Linear Algebra 1.2 Homework

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1.

$$x = -4$$
$$y = 22$$

3.

$$2x + 1 = 5$$
$$x = \frac{5-1}{2} = 2$$
$$3y - 5 = 4$$
$$y = \frac{4+5}{3} = 9$$

5. (a)

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -2 & -1 \end{bmatrix}$$

(c)

$$2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 0 & 0 \end{bmatrix}$$

(e)

$$\begin{bmatrix} .5 & 1 \\ 1 & .5 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -2.5 & -1 \\ 5 & 2.5 \end{bmatrix}$$

7. (a)

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix}$$

(c)

$$2\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 5 & -2 \\ 1 & -3 & 10 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 1 & .5 & .5 \\ -.5 & -.5 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2.5 & 4.5 \\ -3.5 & .5 & 0 \end{bmatrix}$$

9. (a)

$$\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

(b)

$$\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} - \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

(c)

$$2\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 6 \\ -2 & -8 & 0 \end{bmatrix}$$

(d)

$$2\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} - \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

(e)

$$\frac{1}{2}\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

11. (a) $c_{21} = 2(-3) - 3(0) = -6$

    (b) $c_{13} = 2(4) - 3(-7) = 29$

13. $\begin{cases} 4x = 2y + 8 \\ 4y = 2z + 2x \\ 4z = -2x + 10 \\ -4 = 2 - 2x \end{cases}$

$$\begin{aligned} -x &= -3 \\ x &= 3 \\ 4z &= -2(3) + 10 \\ z &= \frac{4}{4} \\ z &= 1 \\ 4y &= 2\,(1) + 2(3) \\ y &= \frac{8}{4} \\ y &= 2 \end{aligned}$$

15. (a)

$$\begin{bmatrix} 1(2) + 2(-1) & 1(-1) + 2(8) \\ 4(2) + 2(-1) & 4(-1) + 2(8) \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 6 & 12 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2(1) - (4) & 2(2) - (2) \\ -1 + 8(4) & -2 + 8(2) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 31 & 14 \end{bmatrix}$$

17. (a)

$$\begin{bmatrix} 2(0) + 4 + 3(-4) & 2(1) - 1 - 3 & 2(2) - 3 + 3(-2) \\ 5(0) - 4 - 2(-4) & 5 + 1 + 2 & 5(2) + 3 - 2(-2) \\ 2(0) + 2(-4) + 3(-4) & 2 + 2 - 3 & 2(2) + 2(3) + 3(-2) \end{bmatrix} = \begin{bmatrix} -8 & -2 & -5 \\ 4 & 8 & 17 \\ -20 & 1 & 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0(2) + 1(5) + 2(2) & 0(-1) + 1(1) + 2(2) & 0(3) + 1(-2) + 2(3) \\ -4(2) + 5 + 3(2) & 4 + 1 + 3(2) & -4(3) - 2 + 3(3) \\ -4(2) - 5 - 2(2) & 4 - 1 - 2(2) & -4(3) + 2 - 2(3) \end{bmatrix} = \begin{bmatrix} 9 & 5 & 4 \\ 3 & 11 & -5 \\ -17 & -1 & -16 \end{bmatrix}$$

20. (a)

$$\begin{bmatrix} 3+2(2)+1 & 3(2)-2-2 \\ -3+0(2)+4 & -3(2)+0(-1)+4(-2) \\ 4-2(2)-4 & 4(2)+2-4(-2) \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 1 & -14 \\ -4 & 18 \end{bmatrix}$$

(b) Not Possible!

21. (a)

$$\begin{bmatrix} 3(2)+2(3)+1(0) \end{bmatrix} = \begin{bmatrix} 12 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2(3) & 2(2) & 2 \\ 3(3) & 3(2) & 3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 \\ 9 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

28. (a) Not Possible!

(b)

$$\begin{bmatrix} 1+6(6) & 6(13) & 3+6(8) & -2+6(-17) & 4+6(20) \\ 4+2(6) & 2(13) & 4(3)+2(8) & 4(-2)+2(-17) & 4(4)+2(20) \end{bmatrix} =$$

$$\begin{bmatrix} 37 & 78 & 51 & -104 & 124 \\ 16 & 26 & 28 & -42 & 56 \end{bmatrix}$$

29. Defined, 3x4

31. Defined, 4x2

33. Defined, 3x2

35. Undefined, the two can not be added because they have different dimensions

37.

$$\left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 1 & -2 & 2 & 0 \end{array}\right]$$

$$R_1 - R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 1 & -2 & 2 & 0 \end{array}\right]$$

$$R_2 - R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & -3 & 5 & 0 \end{array}\right]$$

$$R_2 + \frac{1}{3} R_1 \widetilde{\rightarrow} R_1 \text{ and } \frac{1}{3} R_2 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & \frac{5}{3} & 0 \end{array}\right]$$

$$z = t$$

$$x = \frac{4}{3} t$$

$$y = -\frac{5}{3} t$$

45. $$\begin{bmatrix} 1 & -5 & 2 \\ -3 & 1 & -1 \\ 0 & -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ 8 \\ -16 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ -3 & 1 & -1 & 8 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$R_2 + 3R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ 0 & -14 & 5 & -52 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$R_2 - R_3 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ 0 & -12 & 0 & -36 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$-\frac{1}{12} R_2 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & 5 & -16 \end{array}\right]$$

$$R_1 + 5R_2 \widetilde{\rightarrow} R_1 \text{ and } R_3 + 2R_2 \widetilde{\rightarrow} R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

$$\frac{1}{5} R_3 \widetilde{\rightarrow} R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$R_1 - 2R_3 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

49. $-\begin{bmatrix} 1 \\ 3 \end{bmatrix} - 4\begin{bmatrix} -1 \\ -3 \end{bmatrix} - 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

51. $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 0\begin{bmatrix} -5 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

53.

$$\mathbf{A} = \left(\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\right)^{-1}$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right]$$

$$R_2 - 3R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array}\right]$$

$$R_1 + 2R_2 \widetilde{\rightarrow} R_1 \text{ and } -R_2 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array}\right]$$

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

55. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{cases} a + 2c = 6 \\ b + 2d = 3 \\ 3a + 4c = 19 \\ 3b + 4d = 2 \end{cases}$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 6 \\ 0 & 1 & 0 & 2 & 3 \\ 3 & 0 & 4 & 0 & 19 \\ 0 & 3 & 0 & 4 & 2 \end{array}\right]$$

$$R_4 - 2R_2 \widetilde{\rightarrow} R_2 \text{ and } R_3 - 2R_1 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -4 \\ 3 & 0 & 4 & 0 & 19 \\ 0 & 3 & 0 & 4 & 2 \end{array}\right]$$

$$R_3 - 3R_1 \widetilde{\rightarrow} R_3 \text{ and } R_4 - 3R_2 \widetilde{\rightarrow} R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 4 & 0 & -2 \\ 0 & 0 & 0 & 4 & 14 \end{array}\right]$$

$$.25R_3 \widetilde{\rightarrow} R_3 \text{ and } .25R_4 \widetilde{\rightarrow} R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{7}{2} \end{array}\right]$$

$$S = \left\{\left(7, -4, -\frac{1}{2}, \frac{7}{2}\right)\right\}$$

57. $\mathbf{A}^2 = \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right)^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

59. For a diagonal matrix, $\mathbf{AB} = \mathbf{BA} = \begin{bmatrix} -10 & 0 \\ 0 & -12 \end{bmatrix}$

61. (a)

$$\begin{aligned} \text{For } \mathbf{AB} \text{ and } \mathbf{BA}\mathbf{A} &= [a_i j] \\ \mathbf{B} &= [b_i j] \end{aligned}$$

(b)

$$\text{For } \mathbf{AB} c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} \text{For } \mathbf{BA} c_{ij} = \sum_{k=1}^{n} b_{ik} a_{kj}$$

(c)

$$\begin{aligned} &\text{For } \mathbf{AB} \\ &\text{When } i \neq j \\ a_{ik} b_{kj} &= 0 \text{ because it is not on the diagonal} \\ &\text{When } i = j \\ a_{ik} b_{kj} &\neq 0 \\ &\text{For } \mathbf{BA} \\ &\text{When } i \neq j \\ b_{ik} a_{kj} &= 0 \text{ because it is not on the diagonal} \\ &\text{When } i = j \\ b_{ik} a_{kj} &\neq 0 \\ \hline \mathbf{AB} = \mathbf{BA} \text{ if } b_{ik} a_{kj} &= a_{ik} b_{ij} \\ \text{Because } i = j,\ k &= i = j \\ \therefore b_{kk} a_{kk} &= a_{kk} b_{kk} \end{aligned}$$

63. $1 - 2 + 3 = 2$

67. (a)

$$\begin{aligned} \text{Tr}(\mathbf{A}) &= \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = a_1 + d_1 \\ \text{Tr}(\mathbf{B}) &= \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = a_2 + d_2 \\ \text{Tr}(\mathbf{A}) + \text{Tr}(\mathbf{B}) &= a_1 + a_2 + d_1 + d_2 \\ \text{Tr}(\mathbf{A} + \mathbf{B}) = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix} &= a_1 + a_2 + d_1 + d_2 \\ \therefore \text{Tr}(\mathbf{A} + \mathbf{B}) &= \text{Tr}(\mathbf{A}) + \text{Tr}(\mathbf{B}) \end{aligned}$$

(b)

$$\mathbf{A} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$
$$\mathrm{Tr}(c\mathbf{A}) = c(x + w)$$
$$c\mathrm{Tr}(\mathbf{A}) = c(x + w)$$
$$\therefore \mathrm{Tr}(c\mathbf{A}) = c\mathrm{Tr}(\mathbf{A})$$

69. Multiplication of a similar matrix always, no matter the order, results in the same product matrix. Therefore, $x = -y$, and $w = z = c$, where c is any real constant

71. $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1}$

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1}$$
$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right]$$
$$R_1 - R_2 \rightarrow R_1$$
$$\left[\begin{array}{cc|cc} 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{array} \right]$$

R_1 became zero, so this matrix is not invertible