Linear Algebra 2.4 Participation Problem

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a) Write $\mathbf{A} = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$ as a product of elementary matrices

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$E_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$E_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$
$$E_4 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$
$$E_5 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$
$$E_5E_4E_3E_2E_1 = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$$

b) Write $\mathbf{A}^{-1}$ as a product of elementary matrices

$$\mathbf{A} = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$E_4 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$E_4E_3E_2E_1 = \begin{bmatrix} \frac{2}{3} & -\frac{5}{3} \\ -\frac{1}{3} & \frac{4}{3} \end{bmatrix}$$

$$E_4E_3E_2E_1\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore E_4E_3E_2E_1 = \mathbf{A}^{-1}$$