

# The Dirac Delta Function

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- The Dirac Delta “Function” may be modeled by (1)

$$\delta(t - t_0) = \begin{cases} 0, & 0 \leq t < t_0 - a \\ \frac{1}{2a}, & t_0 - a \leq t < t_0 + a \\ 0, & t \geq t_0 + a \end{cases} \quad (1)$$

- This function could serve as a model for a big force exerted over little time (an impulse)
- The impulse (force over time), or the area under the graph of the force always equals 1. This means that, for a shorter period of time, the force is greater
- The function  $\delta_a(t - t_0)$  is called the unit impulse, or the Dirac Delta Function. The limit that approximates this is defined as (2)

$$\delta(t - t_0) = \lim_{a \rightarrow 0} \delta_a(t - t_0) \quad (2)$$

- The Laplace transform is defined as (3)

$$\mathcal{L}\{\delta(t - t_0)\} = e^{-st_0} \quad (3)$$

- The integral of any function multiplied by the Dirac delta function is the function evaluated at the point  $t_0$ , or (4)

$$\int_0^\infty f(t)\delta(t - t_0) dt = f(t_0) \quad (4)$$