Diagonalization

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- An $n \times n$ matrix $\mathbf{A}$ is diagonalizable if there exists an invertible matrix $\mathbf{P}$, such that $\mathbf{D} = \mathbf{PAP}^{-1}$ is a diagonal matrix. The matrix $\mathbf{P}$ is said to diagonalize $\mathbf{A}$.

- An $n \times n$ matrix is diagonalizable if and only if $\mathbf{A}$ has n linearly independent eigenvectors.

- To Diagonalize a Matrix:

  1. Find n linearly independent eigenvectors $\overrightarrow{p}_1, \overrightarrow{p}_2, \dots, \overrightarrow{p}_n$
  2. Form $\mathbf{P} = \begin{bmatrix} \overrightarrow{p}_1 & \overrightarrow{p}_2 & \dots & \overrightarrow{p}_n \end{bmatrix}$
  3. Form $\mathbf{D} = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}$

- If an $n \times n$ matrix has n distinct eigenvalues, then corresponding eigenvectors are linearly independent, and $\mathbf{A}$ is diagonalizable