$$W(f_1, f_2, f_n) = \begin{vmatrix} f_1 & f_2 & f_n \\ f_1' & f_2' & f_n' \\ \vdots & \vdots & \vdots \\ f_1^{n-1} & f_2^{n-1} & f_n^{n-1} \end{vmatrix} \tag{1}$$

If $W(f) \neq 0$, f is linearly independent

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y_1(x))^2}\,dx \tag{2}$$

Where $y_1(x)$ is a known function

And $P(x)$ is the coefficient of y'

$$\begin{aligned} \alpha \pm \beta i &\Rightarrow e^{\alpha x}\left(c_1 \cos(\beta x) + c_2 \sin(\beta x)\right) \\ \beta i &\Rightarrow c_1 \cos(\beta x) + c_2 \sin(\beta x) \\ \alpha, \beta &\Rightarrow c_1 e^{\alpha x} + c_2 e^{\beta x} \\ \alpha &\Rightarrow c_1 e^{\alpha x} + c_2 x e^{\alpha x} \end{aligned} \tag{3}$$

$$\begin{gathered} D^n;\ x^{n-1} \\ (D-\alpha)^n;\ x^{n-1}e^{\alpha x} \\ [D^2 - 2\alpha D + \alpha^2 + \beta^2]^n;\ x^{n-1}e^{\alpha x}\cos\beta x \end{gathered} \tag{4}$$

$$\begin{aligned} u_1' &= \frac{W_1}{W} = -\frac{y_2 f(x)}{W} \\ u_2' &= \frac{W_2}{W} = -\frac{y_1 f(x)}{W} \\ W &= \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \\ W_1 &= \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} \\ W_2 &= \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} \end{aligned} \tag{5}$$

$$\begin{aligned} u_1(x) &= -\int_{x_0}^{x} \frac{y_2(t)f(t)}{W(t)}\,dt \\ u_2(x) &= \int_{x_0}^{x} \frac{y_1(t)f(t)}{W(t)}\,dt \\ y_p(x) &= u_1(x)y_1(x) + u_2(x)y_2(x) \end{aligned} \tag{6}$$

$y_1(x)$ is complementary function 1

$y_2(x)$ is complementary function 2

$f(x)$ is the right-side function of x

$$\begin{aligned} \lambda^2 - \omega^2 &> 0 \\ m_{1,2} &= -\lambda \pm \sqrt{\lambda^2 - \omega^2} \\ x(t) &= e^{-\lambda t}\left(c_1 e^{\sqrt{\lambda^2-\omega^2}t} + c_2 e^{-\sqrt{\lambda^2-\omega^2}}\right) \end{aligned} \tag{7}$$

$$\begin{aligned} \lambda^2 - \omega^2 &= 0 \\ m_{1,2} &= 0 \\ x(t) &= e^{-\lambda t}(c_1 + c_2 t) \end{aligned} \tag{8}$$

$$\begin{aligned} \lambda^2 - \omega^2 &< 0 \\ m_{1,2} &= -\lambda \pm \sqrt{\omega^2 - \lambda^2}i \\ A &= \sqrt{\omega^2 - \lambda^2} \\ x(t) &= e^{-\lambda t}\left(c_1 \cos At + c_2 \sin At\right) \end{aligned} \tag{9}$$

$$\begin{aligned} \text{Spring Parallel: } k_eff &= k_1 + k_2 \\ \text{Spring Series: } k_eff &= \frac{k_1 k_2}{k_1 + k_2} \\ \frac{d^2x}{dt^2} + \omega^2 x &= 0 \\ x(t) &= c_1 \cos\omega t + c_2 \sin\omega t \\ &A\sin(\omega t + \phi) \end{aligned} \tag{10}$$

$$\begin{aligned}\text{Inductor (henries): } & L\frac{di}{dt}\\ \text{Resistor (ohms): } & iR\\ \text{Capacitor (farads): } & q\frac{1}{C}\\ L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+q\frac{1}{C} &= E(t)\\ R\neq 0 \text{ then } & q_c(t) \text{ is transient}\\ E(t) \text{ is } \cos,\sin,c \to & \, q_p(t) \text{ steady-state}\end{aligned} \tag{11}$$

$$\begin{aligned}&\text{Overdamped if:}\\ &R^2-\frac{4L}{C}>0\\ &\text{Critically Damped if:}\\ &R^2-\frac{4L}{C}=0\\ &\text{Underdamped if:}\\ &R^2-\frac{4L}{C}<0\end{aligned} \tag{12}$$

$$\begin{aligned}\text{Embedded: } & y=0,\ y'=0\\ \text{Free: } & y''=0,\ y'''=0\\ \text{Simply Supported: } & y=0,\ y''=0\\ \text{Horizontal Column: } & EI\frac{d^4y}{dx^4}=\omega(x)\\ \text{Vertical Column: } & EI\frac{d^2y}{dx^2}+Py=0\\ \text{Euler Load: } & \frac{\pi^2 EI}{L}\end{aligned} \tag{13}$$

$$\begin{aligned}&y(0)=0,\ y(L)=0:\\ &\lambda_n=\frac{\pi^2n^2}{L^2}\\ &y_n(x)=\sin\left(\frac{n\pi}{L}x\right)\\ &y(0)=0,\ y'(L)=0\\ &\lambda_n=\frac{(2n-1)^2\pi^2}{4L^2}\\ &y_n(x)=\sin\left(\frac{(2n-1)\pi x}{2L}\right)\end{aligned} \tag{14}$$

$$\begin{aligned}&\text{For a twirling rope:}\\ &T\frac{d^2y}{dx^2}+\rho\omega^2y=0\\ &\text{Where } T \text{ is the tension force}\\ &\rho \text{ is the density per unit length}\\ &\text{and } \omega \text{ is the angular velocity}\end{aligned} \tag{15}$$

$$\begin{aligned}y&=\sum_{n=0}c_nx^n\\ y'&=\sum_{n=1}nc_nx^{n-1}\\ y''&=\sum_{n=2}n(n-1)c_nx^{n-2}\end{aligned} \tag{16}$$