Basis and Dimension

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- $B = \left\{ \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n \right\}$ from vector space $\mathbf{V}$ forms basis for $\mathbf{V}$ if
  1. B is linearly independent
  2. $\text{span}(B) = \mathbf{V}$
  3. $\vec{\mathbf{v}}_i$ called basis vectors
  4. Examples:
     (a) $\hat{i}$ and $\hat{j}$ are basis vectors in $\mathbb{R}^2$
     (b) $\hat{i}$, $\hat{j}$, and $\hat{k}$ are basis vectors in $\mathbb{R}^3$
     (c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are basis vectors in $M_{2,2}$

- Show $\{1, x, x^2\}$ basis for P_2
  1. Trivial Solution only (so it is linearly independent)
  2. Spans $P_2 = c_1 + c_2 x + c_3 x^2$

- If B is basis there is only one set of scalars $c_1, c_2, \ldots, c_n$ such that $\vec{\mathbf{w}} = c_1 \vec{\mathbf{v}}_1 + c_2 \vec{\mathbf{v}}_2 + \cdots + c_n \vec{\mathbf{v}}_n$

- Dimension is the number of basis vectors:
  1. $\mathbb{R}^n - n$
  2. $P_n - n + 1$
  3. $M_{m,n}$ − m·n

- If $S = \left\{ \vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \ldots, \vec{\mathbf{v}}_n \right\}$ is basis for vector space $\mathbf{V}$, then every set containing more than n vectors will be linearly dependent