Length and Magnitude vectors in $\mathbb{R}^n$

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- The length (norm) — Let $\overrightarrow{v} \in \mathbb{R}^n$ such that $\overrightarrow{v} = (v_1, v_2, \ldots, v_n)$, then $||\overrightarrow{v}|| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}$
- A unit vector in the direction of $\overrightarrow{u}$ can be found using $\frac{\overrightarrow{u}}{||\overrightarrow{u}||}$
- Let $\overrightarrow{v} \in \mathbb{R}^n$ and c be a scalar. Then $||c\overrightarrow{v}|| = |c|||\overrightarrow{v}||$
- Distance between vectors — For two vectors $\overrightarrow{u} = (u_1, u_2, \ldots, u_n)$ and $\overrightarrow{v} = (v_1, v_2, \ldots, v_n)$, the distance between the two is given by $\sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + \cdots + (u_n - v_n)^2}$
- Properties:
  1. $d(\overrightarrow{u}, \overrightarrow{v}) \geq 0$
  2. $d(\overrightarrow{u}, \overrightarrow{v}) = 0$ iff $\overrightarrow{u} = \overrightarrow{v}$
  3. $d(\overrightarrow{u}, \overrightarrow{v}) = d(\overrightarrow{v}, \overrightarrow{u})$
- Dot Product — $\overrightarrow{u} \cdot \overrightarrow{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$
- Properties:
  1. $\overrightarrow{u} \cdot \overrightarrow{v} = \overrightarrow{v} \cdot \overrightarrow{u}$
  2. $\overrightarrow{u}(\overrightarrow{v} + \overrightarrow{w}) = \overrightarrow{u}\overrightarrow{v} + \overrightarrow{u}\overrightarrow{w}$
  3. $c(\overrightarrow{u}\overrightarrow{v}) = (c\overrightarrow{u})\overrightarrow{v}$
  4. $\overrightarrow{v} \cdot \overrightarrow{v} = ||\overrightarrow{v}||^2$
  5. $\overrightarrow{v} \cdot \overrightarrow{v} \geq 0$ or $\overrightarrow{v} \cdot \overrightarrow{v} = 0$ iff $\overrightarrow{v} = 0$
- Angle Between Vectors:

$$\cos(\theta) = \frac{\overrightarrow{u} \cdot \overrightarrow{v}}{||\overrightarrow{u}||||\overrightarrow{v}||}, \ \ 0 \leq \theta \leq \pi$$

  1. $\overrightarrow{u} \cdot \overrightarrow{v} > 0 \Rightarrow 0 \leq \theta \leq \pi$
  2. $\overrightarrow{u} \cdot \overrightarrow{v} < 0 \Rightarrow \frac{\pi}{2} \leq \theta \leq \pi$
  3. $\overrightarrow{u} \cdot \overrightarrow{v} = 0 \Rightarrow \theta = \frac{\pi}{2}$ (orthogonal)