Kernel and Range

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- Let $T : V \to W$ be a linear transformation. Then the set of all vectors $\overrightarrow{v}$ in V that satisfy $T(\overrightarrow{v}) = \mathbf{0}$ is the kernel of T and is denoted by $\ker(T)$ or $\ker(T) = \left\{\overrightarrow{v} \in V \,\middle|\, T(\overrightarrow{v}) = \mathbf{0}\right\}$
- $T : \mathbb{R}^n \to \mathbb{R}^m$ matrix transformation, then $T(\overrightarrow{v}) = A\overrightarrow{v}$, then $\ker(T) = A\overrightarrow{v} = \mathbf{0}$
- Let $T : V \to W$ is a linear transformation, then $\ker(T)$ is a subspace of V
- Range — Let $T : V \to W$ be a linear transformation. The set of all vectors in W that are images under T of vectors in V are called the Range of T. $\text{range}(T) = \left\{\overrightarrow{w} \in W \,\middle|\, T(\overrightarrow{v}) = \overrightarrow{w}, \overrightarrow{v} \in W\right\}$
- Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation defined by $T(\overrightarrow{v}) = A\overrightarrow{v}$, then the $\text{range}(T) =$ the column space of A
- Rank and Nullity — Let $T : V \to W$ be a linear transformation. Then $\text{nullity}(T) = \dim(\ker(T))$, $\text{rank}(T) = \dim(\text{range}(T))$, and $\text{rank}(T) + \text{nullity}(T) = \dim(V)$
- $\text{rank}(T) + \text{nullity}(T) = n$