Linear Algebra 7.1 Homework

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1-5 odd, 9, 12, 15-27 odd, 41, 47, 60, 78

1. $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

3. $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} = 3 \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$

5. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

9. (a) $\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 11 \\ 10 \end{bmatrix}$. Not an eigenvector.

   (b) $\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 8 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. This is an eigenvector.

   (c) $\begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = -3 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. This is an eigenvector.

   (d) Because 2 eigenvectors were already found, this is not an eigenvector.

12. (a) $\begin{bmatrix} 1 & 0 & 5 \\ 0 & -2 & 4 \\ 1 & -2 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$. Not an eigenvector.

    (b) $\begin{bmatrix} 1 & 0 & 5 \\ 0 & -2 & 4 \\ 1 & -2 & 9 \end{bmatrix} \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix} = 0 \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}$. This is an eigenvector.

(c) $\begin{bmatrix}1 & 0 & 5\\ 0 & -2 & 4\\ 1 & -2 & 9\end{bmatrix}\begin{bmatrix}0\\0\\0\end{bmatrix} = \lambda\begin{bmatrix}0\\0\\0\end{bmatrix}$. This is not an eigenvector. The zero vector can not be an eigenvector.

(d) $\begin{bmatrix}1 & 0 & 5\\ 0 & -2 & 4\\ 1 & -2 & 9\end{bmatrix}\begin{bmatrix}2\sqrt{6}-3\\ -2\sqrt{6}+6\\ 3\end{bmatrix} = \begin{bmatrix}2\sqrt{6}+12\\ 4\sqrt{6}\\ 6\sqrt{6}+3\end{bmatrix}$. This is not an eigenvector.

15. (a) $|\lambda I - A| = 0 \Rightarrow (\lambda-6)(\lambda-1)-6 \Rightarrow \lambda^2 - 7\lambda \Rightarrow \lambda(\lambda-7) = 0$

(b) $\lambda_1 = 0 \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{cc|c} -6 & 3 & 0\\ 2 & -1 & 0\end{array}\right] \Rightarrow (t, 2t)$ and $\lambda_2 = 7 \Rightarrow \mathbf{x}_{\lambda_2} = \left[\begin{array}{cc|c} 1 & 3 & 0\\ 2 & 6 & 0\end{array}\right] \Rightarrow$ $(-3t, t)$

17. (a) $(\lambda-1)^2 - 4 \Rightarrow \lambda^2 - 2\lambda - 3 \Rightarrow (\lambda-3)(\lambda+1) = 0$

(b) $\lambda_1 = 3 \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{cc|c} 2 & 2 & 0\\ 2 & 2 & 0\end{array}\right] \Rightarrow (t, -t)$ and $\lambda_2 = -1 \Rightarrow \mathbf{x}_{\lambda_2} = \left[\begin{array}{cc|c} -2 & 2 & 0\\ 2 & -2 & 0\end{array}\right] \Rightarrow$ (t, t)

19. (a) $(\lambda-1)(\lambda+1) + \frac{3}{4} \Rightarrow \lambda^2 - \frac{1}{4} \Rightarrow \left(\lambda - \frac{1}{2}\right)\left(\lambda + \frac{1}{2}\right) = 0$

(b) $\lambda_1 = \frac{1}{2} \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{cc|c} -.5 & 1.5 & 0\\ -.5 & 1.5 & 0\end{array}\right] \Rightarrow (3t, t)$ and $\lambda_2 = -\frac{1}{2} \Rightarrow \mathbf{x}_{\lambda_2} = \left[\begin{array}{cc|c} -1.5 & 1.5 & 0\\ -.5 & .5 & 0\end{array}\right] \Rightarrow$ (t, t)

21. (a) $(\lambda-2)\left[(\lambda-3)(\lambda-2)-2\right] \Rightarrow (\lambda-2)\left[\lambda^2 - 5\lambda + 4\right] \Rightarrow (\lambda-2)(\lambda-4)(\lambda-1) = 0$

(b) $\lambda_1 = 1 \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{ccc|c} -1 & 2 & -3 & 0\\ 0 & -2 & 2 & 0\\ 0 & 1 & -1 & 0\end{array}\right] \Rightarrow (-t, t, t)$ and $\lambda_2 = 2 \Rightarrow \mathbf{x}_{\lambda_2} =$ $\left[\begin{array}{ccc|c} 0 & 2 & -3 & 0\\ 0 & 1 & 2 & 0\\ 0 & 1 & 0 & 0\end{array}\right] \Rightarrow (t, 0, 0)$ and $\lambda_3 = 4 \Rightarrow \mathbf{x}_{\lambda_3} = \left[\begin{array}{ccc|c} 2 & 2 & -3 & 0\\ 0 & 1 & 2 & 0\\ 0 & 1 & 2 & 0\end{array}\right] \Rightarrow (3.5t, -2t, t)$

23. (a) $(\lambda-1)\left[(\lambda-5)(\lambda+3)+12\right] + 2\left[2(\lambda+3)-12\right] + 2\left[-12+6(\lambda-5)\right] \Rightarrow (\lambda-1)(\lambda-3)(\lambda+1) + 4\lambda - 12 + 12\lambda - 84 \Rightarrow (\lambda-1)(\lambda-3)(\lambda+1) + 16\lambda - 96 \Rightarrow (\lambda^2-1)(\lambda-3) + 16\lambda - 96 \Rightarrow \lambda^3 - 3\lambda^2 - \lambda + 3 + 16\lambda - 96 \Rightarrow \lambda^3 - 3\lambda^2 + 15\lambda - 99 = 0 \Rightarrow (\lambda-3)^2(\lambda+3) = 0$

(b) $\lambda_1 = 3 \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{ccc|c} 2 & -2 & 2 & 0\\ 2 & -2 & 2 & 0\\ 6 & -6 & 6 & 0\end{array}\right] \Rightarrow (s-t, s, t)$ and $\lambda_2 = -3 \Rightarrow \mathbf{x}_{\lambda_2} =$ $\left[\begin{array}{ccc|c} -4 & -2 & 2 & 0\\ 2 & -8 & 2 & 0\\ 6 & -6 & 0 & 0\end{array}\right] \Rightarrow (t, t, 3t)$

25. (a) $(\lambda-4)\left[\lambda(\lambda-4)-12\right] \Rightarrow (\lambda-4)(\lambda-6)(\lambda+2) = 0$

(b) $\lambda_1 = 4 \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{ccc|c} 4 & 3 & -5 & 0 \\ 4 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow (-\frac{5}{2}t, 5t, t)$ and $\lambda_2 = 6 \Rightarrow \mathbf{x}_{\lambda_2} = \left[\begin{array}{ccc|c} 6 & 3 & -5 & 0 \\ 4 & 2 & 10 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow$
$(t, -2t, 0)$ and $\lambda_3 = -2 \Rightarrow \mathbf{x}_{\lambda_3} = \left[\begin{array}{ccc|c} -2 & 3 & -5 & 0 \\ 4 & -6 & 10 & 0 \\ 0 & 0 & -6 & 0 \end{array}\right] \Rightarrow (\frac{3}{2}t, t, 0)$

27. (a) $(\lambda-2)(\lambda-2)\left[\lambda(\lambda-3)-4\right] \Rightarrow (\lambda-2)^2(\lambda-4)(\lambda+1)$

(b) $\lambda_1 = 2 \Rightarrow \mathbf{x}_{\lambda_1} = \left[\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & -4 & 2 & 0 \end{array}\right] \Rightarrow (s, t, 0, 0)$ and $\lambda_2 = 4 \Rightarrow \mathbf{x}_{\lambda_2} =$
$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -4 & 4 & 0 \end{array}\right] \Rightarrow (0, 0, t, t)$ and $\lambda_3 = -1 \Rightarrow \mathbf{x}_{\lambda_3} = \left[\begin{array}{cccc|c} -3 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & -4 & -1 & 0 \\ 0 & 0 & -4 & -1 & 0 \end{array}\right] \Rightarrow$
$(0, 0, t, -4t)$

41. $\lambda_i = 2, 3, 1$

47. (a) $(\lambda+1)\left[\lambda(\lambda-3)+2\right] \Rightarrow (\lambda+1)(\lambda-2)(\lambda-1)$, so $\lambda = -1, 2, 1$

(b) $\lambda = -1 \Rightarrow \left[\begin{array}{ccc|c} -1 & -2 & 1 & 0 \\ 1 & -4 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow B_1 = \{(1,0,1)\}$ and $\lambda = 1 \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \Rightarrow$
$B_2 = \{2, 1, 0\}$ and $\lambda = 2 \Rightarrow \left[\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] \Rightarrow B_3 = \{(1,1,0)\}$

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

60. $\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u}\cdot\mathbf{v}}{\mathbf{v}\cdot\mathbf{v}}\mathbf{v}$

78. $|\lambda I - A| = (\lambda - \cos\theta)(\lambda - \cos\theta) + \sin^2\theta = \lambda^2 - 2\lambda\cos\theta + \cos^2\theta + \sin^2\theta =$
$\lambda^2 - 2\lambda\cos\theta + 1 \Rightarrow \theta = \cos^{-1}\left(\frac{\lambda^2+1}{2\lambda}\right)$