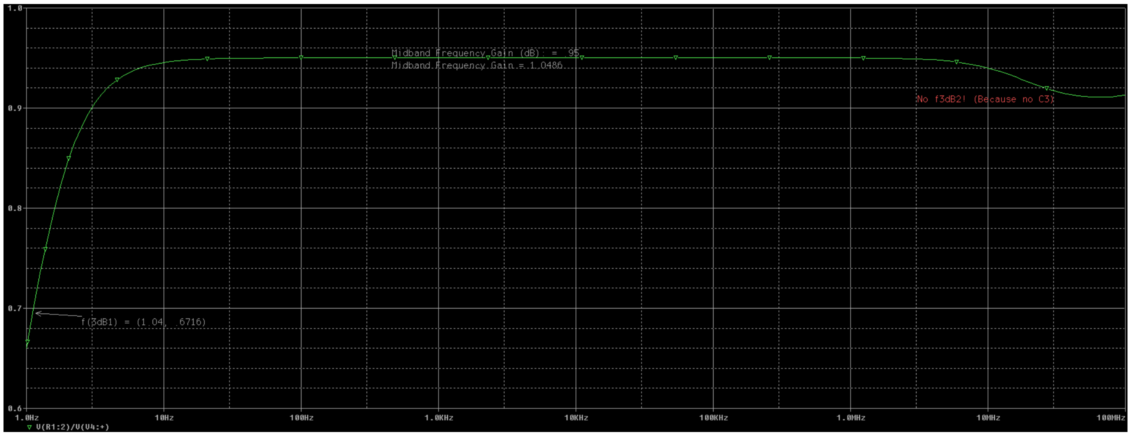

Figure 12: AC Simulation, Same Parameters as Above, No C_3 Capacitor

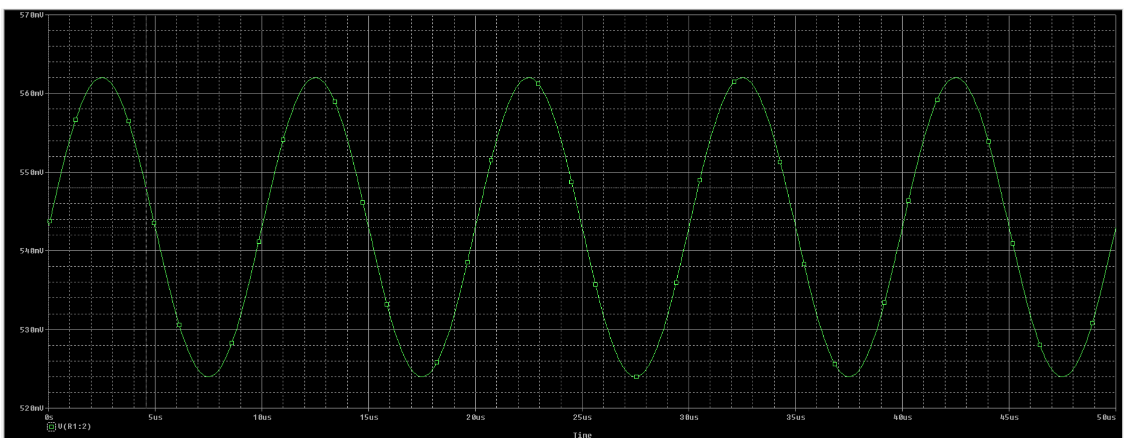

Figure 13: Transient Simulation, Same 20[mV] Parameters as Above, No C_3 Capacitor

Based on the output from above, we may conclude that the gain is (based on a peak voltage of 562[mV]):

$$\boxed{A_v = \frac{562}{20} = 28.1}$$