Linear Algebra 1.2 Homework

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1. 3x3

5. 4x5

9. $R_2 \to R_1$
   $R_1 \to R_2$
   $R_3 + 3R_2 \to R_3$

13.

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$x_3 = -1$$
$$x_2 - 2(-1) = 1$$
$$x_2 = -1$$
$$x_1 - (-1) = 3$$
$$x_1 = 2$$
$$\text{The solution is } S = \{(2, -1, -1)\}$$

17.

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$x_4 = 4$$
$$x_3 + 2(4) = 1$$
$$x_3 = -7$$
$$x_2 + 2(-7) + 4 = 3$$
$$x_2 = 13$$
$$x_1 + 2(13) + 4 = 4$$
$$x_1 = -26$$
$$\text{The solution is } S = \{(-26, 13, -7, 4)\}$$

21. It is not in row or reduced row echelon form

25.

$$\left[\begin{array}{cc|c} 1 & 3 & 11 \\ 3 & 1 & 9 \end{array}\right]$$

$$R_2 - 3R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 11 \\ 0 & -8 & -24 \end{array}\right]$$

$$-\frac{1}{8}R_2 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 11 \\ 0 & 1 & 3 \end{array}\right]$$

$$R_1 - 3R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

The solution is $S = \{(2, 3)\}$

29.

$$\left[\begin{array}{cc|r} -3 & 5 & -22 \\ 3 & 4 & 4 \\ 4 & -8 & 32 \end{array}\right]$$

$$R_1 + R_3 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cc|r} 1 & -3 & 10 \\ 3 & 4 & 4 \\ 4 & -8 & 32 \end{array}\right]$$

$$R_2 - 3R_1 \widetilde{\rightarrow} R_2 \text{ and } R_3 - 4R_1 \widetilde{\rightarrow} R_3$$

$$\left[\begin{array}{cc|r} 1 & -3 & 10 \\ 0 & 13 & -26 \\ 0 & 4 & -8 \end{array}\right]$$

$$\frac{1}{4}R_3 - \frac{1}{13}R_2 \widetilde{\rightarrow} R_3 \text{ and } R_1 + \frac{3}{13}R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cc|r} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

The solution is $S = \{(4, -2)\}$

33.

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 4 & -3 & 7 & 5 \\ 8 & -9 & 15 & 10 \end{array}\right]$$

$$R_3 - 3R_2 + 2R_1 \widetilde{\rightarrow} R_3$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 3 & 3 \\ 4 & -3 & 7 & 5 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

No solution! ($\emptyset$)

35.

$$\left[\begin{array}{cccc|c} 4 & 12 & -7 & -20 & 22 \\ 3 & 9 & -5 & -28 & 30 \end{array}\right]$$

$$R_1 - R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 8 & -8 \\ 3 & 9 & -5 & -28 & 30 \end{array}\right]$$

$$R_2 - 3R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{cccc|c} 1 & 3 & -2 & 8 & -8 \\ 0 & 0 & 1 & -52 & 54 \end{array}\right]$$

$$R_1 + 2R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & -96 & 100 \\ 0 & 0 & 1 & -52 & 54 \end{array}\right]$$

$$y \to s$$

$$w \to t$$

The solution is $S = \{(100 - 3s + 96t, s, 54 + 52t, t)\}$

37.

$$\left[\begin{array}{ccc|c} 3 & 3 & 12 & 6 \\ 1 & 1 & 4 & 2 \\ 2 & 5 & 20 & 10 \\ -1 & 2 & 8 & 4 \end{array}\right]$$

$$R_3 - 2R_2 \widetilde{\rightarrow} R_3$$

$$\left[\begin{array}{ccc|c} 3 & 3 & 12 & 6 \\ 1 & 1 & 4 & 2 \\ 0 & 3 & 12 & 6 \\ -1 & 2 & 8 & 4 \end{array}\right]$$

$$R_1 - R_3 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 0 \\ 1 & 1 & 4 & 2 \\ 0 & 3 & 12 & 6 \\ -1 & 2 & 8 & 4 \end{array}\right]$$

$$x = 0$$

$$R_4 - 2R_2 \widetilde{\rightarrow} R_4 \text{ and } R_3 - 3R_2 \widetilde{\rightarrow} R_4$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$z \to t$$

The solution is $S = \{(0, 2 - 4t, t)\}$

43.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Already in reduced row-echelon form

$$x = 0$$

$$z \to t$$

The solution is $S = \{(0, -t, t)\}$

45.

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Already in reduced row-echelon form

$$x_3 = 0$$

$$x_2 \to s$$

$$x_4 \to t$$

The solution is $S = \{(-t, s, 0, t)\}$

49.

Augmented

$$\left[\begin{array}{cc|c} 1 & k & 2 \\ -3 & 4 & 1 \end{array}\right]$$

Two equations, two variables

$$-\frac{1}{3}R_2 \to \left[\begin{array}{cc|c} 1 & -\frac{4}{3} & -\frac{1}{3} \end{array}\right]$$

$$k \neq -\frac{4}{3}$$

Coefficient

$$\left[\begin{array}{ccc|c} 1 & k & 2 & 0 \\ -3 & 4 & 1 & 0 \end{array}\right]$$

Two equations, three variables

k can be any real number

51. (a) A unique solution if $a = b = c = 0$

    (b) No solution if $a = b = 1$ and $c = 0$

    (c) Infinite solutions is not possible in the given scenario

53. (a)

$$\left[\begin{array}{ccc|c} 4 & -2 & 5 & 16 \\ 1 & 1 & 0 & 0 \end{array}\right]$$

$$R_1 + 2R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{ccc|c} 6 & 0 & 5 & 16 \\ 1 & 1 & 0 & 0 \end{array}\right]$$

$$6R_2 - R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 16 \\ 0 & 6 & -5 & -16 \end{array}\right]$$

$$z = t$$

$$y = \frac{5t - 16}{6}$$

$$x = \frac{-5t + 16}{6}$$

The solution is $S = \left\{\left(\frac{-5t + 16}{6}, \frac{5t - 16}{6}, t\right)\right\}$

(b)

$$\left[\begin{array}{ccc|c} 4 & -2 & 5 & 16 \\ -1 & -3 & 2 & 6 \end{array}\right]$$

$$3R_1 - 2R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{ccc|c} 14 & 0 & 11 & 36 \\ -1 & -3 & 2 & 6 \end{array}\right]$$

$$14R_2 + R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 14 & 0 & 11 & 36 \\ 0 & -42 & 39 & 120 \end{array}\right]$$

$$z = t$$

$$y = \frac{39t - 120}{42}$$

$$x = \frac{36 - 11t}{14}$$

The solution is $S = \left\{\left(\frac{36 - 11t}{14}, \frac{39t - 120}{42}, t\right)\right\}$

(c)

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ -1 & -3 & 2 & 6 \end{array}\right]$$

$$R_2 + R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 2 & 6 \end{array}\right]$$

$$\frac{1}{2} R_2 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 3 \end{array}\right]$$

$$y = t$$

$$x = -t$$

$$z = 3 + t$$

$$\text{The solution is } S = \{(-t, t, 3 + t)\}$$

(d) All of them have infinitely many solutions

57. There are four possible combinations: the identity matrix, **I**, the zero matrix, a matrix with $a_{11} = 1$ and $a_{12} = c$, where c is a constant, with the rest equaling zero, and a matrix with $a_{12} = 1$, with the rest equaling zero.

59. (a) This is true, the dimension convention is "row by columns", meaning it is six rows by three columns

 (b) This is true, as is stated in the book

 (c) This is false, as that means that x_1 has a coefficient of one, not that the system is inconsistent

 (d) This is true, as, when there are more variables than equations, there are infinite solutions

63. As long as $ad - bc \neq 0$, it is row equivalent

65. As long as the system is made up of parallel lines, it will have nontrivial solutions. This means that $\lambda = 1, 3$