

# Linear Algebra 4.5 Homework

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Problems 3, 7, 9, 11\*, 17\*, 23, 29, 35, 39, 41, 47

3. (a)  $(4, 3, 1), (1, -4, 0)$

(b)  $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

7.  $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 \\ 0 & 14 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & .5 \\ 0 & 1 & -.5 \end{bmatrix}$

(a)  $\{(1, 0, .5), (0, 1, -.5)\}$

(b) Two nonzero rows, so  $\text{rank}(A) = 2$

9.  $\begin{bmatrix} 1 & 6 & 18 \\ 7 & 40 & 116 \\ -3 & -12 & -27 \end{bmatrix} \sim \begin{bmatrix} 1 & 6 & 18 \\ 0 & -2 & -10 \\ 0 & 6 & 27 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 5 \\ 0 & 0 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a)  $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(b) Three non-zero rows, so  $\text{rank}(A) = 3$

11.  $\begin{bmatrix} -2 & -4 & 4 & 5 \\ 3 & 6 & -6 & -4 \\ -2 & -4 & 4 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & 2.5 \\ 0 & 0 & 0 & 3.5 \\ 0 & 0 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a)  $\{(1, 2, 2, 0), (0, 0, 0, 1)\}$

(b) Two non-zero rows, so  $\text{rank}(A) = 2$

17.  $\begin{bmatrix} 2 & 9 & -2 & 53 \\ -3 & 2 & 3 & -2 \\ 8 & -3 & -8 & 17 \\ 0 & -3 & 0 & 15 \end{bmatrix} \sim \begin{bmatrix} 1 & 4.5 & -1 & 26.5 \\ 0 & 15.5 & 0 & 76.5 \\ 0 & -39 & 0 & 195 \\ 0 & -1 & 0 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \{(1, 0, -1, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$

23.  $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1.5 \\ 0 & 1 & \frac{5}{4} \end{bmatrix}$

(a)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

(b) Two non-zero rows, so  $\text{rank}(A) = 2$

29.  $\begin{bmatrix} 1 & 2 & 3 & | & 0 \end{bmatrix} \Rightarrow x_1 = -2s - 3t, x_2 = s, x_3 = t \Rightarrow \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

35.  $\begin{bmatrix} 5 & 2 & | & 0 \\ 3 & -1 & | & 0 \\ 2 & 1 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix} \Rightarrow x_1 = 0, x_2 = 0 \Rightarrow \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

39.  $\begin{bmatrix} 2 & 6 & 3 & 1 & | & 0 \\ 2 & 1 & 0 & -2 & | & 0 \\ 3 & -2 & 1 & 1 & | & 0 \\ 0 & 6 & 2 & 0 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1.5 & .5 & | & 0 \\ 0 & -5 & -3 & -3 & | & 0 \\ 0 & -11 & -3.5 & -.5 & | & 0 \\ 0 & 6 & 2 & 0 & | & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & .5 & .5 & | & 0 \\ 0 & 1 & -1 & -3 & | & 0 \\ 0 & 0 & 3 & 5 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix} \sim$   
 $\begin{bmatrix} 1 & 0 & 0 & -\frac{1}{3} & | & 0 \\ 0 & 1 & 0 & -\frac{1}{3} & | & 0 \\ 0 & 0 & 1 & \frac{2}{3} & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix} \Rightarrow x_1 = \frac{1}{3}x_4, x_2 = \frac{4}{3}x_4, x_3 = -\frac{5}{3}x_4, x_4 = 0 \Rightarrow \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

41. (a) Three non-zero rows, so  $\text{rank}(A)=3$ . 5 columns, so nullity =  $5 - 3 = 2$

(b)  $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = s \begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 4 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \Rightarrow \left\{ \begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

(c) Rows  $r_1, r_2$ , and  $r_3 \Rightarrow \{(1, 0, 3, 0, -4), (0, 1, -1, 0, 2), (0, 0, 0, 1, -2)\}$

(d) Columns  $a_1, a_2$ , and  $a_4$ , so:  $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 7 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

(e) According to (b), it is linearly dependent

(f) i.  $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$ , so it is linearly independent

ii.  $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$ , so linearly dependent

iii.  $\begin{vmatrix} 1 & 0 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{vmatrix} = -2$ , so it is linearly independent

$$47. \begin{bmatrix} 9 & -4 & -2 & -20 \\ 12 & -6 & -4 & -29 \\ 3 & -2 & 0 & -7 \\ 3 & -2 & -1 & -8 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 0 & -4 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 3 & -2 & -1 & -8 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 0 & -4 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} =$$

$$t \begin{bmatrix} \frac{4}{3} \\ -\frac{3}{2} \\ -1 \\ 1 \end{bmatrix}$$

(a)  $\{(\frac{4}{3}, -\frac{3}{2}, -1, 1)\}$

(b) One solution term, so dim= 1