Homework 4

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1. Using our formula:

$$\frac{\Delta T}{T} = \frac{v}{c}$$

We may use $T = 2.725[\text{K}]$, the given value of ΔT, and the speed of light as $c = 3 \cdot 10^5[\text{km s}^{-1}]$ to get:

$$v = \frac{3.36 \cdot 10^{-3}}{2.725}(3 \cdot 10^5)$$
$$\boxed{v = 369.908[\text{km s}^{-1}]}$$

As a fraction of the speed of light, we may write this as:

$$\boxed{v = .001233c}$$

2. (a)
   (b)
   (c)

3. (a) Given one second since the Big Bang, we know:
   Integration leads us to find:

$$t(a) = \frac{a^2}{2H_o\sqrt{\Omega_{r,0}}}$$

We rearrange in terms of a to write:

$$a = \sqrt{2tH_o\sqrt{\Omega_{r,0}}}$$

Using our known values, we get:

$$a = \sqrt{2(1)\left(2.27 \cdot 10^{-18}\right)\sqrt{9 \cdot 10^{-5}}}$$

$$\boxed{a = 2.0753 \cdot 10^{-10}}$$

From here, we know:

$$a = \frac{1}{1+z}$$

This gives us the redshift as:

$$z = \frac{1}{a} - 1$$

$$z = \frac{1}{2.0753 \cdot 10^{-10}} - 1$$

$$\boxed{z = 4.8186 \cdot 10^{9}}$$

Using the scale factor, we know that the temperature is simply the current CMB over the factor:

$$T(a) = a^{-1}T_o$$

$$T(a) = \left(2.0753 \cdot 10^{-10}\right)^{-1}(2.725[\text{K}])$$

$$\boxed{T(a) = 1.3131 \cdot 10^{10}[\text{K}]}$$

Using the standard units, we know that the energy is proportional to the temperature; however, we need to adjust our units to electron-volts. This gives us:

$$E \approx (1.3131 \cdot 10^{10})(8.617 \cdot 10^{-5})$$

$$E = 1.1315 \cdot 10^{6}[\text{eV}]$$

$$\boxed{E = 1.1315[\text{MeV}]}$$

With standard units, mass is equal to the energy, which lets us simply convert units to say:

$$\boxed{m = 2.017 \cdot 10^{-30}\,[\text{kg}]}$$

(b) We may begin by converting to Temperature (Kelvin):

$$T = \frac{13 \cdot 10^{12}}{8.617 \cdot 10^{-5}}$$

$$\boxed{T = 1.5086 \cdot 10^{17}[\text{K}]}$$

We can then find the scale factor:

$$a = \frac{T_o}{T}$$

$$a = \frac{2.725}{1.5086 \cdot 10^{17}}$$

$$\boxed{a = 1.8063 \cdot 10^{-17}}$$

Using the time formula obtained in (a), we get:

$$t = \frac{a^2}{2H_o\sqrt{\Omega_{r,0}}}$$

$$t = \frac{(1.8063 \cdot 10^{-17})^2}{2(2.27 \cdot 10^{-18})\sqrt{9 \cdot 10^{-5}}}$$

$$\boxed{t = 7.58 \cdot 10^{-15}[\mathrm{s}]}$$

The redshift may be found as:

$$z = \frac{1}{a} - 1$$

$$z = \frac{1}{1.8063 \cdot 10^{-17}} - 1$$

$$\boxed{z = 5.536 \cdot 10^{16}}$$

The mass can finally be obtained as:

$$m = \left(1.78266 \cdot 10^{-30}\right)\left(13 \cdot 10^{6}\right)$$

$$\boxed{m = 2.317 \cdot 10^{-23}[\mathrm{kg}]}$$

(c) With this mass, we may begin by calculating the energy:

$$10^{-9}[\mathrm{g}] = 10^{-12}[\mathrm{kg}]$$

$$E = \frac{10^{-12}}{1.78266 \cdot 10^{-30}}$$

$$\boxed{E = 5.609 \cdot 10^{11}[\mathrm{TeV}]}$$

The temperature then becomes:

$$T = \frac{5.609 \cdot 10^{23}}{8.617 \cdot 10^{-5}}$$

$$\boxed{T = 6.509 \cdot 10^{27}[\mathrm{K}]}$$

We may obtain the scale factor:

$$a = \frac{2.725}{6.509 \cdot 10^{27}}$$

$$\boxed{a = 4.1864 \cdot 10^{-28}}$$

This gives us the redshift as:

$$z = \frac{1}{a} - 1$$

$$z = \frac{1}{4.1864 \cdot 10^{-28}} - 1$$

$$\boxed{z = 2.389 \cdot 10^{27}}$$

And finally, we may find the time since the Big Bang as:

$$t = \frac{(4.1864 \cdot 10^{-28})^2}{2(2.27 \cdot 10^{-18})\sqrt{9 \cdot 10^{-5}}}$$

$$\boxed{t = 4.069 \cdot 10^{-36}[\mathrm{s}]}$$

(d) In the case of temperature being given, we need to consider radiation and matter. This gives us:

$$\int \frac{1}{H_o\sqrt{\Omega_r a^{-4} + (\Omega_m - \Omega_r)a^{-3}}}\, da = t$$

Using a numerical solver gets us:

$$t = \frac{2}{(\Omega_m - \Omega_r)^2}\left[\frac{1}{3}\left(\Omega_r + (\Omega_m - \Omega_r)\, a\right)^{\frac{3}{2}} - \Omega_r\sqrt{\Omega_r + (\Omega_m - \Omega_r)\, a}\right]$$

We may find the scale factor as:

$$a = \frac{2.725}{3000}$$

$$\boxed{a = .000908}$$

We then plug this into the above to get time:

$$t = \frac{2}{(.31 - 9 \cdot 10^{-5})^2}\left[\frac{1}{3}\left(9 \cdot 10^{-5} + \left(.31 - 9 \cdot 10^{-5}\right)(.000908)\right)^{\frac{3}{2}} - \right.$$
$$\left.\left(9 \cdot 10^{-5}\right)\sqrt{9 \cdot 10^{-5} + (.31 - 9 \cdot 10^{-5})\,(.000908)}\right]$$

$$\boxed{t = .000014[\mathrm{s}]}$$

We can then proceed to find the rest of the values as usual:

$$z = \frac{1}{a} - 1$$
$$z = \frac{1}{.000908} - 1$$
$$\boxed{z = 1099.92}$$

We convert the temperature to energy:

$$E = 3000(8.617\dot{1}0^{-5})$$
$$\boxed{E = .25851}$$

And finally we get the mass as:

$$m = (.25851 \cdot 10^{-6})(1.78266 \cdot 10^{-30})$$
$$\boxed{m = 4.6084 \cdot 10^{-37}[\text{kg}]}$$

4. (a)
   (b)
5. (a)
   (b)
   (c)