Lecture 1

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- The Equivalence Principle (for all freely-falling reference frames)
  - General Relativity > Special Relativity
- Geodesic — The shortest realizable line between two points
- Beyond Newton — Gravity as Geometry
  - Einstein's Equation

$$\nabla^2\Phi = 4\pi G\rho \rightarrow G_{\mu v} = R_{\mu v} - \frac{1}{2}Rg_{\mu v} = 8\pi G T_{\mu v}$$

- Accelerating Universe (Cosmological Constant? Dark Energy?)

$$R_{\mu v} - \frac{1}{2}Rg_{\mu v} + \Lambda g_{\mu v} = 8\pi G T_{\mu v}$$

- Dark Matter
- Cosmic Microwave Background (CMB)
- Galaxy Clustering
- Gravitational Lensing
- Gravitational Waves
- Special Relativity Review
  - Main Tenets (Einstein's Postulates)
    * Laws of Physics are the same in all inertial reference frames
    * The speed of light is invariable (always c)
      · c is the maximum observable speed

- Phenomena Explained by Special Relativity
  - Loss of Simultaneity
  - Length contraction/dilation
  - Time contraction/dilation (Twin Paradox)
  - Magnetic $\leftrightarrow$ Electric fields
  - Velocity addition ($v_t \neq v_1 + v_2$)
- Paradoxes
  - Twin Paradox
    - Two twins, one who stays on Earth, and one who flies to space and returns. The twin who stayed on Earth ages faster relative to the twin in space. To both, it appears that the other ages slower
    - Acceleration means the twin who "goes" is not always in an inertial frame
  - Pole-vaulter Paradox
    - A pole-vaulter with a pole longer than a barn can make it through the barn, even when the doors appear to open and close at the same time for a stationary observer. This is because of the loss of simultaneity; that is, for the vaulter the doors open and close at different times

- Light clock derivation of time dilation
  - To keep time, light is sent from one mirror to another. A counter counts each time light hits the bottom mirror. The two mirrors are separated by length L
  - Time is kept with "cycles" of how long it takes light to travel from one mirror to the other
  - When stationary, a cycle is:

$$T = \frac{2L}{c}$$

  - If a stationary observer sees the light clock move, with some velocity v, the light appears to form a triangle pattern, shown in Figure 1 below:

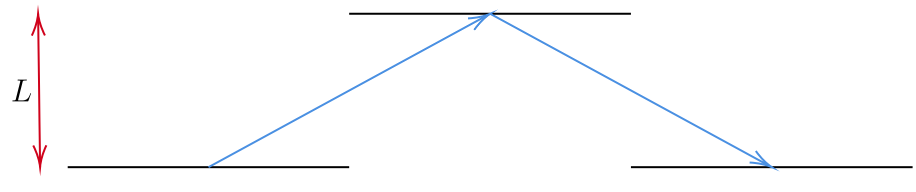

Figure 1: A Moving Light Clock from Stationary Reference Frame

* From the set up, it can be determined that the observed change in time for one individual can be expressed as a function of the change in time observed by the moving individual, times γ, where:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$
$$\Delta t' = \gamma \Delta t$$

* Thus, we see that at $v = 0 : \gamma = 1$ and at $v \to c : \gamma \to \infty$

- Lorentz Transformations and Rotations

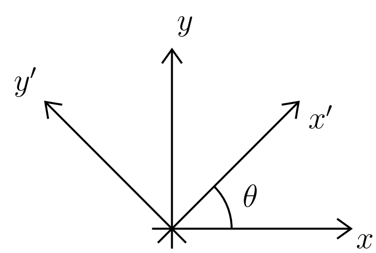

Figure 2: Rotation of Coordinate System

  - Following rotation, we get:

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta)$$

  - For boosting in a single direction ("rotation" between space and time):

$$x' = (x - vt)\gamma$$
$$y' = y$$
$$z' = z$$
$$t' = (t - vx)\gamma$$

  - Space and time are no longer separate $\Rightarrow$ single spacetime
  - A translation would involve movement of the origin

- Spacetime Diagram

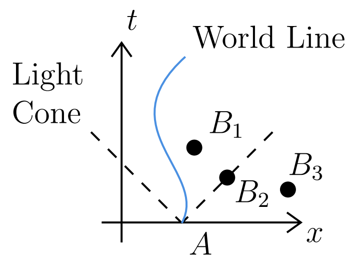

Figure 3: Example Spacetime Diagram

- Points represent events
- Light always travels at 45°
- Cone of causality: outside cone is causally separate (events inside cone can not affect events outside of it)
- Inside a light cone, it is possible to perform a boost such that events happen in the same place
- Lorentz transformation "squeezes" axes

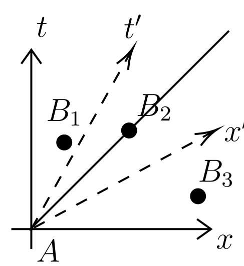

Figure 4: Lorentz Transformation from O to O'

- A to B_1: Δt and $\Delta t'$ are positive
- A to B_2: $\Delta x = \Delta t$, $\Delta x' = \Delta t'$
- A to B_3: Δx, $\Delta x'$ are positive
- Distances are invariant under translations and rotations (in standard Euclidean space)
- Distances in spacetime: spacetime interval
  * For flat space (Minkowski space)

$$\Delta s^2 = -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

    · Has a $-+++$ signature, versus a $+---$ signature

      - If $\Delta s^2 < 0$: time-like separation
      - If $\Delta s^2 = 0$: light-like (null) separation
      - If $\Delta s^2 > 0$: space-like separation
      - With proper time τ:

$$\Delta\tau^2 = -\Delta s^2$$

- Changing between inertial reference frams is a Lorentz transformation
  - Δs is invariant under Lorentz transformations
  - General Relativity involves generalizing this to curved space $\Rightarrow$ differential geometry