Digital Logic Circuits

Michael Brodskiy
Professor: S. Shazli

January 18, 2023

- About a dozen logical operations
  - Similar to algebraic operators (+, *, -, /)
- In the following examples:
  - $p =$ "Today is Friday"
  - $q =$ "Today is my birthday"
- A not operation switches (negates the truth value)
- Symbol: $\neg$, $\sim$, '
- In C and C++ the operand is !
- Ex. $\neg p =$ "Today is not Friday"
- $\neg p = p'$
- An and operation is true if both operands are true
- Symbol: $\wedge$, $\bullet$
  - It's like the "A" in And
- In C and C++, the operand is &&
- $p \wedge q =$ "Today is Friday and today is my birthday"
- $A \wedge B = A \bullet B = AB$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- An or operation is true if either operand is true
- Symbol: $\vee, +$
- In C and C++, the operand is ||

- $p \vee q =$ "Today is Friday or today is my birthday (or possible both)"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- An exclusive or operation is true if one of the operands are true, but false if both are true
- Symbol: $\oplus$
- Often called XOR
- $p \oplus q = (p \vee q) \wedge \neg(p \wedge q)$
- $p \oplus q =$ "Today is Friday or today is my birthday, but not both"

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

- Logical Operator Summary Table:

		not	not	and	or	xor	nand	nor
p	q	$\neg p$	$\neg q$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \vert q$	$p \downarrow q$
T	T	F	F	T	T	F	F	F
T	F	F	T	F	T	T	T	F
F	T	T	F	F	T	T	T	F
F	F	T	T	F	F	F	T	T

- Precedence Order (from highest to lowest):

$$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$$

- Not is always performed before any other operation
- Tautology is a statement that is always true:

$$p \vee \neg p \text{ will always be true}$$
$$p \wedge \neg p \text{ will always be false}$$

- $p \wedge T \equiv p$ — Identity Law

p	T	$p \wedge T$
T	T	T
F	T	F

- $p \wedge F \equiv F$ — Domination Law

p	F	$p \wedge F$
T	F	T
F	F	F

- $p \wedge p \equiv p$ — Idempotent Law

p	p	$p \wedge p$
T	T	T
F	F	F

- $p \wedge q \equiv q \wedge p$ — Commutative Law

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ — Associative Law

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	T	F	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

- $p \vee T \equiv T$ — Identity Law
- $p \vee F \equiv p$ — Domination Law
- $p \vee p \equiv p$ — Idempotent Law
- $p \vee q \equiv q \vee p$ — Commutative Law
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$ — Associative Law