Linear Algebra 2.2 Homework

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1.

$$\begin{aligned}\begin{bmatrix} -5 & 0 \\ 3 & -6 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ -2 & -1 \end{bmatrix} + \begin{bmatrix} -10 & -8 \\ 14 & 6 \end{bmatrix} &= \\ \begin{bmatrix} -5+7-10 & 1-8 \\ 3-2+14 & -6-1+6 \end{bmatrix} &= \\ \begin{bmatrix} -8 & -7 \\ 15 & -1 \end{bmatrix}\end{aligned}$$

3.

$$\begin{aligned}4\left(\begin{bmatrix} -4 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -2 \\ 3 & -6 & 0 \end{bmatrix}\right) &= \\ 4\begin{bmatrix} -6 & -1 & 3 \\ -3 & 8 & 3 \end{bmatrix} &= \\ \begin{bmatrix} -24 & -4 & 12 \\ -12 & 32 & 12 \end{bmatrix}\end{aligned}$$

5.

$$\begin{aligned}-3\left(\begin{bmatrix} 0 & -3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -6 & 3 \\ 8 & 1 \end{bmatrix}\right) - 2\begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix} &= \\ \begin{bmatrix} 18 & 0 \\ -45 & -9 \end{bmatrix} - 2\begin{bmatrix} 4 & -4 \\ 7 & -9 \end{bmatrix} &= \\ \begin{bmatrix} 10 & 8 \\ -59 & 9 \end{bmatrix}\end{aligned}$$

7.

$$\begin{aligned}3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 4\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} &= \\ \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} + \begin{bmatrix} 0 & -4 \\ 4 & -8 \end{bmatrix} &= \\ \begin{bmatrix} 3 & 2 \\ 13 & 4 \end{bmatrix}\end{aligned}$$

9.

$$(-4)(3)\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} =$$
$$\begin{bmatrix} 0 & -12 \\ 12 & -24 \end{bmatrix}$$

11.

$$(3-(-4))\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}\right) =$$
$$7\begin{bmatrix} 1 & 1 \\ 4 & 2 \end{bmatrix} =$$
$$\begin{bmatrix} 7 & 7 \\ 28 & 14 \end{bmatrix}$$

13. (a)

$$\mathbf{X} = \frac{1}{3}\mathbf{B} - \frac{2}{3}\mathbf{A}$$
$$\frac{1}{3}\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} - \frac{2}{3}\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} =$$
$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{1}{3} \\ \frac{4}{3} & \frac{4}{3} \end{bmatrix} + \begin{bmatrix} \frac{8}{3} & 0 \\ -\frac{2}{3} & \frac{10}{3} \\ 2 & -\frac{4}{3} \end{bmatrix} =$$
$$\begin{bmatrix} 3 & \frac{2}{3} \\ -\frac{4}{3} & \frac{11}{3} \\ \frac{10}{3} & 0 \end{bmatrix}$$

(b)

$$\mathbf{X} = -\frac{5}{3}\mathbf{B} + \frac{2}{3}\mathbf{A}$$
$$-\frac{5}{3}\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} =$$
$$\begin{bmatrix} -\frac{5}{3} & -\frac{10}{3} \\ \frac{10}{3} & -\frac{5}{3} \\ -\frac{20}{3} & -\frac{20}{3} \end{bmatrix} + \begin{bmatrix} -\frac{8}{3} & 0 \\ \frac{2}{3} & -\frac{10}{3} \\ -2 & \frac{4}{3} \end{bmatrix} =$$
$$\begin{bmatrix} -\frac{13}{3} & -\frac{10}{3} \\ 4 & -5 \\ -\frac{26}{3} & -\frac{16}{3} \end{bmatrix}$$

(c)

$$\mathbf{X} = -2\mathbf{B} + 3\mathbf{A}$$

$$-2\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} + 3\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} -2 & -4 \\ 4 & -2 \\ -8 & -8 \end{bmatrix} + 3\begin{bmatrix} -12 & 0 \\ 3 & -15 \\ -9 & 6 \end{bmatrix} =$$

$$\begin{bmatrix} -14 & -4 \\ 7 & -17 \\ -17 & -2 \end{bmatrix}$$

(d)

$$\mathbf{X} = \frac{1}{2}\mathbf{B} + \frac{2}{3}\mathbf{A}$$

$$\frac{1}{2}\begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{2} & 1 \\ -1 & \frac{1}{2} \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -\frac{8}{3} & 0 \\ \frac{2}{3} & -\frac{10}{3} \\ -2 & \frac{4}{3} \end{bmatrix} =$$

$$\begin{bmatrix} -\frac{13}{6} & 1 \\ -\frac{1}{3} & -\frac{17}{6} \\ 0 & \frac{10}{3} \end{bmatrix}$$

15.

$$\mathbf{BA} = \begin{bmatrix} 1 & 5 & 0 \\ -1 & 0 & -5 \end{bmatrix}$$

$$c\mathbf{BA} = \begin{bmatrix} -2 & -10 & 0 \\ 2 & 0 & 10 \end{bmatrix}$$

17.

$$\mathbf{CA} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & -3 \end{bmatrix}$$

$$\mathbf{B}(\mathbf{CA}) = \begin{bmatrix} -3 & -5 & -10 \\ -2 & -5 & -5 \end{bmatrix}$$

19.

$$\mathbf{B} + \mathbf{C} = \begin{bmatrix} 1 & 4 \\ -2 & 2 \end{bmatrix}$$

$$(\mathbf{B} + \mathbf{C})\mathbf{A} = \begin{bmatrix} 1 & 6 & -1 \\ -2 & -2 & -8 \end{bmatrix}$$

21.

$$2\mathbf{C} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\mathbf{B}(2\mathbf{C}) = \begin{bmatrix} -6 & 2 \\ -4 & -2 \end{bmatrix}$$

$$-2\mathbf{B}(2\mathbf{C}) = \begin{bmatrix} 12 & -4 \\ 8 & 4 \end{bmatrix}$$

23. (a)

$$\mathbf{AB} = \begin{bmatrix} 4 & 7 \\ 8 & 15 \end{bmatrix}$$

$$(\mathbf{AB})\mathbf{C} = \begin{bmatrix} 12 & 7 \\ 24 & 15 \end{bmatrix}$$

(b)

$$\mathbf{BC} = \begin{bmatrix} 0 & 1 \\ 6 & 3 \end{bmatrix}$$

$$\mathbf{A}(BC) = \begin{bmatrix} 12 & 7 \\ 24 & 15 \end{bmatrix}$$

25.

$$\mathbf{AB} = \begin{bmatrix} -9 & 2 \\ 3 & 6 \end{bmatrix}$$

$$\mathbf{BA} = \begin{bmatrix} -8 & 4 \\ 2 & 5 \end{bmatrix}$$

$$\therefore \mathbf{AB} \neq \mathbf{BA}$$

27.

$$\mathbf{AC} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$\mathbf{BC} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$\therefore \mathbf{AC} = \mathbf{BC}$$

Because **C** has the same rows

29.

$$\mathbf{AB} = \begin{bmatrix} 3-3 & 3-3 \\ 4-4 & 4-4 \end{bmatrix}$$

$$\mathbf{AB} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

31.

$$\mathbf{IA} = \mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

33.

$$\mathbf{A} + \mathbf{I} = \begin{bmatrix} 1+1 & 2+0 \\ 0+0 & -1+1 \end{bmatrix}$$
$$\mathbf{A} + \mathbf{I} = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$
$$\mathbf{A}(\mathbf{A} + \mathbf{I}) = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$

41.

$$\mathbf{A}^\intercal = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}$$
$$\mathbf{B}^\intercal = \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$
$$\mathbf{B}^\intercal\mathbf{A}^\intercal = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$$
$$\mathbf{AB} = \begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$$
$$(\mathbf{AB})^\intercal = \begin{bmatrix} 2 & -5 \\ 4 & -1 \end{bmatrix}$$

45.

$$\mathbf{A}^\intercal = \begin{bmatrix} 4 & 0 \\ 2 & 2 \\ 1 & -1 \end{bmatrix}$$
$$\mathbf{A} \cdot \mathbf{A}^\intercal = \begin{bmatrix} 21 & 3 \\ 3 & 5 \\ 1 & -1 \end{bmatrix}$$

The 3s make the matrix symmetric

$$\mathbf{A}^\intercal\mathbf{A} = \begin{bmatrix} 16 & 8 & 4 \\ 8 & 8 & 0 \\ 4 & 0 & 2 \end{bmatrix}$$

The non-diagonal numbers are symmetric

49.

$$\mathbf{A}^{16} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Because it is a diagonal matrix, each term is raised to the (even) exponent

55. (a) True. Each term is added to its corresponding ij term, meaning that the order does not matter.
    (b) False. Problem (45) is an example of this.
    (c) True. The product of a matrix and its transpose is always symmetric.

56. (a) False. Problem (25) is an example of this.
    (b) False. If $\mathbf{A} = \mathbf{O}$, this is not necessarily true.
    (c) True. In this case, the same terms are added, they are just located in different locations.
    (d)

57. (a)

$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

b must be -1

Then $a = 3$

(b)

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

b must be 1

Then a must be 1

This does not result in the desired matrix, not possible

(c)

$$c \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + a \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 0$$

$$\begin{cases} a + b + c = 0 \\ \phantom{a + {}} b + c = 0 \\ a \phantom{{} + b} + c = 0 \end{cases}$$

There is no solution! $\emptyset$

(d)

The same but negative coefficients from (57a) should be used

$$a = -3,\ b = 1,\ c = 1$$

60.

$$-10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix} +$$

$$5\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 10 & 5 & -5 \\ 5 & 0 & 10 \\ -5 & 5 & 15 \end{bmatrix} +$$

$$-2\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}^2 = \begin{bmatrix} -12 & -2 & 6 \\ 0 & -6 & -10 \\ 8 & -4 & -24 \end{bmatrix} +$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 0 & 2 \\ -1 & 1 & 3 \end{bmatrix}^3 = \begin{bmatrix} 16 & 3 & -13 \\ -2 & 5 & 21 \\ -18 & 8 & 44 \end{bmatrix} =$$

$$\begin{bmatrix} 16 & 7 & -13 \\ 4 & -1 & 23 \\ -16 & 10 & 38 \end{bmatrix}$$

61. (a)

$$\mathbf{A},\ \mathbf{B},\ \mathbf{C} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & a_{3n} \\ a_{m1} & a_{m2} & a_{m3} & a_{mn} \end{bmatrix}$$

(b)

$$\mathbf{B} + \mathbf{C} = b_{ij} + c_{ij}$$
$$\mathbf{A} + \mathbf{B} = a_{ij} + b_{ij}$$

(c)

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = a_{ij} + (b_{ij} + c_{ij})$$
$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = (a_{ij} + b_{ij}) + c_{ij}$$

(d)

$$a_{ij} + (b_{ij} + c_{ij}) = (a_{ij} + b_{ij}) + c_{ij}$$
$$\therefore \mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

64.

$$\begin{aligned} \mathbf{A} &= [a_{ij}] \\ (c+d)a_{ij} &= ca_{ij} + da_{ij} \end{aligned}$$

73. It is symmetric

77. (a) For a sum of matrices, $\mathbf{A} + \mathbf{A}^\intercal = a_{ij} + a_{ji}$

$$\begin{aligned} \mathbf{A} + \mathbf{A}^\intercal &= a_{ij} + a_{ji} \\ c(a_{ij} + a_{ji}) &= ca_{ij} + ca_{ji} \\ \mathbf{B} = \frac{1}{2}(\mathbf{A} + \mathbf{A}^\intercal) &= \frac{1}{2}\begin{bmatrix} 2a_{11} & a_{12} + a21 & \dots & a_{1j} + a_{j1} \\ a_{21} + a_{12} & 2a_{22} & \dots & a_{2j} + a_{j2} \\ \vdots & \vdots & \ddots & a_{3j} + a_{j3} \\ a_{i1} + a_{1i} & a_{i2} + a_{2i} & a_{i3} + a_{3i} & 2a_{ij} \end{bmatrix} \\ \mathbf{B}^\intercal = \frac{1}{2}(\mathbf{A} + \mathbf{A}^\intercal) &= \frac{1}{2}\begin{bmatrix} 2a_{11} & a_{12} + a21 & \dots & a_{1j} + a_{j1} \\ a_{21} + a_{12} & 2a_{22} & \dots & a_{2j} + a_{j2} \\ \vdots & \vdots & \ddots & a_{3j} + a_{j3} \\ a_{i1} + a_{1i} & a_{i2} + a_{2i} & a_{i3} + a_{3i} & 2a_{ij} \end{bmatrix} \\ \text{Because } \mathbf{B} &= \mathbf{B}^\intercal\text{, it is symmetric} \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{A} - \mathbf{A}^\intercal &= a_{ij} - a_{ji} \\ c(a_{ij} - a_{ji}) &= ca_{ij} - ca_{ji} \\ \mathbf{B} = \frac{1}{2}(\mathbf{A} - \mathbf{A}^\intercal) &= \frac{1}{2}\begin{bmatrix} 0 & a_{12} - a21 & \dots & a_{1j} - a_{j1} \\ a_{21} - a_{12} & 0 & \dots & a_{2j} - a_{j2} \\ \vdots & \vdots & \ddots & a_{3j} - a_{j3} \\ a_{i1} - a_{1i} & a_{i2} - a_{2i} & a_{i3} - a_{3i} & 0 \end{bmatrix} \\ \mathbf{B}^\intercal &= \frac{1}{2}\begin{bmatrix} 0 & a_{12} - a21 & \dots & a_{1j} - a_{j1} \\ a_{21} - a_{12} & 0 & \dots & a_{2j} - a_{j2} \\ \vdots & \vdots & \ddots & a_{3j} - a_{j3} \\ a_{i1} - a_{1i} & a_{i2} - a_{2i} & a_{i3} - a_{3i} & 0 \end{bmatrix} \\ \text{Because } \mathbf{B} &= -\mathbf{B}^\intercal\text{, it is \underline{skew} symmetric} \end{aligned}$$