Linear Algebra 2.4 Homework

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1. The matrix is elementary, $2R_2 \widetilde{\rightarrow} R_2$

3. The matrix is elementary, $2R_1 + R_2 \widetilde{\rightarrow} R_2$

5. The matrix is not elementary

7. The matrix is elementary, $R_3 - 5R_2 \widetilde{\rightarrow} R_3$

9. $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

11. $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

15. $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 6 & 0 & 1 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$, $E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_3E_2E_1\mathbf{A} = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

17. $E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$E_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_6 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{10} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_7 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_9 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{10} = \begin{bmatrix} -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{11} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$E_{11}E_{10}E_9E_8E_7E_6E_5E_4E_3E_2E_1\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

19.

$$\frac{1}{0-1}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

21.

$$\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 0 & 0 & 1^{-1} \\ 0 & 1^{-1} & 0 \\ 1^{-1} & 0 & 0 \end{bmatrix}$$
$$\mathbf{A}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

23.

$$\left(\begin{bmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)^{-1} = \begin{bmatrix} k^{-1} & 0 & 0 \\ 0 & 1^{-1} & 0 \\ 0 & 0 & 1^{-1} \end{bmatrix}$$
$$\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{k} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ k \neq 0$$

25. $E_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$E_4 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$

$$E_4E_3E_2E_1\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$E_4E_3E_2E_1 = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$
$$\therefore E_4E_3E_2E_1 = \mathbf{A}^{-1}$$

27. $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_4E_3E_2E_1\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$E_4E_3E_2E_1 = \begin{bmatrix} 1 & 0 & \frac{1}{4} \\ 0 & \frac{1}{6} & \frac{1}{24} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$
$$\therefore E_4E_3E_2E_1 = \mathbf{A}^{-1}$$

31. $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\mathrm{E}_3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$E_4 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$E_4E_3E_2E_1 = \begin{bmatrix} 4 & -1 \\ 3 & -1 \end{bmatrix}$$

33. $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_2 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$E_5E_4E_3E_2E_1 = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

37. It is **NOT** always elementary. This is evident because elementary matrices may be used to come up with non-elementary matrices, such as inverses, which means that the product is not always elementary.

40.

$$\mathbf{A} = \begin{bmatrix} 1+ab & a & 0 \\ b & 1 & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & -a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ -b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

$$E_3E_2E_1 = \begin{bmatrix} 1 & -a & 0 \\ -b & ab+1 & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

$$E_3E_2E_1\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

41. (a) True. It has one or less modifications done to it, with respect to the identity matrix.
    (b) False. This modification can possibly affect more than one number, which would mean there is more than one modification done, which means it is no longer an elementary matrix.
    (c) True. An inversed elementary matrix produces a matrix with one or less modification, meaning that it is an elementary matrix itself.