Number System Conversion
Binary Arithmetic

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- Binary Numbers

  - All computers work with 0's and 1's so it is like learning alphabets before learning a language

- Number Systems

  - There is more than one way to express a number in binary, so 1010 could be -2, -5, or -6, and it is necessary to know which one

- A/D and D/A Conversion

  - Real world signals come in continuous/analog format, and it is good to how they become 0's and 1's (and vice versa)

- Digital = Discrete

  - Binary Codes
    * Represent symbols using binary digits (bits)
  - Decimal digits 0-9

- Digital Computers

  - I/O is digital
  - Internal representation is binary
  - Sometimes use hexadecimal (base 16) for large binary numbers

- Bases we will use

  - Binary: Base 2

- Octal: Base 8

- Decimal: Base 10

- Hexadecimal: Base 16

- Positional Number System

   - $101_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$
   - $63_8 = 6 \cdot 8^1 + 3 \cdot 8^0$
   - $A1_{16} = 10 \cdot 16^1 + 1 \cdot 16^0$

- Conversion from binary to octal/hex

   - Binary: 10011110001
   - Octal: $10|011|110|001 = 2361_8$
   - Hex: $100|1111|0001 = 4F1_{16}$

- Conversion from binary to decimal

   - $101_2 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 5_{10}$
   - $63.4_8 = 6 \cdot 8^1 + 3 \cdot 8^0 + 4 \cdot 8^{-1} = 51.5_{10}$
   - $A1_{16} = 10 \cdot 16^1 + 1 \cdot 16^0 = 161_{10}$

- Decimal to binary/octal/hex

   - Binary: 56 becomes 111000_2

	Quotient	Remainder
$56/2 =$	28	0
$28/2 =$	14	0
$14/2 =$	7	0
$7/2 =$	3	1
$3/2 =$	1	1
$1/2 =$	0	1

   - Octal: 56 becomes 70_8

	Quotient	Remainder
$56/8 =$	7	0
$7/8 =$	0	7

   - Each successive divide releases another LSB (least significant bit)

- Negative Numbers

   - Sign-and-magnitude

- ∗ The most-significant bit (MSB) is the sign digit, where 0 is positive and 1 is negative
- ∗ The remaining bits are the numbers magnitude

– Ones-complement

- ∗ Negative number: a bitwise complement positive number

– Twos-complement

- ∗ Most important option
- ∗ Simplifies arithmetic
- ∗ Used almost universally
- ∗ Negative number: a bitwise complement plus one
  - · $0011 = 3_{10}$
  - · $1101 = -3_{10}$
- ∗ Only one zero
- ∗ MSB is the sign digit, where 0 is positive and 1 is negative