Vectors in $\mathbb{R}^n$

Michael Brodskiy

Professor: Lynn Knight

March 3, 2021

- Vector $-$ Quantity described with length and direction

  1. Written bold and lowercase ($\vec{\mathbf{x}}$)
  2. Vectors are position free
  3. Standard Position $-$ When the initial point is at the origin and the terminal point is somewhere in the plane

- Algebraic work with vectors:

  1. Addition: $\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle = \langle u_1 + v_1, u_2 + v_2 \rangle$
  2. Subtraction: $\langle u_1, u_2 \rangle - \langle v_1, v_2 \rangle = \langle u_1 - v_1, u_2 - v_2 \rangle$
  3. Scalar Multiplication: $c\langle u_1, u_2 \rangle = \langle cu_1, cu_2 \rangle$
  4. $\vec{\mathbf{u}} = \vec{\mathbf{v}}$ iff $u_1 = v_1$ and $u_2 = v_2$

- Zero Vector $-$ $\vec{\mathbf{o}} = \langle 0, 0, \ldots, 0 \rangle$

- Properties of vectors in $\mathbb{R}^2$

  1. $\vec{\mathbf{u}} + \vec{\mathbf{v}} \in \mathbb{R}$ (two vectors in $\mathbb{R}^2$ remain in $\mathbb{R}^2$)
  2. $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}$
  3. $(\vec{\mathbf{u}} + \vec{\mathbf{v}}) + \vec{\mathbf{w}} = \vec{\mathbf{u}} + (\vec{\mathbf{v}} + \vec{\mathbf{w}})$
  4. $\vec{\mathbf{u}} + \vec{\mathbf{o}} = \vec{\mathbf{u}}$
  5. $\vec{\mathbf{u}} + (-\vec{\mathbf{u}}) = \vec{\mathbf{o}}$
  6. $c\vec{\mathbf{u}} \in \mathbb{R}$ (scalar times a vector in $\mathbb{R}^2$ remains a vector in $\mathbb{R}^2$)
  7. $c(\vec{\mathbf{u}} + \vec{\mathbf{v}}) = c\vec{\mathbf{u}} + c\vec{\mathbf{v}}$
  8. $(c + d)\vec{\mathbf{u}} = c\vec{\mathbf{u}} + d\vec{\mathbf{u}}$
  9. $c(d\vec{\mathbf{u}}) = (cd)\vec{\mathbf{u}}$
  10. $1(\vec{\mathbf{u}}) = \vec{\mathbf{u}}$

- Vectors in $\mathbb{R}^n$

  1. All properties for vectors in $\mathbb{R}^2$ apply to $\mathbb{R}^n$ as well
  2. Addition: $\langle u_1, u_2, \ldots, u_n \rangle + \langle v_1, v_2, \ldots, v_n \rangle = \langle u_1 + v_1, u_2 + v_2, \ldots, u_n + v_n \rangle$
  3. Subtraction: $\langle u_1, u_2, \ldots, u_n \rangle - \langle v_1, v_2, \ldots, v_n \rangle = \langle u_1 - v_1, u_2 - v_2, \ldots, u_n - v_n \rangle$
  4. Scalar Multiplication: $c\langle u_1, u_2, \ldots, u_n \rangle = \langle cu_1, cu_2, \ldots, cu_n \rangle$
  5. Equality: $\vec{u} = \vec{v}$ iff $u_1 = v_1$, $u_2 = v_2$, $\ldots$, and $u_n = v_n$

- $\vec{x}$ is a linear combination of $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ $\exists$ $c_1, c_2, \ldots, c_n$ such that $\vec{x} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \cdots + c_n \vec{v}_n$