Linear Algebra 4.4 Homework

Michael Brodskiy
Instructor: Prof. Knight

1. (a) $(-1,-2,2)=2(2,-1,3)-(5,0,4)$

3. (a)

$$\left[\begin{array}{ccc|c} 2 & 2 & 2 & -1 \\ 0 & 4 & -12 & 5 \\ 7 & 5 & 13 & -6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & -\frac{1}{2} \\ 0 & 1 & -3 & \frac{5}{4} \\ 0 & -2 & 6 & \frac{5}{2} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 4 & -\frac{7}{4} \\ 0 & 1 & -3 & \frac{5}{4} \\ 0 & 0 & 0 & 5 \end{array}\right]$$

$$S=\left\{-\frac{7}{4}-t,\frac{5}{4}+t,t\right\}$$

$$(-1,5,-6)=-\frac{7}{4}(2,0,7)+\frac{5}{4}(2,4,5)+0(2,-12,13)$$

5. $\begin{bmatrix} 6 & -19 \\ 10 & 7 \end{bmatrix}=3\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}-2\begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix}$

7. $\begin{bmatrix} -2 & 23 \\ 0 & -9 \end{bmatrix}=-\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}+4\begin{bmatrix} 0 & 5 \\ 1 & -2 \end{bmatrix}$

9. $\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}=5$, so it does span $\mathbb{R}^2$

13. It does not span $\mathbb{R}^2$. It is a line.

15. $\begin{vmatrix} -1 & 2 \\ 2 & -4 \end{vmatrix}=0$, so it does not span $\mathbb{R}^2$

19.

$$\begin{vmatrix} 4 & -1 & 2 \\ 7 & 2 & -3 \\ 3 & 6 & 5 \end{vmatrix}$$
$$\begin{vmatrix} 1 & -7 & -3 \\ 0 & -3 & -10 \\ 3 & 6 & 5 \end{vmatrix}$$
$$\begin{vmatrix} 1 & -7 & -3 \\ 0 & -3 & -10 \\ 0 & 0 & -76 \end{vmatrix} = 228$$
$$\det(\mathbf{A}) \neq 0, \text{ so it spans } \mathbb{R}^3$$

21. It does not span $\mathbb{R}^3$, but S spans a plane

25.

$$\begin{vmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$$
$$\text{It does not span } P_2$$

26.

$$\begin{vmatrix} 0 & 8 & 0 & -4 \\ -2 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 0 \end{vmatrix} = 2(-8-4) = -24$$
$$\text{It does span } P_3$$

31. Linearly independent

37. Linearly dependent

44. Linearly independent

47. $\begin{vmatrix} 7 & 6 & 1 \\ -3 & 2 & -8 \\ 4 & -1 & 5 \end{vmatrix} = 7(2) - 6(-15+32) + 1(-5) = 403$, so it is linearly independent

49. $2\mathbf{A} - \mathbf{B} + \mathbf{C} = 0$, so it is linearly dependent

51. The system only has a trivial solution, so it is linearly independent

55. $(1,1,1) - (1,1,0) + 0(0,1,1) - (0,0,1) = 0 \Rightarrow (1,1,1) = (1,1,0) + (0,0,1) + 0(0,1,1)$

57. (a) $\begin{vmatrix} t & 1 & 1 \\ 1 & t & 1 \\ 1 & 1 & t \end{vmatrix} = t(t^2-1)-(t-1)+(1-t) = t^3-3t+2 \Rightarrow t = 1, -2$, so the set is linearly independent for $t \neq 1, -2$

(b) $\begin{vmatrix} t & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 3t \end{vmatrix} = t(-1)-(3t-1)+1 = -4t+2 \Rightarrow t = \frac{1}{2}$, so the set is linearly independent for $t \neq \frac{1}{2}$

61.

$$S_1 = \begin{bmatrix} 1 & 2 & -1 & \\ & 0 & 1 & 1 \\ 2 & 5 & -1 & \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 1 & 0 & -3 & \\ & 0 & 1 & 1 \\ 0 & 1 & 1 & \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 1 & 0 & -3 & \\ & 0 & 1 & 1 \\ 0 & 0 & 0 & \end{bmatrix}$$

$$S_2 = \begin{bmatrix} -2 & -6 & 0 \\ 1 & 1 & -2 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & -2 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\therefore S_1 = S_2$$

65. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \neq 0$, so it is linearly independent and spans $\mathbb{R}^3$

67. (a) S is linearly independent, so it only has the trivial solution. T is a subset of S.
(b) $T = \{v_1, v_2, \ldots, v_n\}$ and has a solution $c_1v_1 + c_2v_2 + \cdots + c_nv_n = 0$.
(c) v_n is in S, so it is impossible for T to be linearly dependent.

69. If a set contains the zero vector, then it can be said that $0v_1 + 0v_2 + \cdots + 0v_n = \mathbf{0}$, which is not trivial because it contains the zero vector, so any such set must be linearly dependent

73. $c_1(\mathbf{u}+\mathbf{v}) + c_2(\mathbf{u}-\mathbf{v}) = 0 \Rightarrow (c_1+c_2)\mathbf{u} + (c_1-c_2)\mathbf{v} = 0$ Because $\mathbf{u}$ and $\mathbf{v}$ were already determined to be linearly dependent, then so is $S = \{\mathbf{u}+\mathbf{v}, \mathbf{u}-\mathbf{v}\}$

74. $\begin{vmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{vmatrix} = -1(1) - 0(-1) + 1(1) = 0$, so it must be linearly dependent