Linear Algebra 6.1 Homework

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Instructor: Prof. Knight

1, 6, 11, 13, 16, 19, 21, 23, 25, 29, 33, 36, 37, 39, 45, 48, 55, 56, 57, 58, 63, 65, 69

1. (a) $(3+(-4),3-(-4)) = (-1,7)$

   (b) $\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 1 & -1 & 19 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -2 & 16 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 11 \\ 0 & 1 & -8 \end{array}\right] \Rightarrow (11,-8)$

6. (a) $(2(2)+1,2-1) = (5,1)$

   (b) $\left[\begin{array}{cc|c} 2 & 1 & -1 \\ 1 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & -3 \\ 1 & -1 & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & -3 \\ 0 & -3 & 5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{5}{3} \end{array}\right] \Rightarrow \left(\frac{1}{3}, -\frac{5}{3}, c\right)$

11. This is a linear transformation

    (a) $T(\overrightarrow{u}) + T(\overrightarrow{v}) = (u_1+u_2, u_1-u_2, u_3) + (v_1+v_2, v_1-v_2, v_3) = (u_1+v_1+u_2+v_2, u_1+v_1-u_2-v_2, u_3+v_3) = T(\overrightarrow{u}+\overrightarrow{v})$ ✓

    (b) $cT(\overrightarrow{u}) = c(u_1+u_2, u_1-u_2, u_3) = (cu_1+cu_2, cu_1-cu_2, cu_3) = T(c\overrightarrow{u})$ ✓

13. This is not a linear transformation

    (a) The second axiom fails: $cT(\overrightarrow{u}) = (c\sqrt{u_1}, cu_1u_2, c\sqrt{u_2}) \neq (\sqrt{cu_1}, c^2u_1u_2, \sqrt{cu_2}) = T(c\overrightarrow{u})$ ✗

16. This is a linear transformation

    (a) $T(\mathbf{A}) + T(\mathbf{B}) = a+b+c+d+e+f+g+h = T(\mathbf{A}+\mathbf{B})$ ✓

    (b) $kT(\mathbf{A}) = k(a+b+c+d) = ka+kb+kc+kd = T(k\mathbf{A})$

19. This is a linear transformation

    (a) $T(\mathbf{A}) + T(\mathbf{B}) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \mathbf{A} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} (\mathbf{A}+\mathbf{B}) = T(\mathbf{A}+\mathbf{B})$ ✓

(b) $cT(\mathbf{A}) = \begin{bmatrix} 0 & 0 & c \\ 0 & c & 0 \\ c & 0 & 0 \end{bmatrix} \mathbf{A} = T(c\mathbf{A})$ ✓

21. This is a linear transformation

    (a) $T(a_0+a_1x+a_2x^2)+T(b_0+b_1x+b_2x^2) = (a_0+a_1+a_2)+(a_1+a_2)x+a_2x^2+(b_0+b_1+b_2)+(b_1+b_2)x+b_2x^2 = (a_0+b_0+a_1+b_1+a_2+b_2)+(a_1+b_1+a_2+b_2)x+(a_2+b_2)x^2 = T((a_0+b_0)+(a_1+b_1)x+(a_2+b_2)x^2)$ ✓

    (b) $cT(a_0+a_1x+a_2x^2) = c((a_0+a_1+a_2)+(a_1+a_2)x+a_2x^2) = (ca_0+ca_1+ca_2)+(ca_1+ca_2)x+ca_2x^2 = T(ca_0+ca_1x+ca_2x^2)$ ✓

23. (a) $T(1,4) = T(1,0)+4T(0,1) = (1,1)+4(-1,1) = (-3,5)$

    (b) $T(-2,1) = -2T(1,0)+T(0,1) = -2(1,1)+(-1,1) = (-3,-1)$

25. $T(1,-3,0) = T(1,0,0)-3T(0,1,0) = (2,4,-1)-3(1,3,-2) = (-1,-5,5)$

29. $T(4,2,0) = 4T(1,0,1)-2T(0,-1,2) = 4(1,1,0)-2(-3,2,-1) = (10,0,2)$

33. $\mathbf{A}$ is 2×2, so $m=n=2$ ($T:\mathbb{R}^2\to\mathbb{R}^2$)

36. $\mathbf{A}$ is 4×4, so $m=n=4$ ($T:\mathbb{R}^4\to\mathbb{R}^4$)

37. $\mathbf{A}$ is 2×5, so $n=5$, and $m=2$ ($T:\mathbb{R}^5\to\mathbb{R}^2$)

39. (a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}(1,1) = (-1,-1)$

    (b) $\left[\begin{array}{cc|c} 0 & -1 & 1 \\ -1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} -1 & 0 & 1 \\ 0 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -1 \end{array}\right] \Rightarrow (-1,-1)$

    (c) The preimage of $(0,0)$ is $(0,0)$

45. (a) $(4\cos(45)-4\sin(45), 4\sin(45)+4\cos(45)) = (0, 4\sqrt{2})$

    (b) $(4\cos(30)-4\sin(30), 4\sin(30)+4\cos(30)) = (2\sqrt{3}-2, 2\sqrt{3}+2)$

    (c) $(5\cos(120), 5\sin(120)) = \left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

48. $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}(12,5) = (13,0) \Rightarrow \begin{cases} 12a-5b=13 \\ 12b+5a=0 \end{cases} \Rightarrow \left[\begin{array}{cc|c} 12 & -5 & 13 \\ 5 & 12 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & -29 & 13 \\ 1 & \frac{12}{5} & 0 \end{array}\right] \sim$
$\left[\begin{array}{cc|c} 1 & -\frac{29}{2} & \frac{13}{2} \\ 0 & \frac{169}{10} & -\frac{13}{2} \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -\frac{29}{2} & \frac{13}{2} \\ 0 & 1 & -\frac{5}{13} \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & \frac{12}{13} \\ 0 & 1 & -\frac{5}{13} \end{array}\right] \Rightarrow a = \frac{12}{13}, b = -\frac{5}{13}$

55. $2T(1)-6T(x)+T(x^2) = x^2-3x-5$

56. $T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) + 3T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) - T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) + 4T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 12 & -1 \\ 7 & 4 \end{bmatrix}$

57. The statement is true. The differential operator is a linear transformation, so this holds true.

58. The statement is true. As with (57), the differential operator is a linear transformation, so it can be broken up as shown.

63. $D_x(f) = \sin(x)$, so the preimage is $\int \sin(x)\,dx \Rightarrow F(x) = -\cos(x) + c$

65. (a) $\int_0^1 -2 + 3x^2\,dx = (-2x + x^3)\Big|_0^1 = -1$

    (b) $\int_0^1 x^3 - x^5\,dx = \left(\frac{x^4}{4} - \frac{x^6}{6}\right)\Big|_0^1 = \frac{1}{12}$

    (c) $\int_0^1 -6 + 4x\,dx = (-6x + 2x^2)\Big|_0^1 = -4$

69. (a) $T(x, y) = xT(1, 0) + yT(0, 1) = (x, 0)$

    (b) T projects onto the x-axis