Linear Algebra 4.7 Homework

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Problems 3, 9, 11, 13, 17, 19, 21, 23, 35, 37, 43, 45, 51

3. $\begin{bmatrix} 7 \\ -4 \\ -1 \\ 2 \end{bmatrix}$

9. $\begin{bmatrix} -1(1) \\ 3(1) - 1(1) \\ -2(1) + 3(1) - 1(1) \\ 1(1) - 2(1) + 3(1) - 1(1) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

11. $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

13. $\left[\begin{array}{ccc|ccc} 8 & 7 & 1 & 1 & 0 & 0 \\ 11 & 0 & 4 & 0 & 1 & 0 \\ 0 & 10 & 6 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & \frac{7}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 \\ 0 & -\frac{77}{8} & \frac{21}{8} & -\frac{11}{8} & 1 & 0 \\ 0 & 10 & 6 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & \frac{7}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 \\ 0 & 1 & -\frac{21}{77} & \frac{1}{7} & -\frac{8}{77} & 0 \\ 0 & 0 & \frac{96}{11} & -\frac{10}{7} & \frac{80}{77} & 1 \end{array}\right] \sim$

$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{4}{11} & 0 & \frac{1}{11} & 0 \\ 0 & 1 & -\frac{21}{77} & \frac{1}{7} & -\frac{8}{77} & 0 \\ 0 & 0 & 1 & -\frac{55}{336} & \frac{5}{42} & \frac{11}{96} \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{5}{84} & \frac{1}{21} & -\frac{1}{24} \\ 0 & 1 & 0 & \frac{11}{112} & -\frac{1}{14} & \frac{1}{32} \\ 0 & 0 & 1 & -\frac{55}{336} & \frac{5}{42} & \frac{11}{96} \end{array}\right] \Rightarrow \begin{bmatrix} \frac{5}{84} & \frac{1}{21} & -\frac{1}{24} \\ \frac{11}{112} & -\frac{1}{14} & \frac{1}{32} \\ -\frac{55}{336} & \frac{5}{42} & \frac{11}{96} \end{bmatrix} \begin{bmatrix} 3 \\ 19 \\ 2 \end{bmatrix} =$

$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

17. $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 3 & -1 \\ 0 & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1.5 & -.5 \\ 0 & 1 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.5 & -.5 \\ -2 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 4 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

21. $\begin{bmatrix} 0 & 1 & 5 & -1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 2 & 0 & 2 & 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 0 & 0 & 5 & -1 & -\frac{1}{4} & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 2 & 0 & 2 & 0 & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & \frac{1}{5} & \frac{1}{20} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & -\frac{1}{5} & -\frac{1}{20} & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{5} & \frac{1}{20} & -\frac{1}{2} \\ 0 & \frac{1}{4} & 0 \\ -\frac{1}{5} & -\frac{1}{20} & 0 \end{bmatrix}$

23. $\begin{bmatrix} 1 & 0 & 0 & 3 & -2 & 1 \\ 0 & 1 & 0 & 4 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 & -2 & 1 \\ 4 & -1 & 0 \\ 0 & 1 & -3 \end{bmatrix}$

35. $P = \begin{bmatrix} 1 & -\frac{3}{11} & \frac{5}{11} & 0 & -\frac{7}{11} \\ 0 & -\frac{2}{11} & \frac{3}{22} & 0 & -\frac{1}{11} \\ -\frac{5}{4} & \frac{9}{22} & -\frac{19}{44} & -\frac{1}{4} & \frac{21}{22} \\ -\frac{3}{4} & \frac{1}{2} & -\frac{1}{4} & \frac{1}{4} & \frac{1}{2} \\ 0 & -\frac{1}{11} & -\frac{2}{11} & 0 & \frac{5}{11} \end{bmatrix}$

37. (a) $\begin{bmatrix} -12 & -4 & 1 & -2 \\ 0 & 4 & 3 & -2 \end{bmatrix} \sim \begin{bmatrix} -12 & 0 & 4 & -4 \\ 0 & 1 & \frac{3}{4} & -\frac{1}{2} \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{3}{4} & -\frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{3}{4} & -\frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 & -12 & -4 \\ 3 & -2 & 0 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -12 & -4 \\ 0 & 4 & 36 & 16 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 6 & 4 \\ 0 & 1 & 9 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 4 \\ 9 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{3}{4} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 6 & 4 \\ 9 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

(d) $\begin{bmatrix} 6 & 4 \\ 9 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

43. (a) $\begin{bmatrix} \frac{19}{39} & -\frac{9}{13} & \frac{44}{39} \\ -\frac{3}{13} & -\frac{6}{13} & -\frac{9}{13} \\ -\frac{23}{39} & \frac{2}{13} & -\frac{4}{39} \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{2}{7} & -\frac{4}{21} & -\frac{13}{7} \\ -\frac{5}{7} & -\frac{8}{7} & -\frac{1}{7} \\ \frac{4}{7} & -\frac{13}{21} & \frac{5}{7} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{19}{39} & -\frac{9}{13} & \frac{44}{39} \\ -\frac{3}{13} & -\frac{6}{13} & -\frac{9}{13} \\ -\frac{23}{39} & \frac{2}{13} & -\frac{4}{39} \end{bmatrix} \begin{bmatrix} -\frac{2}{7} & -\frac{4}{21} & -\frac{13}{7} \\ -\frac{5}{7} & -\frac{8}{7} & -\frac{1}{7} \\ \frac{4}{7} & -\frac{13}{21} & \frac{5}{7} \end{bmatrix} = I$ ✓

(d) $\begin{bmatrix} \frac{22}{7} \\ \frac{6}{7} \\ \frac{19}{7} \end{bmatrix}$

45. $\begin{bmatrix} 1 \\ 5 \\ -2 \\ 1 \end{bmatrix}$

51. $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$