

# Subspaces of Vector Spaces

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- A non-empty subset  $\mathbf{W}$  of vector space  $\mathbf{V}$  is called a subspace of  $\mathbf{V}$  if  $\mathbf{W}$  is itself a vector space under addition and scalar multiplication defined on  $\mathbf{V}$ 
  1. If  $\vec{\mathbf{u}}$  and  $\vec{\mathbf{v}}$  are in  $\mathbf{W}$ , then  $\vec{\mathbf{u}} + \vec{\mathbf{v}}$  is in  $\mathbf{W}$
  2. If  $\vec{\mathbf{u}}$  is in  $\mathbf{W}$ , and  $c$  is any scalar, then  $c\vec{\mathbf{u}}$  is in  $\mathbf{W}$
  3. Ex.  $\mathbb{R}^2$  is a vector space, then  $\{(x, 2x)\}$  is a subspace of  $\mathbb{R}^2$ , but  $\{(x, 2x + 1)\}$  is not
- Verify that  $\mathbf{W}$  is a subspace of  $\mathbf{V}$ 
  1.  $\mathbf{W} \leq \mathbf{V}$
  2.  $\mathbf{W}$  not empty
  3.  $\vec{\mathbf{u}} + \vec{\mathbf{v}} \in \mathbf{W}$
  4.  $c\vec{\mathbf{u}} \in \mathbf{W}$