

Basis and Dimension

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- $B = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ from vector space \mathbf{V} forms basis for \mathbf{V} if
 1. B is linearly independent
 2. $\text{span}(B) = \mathbf{V}$
 3. \vec{v}_i called basis vectors
 4. Examples:
 - (a) \hat{i} and \hat{j} are basis vectors in \mathbb{R}^2
 - (b) \hat{i} , \hat{j} , and \hat{k} are basis vectors in \mathbb{R}^3
 - (c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ are basis vectors in $M_{2,2}$
- Show $\{1, x, x^2\}$ basis for P_2
 1. Trivial Solution only (so it is linearly independent)
 2. Spans $P_2 = c_1 + c_2x + c_3x^2$
- If B is basis there is only one set of scalars c_1, c_2, \dots, c_n such that $\vec{w} = c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_n\vec{v}_n$
- Dimension is the number of basis vectors:
 1. $\mathbb{R}^n - n$
 2. $P_n - n + 1$
 3. $M_{m,n} - m \cdot n$
- If $S = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ is basis for vector space \mathbf{V} , then every set containing more than n vectors will be linearly dependent