Linear Transformations

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- $T : V \to W$
- Ex. $T(\overrightarrow{v}) = (x + y, y + z)$, $\overrightarrow{v} = (1, 2, 3)$. In this case, $T(\overrightarrow{v} = (1 + 2, 2 + 3) \to (3, 5)$
- $T(\overrightarrow{u} + \overrightarrow{v}) = T(\overrightarrow{u}) + T(\overrightarrow{v})$
- $T : V \to W$ is called a linear transformation if:
  1. $T(\overrightarrow{u} + \overrightarrow{v}) = T(\overrightarrow{u}) + T(\overrightarrow{v})\ \forall \overrightarrow{u}, \overrightarrow{v} \in V$
  2. $T(c\overrightarrow{u} = cT(\overrightarrow{u})\ \forall c$
- Matrix Transformation — $T(\overrightarrow{v}) = A\overrightarrow{v}$, where A is an $m \times n$ matrix, and $\overrightarrow{v} = n \times 1$ and $T : \mathbb{R}^n \to \mathbb{R}^m$
- Linear Operator: $T : \overrightarrow{v} \to \overrightarrow{v}$
- Differential Operator: $C'[a, b]$ = set of all functions whose derivatives are continuous on $[a, b]$. $D_x \cdot C'[a, b] \to C[a, b]$. $D_x(f) = \frac{d}{dx}(f),\ f \in C'[a, b]$
- Properties of Linear Transformations:
  1. $T(\overrightarrow{0}) = \overrightarrow{0}$
  2. $T(-\overrightarrow{u}) = -T(\overrightarrow{u})$
  3. $T(\overrightarrow{u} - \overrightarrow{v}) = T(\overrightarrow{u}) - T(\overrightarrow{v})$
  4. If $\overrightarrow{v} = c_1\overrightarrow{v}_1 + c_2\overrightarrow{v}_2 + \cdots + c_n\overrightarrow{v}_n$, then $T(\overrightarrow{v}) = T(c_1\overrightarrow{v}_1 + c_2\overrightarrow{v}_2 + \cdots + c_n\overrightarrow{v}_n) = c_1T(\overrightarrow{v}_1) + c_2T(\overrightarrow{v}_2) + \cdots + c_nT(\overrightarrow{v}_n)$