Linear Algebra 4.2 Homework

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1. $(0,0,0,0)$

3. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

5. 0 or $0+0x+0x^2+0x^3$

7. $(a,b,c)+(-a,-b,-c)=(0,0,0)$

9. $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}+\begin{bmatrix} -a & -b & -c \\ -d & -e & -f \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

11. $a+bx+cx^2+dx^3+ex^4+(-a-bx-cx^2-dx^3-ex^4)=0$

13. $M_{4,6}$ meets all axioms, and, therefore, is a vector space

15. P_3 does not meet axiom one, and, therefore, is not a vector space. (For example, if $v_1=1-x^3$ and $v_2=1+x^2+x^3$, then $v_1+v_2=2+x^2$, and is not in P_3)

21. The set $\{(x,y): x \geq 0, y \text{ is a real number}\}$ is not a vector space because it fails axiom six. If $(x,y)=(1,1)$, and $c=-1$, then $c(x,y)$ is not in the vector space

24. The set $\left\{\left(x, \frac{1}{2}x\right): x \text{ is a real number}\right\}$ meets all 10 axioms, and is therefore a vector space

26. The set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}$ fails axiom six, and, therefore, is not a vector space (ex. $k=-1 \Rightarrow k\begin{bmatrix} a & b \\ c & 1 \end{bmatrix}=\begin{bmatrix} -a & -b \\ -c & -1 \end{bmatrix}$, and is no longer in $\mathbf{V}$)

27. The set of all 3×3 matrices of the form $\begin{bmatrix} 0 & a & b \\ c & 0 & d \\ e & f & 0 \end{bmatrix}$ meets all 10 axioms and is, therefore, a vector space

34. The set of all 3×3 upper triangular matrices meet all 10 axioms, and, therefore, are vector spaces

35. $C[0,1]$, the set of all continuous functions defined on the interval $[0,1]$ meet all 10 axioms, and, therefore, are vector spaces

36. $C[-1,1]$, the set of all continuous functions defined on the interval $[-1,1]$ meet all 10 axioms, and, therefore, are vector spaces

37. By the definition given, it is not a vector space, because it fails axioms 4,5,7, and 8.

    (a) $x_1 + y_1 = x_1 y_1$ is in $\mathbf{V}$ ✓
    (b) $x_1 + y_1 = x_1 y_1 = y_1 x_1 = y_1 + x_1$ is true ✓
    (c) $x_1 + (y_1 + z_1) = x_1 + y_1 z_1 = x_1 y_1 z_1 = x_1 y_1 + z_1 = (x_1 + y_1) + z_1$ is true ✓
    (d) $x_1 + 0 = x_1 \cdot 0$ is not true ✗
    (e) $x_1 + (-a) = 0 \Rightarrow -a x_1$ is not true ✗
    (f) $c x_1 = x_1^c$ is in $\mathbf{V}$ ✓
    (g) $c(x_1 + y_1) \neq c x_1 + c y_1$ ✗
        i. $c(x_1 + y_1) = (x_1 + y_1)^c$
        ii. $c x_1 + c y_1 = x_1^c + y_1^c$
    (h) $(c + d) x_1 \neq c x_1 + d x_1$ ✗
        i. $(c + d) x_1 = x_1^{(c+d)}$
        ii. $c x_1 + d x_1 = x_1^c + x_1^d$
    (i) $c(d x_1) = (cd) x_1$ ✓
        i. $c(d x_1) = c x_1^d = x_1^{cd}$
        ii. $(cd) x_1 = x_1^{cd}$
    (j) $1 x_1 = x_1^1$ is true ✓

40. $M_{2,2}$ is a vector space because:

    (a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$ is in $\mathbf{V}$ ✓
    (b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is true ✓
    (c) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \left(\begin{bmatrix} e & f \\ g & h \end{bmatrix} + \begin{bmatrix} i & j \\ k & l \end{bmatrix} \right) = \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right) + \begin{bmatrix} i & j \\ k & l \end{bmatrix}$ is true ✓
    (d) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ exists ✓
    (e) $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -a & -b \\ -c & -d \end{bmatrix} = 0$ exists ✓

(f) $k \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$ is in $\mathbf{V}$ ✓

(g) $k \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right) = k \begin{bmatrix} a & b \\ c & d \end{bmatrix} + k \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ is true ✓

(h) $(k+l) \begin{bmatrix} a & b \\ c & d \end{bmatrix} = k \begin{bmatrix} a & b \\ c & d \end{bmatrix} + l \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is true ✓

(i) $k \left(l \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (kl) \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is true ✓

(j) $1 \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is true ✓

41. (a) It is not a vector space because axiom eight fails. For example, if $c = 5$ and $d = 10$, $(5+10)(x_1, y_1) = (15x_1, y_1)$, while $5(x_1, y_1) + 10(x_1, y_1) = (15x_1, 2y_1)$

42. (d) By this $\mathbb{R}^3$ definition, it is a vector space

i. $(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2 + 1, y_1 + y_2 + 1, z_1 + z_2 + 1)$ is in $\mathbf{V}$ ✓

ii. $\overrightarrow{\mathbf{v}}_1 + \overrightarrow{\mathbf{v}}_2 = \overrightarrow{\mathbf{v}}_2 + \overrightarrow{\mathbf{v}}_1$ is true ✓

iii. $\overrightarrow{\mathbf{v}}_1 + \left(\overrightarrow{\mathbf{v}}_2 + \overrightarrow{\mathbf{v}}_3\right) = (\overrightarrow{\mathbf{v}}_1 + \overrightarrow{\mathbf{v}}_2) + \overrightarrow{\mathbf{v}}_3$ is true ✓

iv. $(x_1, y_1, z_1) + \overrightarrow{\mathbf{o}} = (x_1 + o_1 + 1, y_2 + o_2 + 1, z_1 + o_3 + 1) \Rightarrow \overrightarrow{\mathbf{o}} = (-1, -1, -1)$ ✓

v. $(x_1, y_1, z_1) + (a, b, c) = (-1, -1, -1) \Rightarrow \begin{matrix} a & = & -x_1 & - & 2 \\ b & = & -y_1 & - & 2 \\ c & = & z_1 & - & 2 \end{matrix}$ ✓

vi. $c\,(x_1, y_1, z_1) = (cx + c - 1, cy + x - 1, cz + c - 1)$ is in $\mathbf{V}$ ✓

vii. $c(\overrightarrow{\mathbf{v}}_1 + \overrightarrow{\mathbf{v}}_2) = c\overrightarrow{\mathbf{v}}_1 + c\overrightarrow{\mathbf{v}}_2$ ✓

A. $c((x_1, y_1, z_1) + (x_2, y_2, z_2)) = c\,(x_1 + x_2 + 1, y_1 + y_2 + 1, z_1 + z_2 + 1) \Rightarrow (c(x_1 + x_2 + 1) + c - 1, c(y_1 + y_2 + 1) + c - 1, c(z_1 + z_2 + 1) + c - 1)$

B. $c\,(x_1, y_1, z_1) + c\,(x_2, y_2, z_2) =$
$(cx_1 + c - 1, cy_1 + c - 1, cz_1 + c - 1) + (cx_2 + c - 1, cy_2 + c - 1, cz_2 + c - 1) \Rightarrow$
$(cx_1 + cx_2 + 2c - 2 + 1, cy_1 + cy_2 + 2c - 2 + 1, cz_1 + cz_2 + 2c - 2 + 1) =$
$(cx_1 + cx_2 + 2c - 1, cy_1 + cy_2 + 2c - 1, cz_1 + cz_2 + 2c - 1)$

viii. $(c+d)\overrightarrow{\mathbf{v}}_1 = c\overrightarrow{\mathbf{v}}_1 + d\overrightarrow{\mathbf{v}}_1$ ✓

A. $(c+d)(x_1, y_1, z_1) =$
$((c+d)x_1 + (c+d) - 1, (c+d)y_1 + (c+d) - 1, (c+d)z_1 + (c+d) - 1)$

B. $c(x_1, y_1, z_1) + d(x_1, y_1, z_1) =$
$(cx_1 + c - 1, cy_1 + c - 1, cz_1 + c - 1) + (dx_1 + d - 1, dy_1 + d - 1, dz_1 + d - 1) \Rightarrow$
$((c+d)x_1 + (c+d) - 1, (c+d)y_1 + (c+d) - 1, (c+d)z_1 + (c+d) - 1)$

ix. $c(d\overrightarrow{\mathbf{v}}_1) = (cd)\overrightarrow{\mathbf{v}}_1$ ✓

A. $(cd)\overrightarrow{\mathbf{v}}_1 = (cdx_1 + cd - 1, cdy_1 + cd - 1, cdz_1 + cd - 1)$

B. $c(d\overrightarrow{\mathbf{v}}_1) = c\,(dx_1 + d - 1, dy_1 + d - 1, dz_1 + d - 1) =$
$(c(dx_1 + d - 1) + c - 1, c(dy_1 + d - 1) + c - 1, c(dz_1 + d - 1) + c - 1) \Rightarrow$
$(cdx_1 + cd - 1, cdy_1 + cd - 1, cdz_1 + cd - 1)$

x. $1(x_1, y_1, z_1) = (1x_1 + 1 - 1, 1y_1 + 1 - 1, 1z_1 + 1 - 1) = (x_1, y_1, z_1)$ ✓