Linear Algebra 5.1 Homework

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<u>Problems 1, 5, 9, 13, 17, 18, 21, 25, 27, 33, 41, 45, 49, 53, 57, 67, 71, 73, 77, 83, 84</u>

1. $\sqrt{3^2+4^2}=5$

5. (a) $\sqrt{(-1)^2+\left(\frac{1}{4}\right)^2}=\frac{\sqrt{17}}{4}$

   (b) $\sqrt{(4)^2+\left(-\frac{1}{8}\right)^2}=\sqrt{\frac{1025}{64}}=\frac{5\sqrt{41}}{8}$

   (c) $\sqrt{(-1+4)^2+\left(\frac{1}{8}\right)^2}=\sqrt{\frac{577}{64}}=\frac{\sqrt{577}}{8}$

9. (a) $||\overrightarrow{u}||=13\Rightarrow\langle-\frac{5}{13},\frac{12}{13}\rangle$; $\left(-\frac{5}{13}\right)^2+\left(\frac{12}{13}\right)^2=1$

   (b) Opposite Direction: $-\langle-\frac{5}{13},\frac{12}{13}\rangle=\langle\frac{5}{13},-\frac{12}{13}\rangle$; $\left(\frac{5}{13}\right)^2+\left(-\frac{12}{13}\right)^2=1$

13. $4\left(\langle 1,1\rangle\right)\cdot\frac{1}{||\overrightarrow{u}||}=\langle 2\sqrt{2},2\sqrt{2}\rangle$

17. (a) $\frac{1}{2}\overrightarrow{v}=\langle-\frac{1}{2},\frac{3}{2},0,2\rangle$

    (b) $-2\overrightarrow{v}=\langle 2,-6,0,-8\rangle$

18. $\sqrt{(c)^2+(2c)^2+(3c)^2}=1\Rightarrow c\sqrt{14}=\pm1\Rightarrow c=\frac{\sqrt{14}}{14}$

21. $\sqrt{(1-(-1))^2+(4-2)^2+(1-0)^2}=3$

25. (a) $\overrightarrow{u}\cdot\overrightarrow{v}=2(2)+(-1)(-2)+(-6)=0$

    (b) $\overrightarrow{v}\cdot\overrightarrow{v}=(2)^2+(-1)^2+(-6)^2=41$

    (c) $\left(\sqrt{(2)^2+(-2)^2+(1)^2}\right)^2=9$

    (d) $(\overrightarrow{u}\cdot\overrightarrow{v})\overrightarrow{v}=\langle 0,0\rangle$

    (e) $\overrightarrow{u}\cdot(5\overrightarrow{v}=5(\overrightarrow{u}\cdot\overrightarrow{v})=5(0)=0$

27. $(\overrightarrow{u}+\overrightarrow{v})\cdot(2\overrightarrow{u}-\overrightarrow{v})\Rightarrow 2(\overrightarrow{u}\cdot\overrightarrow{u})-\overrightarrow{u}\cdot\overrightarrow{v}+2(\overrightarrow{u}\cdot\overrightarrow{v})-\overrightarrow{v}\cdot\overrightarrow{v}=2(4)-(-5)+2(-5)-(10)=-7$

33. (a) $||\overrightarrow{u}||\approx 3.464$; $||\overrightarrow{v}||\approx 3.317$

(b) $\langle -.603, .426, -.522, -.426\rangle$

(c) $\langle -.577, -.5, -.408, -.5\rangle$

(d) -6.45

(e) 12

(f) 11

41. $\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{||\vec{u}||||\vec{v}||}\right)$; $\vec{u}\cdot\vec{v} = \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{6}+\sqrt{2}}{4}$; $||\vec{u}|| = \frac{5}{4}, ||\vec{v}|| = \frac{3}{4}$; $\theta = 106°$

45. $||\vec{u}|| = \sqrt{2}$, $||\vec{v}|| = 6$; $\vec{u}\cdot\vec{v} = 6 \Rightarrow \theta = \cos^{-1}\left(\frac{6}{6\sqrt{2}}\right) = 45°$

49. $-\frac{1}{3}(2)+\frac{2}{3}(-4) = -\frac{10}{3} \Rightarrow ||\vec{u}|| = \frac{\sqrt{5}}{3}, ||\vec{v}|| = 2\sqrt{5} \Rightarrow \cos^{-1}(-1) = \pi$, so parallel

53. $\frac{1}{4}(-2)-\frac{5}{4}(5) = -\frac{27}{4} \Rightarrow ||\vec{u}|| = \sqrt{30}, ||\vec{v}|| = \sqrt{\frac{21}{8}}$, so neither

57. $2a-b+c = 0 \Rightarrow b = s, c = t \Rightarrow \langle\frac{s-t}{2}, s, t\rangle$

67. (a) $\begin{bmatrix}3 & 4\end{bmatrix}\begin{bmatrix}2\\-3\end{bmatrix} = -6$

(b) $\begin{bmatrix}2 & -3\end{bmatrix}\begin{bmatrix}2\\-3\end{bmatrix} = 13$

(c) $\begin{bmatrix}3 & 4\end{bmatrix}\begin{bmatrix}3\\4\end{bmatrix} = 25$

(d) $\left(\begin{bmatrix}3 & 4\end{bmatrix}\begin{bmatrix}2\\-3\end{bmatrix}\right)\begin{bmatrix}2 & -3\end{bmatrix} = \begin{bmatrix}-12 & 18\end{bmatrix}$

(e) $\begin{bmatrix}3 & 4\end{bmatrix}\left(5\begin{bmatrix}2\\-3\end{bmatrix}\right) = 5\left(\begin{bmatrix}3 & 4\end{bmatrix}\begin{bmatrix}2\\-3\end{bmatrix}\right) = -30$

71. $\vec{u}\cdot\vec{v} = (\cos\theta)(\sin\theta)+(\sin\theta)(-\cos\theta) = 0$, so the two are orthogonal

73. (a) False. The norm is defined as the square root of the sum of all components squared.

(b) False. The dot product always becomes a scalar.

77. $v_1v_2+(v_2)(-v_1) = 0$; $||\vec{v}|| = 13 \Rightarrow \langle -\frac{12}{13}, \frac{5}{13}\rangle, \langle\frac{12}{13}, -\frac{5}{13}\rangle$

83. $\vec{u}\cdot(c\vec{v}+d\vec{w}) = c(\vec{u}\cdot\vec{v})+d(\vec{u}\cdot\vec{w}) = c(0)+d(0) = 0$, so it is orthogonal

84. $\vec{u}\cdot\vec{v} = \frac{1}{4}||\vec{u}+\vec{v}||^2-\frac{1}{4}||\vec{u}-\vec{v}||^2 \Rightarrow \frac{1}{4}(\cancel{||\vec{u}||^2}+2(\vec{u}\cdot\vec{v})+\cancel{||\vec{v}||^2}\cancel{-||\vec{u}||^2}+2(\vec{u}\cdot\vec{v})\cancel{-||\vec{v}||^2}) = \frac{1}{4}(4\vec{u}\cdot\vec{v}) = \vec{u}\cdot\vec{v}$ ✓