Linear Algebra 3.3 Homework

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1. (a) $|\mathbf{A}| = -2(-2) - 1(4) = 0$
   (b) $|\mathbf{B}| = -1(1) - 0(1) = -1$
   (c) $\mathbf{AB} = \begin{bmatrix} -2 & -3 \\ 4 & 6 \end{bmatrix}$
   (d) $|\mathbf{AB}| = -2(6) - 4(3) = 0$

   $|\mathbf{A}||\mathbf{B}| = |\mathbf{AB}|$ ✓

3. (a) $|\mathbf{A}| = -1(-1) - 2(0) + 1(1) = 2$
   (b) $|\mathbf{B}| = -1(2)(3) = -6$
   (c) $\mathbf{AB} = \begin{bmatrix} 1 & 4 & 3 \\ -1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}$
   (d) $|\mathbf{AB}| = -12$

   $|\mathbf{A}||\mathbf{B}| = |\mathbf{AB}|$ ✓

5. (a) $|\mathbf{A}| = 3$
   (b) $|\mathbf{B}| = 6$
   (c) $\mathbf{AB} = \begin{bmatrix} 6 & 3 & -2 & 2 \\ 2 & 1 & 0 & -1 \\ 9 & 4 & -3 & 8 \\ 8 & 5 & -4 & 5 \end{bmatrix}$
   (d) $|\mathbf{AB}| = 18$

   $|\mathbf{A}||\mathbf{B}| = |\mathbf{AB}|$ ✓

7. $5^2\left(1(-4) - 2(3)\right) = -250$

9. $3^3\left(-1(15 - 16) - 2(10 - 12) + 3(8 - 9)\right) = 54$

11. $2^3\left((15-16)+2(-10+12)+3(8-9)\right)=0$

13. $5^4\left(\begin{vmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{vmatrix}\right) = 5^4\left(\begin{vmatrix} -5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & -4 & 0 & 1 \end{vmatrix}\right) = -3125$

19. $5(8)-4(10)=0$. The matrix is singular.

23. $1(8)(0)(2)=0$. The matrix is singular.

25. (a) $\mathbf{A}^{-1}=\frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}=\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix} \Rightarrow \frac{8}{25}-\frac{3}{25}=\frac{1}{5}$

(b) $\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}=4(2)-3(1)=5 \Rightarrow \frac{1}{5}$

$$|\mathbf{A}^{-1}|=\tfrac{1}{|\mathbf{A}|}\ \checkmark$$

29. (a)

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 1 & 0 & 0 & 0 \\ 1 & 0 & 3 & -2 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & -3 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & -5 & -1 & 1 & 0 & 0 \\ 0 & 0 & 4 & -7 & -2 & 0 & 1 & 0 \\ 0 & -3 & 2 & -1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & -1 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{7}{4} & -\frac{1}{2} & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & \frac{5}{4} & \frac{1}{2} & 0 & \frac{1}{4} & 0 \\ 0 & 1 & 0 & -\frac{5}{6} & 0 & 0 & \frac{1}{6} & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{3}{8} & \frac{7}{8} & -\frac{5}{8} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{1}{8} & -\frac{5}{8} & \frac{7}{8} & 0 \\ 0 & 1 & 0 & 0 & \frac{5}{12} & \frac{5}{12} & -\frac{1}{4} & -\frac{1}{3} \\ 0 & 0 & 1 & 0 & \frac{3}{8} & \frac{7}{8} & -\frac{5}{8} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right]$$

$$\det\left(\mathbf{A}^{-1}\right)=-\frac{1}{3}\left[-\frac{1}{8}\left(-\frac{7}{16}+\frac{5}{16}\right)+\frac{5}{8}\left(-\frac{3}{16}+\frac{5}{16}\right)+\frac{7}{8}\left(\frac{3}{16}-\frac{7}{16}\right)\right]=\frac{1}{24}$$

(b) $|\mathbf{A}|=-3\left[2(2-9)-(1-6)+(-3+4)\right]=24 \Rightarrow \frac{1}{24}$

$$|\mathbf{A}^{-1}|=\tfrac{1}{|\mathbf{A}|}\ \checkmark$$

31.

33.

37.

41.

47.

65.

75.

80.