Digital Logic Minimization

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- Definitions
  - Literal: x_i, x_i'
  - Product Term: $x_2 x_1' x_0$
  - Sum Term: $x_2 + x_1' + x_0$
- Minterm of n variables
  - A product of n literals in which every variable appears exactly once
  - $f(a, b, c, d)$: $ab'cd'$, $a'bc'd'$
- Maxterm of n variables
  - A sum of n literals in which every variable appears exactly once
  - $f(a, b, c, d)$: $a' + b + c + d$, $a' + b' + c + d$
- Input:
  - Boolean expression of n binary variables
- Goal
  - Simplification of the expression
  - We want to minimize # terms and # literals
- Applications
  - Logic: rule reduction
  - Hardware Design: cost and performance optimization
  - Cost (wires, gates): # literals, product terms, sum terms
  - Performance: speed, reliability
- Boolean Optimization
  - Function can be represented by sum of minterms:

$$f(A, B) = A'B + AB' + AD$$

  - This is not minimal
  - We want to minimize the number of literals and terms
  - We can factor out common terms

$$\begin{aligned} A'B + AB' + AB &= A'B + AB' + AB + AB \\ &= (A' + A)B + A(B' + B) = B + A \end{aligned}$$

  - Thus, we have $f(A, B) = A + B$

- Karnaugh Maps (K-Maps)
  - Developed by Maurice Karnaugh
  - Essentially a truth table in 2 dimensions
  - For $f(A, B) = A + B$

ID	A	B	f	
0	0	0	0	
1	0	1	1	$A'B$
2	1	0	1	AB'
3	1	1	1	AB

  - Is converted to

	$B = 0$	$B = 1$
$A = 0$	0	1
$A = 1$	1	1

  - The bottom left corresponds to AB', the bottom right to AB, and the top right to $A'B$

- Representation of k-Variable Functions
  - Boolean Expression
  - Truth Table
  - Cube
  - K Map
  - Binary Decision Diagram

- Boolean expressions can be minimized by combining terms

- K-Map Grouping Rules
  1. Group all adjacent 1's
  2. Group only in powers of 2 (*i.e.* 2^n cells)
  3. Use rectangles or squares to cover 1's in adjacent entries
  4. Rectangles can overlap but should not include 0's
  5. Groups should be as large as possible (avoid redundant groups)
  6. Adjacent cells extend to:
     - Leftmost edge and rightmost edge are adjacent
     - Top and bottom rows are adjacent
     - The four corners are also adjacent

  - "Don't Cares" (X) don't matter at input or output, and can be assigned a 1 or 0 value