Modeling with Systems of First-Order DEs

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- Say we are given two populations which interact, $x(t)$ and $y(t)$. The two differential equations (1) can be used to model population growth. A **linear system** would be of form (2), where c_i could depend on t. Any model of another form is said to be nonlinear.

$$\frac{dx}{dt}\frac{dy}{dt} \tag{1}$$

$$\begin{aligned} g_1(t,x,y) &= c_1x + c_2y + f_1(t) \\ g_2(t,x,y) &= c_3x + c_4y + f_2(t) \end{aligned} \tag{2}$$

- Given different elements with radioactive decay, where y is gaining atoms from decay of x and itself decaying, it depends on x and y the differential equations (3)

$$\begin{aligned} \frac{dx}{dt} &= -\lambda_1 x \\ \frac{dy}{dt} &= \lambda_1 x - \lambda_2 y \\ \frac{dz}{dt} &= \lambda_2 y \end{aligned} \tag{3}$$

- Say $x(t)$ and $y(t)$ are fox and rabbit populations, respectively. The model for the fox population, without rabbits, may be found using equation 4 (4). If there are rabbits in the system, there could be a better model, equation 5 (5).

$$\frac{dx}{dt} = -ax \tag{4}$$

$$\frac{dx}{dt} = -ax + bxy \tag{5}$$