Momentum and Energy Challenge Lab
Finding the Spring Constant
Physics 1 AP

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1 Objective

To find the spring constant, k, of a cart, by designing our own lab with a limited amount of money

1.1 Definitions

Momentum Momentum, p is equal to the mass, m, times velocity, v.

Potential Spring Energy The Potential Spring Energy is equal to one half times the spring constant, k, times the compression distance, x, squared

Kinetic Energy The Kinetic Energy is equal to one half times the mass, m, times the velocity, v, squared, or the antiderivative of momentum

Conservation of Momentum Conservation of Momentum states that the Total Mechanical Energy, or all of the energies added together, is conserved. This means that we can set the kinetic energy equal to the potential spring energy, yielding:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

2 Experimental Data

Mass of Cart with Flag	276.4 g
Mass of Cart with Flag and One Mass	589 g
Mass of Cart with Flag and Two Masses	781.8 g
Mass of Cart with Flag and Three Masses	1033.7 g
Mass of Cart with Flag and Four Masses	1286.4 g
Cart used	#7
Balance Used	#2
Spring Compression Distance x	51.61 mm
Flag Height	25mm

3 Purchases

Photogate		\$20
iPad		\$20
Unlimited Balance Access		\$10
Flag		\$5
Calipers		\$5
Level		\$3
Four Masses	\$5 each −	\$20
Tape		\$2
Total		\$85

With the purchases, a system was set up which looked as follows:

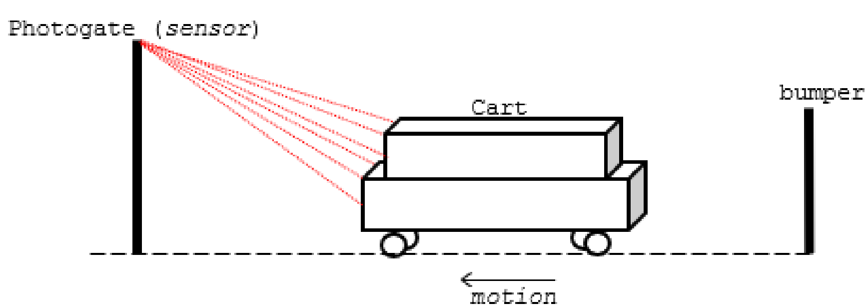

Figure 1: The Lab Setup

The system can be used in tandem with the known variables, x, m, and v, to solve for k as follows:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \implies mv^2 = kx^2$$

$$\therefore k = \frac{mv^2}{x^2}$$

The velocity can be computed by using the photogate, the compression distance can be measured, and the mass can be weighed.

4 Collected Data

	276.4 g	589 g	781.8 g	1033.7 g	1286.4 g
Trial 1	1.23 $\frac{m}{s}$	0.93 $\frac{m}{s}$	0.74 $\frac{m}{s}$	0.66 $\frac{m}{s}$	0.61 $\frac{m}{s}$
Trial 2	1.25 $\frac{m}{s}$	0.92 $\frac{m}{s}$	0.77 $\frac{m}{s}$	0.68 $\frac{m}{s}$	0.62 $\frac{m}{s}$
Trial 3	1.2 $\frac{m}{s}$	0.87 $\frac{m}{s}$	0.71 $\frac{m}{s}$	0.69 $\frac{m}{s}$	0.62 $\frac{m}{s}$
Average	1.2267 $\frac{m}{s}$	0.9067 $\frac{m}{s}$	0.74 $\frac{m}{s}$	0.6767 $\frac{m}{s}$	0.6167 $\frac{m}{s}$

5 Graphing

In order to produce a linear graph, the mathematical equation needs to be rearranged as follows:

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \implies mv^2 = kx^2 \implies v^2 = \frac{kx^2}{m}$$

$$\therefore v = \sqrt{\frac{kx^2}{m}}$$

To then obtain the slope, the variables must be separated into:

$$v = \sqrt{kx^2} \cdot \frac{1}{\sqrt{m}}$$

The slope is given by $\sqrt{kx^2}$, which means the graph will be Average Velocity (v) plotted against Inverse Root Mass $\left(\frac{1}{\sqrt{m}}\right)$

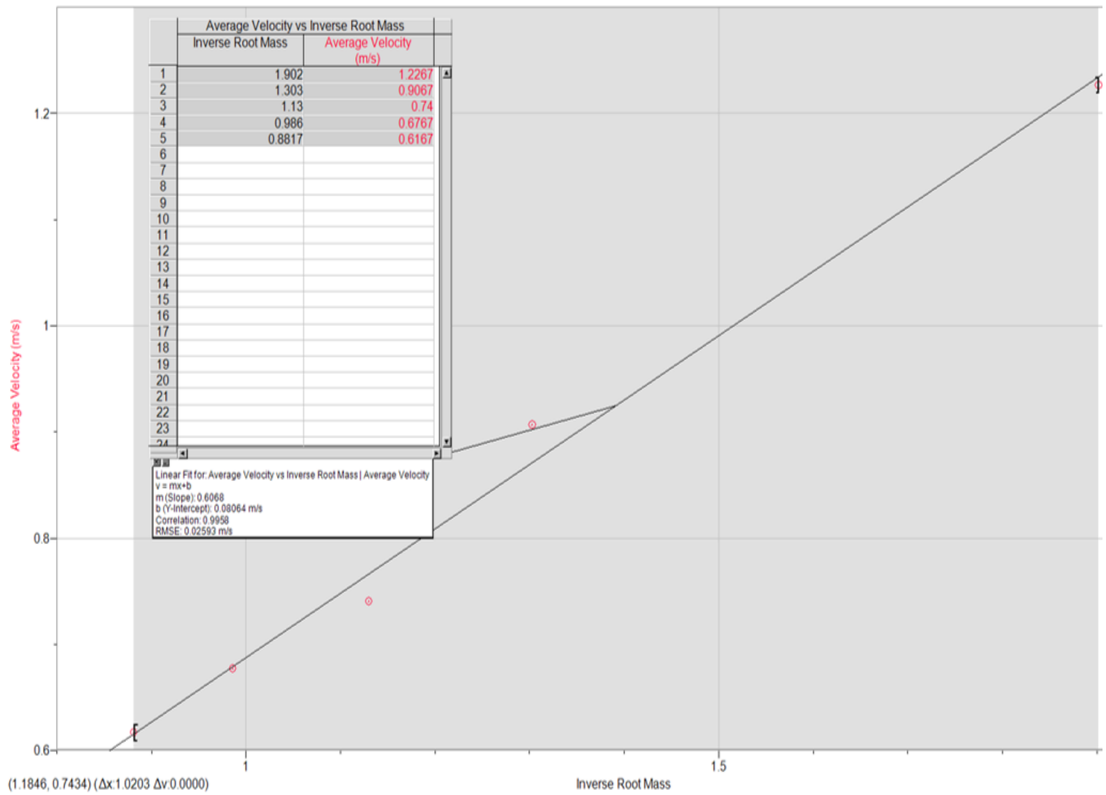

Figure 2: `Average Velocity vs Inverse Root Mass`

Linear Fit for: Average Velocity vs Inverse Root Mass | Average Velocity
v = mx+b
m (Slope): 0.6068
b (Y-Intercept): 0.08064 m/s
Correlation: 0.9958
RMSE: 0.02593 m/s

Figure 3: `Slope Specifications`

6 Analysis & Quality Control

To find the spring constant, the line of best fit's slope must be set equal to the slope equation:

$$.6068 = \sqrt{kx^2} \Longrightarrow k = \frac{(.6068)^2}{(.05161)^2}$$

$$\therefore k = 138.237 \frac{\mathrm{N}}{\mathrm{m}}$$

This value can then be applied to find the initial velocity of a marble of mass $65.36\,\mathrm{g}$

$$138.237 \cdot (.05161)^2 = .06536v^2 \Longrightarrow v = \sqrt{\frac{138.237 \cdot (.05161)^2}{.06536}}$$

$$\therefore v = 2.3735\,\frac{\mathrm{m}}{\mathrm{s}}$$

This can then be applied to a kinetmatics equation to solve for the range:

$$\Delta x = v_{ox}t + \frac{1}{2}a_x t^2$$

The acceleration can be set to zero, because there is none in the horizontal direction, resulting in:

$$\Delta x = v_{ox}t$$

The time must be solved for using the vertical dimension:

$$v_{fy} = v_{oy} + a_y t \Longrightarrow 0 = v_{oy} + a_y t \Longrightarrow \frac{-v_{oy}}{a_y} = t$$

$$\therefore t = \frac{-2.3735}{-9.80665} = 0.242\,03\,\mathrm{s}$$

By plugging in the time value, the range of the projectile is found to be:

$$\Delta x = (2.3735)(.24203)$$

$$\Delta x = 0.574\,458\,\mathrm{m}$$

7 Error Analysis

7.1 Wear and Tear — With every use, the spring becomes more worn, infinitesimally. This wear would cause the spring constant value to decrease, as the potential energy would be affected by the weakened spring.

7.2 Frictional Forces — Throughout the experimentation there exist many frictional forces which are not accounted for. One such force is the force of friction exerted upon the wheels of the cart from the track. This would result in a lesser velocity value, and thus, a lower spring constant value. Another such force that would affect the launched ball is the frictional force from the air. This would cause the velocity to decrease at a faster rate, causing the range to be shorter than the calculated one.