Lecture 2 — Introduction to Differential Geometry

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- Metric → measuring things ("meter")
- In differential geometry, a metric defines how we calculate distance

$$\begin{aligned}\Delta s^2 &= -\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2 \qquad \text{(the metric in Minkowski space)} \\ &= \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu\end{aligned}$$

  - A repeated index (up and down) → sum
    - Greek: 0-3

$$\begin{array}{ll} \Delta x^\mu & \Delta x^0 \\ & \Delta x^1 \\ & \Delta x^2 \\ & \Delta x^3 \end{array}$$

    - Latin: 1-3

$$\begin{array}{ll} \Delta x^i & \Delta x^1 \\ & \Delta x^2 \\ & \Delta x^3 \end{array}$$

$$\eta_{00}\Delta x^0\Delta x^0 + \eta_{01}\Delta x^0\Delta x^1 + \eta_{02}\Delta x^0\Delta x^2 + \eta_{10}\Delta x^1\Delta x^0 + \eta_{11}\Delta x^1\Delta x^1 + \cdots$$

  - This can be written as:

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \eta_{00}\Delta x^0\Delta x^0 + = \eta_{11}\Delta x^1\Delta x^1 + = \eta_{22}\Delta x^2\Delta x^2 + = \eta_{33}\Delta x^3\Delta x^3$$