Digital Logic Circuits Cont'd

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- Logical Equivalences of And
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ — Associative Law

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

- Logical Equivalences of Or
  - $p \vee T \equiv T$ — Identity Law
  - $p \vee F \equiv p$ — Domination Law
  - $p \vee p \equiv p$ — Idempotent Law
  - $p \vee q \equiv q \vee p$ — Commutative Law
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$ — Associative Law
  - $\sim (p \vee q) \equiv (\sim p) \wedge (\sim q)$ — De Morgan's Law
- Can implement any truth table with AND, OR, and NOT
  1. AND combinations that yield a "1" in the truth table
  2. OR the results of the AND gates
- Sum of Products Form: Key Idea
  - Assume we have the truth table of a boolean function
  - How do we express the function in terms of the inputs in a standard manner?
    * Idea: Sum of Products Form
    * Express the truth table as a two-level Boolean expression
    * If ANY of the combinations of input variables that results in a 1 is TRUE, then the output is 1
    * F = OR of all input variable combinations that result in a 1
  - Complement: Variable with a bar over it

$$\bar{A}, \bar{B}, \bar{C}$$

- Literal: Variable or its complement

$$A,\ \bar{A},\ B,\ \bar{B},\ C,\ \bar{C}$$

- Implicant: Product (AND) of literals

$$(A \bullet B \bullet \bar{C}),\ (\bar{A} \bullet C),\ (B \bullet \bar{C})$$

- Minterm: Product (AND) that includes all input variables

$$(A \bullet B \bullet \bar{C}),\ (\bar{A} \bullet \bar{B} \bullet C),\ (\bar{A} \bullet B \bullet \bar{C})$$

- Maxterm: Sum (OR) that includes all input variables

$$(A \vee B \vee \bar{C}),\ (\bar{A} \vee \bar{B} \vee C),\ (\bar{A} \vee B \vee \bar{C})$$

- Canonical Form: Standard form for a boolean expression
- Sum of Products Form (SOP)
  - Also known as a disjunctive normal form or minterm expansion
  - Each row in a truth table has a minterm
  - A minterm is a product (AND) of literals
  - Each minterm is TRUE for that row (and only that row)
  - All boolean equations can be written in SOP form
  - Standard "shorthand" notation
    * If the order of variables in the rows of a truth table are agreed upon, then one may write M<ROW> as shorthand, for example, M4 for row 4
    * This can be be rewritten as a sum of products or summation notation:

$$m3 + m4 + m5 + m6 + m7 = \sum m(3, 4, 5, 6, 7)$$

    * The canonical form is not always the minimal form
- Product of Sums (POS) Form
  - Each sum term represents one of the "zeros" of the function
  - Write the inverses of the zero functions as each term
    * Ex. If a truth table gives $\begin{matrix} A & B & C \\ 0 & 1 & 0 \end{matrix}$, then a term of the product of sums would $(A \vee \bar{B} \vee C)$
    * Multiply (AND) each of these terms
  - This is also known as maxterm form or conjunctive normal form

1. Find truth table rows where F is 0
2. 0 in input column → true literal
3. 1 in input column → complemented literal
4. OR the literals to get a maxterm
5. AND together all the maxterms

- Standard "shorthand" notation
  - If the order of variables in the rows of a truth table are agreed upon, then one may write M<ROW> as shorthand, for example, M4 for row 4
  - This can be be rewritten as a product of the sums or product notation:

$$(M0)(M1)(M2) = \prod M(0, 1, 2)$$

  - The canonical form is not always the minimal form

- Minterm-Maxterm Conversion
  - Rewrite minterm shorthand using maxterm shorthand, replacing minterm indices with the indices not already used, and vice versa:

$$\sum m(3, 4, 5, 6, 7) = \prod M(0, 1, 2)$$