Coordinates and Change of Basis

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- Suppose $\overrightarrow{u} = (3,4)$ and $B = \{(1,0),(0,1)\}$. Then you could say $\overrightarrow{u} = 3(1,0) + 4(0,1)$. This could be written as $[\overrightarrow{u}]_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

- Let the set of vectors $\{\overrightarrow{v}_1, \overrightarrow{v}_2, \ldots, \overrightarrow{v}_n\}$ be the basis for vector space $\mathbf{V}$, and $c_1, c_2, \ldots, c_n$ be scalars, where $\overrightarrow{u} \in \mathbf{V}$ such that $\overrightarrow{u} = c_1\overrightarrow{v}_1 + c_2\overrightarrow{v}_2 + \cdots + c_v\overrightarrow{v}_n$. This can be written as a coordinate matrix of vector $\overrightarrow{u}$ with respect to basis B:

$$[\overrightarrow{u}]_B = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

- Change of Basis — Suppose you have basis $B = \left\{\overrightarrow{b}_1, \overrightarrow{b}_2\right\}$ and $C = \{\overrightarrow{c}_1, \overrightarrow{c}_2\}$ of vector space V. Then you could write $\begin{matrix} \overrightarrow{b}_1 = a\overrightarrow{c}_1 + b\overrightarrow{c}_2 \\ \overrightarrow{b}_2 = c\overrightarrow{c}_1 + d\overrightarrow{c}_2 \end{matrix}$, and, given $[\overrightarrow{v}]_B = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$, you could find $[\overrightarrow{v}]_C$ through the following method:

$$\begin{aligned} \overrightarrow{v} &= k_1\overrightarrow{b}_1 + k_2\overrightarrow{b}_2 \\ \overrightarrow{v} &= k_1(a\overrightarrow{c}_1 + b\overrightarrow{c}_2) + k_2(c\overrightarrow{c}_1 + d\overrightarrow{c}_2) \\ [\overrightarrow{v}]_c &= \begin{bmatrix} ak_1 + ck_2 \\ bk_1 + dk_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \\ \text{Where } P[\overrightarrow{v}]_B &= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is a transition matrix} \end{aligned}$$

- For $B \to C$: $[\overrightarrow{v}]_C = P[\overrightarrow{v}]_B$

- For $C \to B$: $[\overrightarrow{v}]_B = P[\overrightarrow{v}]_C$