Linear Algebra 5.3 Homework

Michael Brodskiy

Instructor: Prof. Knight

5, 7, 11, 15, 17, 19, 43, 48, 56, 61, 64

5. (a) $(4)(-1)+(1)(4)=0$, $(4)(-4)+(-17)(-1)+(1)(-1)=0$, $(-1)(-4)+(-1)(4)=0$. The set is orthogonal

   (b) $\sqrt{4^2+(-1)^2+1^2}\neq 1$, so the set is not orthonormal

   (c) $\begin{vmatrix} 4 & -1 & -4 \\ -1 & 0 & -17 \\ 1 & 4 & -1 \end{vmatrix} \sim \begin{vmatrix} 1 & -17 & 0 \\ 0 & 4 & -18 \\ 0 & 21 & -1 \end{vmatrix} = 4(-1)-(21)(-18)=374$, so it is linearly independent. Therefore, it is a basis

7. (a) $\left(-\frac{\sqrt{2}}{6}\right)\left(-\frac{\sqrt{5}}{5}\right)\neq 0$, so it is not orthogonal

   (b) Because it is not orthogonal, it is not orthonormal

   (c) $\begin{vmatrix} \frac{\sqrt{2}}{3} & 0 & \frac{\sqrt{5}}{5} \\ 0 & \frac{2\sqrt{5}}{5} & 0 \\ -\frac{\sqrt{2}}{6} & -\frac{\sqrt{5}}{5} & \frac{1}{2} \end{vmatrix} = \frac{2\sqrt{5}}{5}\left(\left(\frac{\sqrt{2}}{3}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{5}}{5}\right)\left(-\frac{\sqrt{2}}{6}\right)\right)\neq 0$, so it is linearly independent. Therefore, it is a basis

11. (a) $\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)=0$, $4\left(\frac{\sqrt{2}}{2}\right)(0)=0$, $\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)=0$, so it is orthogonal

    (b) $\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2}=1$, $\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)}=1$, $\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2}=1$, so it is orthonormal

    (c) There are more terms than vectors, so it is linearly dependent

15. (a) $(-\sqrt{2})(\sqrt{3})+(\sqrt{2})(\sqrt{3})=0$, so it is orthogonal

    (b) $\frac{1}{\sqrt{(\sqrt{3})^2+(\sqrt{3})^2+(\sqrt{3})^2}}(\sqrt{3},\sqrt{3},\sqrt{3})=\left(\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3}\right)$, $\frac{1}{\sqrt{\left(-\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2}}(-\sqrt{2},0,\sqrt{2})=\left(-\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}\right)\Rightarrow\left\{\left(\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3}\right),\left(-\frac{\sqrt{2}}{2},0,\frac{\sqrt{2}}{2}\right)\right\}$

17. $1\cdot x=0$, $x\cdot x^2=0$, $x^2\cdot x^3=0$, $1\cdot x^2=0$, $1\cdot x^3=0$, $x\cdot x^3=0$, and $||1||=1$, $||x||=1$, $||x^2||=1$, $||x^3||=1$, so it is orthonormal

19. $\begin{bmatrix} -\frac{2\sqrt{13}}{13} & \frac{3\sqrt{13}}{13} \\ \frac{3\sqrt{13}}{13} & \frac{2\sqrt{13}}{13} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{4\sqrt{13}}{13} \\ \frac{7\sqrt{13}}{13} \end{bmatrix}$

43. $\int_{-1}^{1} x\,dx = \left(\frac{x^2}{2}\right)\Big|_{-1}^{1} = 0$ ✓

48. $2\int_0^1 \left(x^2 - \frac{1}{3}\right)^2 dx = 2\int_0^1 x^4 - \frac{2}{3}x^2 + \frac{1}{9}\,dx = \left(\frac{2x^5}{5} - \frac{4x^3}{9} + \frac{2}{9}x\right)\Big|_0^1 = \frac{8}{45}$ ✓

56. (a) True. The requirement for orthonormality is for each vector to be orthogonal to each other, and for each vector to be a unit vector.

    (b) False. Orthogonality does not define linear independence.

61. $\left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{3}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{3}}\right) = 0 \Rightarrow \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$
and $\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1$, so it is orthonormal

64. (a) $\overrightarrow{v} = c_1\overrightarrow{v}_1 + c_2\overrightarrow{v}_2 + \cdots + \overrightarrow{v}_n$

    (b) $\langle\overrightarrow{w}, \overrightarrow{v}\rangle = \overrightarrow{w}_1\overrightarrow{v}_1 + \overrightarrow{w}_2\overrightarrow{v}_2 + \cdots + \overrightarrow{w}_n\overrightarrow{v}_n$

    (c) $\overrightarrow{w}_1\overrightarrow{v}_1 + \overrightarrow{w}_2\overrightarrow{v}_2 + \cdots + \overrightarrow{w}_n\overrightarrow{v}_n = c$

    (d) If $\overrightarrow{w}$ is orthogonal, then $\cancel{\overrightarrow{w}_1\overrightarrow{v}_1 + \overrightarrow{w}_2\overrightarrow{v}_2 + \cdots + \overrightarrow{w}_n\overrightarrow{v}_n} = 0$