

Linear Algebra 6.3 Homework

Michael Brodskiy

Instructor: Prof. Knight

3, 5, 9, 15, 21, 37, 39, 42-49 odd, 51a

$$3. \begin{cases} x + y \\ x - y \\ z - x \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$5. \begin{cases} 3x - 2z \\ 2y - z \end{cases} \Rightarrow \begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & -1 \end{bmatrix}$$

$$9. \begin{bmatrix} 1 & -3 \\ 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -14 \\ 0 \\ 4 \end{bmatrix} \Rightarrow (-14, 0, 4)$$

$$15. (a) \frac{1}{2} \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix}$$

$$(b) \frac{1}{2} \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2\sqrt{2} \end{bmatrix}$$

(c) Draw out a graph

$$21. (a) \mathbf{w} \cdot \mathbf{w} = 10 \Rightarrow \frac{\mathbf{w}}{10} = \left(\frac{3}{10}, \frac{1}{10}\right) \Rightarrow \begin{bmatrix} .9 & .3 \\ .3 & .1 \end{bmatrix}$$

$$(b) \begin{bmatrix} .9 & .3 \\ .3 & .1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{21}{10} \\ \frac{7}{10} \end{bmatrix}$$

(c) Draw out a graph

$$37. (a) \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 4 \end{bmatrix} = (9, 5, 4)$$

$$(b) T(1, -1) = (0, 1, -1) \text{ and } T(0, 1) = (1, 0, 1) \Rightarrow \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 9 \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 4 \end{bmatrix}$$

$$39. \quad (a) \quad \begin{bmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ -3 \\ 3 \end{bmatrix}$$

$$(b) \quad T(2, 0, 1) = (4, 2, 1, 3) \text{ and } T(0, 2, 1) = (0, 2, 3, 1) \text{ and } T(1, 2, 1) = (2, 3, 3, 2) \Rightarrow$$

$$\begin{bmatrix} 4 & 0 & 2 \\ 2 & 2 & 3 \\ 1 & 3 & 3 \\ 3 & 1 & 2 \end{bmatrix} \Rightarrow \left[\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 0 & 2 & 2 & -5 \\ 1 & 1 & 1 & 2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 2 & 0 & 1 & 1 \\ 0 & 1 & 1 & -\frac{5}{2} \\ 1 & 0 & 0 & \frac{9}{2} \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{9}{2} \\ 0 & 1 & 0 & -\frac{11}{2} \\ 0 & 0 & 1 & -8 \end{array} \right] \Rightarrow \begin{bmatrix} 4 & 0 & 2 \\ 2 & 2 & 3 \\ 1 & 3 & 3 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} \frac{9}{2} \\ -\frac{11}{2} \\ -8 \end{bmatrix} =$$

$$\begin{bmatrix} 2 \\ -4 \\ -3 \\ 3 \end{bmatrix}$$

$$43. \quad T(1) = x, T(x) = x^2, T(x^2) = x^3 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$45. \quad T(1) = 0, T(x) = 1, T(e^x) = e^x, \text{ and } T(xe^x) = e^x + xe^x \Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$47. \quad \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -3 \\ -3 \end{bmatrix} = 4 - 3e^x - 3xe^x$$

$$49. \quad (a) \quad T(1) = x, T(x) = \frac{1}{2}x^2, T(x^2) = \frac{1}{3}x^3, \text{ and } T(x^3) = \frac{1}{4}x^4 \Rightarrow \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$(b) \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 8 \\ -4 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -2 \\ 0 \\ \frac{3}{4} \end{bmatrix} = 8x - 2x^2 + \frac{3}{4}x^4$$

$$51. \quad T\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\right) =$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, T\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(a) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$