Vectors in $\mathbb{R}^n$

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- Vector – Quantity described with length and direction
  1. Written bold and lowercase ($\overrightarrow{\mathbf{x}}$)
  2. Vectors are position free
  3. Standard Position – When the initial point is at the origin and the terminal point is somewhere in the plane
- Algebraic work with vectors:
  1. Addition: $\langle u_1, u_2\rangle + \langle v_1, v_2\rangle = \langle u_1 + v_1, u_2 + v_2\rangle$
  2. Subtraction: $\langle u_1, u_2\rangle - \langle v_1, v_2\rangle = \langle u_1 - v_1, u_2 - v_2\rangle$
  3. Scalar Multiplication: $c\langle u_1, u_2\rangle = \langle cu_1, cu_2\rangle$
  4. $\overrightarrow{\mathbf{u}} = \overrightarrow{\mathbf{v}}$ iff $u_1 = v_1$ and $u_2 = v_2$
- Zero Vector – $\overrightarrow{\mathbf{o}} = \langle 0, 0, \dots, 0\rangle$
- Properties of vectors in $\mathbb{R}^2$
  1. $\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}} \in \mathbb{R}$ (two vectors in $\mathbb{R}^2$ remain in $\mathbb{R}^2$)
  2. $\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{v}} + \overrightarrow{\mathbf{u}}$
  3. $(\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}}) + \overrightarrow{\mathbf{w}} = \overrightarrow{\mathbf{u}} + (\overrightarrow{\mathbf{v}} + \overrightarrow{\mathbf{w}})$
  4. $\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{o}} = \overrightarrow{\mathbf{u}}$
  5. $\overrightarrow{\mathbf{u}} + (-\overrightarrow{\mathbf{u}}) = \overrightarrow{\mathbf{o}}$
  6. $c\overrightarrow{\mathbf{u}} \in \mathbb{R}$ (scalar times a vector in $\mathbb{R}^2$ remains a vector in $\mathbb{R}^2$)
  7. $c(\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}}) = c\overrightarrow{\mathbf{u}} + c\overrightarrow{\mathbf{v}}$
  8. $(c + d)\overrightarrow{\mathbf{u}} = c\overrightarrow{\mathbf{u}} + d\overrightarrow{\mathbf{u}}$
  9. $c(d\overrightarrow{\mathbf{u}}) = (cd)\overrightarrow{\mathbf{u}}$
  10. $1(\overrightarrow{\mathbf{u}}) = \overrightarrow{\mathbf{u}}$

- Vectors in $\mathbb{R}^n$

  1. All properties for vectors in $\mathbb{R}^2$ apply to $\mathbb{R}^n$ as well
  2. Addition: $\langle u_1, u_2, \ldots, u_n\rangle + \langle v_1, v_2, \ldots, v_n\rangle = \langle u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n\rangle$
  3. Subtraction: $\langle u_1, u_2, \ldots, u_n\rangle - \langle v_1, v_2, \ldots, v_n\rangle = \langle u_1 - v_1, u_2 - v_2, \ldots, u_n - v_n\rangle$
  4. Scalar Multiplication: $c\langle u_1, u_2, \ldots, u_n\rangle = \langle cu_1, cu_2, \ldots, cu_n\rangle$
  5. Equality: $\overrightarrow{\mathbf{u}} = \overrightarrow{\mathbf{v}}$ iff $u_1 = v_1$, $u_2 = v_2$, $\ldots$, and $u_n = v_n$

- $\overrightarrow{\mathbf{x}}$ is a linear combination of $\overrightarrow{\mathbf{v}}_1, \overrightarrow{\mathbf{v}}_2, \ldots, \overrightarrow{\mathbf{v}}_n$ $\exists$ $c_1, c_2, \ldots, c_n$ such that $\overrightarrow{\mathbf{x}} = c_1\overrightarrow{\mathbf{v}}_1 + c_2\overrightarrow{\mathbf{v}}_2 + \cdots + c_n\overrightarrow{\mathbf{v}}_n$