Augmented Matrices and Elementary Row Operations

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- Matrix $-$ An ordered array of numbers

- Examples of a Matrix

  1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2x2 Matrix)

  2. $\begin{bmatrix} 9 & 0 & e \end{bmatrix}$ (1x3 Matrix)

  3. $\begin{bmatrix} 2 & 4 \\ \pi & 6 \\ 3 & 0 \end{bmatrix}$ (3x2 Matrix)

  4. Augmented Matrix: $\left[\begin{array}{cc|c} 1 & -2 & 6 \\ 2 & 3 & -2 \end{array} \right]$

- Elementary Row Operations:

  1. Interchange Rows $(R_i \leftrightarrow R_j)$
  2. Multiply a Row by a non-zero Constant $(kR_i \rightarrow R_i)$
  3. Add a Multiple of one Row to Another $(kR_i + R_j \rightarrow R_j)$
  4. Notation: $\widetilde{R}$ (Row Equivalent)

- Solving (Get $\widetilde{R}$ so that some entries are 0s)

- Example: $\begin{aligned} x - 2y &= 6 \\ 2x + 3y &= -2 \end{aligned}$

$$\left[\begin{array}{cc|c} 1 & -2 & 6 \\ 2 & 3 & -2 \end{array}\right]$$

$$-2\widetilde{R_1} \to R_1$$

$$\left[\begin{array}{cc|c} -2 & 4 & -12 \\ 2 & 3 & -2 \end{array}\right]$$

$$R_1 + \widetilde{R_2} \to R_2 \text{ and } -\frac{1}{2}\widetilde{R_1} \to R_1$$

$$\left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 7 & -14 \end{array}\right]$$

$$\frac{1}{7}\widetilde{R_2} \to R_2$$

$$\left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 1 & -2 \end{array}\right]$$

$$y = -2$$

$$x - 2(-2) = 6 \longrightarrow 2$$

$$S = \{(2, -2)\}$$

- Organization

1. $\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= c_1 \quad (E_1) \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= c_2 \quad (E_2) \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= c_3 \quad (E_3) \end{aligned}$

2. $\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array}\right]$

- Example: $\begin{aligned} x_1 - 2x_2 + x_3 &= -1 \\ 2x_1 - 3x_2 + x_3 &= -4 \\ 3x_1 - 4x_2 + 2x_3 &= -3 \end{aligned}$

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$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & -3 & 1 & -4 \\ 3 & -4 & 2 & -3 \end{array}\right]$$

$$-2R_1 + \widetilde{R_2} \to R_2$$
$$-3R_1 + \widetilde{R_3} \to R_3$$
$$-2R_2 + \widetilde{R_3} \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$x_3 = 4$$
$$x_2 = (4) - 2 \to 2$$
$$x_1 = 2(2) - 4 - 1 \to -1$$
$$S = \{(-1, 2, 4)\}$$

- Gaussian Elimination and Gauss Jordan Elimination

    1. Reduced Row-Echelon Form
        (a) If a row does not consist entirely of 0s, and the first non-zero element in the row is a 1, then it is called a leading one.
        (b) For any two successive nonzero rows, the leading 1 in the lower row is farther to the right than the leading 1 in the higher row.
        (c) All the rows consisting entirely of 0s are at the bottom of the matrix. If the fourth property is also satisfied, a matrix is said to be in reduced row-echelon form:
        (d) Each column that contains a leading 1 has 0s everywhere else (above and below)
        (e) If properties $a-c$ are met, but d is not, then the matrix is in just Row Echelon Form

    2. A Matrix in Reduced Row-Echelon Form: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array}\right]$

- For Gaussian Elimination

    1. Put Augmented Matrix in Row-Echelon Form
    2. Back Substitution

- For Gauss-Jordan Elimination

    1. Put Augmented Matrix in Reduced Row-Echelon Form
    2. Back Substitution