

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + q \frac{1}{C} = E(t)$$

If $R \neq 0$ then $q_c(t)$ is transient

$E(t)$ is $\cos, \sin, c \rightarrow q_p(t)$ is steady-state

$$D^n; x^{n-1}$$

$$(D - \alpha)^n; x^{n-1} e^{\alpha x}$$

$$[D^2 - 2\alpha D + \alpha^2 + \beta^2]^n; x^{n-1} e^{\alpha x} \cos \beta x$$

$f(t)$	$F(s)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
$t^{-\frac{1}{2}}$	$\sqrt{\frac{\pi}{s}}$
$t^{\frac{1}{2}}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
$\sin(kt)$	$\frac{k}{s^2 + k^2}$
$\cos(kt)$	$\frac{s}{s^2 + k^2}$
$\sin^2(kt)$	$\frac{2k^2}{s(s^2 + 4k^2)}$
$\cos^2(kt)$	$\frac{s^2 + 2k^2}{s(s^2 + 4k^2)}$
$e^{\alpha t}$	$\frac{1}{s - a}$
$\sinh(kt)$	$\frac{k}{s^2 - k^2}$
$\cosh(kt)$	$\frac{s}{s^2 - k^2}$
$\sinh^2(kt)$	$\frac{2k^2}{s(s^2 - 4k^2)}$
$\cosh^2(kt)$	$\frac{s^2 - 2k^2}{s(s^2 - 4k^2)}$
$t \sin(kt)$	$\frac{2ks}{(s^2 + k^2)^2}$
$t \cos(kt)$	$\frac{s^2 - k^2}{(s^2 + k^2)^2}$
$\sin(kt) + kt \cos(kt)$	$\frac{2ks^2}{(s^2 + k^2)^2}$
$\sin(kt) - kt \cos(kt)$	$\frac{2k^3}{(s^2 + k^2)^2}$
$\delta(t - t_0)$	e^{-st_0}

$f(t)$	$F(s)$
$t \sinh(kt)$	$\frac{2ks}{(s^2 - k^2)^2}$
$t \cosh(kt)$	$\frac{s^2 + k^2}{(s^2 - k^2)^2}$
$\frac{e^{at} - e^{bt}}{a - b}$	$\frac{1}{(s - a)(s - b)}$
$\frac{ae^{at} - be^{bt}}{a - b}$	$\frac{s}{(s - a)(s - b)}$
$1 - \cos(kt)$	$\frac{k^2}{s(s^2 + k^2)}$
$kt - \sin(kt)$	$\frac{k^3}{s^2(s^2 + k^2)}$
$\frac{a \sin(bt) - b \sin(at)}{ab(a^2 - b^2)}$	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$
$\frac{\cos(bt) - \cos(at)}{a^2 - b^2}$	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$
$\sin(kt) \sinh(kt)$	$\frac{2k^2 s}{s^4 + 4k^4}$
$\sin(kt) \cosh(kt)$	$\frac{k(s^2 + 2k^2)}{s^4 + 4k^4}$
$\cos(kt) \sinh(kt)$	$\frac{k(s^2 - 2k^2)}{s^4 + 4k^4}$
$\cos(kt) \cosh(kt)$	$\frac{s^3}{s^4 + 4k^4}$
$\frac{e^{bt} - e^{at}}{t}$	$\ln \left(\frac{s - a}{s - b} \right)$
$e^{at} f(t)$	$F(s - a)$
$\mathcal{U}(t - a)$	$\frac{e^{-as}}{s}$
$f(t - a) \mathcal{U}(t - a)$	$e^{-as} F(s)$
$f * g = \int_0^t f(\tau) g(t - \tau) d\tau$	$F(s) G(s)$
$g(t) \mathcal{U}(t - a)$	$e^{-as} \mathcal{L}\{g(t + a)\}$

$$\lambda^2 - \omega^2 > 0$$

$$m_{1,2} = -\lambda \pm \sqrt{\lambda^2 - \omega^2}$$

$$x(t) = e^{-\lambda t} \left(c_1 e^{\sqrt{\lambda^2 - \omega^2} t} + c_2 e^{-\sqrt{\lambda^2 - \omega^2} t} \right)$$

$$F = kx$$

$$m \frac{d^2 x}{dt^2} + 2\lambda \frac{dx}{dt} + \omega^2 x = f(t)$$

$$\lambda^2 - \omega^2 < 0$$

$$m_{1,2} = -\lambda \pm \sqrt{\omega^2 - \lambda^2} i$$

$$A = \sqrt{\omega^2 - \lambda^2}$$

$$x(t) = e^{-\lambda t} (c_1 \cos At + c_2 \sin At)$$

Given $a_2(x) \frac{d^2 y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x)y = 0$
 Has radius of convergence given by distance
 from analysis point to roots of $a_2(x)$

$$y = \sum_{n=0} c_n x^n$$

$$y' = \sum_{n=1} n c_n x^{n-1}$$

$$y'' = \sum_{n=2} n(n-1) c_n x^{n-2}$$

Even if $f(-x) = f(x)$
 Even = Cosine Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{p} x$$

$$a_0 = \frac{2}{p} \int_0^p f(x) dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos \frac{n\pi}{p} x dx$$

$$b_n = 0$$

$$\lambda^2 - \omega^2 = 0$$

$$m_{1,2} = 0$$

$$x(t) = e^{-\lambda t} (c_1 + c_2 t)$$

$$W = mg$$

Spring Parallel: $k_e f f = k_1 + k_2$

Spring Series: $k_e f f = \frac{k_1 k_2}{k_1 + k_2}$

$$\frac{d^2 x}{dt^2} + \omega^2 x = 0$$

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t$$

$$= A \sin(\omega t + \phi)$$

$$k_1 = f(x, y)$$

$$k_2 = f\left(x + \frac{h}{2}, y + \frac{h \cdot k_1}{2}\right)$$

$$k_3 = f\left(x + \frac{h}{2}, y + \frac{h \cdot k_2}{2}\right)$$

$$k_4 = f(x + h, y + h \cdot k_3)$$

$$RK4 = y + h \left(\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} \right)$$

$$O(h^n) = y^{(n)}(c) \frac{h^n}{n!}$$

$O(h^4)$ for global truncation on RK4
 $O(h^5)$ for local truncation on RK4

Odd if $f(-x) = -f(x)$
 Odd = Sine Series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{p} x$$

$$a_0 = 0$$

$$a_n = 0$$

$$b_n = \frac{2}{p} \int_0^p f(x) \sin \frac{n\pi}{p} x dx$$

Good luck. May the +c be with you.