

Inverse Transforms and Transforms of Derivatives

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- Some known values for inverse Laplace transforms, $\mathcal{L}^{-1}\{f(t)\}$, are shown:

$$\begin{aligned} 1 &= \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} \\ t^n &= \mathcal{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} & e^{at} &= \mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} \\ \sin kt &= \mathcal{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} & \cos kt &= \mathcal{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} \\ \sinh kt &= \mathcal{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} & \cosh kt &= \mathcal{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} \end{aligned} \tag{1}$$

- If f, f', \dots, f^{n-1} are continuous on $[0, \infty)$, and are of exponential order and if $f^{(n)}(t)$ is piecewise continuous on $[0, \infty)$, then (2) where $F(s) = \mathcal{L}\{f(t)\}$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f^{n-1}(0) - s^{n-2}f^{n-2}(0) - \dots - f^{(n-1)}(0) \tag{2}$$