Linear Algebra 1.2 Participation Problem

Michael Brodskiy
Instructor: Prof. Knight

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Solve the following problem using Gauss-Jordan Elimination and augmented matrices:

$$\begin{gathered} x_1 + x_2 - x_3 - 2x_4 + x_5 = 0 \\ 2x_1 + 3x_2 - x_3 - 7x_4 + 4x_5 = 0 \\ 2x_1 + 4x_2 - 10x_4 + 6x_5 = 0 \end{gathered}$$

$$\begin{gathered} \left[\begin{array}{ccccc|c} 1 & 1 & -1 & -2 & 1 & 0 \\ 2 & 3 & -1 & -7 & 4 & 0 \\ 2 & 4 & 0 & -10 & 6 & 0 \end{array}\right] \\ R_3 - R_2 \to R_2 \\ \left[\begin{array}{ccccc|c} 1 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 2 & 0 \\ 2 & 4 & 0 & -10 & 6 & 0 \end{array}\right] \\ R_3 - 2R_1 \to R_3 \\ \left[\begin{array}{ccccc|c} 1 & 1 & -1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 2 & 0 \\ 0 & 2 & 2 & -6 & 4 & 0 \end{array}\right] \\ R_1 - R_2 \to R_1 \\ \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & -1 & 0 \\ 0 & 1 & 1 & -3 & 2 & 0 \\ 0 & 2 & 2 & -6 & 4 & 0 \end{array}\right] \\ R_3 - 2R_2 \to R_3 \\ \left[\begin{array}{ccccc|c} 1 & 0 & -2 & 1 & -1 & 0 \\ 0 & 1 & 1 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{gathered} \tag{1}$$

The matrix is in reduced row-echelon form

$$\begin{gathered} x_1 - 2x_3 + x_4 - x_5 = 0 \\ x_2 + x_3 - 3x_4 + 2x_5 = 0 \end{gathered}$$

$$x_3 \to r$$
$$x_4 \to s$$
$$x_5 \to t$$
Solution Set is $S = \{(2r - s + t, -r + 3s - 2t, r, s, t)\}$