Linear Algebra 6.2 Homework

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1-27 odd, 31, 33, 39, 41, 44, 45, 47, 54, 55, 57

1. $\ker(T) = \mathbb{R}^3$ because T is the zero transformation

3. $\ker(T)$ is $(0, 0, 0, 0)$, because $T(v) = 0$ only when all terms are 0

5. $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_1 + a_2 \to a_1 + a_2 = 0,$, so $a_1 = -a_2$. Therefore, $\ker(T) = \{a_0 + a_1x - a_1x^2 + a_3,\ a_0, a_1, a_3 \text{ are real}\}$

7. $a_1 + 2a_2x = 0 \to T(a_0 + 0) = T(a_0)$, so $\ker(T) = \{a_0,\ a_0 \text{ is real}\}$

9. $(x + 2y, y - x) = (0, 0) \to \left[\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \to \ker(T) = (0, 0)$

11. (a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \to \begin{cases} x_1 + 2x_2 = 0 \\ 3x_1 + 4x_2 = 0 \end{cases} \to \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 3 & 4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$ $\to \ker(T) = (0, 0)$

    (b) $(0, 0)$ is in $\mathbb{R}^2$

13. (a) $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 4 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} \Rightarrow \operatorname{span}\{(-4, -2, 1)\}$

    (b) The column space of A spans $\mathbb{R}^2$, so the range is $\mathbb{R}^2$

15. (a) $\left[\begin{array}{cc|c} 1 & 3 & 0 \\ -1 & -3 & 0 \\ 2 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \Rightarrow \ker(T) = \{(0, 0)\}$

    (b) The column space simplifies to $\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}$, so $\operatorname{range}(T) = \operatorname{span}\{(1, -1, 0), (0, 0, 1)\}$

17. (a) $\left[\begin{array}{cccc|c} 1 & 2 & -1 & 4 & 0 \\ 3 & 1 & 2 & -1 & 0 \\ -4 & -3 & -1 & -3 & 0 \\ -1 & -2 & 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 4 & 0 \\ 0 & -5 & 5 & -13 & 0 \\ 0 & 5 & -5 & 13 & 0 \\ 0 & 0 & 0 & 5 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & -\frac{6}{5} & 0 \\ 0 & 1 & -1 & \frac{13}{5} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \sim$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \ker(T) = \text{span}\{(-1,1,1,0)\}$

(b) $\text{range}(T) = \text{span}\{(1,0,-1,0),(0,1,-1,0),(0,0,0,1)\}$

19. (a) $\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right] \Rightarrow \ker(T) = \{(0,0)\}$

(b) $\text{nullity}(T) = \dim(\ker(T)) = 0$

(c) The column space spans $\mathbb{R}^2$

(d) $\text{rank}(T) = n - \text{nullity}(T) = n = 2$

21. (a) $\left[\begin{array}{cc|c} 5 & -3 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -7 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \ker(T) = \{(0,0)\}$

(b) $\text{nullity}(T) = \dim(\ker(T)) = 0$

(c) Column space of $A \to \text{span}\{(5,1,1),(-3,1,-1)\}$

(d) $2 - 0 = 2$

23. (a) $\left[\begin{array}{cc|c} .9 & .3 & 0 \\ .3 & .1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & \frac{1}{3} & 0 \\ 1 & \frac{1}{3} & 0 \end{array}\right] \Rightarrow x_2 = -3x_1 \Rightarrow \ker(T) = \{(x_1, -3x_1)\}$

(b) $\text{nullity}(T) = \dim(\ker(T)) = 1$

(c) Column space of $A \to \text{span}\{(3x_2, x_2)\}$

(d) $2 - 1 = 1$

25. (a) $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \ker(T) = \{(-x_3, 0, x_3)\}$

(b) $\text{nullity}(T) = \dim(\ker(T)) = 1$

(c) Column space of $A \to \text{span}\{(s,t,s)\}$

(d) $3 - 1 = 2$

27. (a) $\left[\begin{array}{ccc|c} 4 & -4 & 2 & 0 \\ -4 & 4 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \ker(T) = \text{span}\{(s,t,2t-2s)\}$

(b) $\text{nullity}(T) = \dim(\ker(T)) = 2$

(c) Column space of $A \to \text{span}\{(2t,-2t,t)\}$

(d) $3-2=1$

31. (a) $\left[\begin{array}{ccccc|c} 2 & 2 & -3 & 1 & 13 & 0 \\ 1 & 1 & 1 & 1 & -1 & 0 \\ 3 & 3 & -5 & 0 & 14 & 0 \\ 6 & 6 & -2 & 4 & 16 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & -4 & 0 & 14 & 0 \\ 0 & 0 & 5 & 1 & -15 & 0 \\ 0 & 0 & -8 & -3 & 17 & 0 \\ 0 & 0 & -8 & -2 & 22 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & -4 & 0 & 14 & 0 \\ 0 & 0 & 5 & 1 & -15 & 0 \\ 0 & 0 & -8 & -3 & 17 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 \end{array}\right] \sim$

$$\left[\begin{array}{ccccc|c} 1 & 1 & -4 & 0 & 14 & 0 \\ 0 & 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & -4 & 0 \\ 0 & 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 2 \\ 0 \\ 4 \\ -5 \\ 1 \end{bmatrix}$$

$\ker(T)) = \text{span}\,\{(2t-s, s, 4t, -5t, t)\}$

(b) $\text{nullity}(T) = \dim(\ker(T)) = 2$

(c) Column space of $A \to \text{span}\,\{(2,1,3,6),(1,1,0,4)\}$

(d) $5-2=3$

33. $\text{nullity}(T) = 3 - \text{rank}(T) = 1$. The dimension of the kernel is one, so it must be a line. The dimension of the range is two, so it must be a plane.

39. $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 0 \\ 2 & 4 & 4 & 0 \\ 2 & 4 & 4 & 0 \end{array}\right] \Rightarrow \ker(T) = \text{span}\,\{(-2s-2t, s, t)\}$, so the nullity and dimension of the kernel is two. The kernel must be a plane, and the range must be a line.

41. $4-2=2$

44. $4-2=2$

45. $8-4=4$

47. $|A| = -4$. This means it has trivial solution only, which signifies that $\ker(T) = \{\mathbf{0}\}$. This means it is a one-to-one transformation. The dimension of the kernel is therefore 0, which means the rank is of dimension 2. This equals the dimension of R^2, which means it is onto.

54. A has trivial solution only, so it is one-to-one.

55.

Vector Space	Zero Vector	Standard Basis
$\mathbb{R}^4$	(0,0,0,0)	$\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$
$M_{4,1}$	$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$
$M_{2,2}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
P_3	$0 + 0x + 0x^2 + 0x^3$	$\{1, x, x^2, x^3\}$
V	$(0,0,0,0,0)$	$\{(1,0,0,0,0),(0,1,0,0,0),(0,0,1,0,0),(0,0,0,1,0)\}$

57. $p' = 0 \Rightarrow \int p' \, dx = 0 \Rightarrow p(x) = c$, so any constant number