Linear Algebra 1.1 Homework

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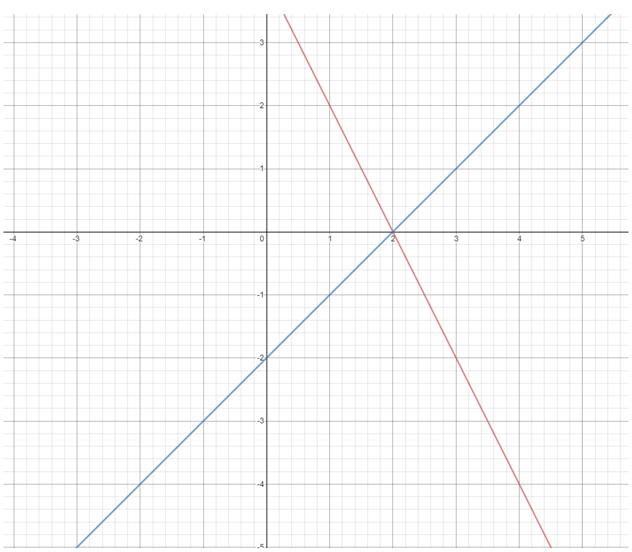
3. Not linear
5. Not linear
9.

$$
\begin{aligned}
y &\to s \\
z &\to t \\
S &= \{(1 - s - t, s, t)\}
\end{aligned}
$$

10.

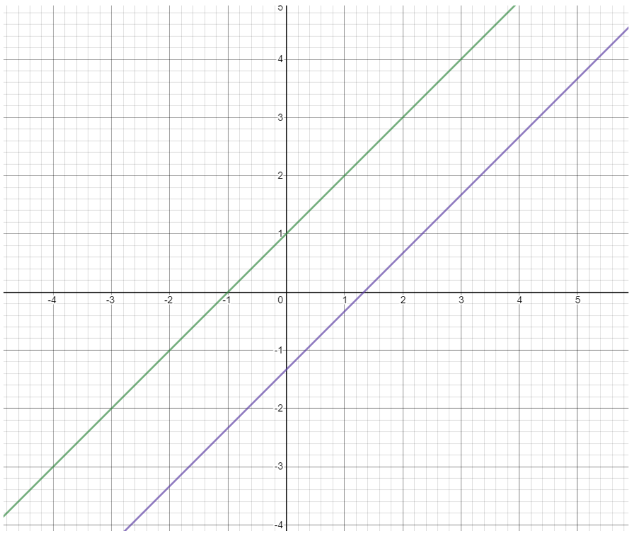
$$
\begin{aligned}
x_2 &\to s \\
x_3 &\to t \\
S &= \{(1 - 2s + 3t, s, t)\}
\end{aligned}
$$

11.



$$
\begin{aligned}
2x + y = 4 \quad & L_1 \\
x - y = 2 \quad & L_2 \\
L_1 - L_2 \to x &= 2 \\
2(2) + y &= 4 \\
y &= 0 \\
\text{The solution is at point } &(2, 0)
\end{aligned}
$$

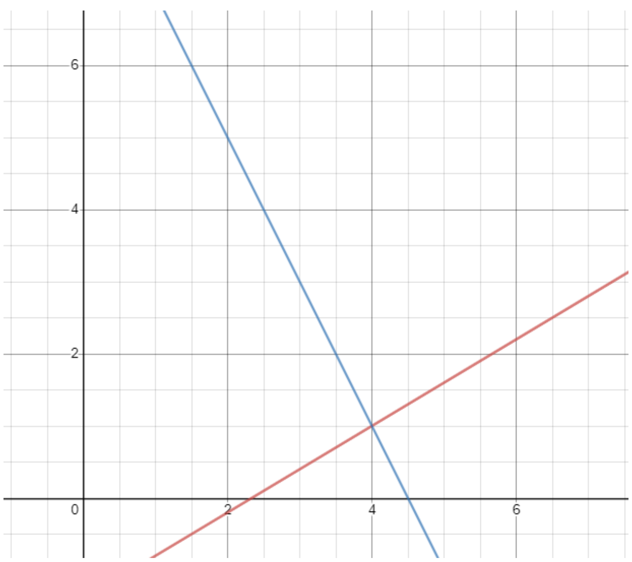
13.



$$-x + y = 1 \quad L_1$$
$$3x - 3y = 4 \quad L_2$$
$$-\frac{1}{3}L_2 \rightarrow -x + y = -\frac{4}{3}$$

No Solution, Lines Parallel

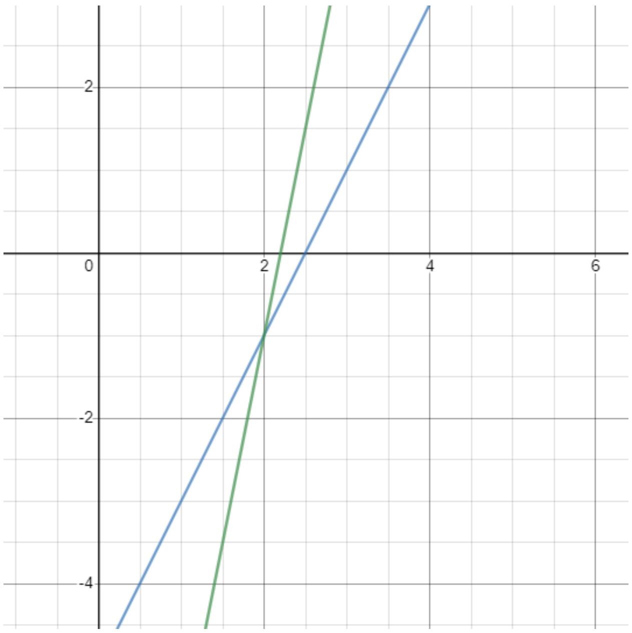
15.



$$3x - 5y = 7 \quad L_1$$
$$2x + y = 9 \quad L_2$$
$$5L_2 + L_1 \rightarrow 13x = 52$$
$$x = 4$$
$$2(4) + y = 9$$
$$y = 1$$

The solution is at point $(4, 1)$

17.



$$\begin{aligned} 2x - y &= 5 \quad L_1 \\ 5x - y &= 11 \quad L_2 \end{aligned}$$

$$L_2 - L_1 \rightarrow 3x = 6$$

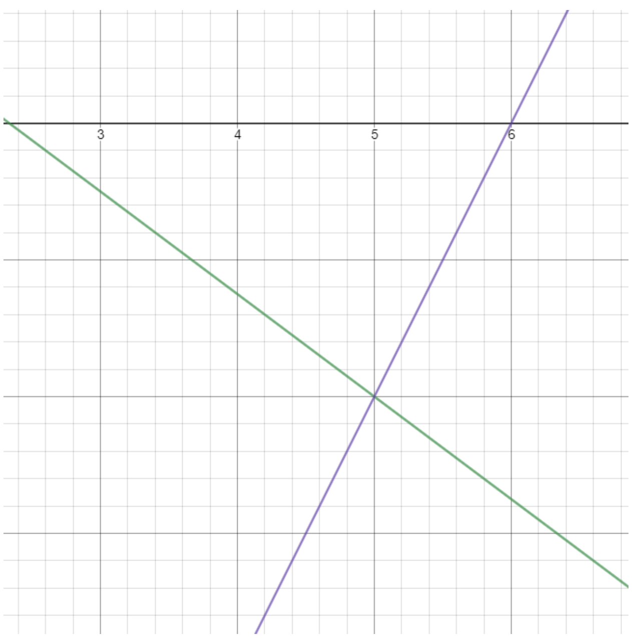
$$x = 2$$

$$2(2) - y = 5$$

$$y = -1$$

The solution is at point $(2, -1)$

19.



$$\begin{aligned} \tfrac{x+3}{4} + \tfrac{y-1}{3} &= 1 \quad L_1 \\ 2x - y &= 12 \quad L_2 \end{aligned}$$

$$12L_1 \rightarrow 3x + 4y = 7$$

$$4L_2 + (3x + 4y = 7) \rightarrow 11x = 55$$

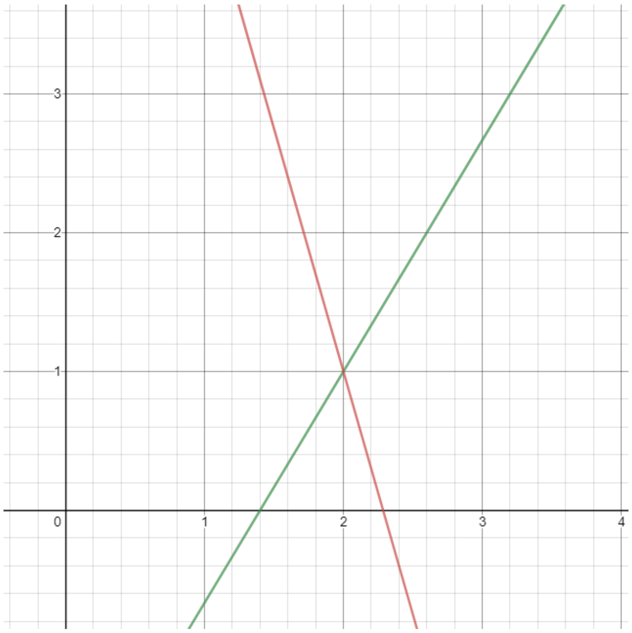
$$x = 5$$

$$2(5) - y = 12$$

$$y = -2$$

The solution is at point $(5, -2)$

21.

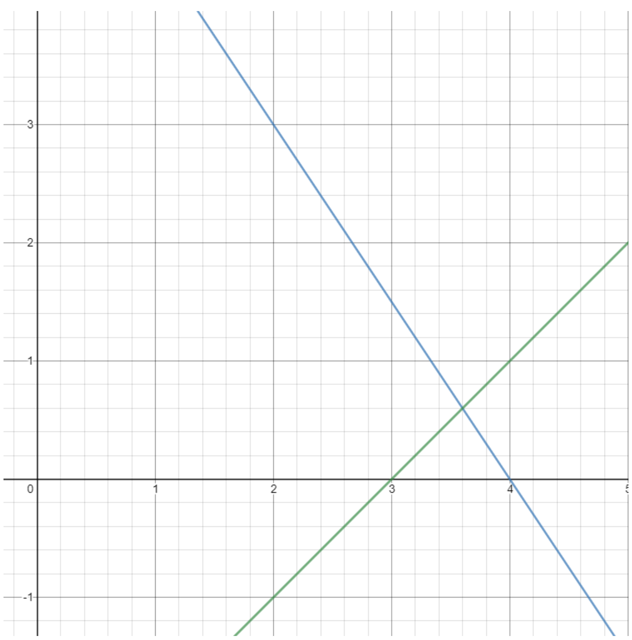


$$
\begin{aligned}
.05x - .03y &= .07 \quad L_1 \\
.07x + .02y &= .16 \quad L_2
\end{aligned}
$$

$$
\begin{aligned}
200L_1 + 300L_2 \rightarrow 31x &= 62 \\
x &= 2 \\
.05(2) - .03y &= .07 \\
y &= 1
\end{aligned}
$$

The solution is at point $(2,1)$

23.



$$
\begin{aligned}
\tfrac{x}{4} + \tfrac{y}{6} &= 1 \quad L_1 \\
x - y &= 3 \quad L_2
\end{aligned}
$$

$$
\begin{aligned}
24L_1 \rightarrow 6x + 4y &= 24 \\
4L_2 + (6x + 4y = 24) \rightarrow 10x &= 36 \\
x &= 3.6 \\
-y &= 3 - 3.6 \\
y &= .6
\end{aligned}
$$

The solution is at point $(3.6, 0.6)$

25. $\boxed{\begin{aligned} x_1 - x_2 &= 2 \\ x_2 &= 3 \end{aligned}} \rightarrow x_1 = 2 + 3 \rightarrow x_1 = 5$

$$S = \{(5,3)\}$$

27. $\boxed{\begin{aligned} -x + y - z &= 0 \\ 2y + z &= 3 \\ \tfrac{1}{2}z &= 0 \end{aligned}} \rightarrow z = 0 \rightarrow 2y = 3 \rightarrow y = \frac{3}{2} \rightarrow -x = -\frac{3}{2} \rightarrow x = \frac{3}{2}$

$$S = \left\{\frac{3}{2}, \frac{3}{2}, 0\right\}$$

29. $\boxed{\begin{aligned} 5x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 &= 0 \end{aligned}} \rightarrow x_1 = -\frac{x_2}{2} \rightarrow x_3 = t \rightarrow x_2 = 2t \rightarrow x_3 = -t$

$$S = \{-t, 2t, t\}$$

39.

$$\begin{aligned} 3u + v &= 240 \quad L_1 \\ u + 3v &= 240 \quad L_2 \end{aligned}$$

$$\begin{aligned} 3L_2 - L_1 \rightarrow 8v &= 480 \\ v &= 60 \\ u &= \frac{240 - 60}{3} \\ u &= 60 \\ \text{The solution is at point } &(60, 60) \end{aligned}$$

41.

$$\begin{aligned} 9x - 3y &= -1 \quad L_1 \\ \tfrac{1}{5}x + \tfrac{2}{5}y &= -\tfrac{1}{3} \quad L_2 \end{aligned}$$

$$\begin{aligned} 45L_2 - L_1 \rightarrow 21y &= -14 \\ y &= -\frac{2}{3} \\ 9x + 2 &= -1 \\ x &= -\frac{1}{3} \\ \text{The solution is at point } &\left(-\frac{1}{3}, -\frac{2}{3}\right) \end{aligned}$$

47.

$$\begin{aligned} &x - y - z = 0 \quad L_1 \\ &x + 2y - z = 6 \quad L_2 \\ &2x - z = 5 \quad L_3 \end{aligned}$$

$$\begin{aligned} x - z &= y \\ 2y + y = 6 \rightarrow y &= 2 \end{aligned}$$

$$\begin{aligned} x - z &= 2 \quad L_4 \\ 2x - z &= 5 \quad L_5 \end{aligned}$$

$$\begin{aligned} L_5 - L_4 \rightarrow x &= 3 \\ z &= 1 \\ \text{The solution is at point } &(3, 2, 1) \end{aligned}$$

49.

$$
\begin{array}{ll}
3x_1 - 2x_2 + 4x_3 = 1 & L_1 \\
x_1 + x_2 - 2x_3 = 3 & L_2 \\
2x_1 - 3x_2 + 6x_3 = 8 & L_3
\end{array}
$$

No solution, planes parallel

51.

$$
\begin{array}{ll}
2x_1 + x_2 - 3x_3 = 4 & L_1 \\
4x_1 + 2x_3 = 10 & L_2 \\
-2x_1 + 3x_2 - 13x_3 = -8 & L_3
\end{array}
$$

$$
\begin{aligned}
L_3 - 3L_1 + 2L_2 \to -18x_3 &= 0 \\
x_3 &= t \\
4x_1 + 2t &= 10 \\
x_1 &= \frac{5-t}{2} \\
5 - t + x_2 - 3t &= 4 \\
x_2 &= 4t - 1 \\
\text{The solution is at } &\left(\frac{5-t}{2}, 4t-1, t\right)
\end{aligned}
$$

53.

$$
\begin{array}{ll}
x - 3y + 2z = 18 & L_1 \\
5x - 15y + 10z = 18 & L_2
\end{array}
$$

No solution, planes parallel

65. The system must have at least one solution because the number of rows is equal to the number of unknowns, and the lines are distinctly different (not parallel). Also, because it is a homogeneous system, x, y, and z equaling zero can always be a solution.

$$
\begin{array}{ll}
5x + 5y - z = 0 & L_1 \\
10x + 5y + 2z = 0 & L_2 \\
5x + 15y - 9z = 0 & L_3
\end{array}
$$

$$
\begin{aligned}
L_2 - L_1 = 5x + 3z &= 0 \\
L_3 - L_1 = 10y - 8z &= 0 \\
z &= t \\
y &= \frac{4}{5}t \\
x &= -\frac{3}{5}t \\
\text{The solution is } &\left(-\frac{3}{5}t, \frac{4}{5}t, t\right)
\end{aligned}
$$

69. (a) This is true because it can always be represented as a dependent, parametrized system.
    (b) This is false because the planes could be parallel
    (c) This is false because a consistent system may only have a single solution

71. One such system is the single equation: $x_1 - \frac{x_2}{3} = \frac{4}{3}$. If $x_1 = t$, then $\frac{x_2}{3} = -\frac{4}{3} + t \rightarrow x_2 = 3t - 4$. Additionally, if $x_2 = t$, then $x_1 = \frac{x_2+4}{3}$.

75.

$$\begin{array}{ll} 2A + B - 3C = 4 & L_1 \\ 4A + 2C = 10 & L_2 \\ -2A + 3B - 13C = -8 & L_3 \end{array}$$

$$\begin{aligned} &\text{Same as (51)} \\ A &= \frac{5-t}{2} \\ B &= 4t - 1 \\ C &= t \\ &\text{This means:} \\ x &= \frac{2}{5-t} \\ y &= \frac{1}{4t-1} \\ z = \frac{1}{t} &\text{Where } t \neq \frac{1}{4}, 5, 0 \end{aligned}$$

77.

$$\begin{array}{ll} (\cos\theta)x + (\sin\theta)y = 1 & L_1 \\ (-\sin\theta)x + (\cos\theta)y = 0 & L_2 \\ x = \frac{\cos\theta}{\sin\theta}y & \\ (\cos^2\theta)y + (\sin^2\theta)y = \sin\theta & \\ (\cos^2\theta + \sin^2\theta)y = \sin\theta & \\ y = \sin\theta & \\ (\cos\theta)x + (\sin^2\theta) = 1 & \\ x = \cos\theta & \\ \text{The solution is } (\cos\theta, \sin\theta) & \end{array}$$

79. For no solution, systems must be parallel

$$\begin{aligned} x + ky = 2 \quad & L_1 \\ kx + y = 4 \quad & L_2 \\ L_1 - \frac{1}{k}L_2 \rightarrow \left(k - \frac{1}{k}\right) y &= 2 - \frac{4}{k} \qquad (1) \\ \left(k^2 - 1\right) y &= 2k - 4 \\ \text{The statement is not possible for } k &= \pm 1 \end{aligned}$$

81. Any n x n system has exactly one solution. This means that, as long as $k \neq 0$, the statement is true

83. Lines must be equal to each other to have infinite solutions

$$
\begin{aligned}
4x + ky &= 6 \quad L_1 \\
kx + y &= -3 \quad L_2 \\
-2L_2 \rightarrow -2kx - 2y &= 6 \\
4x + ky &= -2kx - 2y \\
k &= -2
\end{aligned} \tag{2}
$$

85. It is not possible to find a solution when $k = 1$ because the planes are all parallel. Additionally, the same holds true for $k = -2$, because it is then not possible to solve the system.