

Augmented Matrices and Elementary Row Operations

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- Matrix – An ordered array of numbers

- Examples of a Matrix

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (2x2 Matrix)

2. $\begin{bmatrix} 9 & 0 & e \end{bmatrix}$ (1x3 Matrix)

3. $\begin{bmatrix} 2 & 4 \\ \pi & 6 \\ 3 & 0 \end{bmatrix}$ (3x2 Matrix)

4. Augmented Matrix: $\left[\begin{array}{cc|c} 1 & -2 & 6 \\ 2 & 3 & -2 \end{array} \right]$

- Elementary Row Operations:

1. Interchange Rows ($R_i \leftrightarrow R_j$)

2. Multiply a Row by a non-zero Constant ($kR_i \rightarrow R_i$)

3. Add a Multiple of one Row to Another ($kR_i + R_j \rightarrow R_j$)

4. Notation: \tilde{R} (Row Equivalent)

- Solving (Get \tilde{R} so that some entries are 0s)

- Example:
$$\begin{aligned} x - 2y &= 6 \\ 2x + 3y &= -2 \end{aligned}$$

$$\begin{aligned} & \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 2 & 3 & -2 \end{array} \right] \\ & \quad \widetilde{-2R_1 \rightarrow R_2} \\ & \left[\begin{array}{cc|c} -2 & 4 & -12 \\ 2 & 3 & -2 \end{array} \right] \\ & \quad \widetilde{R_1 + R_2 \rightarrow R_2} \text{ and } \widetilde{-\frac{1}{2}R_1 \rightarrow R_1} \\ & \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 7 & -14 \end{array} \right] \\ & \quad \widetilde{\frac{1}{7}R_2 \rightarrow R_2} \\ & \left[\begin{array}{cc|c} 1 & -2 & 6 \\ 0 & 1 & -2 \end{array} \right] \\ & \quad y = -2 \\ & x - 2(-2) = 6 \longrightarrow 2 \\ & S = \{(2, -2)\} \end{aligned}$$

- Organization

$$\begin{aligned} & a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = c_1 \quad (E_1) \\ 1. & a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = c_2 \quad (E_2) \\ & a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = c_3 \quad (E_3) \\ 2. & \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_1 \\ a_{21} & a_{22} & a_{23} & c_2 \\ a_{31} & a_{32} & a_{33} & c_3 \end{array} \right] \end{aligned}$$

- Example:
$$\begin{aligned} x_1 - 2x_2 + x_3 &= -1 \\ 2x_1 - 3x_2 + x_3 &= -4 \\ 3x_1 - 4x_2 + 2x_3 &= -3 \end{aligned}$$

$$\begin{array}{c}
\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 2 & -3 & 1 & -4 \\ 3 & -4 & 2 & -3 \end{array} \right] \\
\widetilde{-2R_1 + R_2 \rightarrow R_2} \\
\widetilde{-3R_1 + R_3 \rightarrow R_3} \\
\widetilde{-2R_2 + R_3 \rightarrow R_3} \\
\left[\begin{array}{ccc|c} 1 & -2 & 1 & -1 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 4 \end{array} \right] \\
x_3 = 4 \\
x_2 = (4) - 2 \rightarrow 2 \\
x_1 = 2(2) - 4 - 1 \rightarrow -1 \\
S = \{(-1, 2, 4)\}
\end{array}$$

- Gaussian Elimination and Gauss Jordan Elimination

1. Reduced Row-Echelon Form

- (a) If a row does not consist entirely of 0s, and the first non-zero element in the row is a 1, then it is called a leading one.
- (b) For any two successive nonzero rows, the leading 1 in the lower row is farther to the right than the leading 1 in the higher row.
- (c) All the rows consisting entirely of 0s are at the bottom of the matrix. If the fourth property is also satisfied, a matrix is said to be in reduced row-echelon form:
- (d) Each column that contains a leading 1 has 0s everywhere else (above and below)
- (e) If properties $a-c$ are met, but d is not, then the matrix is in just Row Echelon Form

2. A Matrix in Reduced Row-Echelon Form: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & -3 \end{array} \right]$