The Determinant

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- Determinants may be found for square matrices only
- For a 2x2 matrix:
  1. det($\mathbf{A}$)=det$\left(\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}\right)$=$a_{11}a_{22} - a_{12}a_{21}$
- Minors and Cofactors:
  1. Let $\mathbf{A}$ be a square matrix
     (a) Minor $-$ M_{ij} of element a_{ij} is the determinant of the submatrix formed by eliminating row i and column j of matrix $\mathbf{A}$
     (b) Ex. $\mathbf{A} = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{bmatrix}$, then $M_{23} = \begin{vmatrix} 1 & 4 \\ -1 & 4 \end{vmatrix} = 8$
     (a) Cofactor $-$ C_{ij} of entry $a_{ij} = (-1)^{i+j}M_{ij}$
     (b) Ex. $\mathbf{A} = \begin{bmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{bmatrix}$, then $C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 4 \\ -1 & 4 \end{vmatrix} = -8$
- For an nxn matrix
  1. Expand about r_i

$$\begin{aligned} \det(\mathbf{A}) &= a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in} \\ &= \sum_{n=1}^{c} a_{in}C_{in} \end{aligned} \tag{1}$$

2. Expand about column j

$$\begin{aligned} \det(\mathbf{A}) &= a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj} \\ &= \sum_{n=1}^{c} a_{nj}C_{nj} \end{aligned} \tag{2}$$

- Upper Triangular Matrix $- \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$
- Lower Triangular Matrix $- \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$
- If $\mathbf{A}$ is a triangular matrix, its determinant is $\det(\mathbf{A}){=}a_{11}a_{22}\ldots a_{nn}$