Matrices for Linear Transformations

Michael Brodskiy
Professor: Lynn Knight

April 28, 2021

- Standard Matrix for linear transformations:

  1. ex. $T : \mathbb{R}^2 \to \mathbb{R}^3$. $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ 3x_1 + 4x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$, then $T(\overrightarrow{x}) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A\overrightarrow{x}$, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$
  2. Standard Matrix for $T : \mathbb{R}^n \to \mathbb{R}^m$, where T is a linear transformation and $B = \{\overrightarrow{e}_1, \overrightarrow{e}_2, \dots, \overrightarrow{e}_n\}$ is a standard basis for $\mathbb{R}^n$, then $A = \left[T(\overrightarrow{e}_1) \,\middle|\, T(\overrightarrow{e}_2) \,\middle|\, \dots \,\middle|\, T(\overrightarrow{e}_n)\right]$ is a standard matrix for T such that $T(\overrightarrow{x}) = A\overrightarrow{x} \;\; \forall \overrightarrow{x} \in \mathbb{R}^n$
  3. Note: Every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation

- Let V and W be vector spaces with bases B and B', where $B = \{\overrightarrow{v}_1, \overrightarrow{v}_2, \dots, \overrightarrow{v}_n\}$. If $T : V \to W$ is a linear transformation such that:
  $[T(\overrightarrow{v}_1)]_{B'} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{31} \end{bmatrix}, [T(\overrightarrow{v}_2)]_{B'} = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{32} \end{bmatrix}, \dots, [T(\overrightarrow{v}_B)]_{B'} = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$. Then, the $m \times n$ matrix whose n columns correspond to $[T(\overrightarrow{v}_i)]_{B'}$ is such that $[T(\overrightarrow{v})]_{B'} = A[\overrightarrow{v}]_B$