Linear Algebra 7.2 Homework

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<u>3-23 eoo, 25</u>

3. (a) $P^{-1} = \frac{1}{1-4}\begin{bmatrix}1 & -2\\ -2 & 1\end{bmatrix} = \begin{bmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{2}{3} & -\frac{1}{3}\end{bmatrix} \Rightarrow \frac{1}{3}\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}\begin{bmatrix}3 & -2\\ 2 & -2\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & -\frac{2}{3}\\ \frac{4}{3} & -\frac{2}{3}\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & 2\end{bmatrix}$

   (b) $\lambda_i = -1, 2$

7. $(\lambda-6)(\lambda-1)-6 \Rightarrow \lambda(\lambda-7) = 0$, so $\lambda_i = 7, 0 \Rightarrow \left[\begin{array}{cc|c}-6 & 3 & 0\\ 2 & -1 & 0\end{array}\right] \Rightarrow \begin{bmatrix}1\\ 2\end{bmatrix}$ and $\left[\begin{array}{cc|c}1 & 3 & 0\\ 2 & 6 & 0\end{array}\right] \Rightarrow \begin{bmatrix}3\\ -1\end{bmatrix} \Rightarrow P = \begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}$ and $P^{-1} = \frac{1}{7}\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix} \Rightarrow \frac{1}{7}\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix}\begin{bmatrix}6 & -3\\ -2 & 1\end{bmatrix}\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 3\\ 2 & -1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 7\end{bmatrix}$ $\checkmark$

11. $\lambda I - A = \begin{vmatrix}\lambda-1 & -2 & 2\\ 2 & \lambda-5 & 2\\ 6 & -6 & \lambda+3\end{vmatrix} = (\lambda-1)\left[(\lambda-5)(\lambda+3)+12\right] + 2\left[2(\lambda+3)-12\right] + 2\left[-12-6(\lambda-5)\right] \Rightarrow (\lambda^2-1)(\lambda-3) + 4\lambda - 12 + 36 - 12\lambda \Rightarrow \lambda^3 - 3\lambda^2 - \lambda + 3 + 4\lambda - 12 + 36 - 12\lambda \Rightarrow \lambda^3 - 3\lambda^2 - 9\lambda + 27 = 0 \Rightarrow (\lambda+3)(\lambda-3)^2 = 0 \Rightarrow \lambda_i = 3, -3 \Rightarrow \left[\begin{array}{ccc|c}2 & -2 & 2 & 0\\ 2 & -2 & 2 & 0\\ 6 & -6 & 6 & 0\end{array}\right] \Rightarrow \begin{bmatrix}1\\ 1\\ 0\end{bmatrix}, \begin{bmatrix}-1\\ 0\\ 1\end{bmatrix}$ and $\left[\begin{array}{ccc|c}-4 & -2 & 2 & 0\\ 2 & -8 & 2 & 0\\ 6 & -6 & 0 & 0\end{array}\right] \Rightarrow \begin{bmatrix}1\\ 1\\ 3\end{bmatrix} \Rightarrow P = \begin{bmatrix}1 & -1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 3\end{bmatrix} \Rightarrow P^{-1} = \frac{1}{3}\begin{bmatrix}-1 & 4 & -1\\ -3 & 3 & 0\\ 1 & -1 & 1\end{bmatrix} \Rightarrow \frac{1}{3}\begin{bmatrix}-1 & 4 & -1\\ -3 & 3 & 0\\ 1 & -1 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & -2\\ -2 & 5 & -2\\ -6 & 6 & -3\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 3\end{bmatrix} = \begin{bmatrix}-1 & 4 & -1\\ -3 & 3 & 0\\ -1 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ 1 & 0 & 1\\ 0 & 1 & 3\end{bmatrix} = \begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & -3\end{bmatrix}$ $\checkmark$

15. $(\lambda)(\lambda) = 0 \Rightarrow \lambda_i = 0$. There are not n distinct eigenvalues, so it is not diagonalizable.

19. $(\lambda - 1)\left[(\lambda - 1)(\lambda - 2)\right] \Rightarrow (\lambda - 1)^2(\lambda - 2) = 0 \Rightarrow \lambda_i = 1, 2$. There are not n distinct eigenvalues, so it is not diagonalizable.

23. $(\lambda - 1)^2 - 1 \Rightarrow \lambda^2 - 2\lambda = 0 \Rightarrow \lambda_i = 0, 2$, so there are enough distinct eigenvalues.

25. $(\lambda + 3)\left[(\lambda - 4)(\lambda + 5) + 18\right] - 2\left[-3(\lambda + 5) + 9\right] - 3\left[6 + (\lambda - 4)\right] \Rightarrow (\lambda + 3)(\lambda^2 + \lambda - 2) + 6\lambda + 12 - 6 - 3\lambda \Rightarrow \lambda^3 + \lambda^2 - 2\lambda + 3\lambda^2 + 3\lambda - 6 + 6\lambda + 12 - 6 - 3\lambda \Rightarrow \lambda^3 + 4\lambda^2 + 4\lambda = 0 \Rightarrow \lambda_i = 0, -2$, so there are not enough distinct eigenvalues.