Linear Algebra 4.6 Homework

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Problems 3, 7, 9, 11*, 17*, 23, 29, 35, 39, 41, 47

3. (a) $(4,3,1)$, $(1,-4,0)$

   (b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

7. $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 \\ 0 & 14 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & .5 \\ 0 & 1 & -.5 \end{bmatrix}$

   (a) $\{(1,0,.5),(0,1,-.5)\}$

   (b) Two nonzero rows, so $\text{rank}(A) = 2$

9. $\begin{bmatrix} 1 & 6 & 18 \\ 7 & 40 & 116 \\ -3 & -12 & -27 \end{bmatrix} \sim \begin{bmatrix} 1 & 6 & 18 \\ 0 & -2 & -10 \\ 0 & 6 & 27 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 5 \\ 0 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (a) $\{(1,0,0),(0,1,0),(0,0,1)\}$

   (b) Three non-zero rows, so $\text{rank}(A) = 3$

11. $\begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6-6 & -4 & \\ -2 & -4 & 4 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & 2.5 \\ 0 & 0 & 0 & 3.5 \\ 0 & 0 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

   (a) $\{(1,2,2,0),(0,0,0,1)\}$

   (b) Two non-zero rows, so $\text{rank}(A) = 2$

17. $\begin{bmatrix} 2 & 9 & -2 & 53 \\ -3 & 2 & 3 & -2 \\ 8 & -3 & -8 & 17 \\ 0 & -3 & 0 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 4.5 & -1 & 26.5 \\ 0 & 15.5 & 0 & 76.5 \\ 0 & -39 & 0 & 195 \\ 0 & -1 & 0 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \{(1,0,-1,0),(0,0,1,0),(0,0,0,1)\}$

23. $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & \frac{5}{4} \end{bmatrix}$

(a) $\left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$

(b) Two non-zero rows, so rank$(A) = 2$

29. $\begin{bmatrix}1 & 2 & 3 & | & 0\end{bmatrix} \Rightarrow x_1 = -2s - 3t, x_2 = s, x_3 = t \Rightarrow \left\{\begin{bmatrix}-2\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\0\\1\end{bmatrix}\right\}$

35. $\begin{bmatrix}5 & 2 & | & 0\\3 & -1 & | & 0\\2 & 1 & | & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & | & 0\\0 & 1 & | & 0\\0 & 0 & | & 0\end{bmatrix} \Rightarrow x_1 = 0, x_2 = 0 \Rightarrow \left\{\begin{bmatrix}0\\0\end{bmatrix}\right\}$

39. $\begin{bmatrix}2 & 6 & 3 & 1 & | & 0\\2 & 1 & 0 & -2 & | & 0\\3 & -2 & 1 & 1 & | & 0\\0 & 6 & 2 & 0 & | & 0\end{bmatrix} \sim \begin{bmatrix}1 & 3 & 1.5 & .5 & | & 0\\0 & -5 & -3 & -3 & | & 0\\0 & -11 & -3.5 & -.5 & | & 0\\0 & 6 & 2 & 0 & | & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & .5 & .5 & | & 0\\0 & 1 & -1 & -3 & | & 0\\0 & 0 & 3 & 5 & | & 0\\0 & 0 & 0 & 0 & | & 0\end{bmatrix} \sim$

$\begin{bmatrix}1 & 0 & 0 & -\frac{1}{3} & | & 0\\0 & 1 & 0 & -\frac{4}{3} & | & 0\\0 & 0 & 1 & \frac{5}{3} & | & 0\\0 & 0 & 0 & 0 & | & 0\end{bmatrix} \Rightarrow x_1 = \frac{1}{3}x_4, x_2 = \frac{4}{3}x_4, x_3 = -\frac{5}{3}x_4, x_4 = 0 \Rightarrow \left\{\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\right\}$

41. (a) Three non-zero rows, so rank(A)=3. 5 columns, so nullity $= 5 - 3 = 2$

(b) $\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = s\begin{bmatrix}-3\\1\\1\\0\\0\end{bmatrix} + t\begin{bmatrix}4\\-2\\0\\2\\1\end{bmatrix} \Rightarrow \left\{\begin{bmatrix}-3\\1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}4\\-2\\0\\2\\1\end{bmatrix}\right\}$

(c) Rows r_1, r_2, and $r_3 \Rightarrow \{(1,0,3,0,-4),(0,1,-1,0,2),(0,0,0,1,-2)\}$

(d) Columns a_1, a_2, and a_4, so: $\left\{\begin{bmatrix}1\\2\\3\\4\end{bmatrix}, \begin{bmatrix}2\\5\\7\\9\end{bmatrix}, \begin{bmatrix}0\\1\\2\\-1\end{bmatrix}\right\}$

(e) According to (b), it is linearly dependent

(f) i. $\begin{vmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{vmatrix} = 1$, so it is linearly independent

ii. $\begin{bmatrix}1 & 0 & 3\\0 & 1 & -1\\0 & 0 & 0\end{bmatrix} \Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = t\begin{bmatrix}-3\\1\\1\end{bmatrix}$, so linearly dependent

iii. $\begin{vmatrix}1 & 0 & -4\\0 & 1 & 2\\0 & 0 & -2\end{vmatrix} \Rightarrow \begin{vmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -2\end{vmatrix} = -2$, so it is linearly independent

47. $\begin{bmatrix} 9 & -4 & -2 & -20 \\ 12 & -6 & -4 & -29 \\ 3 & -2 & 0 & -7 \\ 3 & -2 & -1 & -8 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 0 & -4 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 3 & -2 & -1 & -8 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 0 & -4 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = t \begin{bmatrix} \frac{4}{3} \\ -\frac{3}{2} \\ -1 \\ 1 \end{bmatrix}$

(a) $\left\{\left(\frac{4}{3}, -\frac{3}{2}, -1, 1\right)\right\}$

(b) One solution term, so dim$= 1$