Gram-Schmidt Orthonormalization Process

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April 14, 2021

- Orthogonal if $\langle \overrightarrow{u}, \overrightarrow{v} \rangle = 0$
- Orthonormal if $\langle \overrightarrow{u}, \overrightarrow{v} \rangle$ and $\langle \overrightarrow{v}, \overrightarrow{v} \rangle = 1$
- The Gram-Schmidt Process — Given basis $B = \{\overrightarrow{v}_1, \overrightarrow{v}_2, \ldots, \overrightarrow{v}_n\}$
  1. $\overrightarrow{w}_1 = \overrightarrow{v}_1$
  2. $\overrightarrow{w}_2 = \overrightarrow{v}_2 - \frac{\langle \overrightarrow{v}_2, \overrightarrow{w}_1 \rangle}{\langle \overrightarrow{w}_1, \overrightarrow{w}_1 \rangle} \overrightarrow{w}_1$
  3. $\vdots$
  4. $\overrightarrow{w}_n = \overrightarrow{v}_n - \frac{\langle \overrightarrow{v}_n, \overrightarrow{w}_1 \rangle}{\langle \overrightarrow{w}_1, \overrightarrow{w}_1 \rangle} \overrightarrow{w}_1 - \frac{\langle \overrightarrow{v}_n, \overrightarrow{w}_2 \rangle}{\langle \overrightarrow{w}_2, \overrightarrow{w}_2 \rangle} \overrightarrow{w}_2 - \cdots - \frac{\langle \overrightarrow{v}_n, \overrightarrow{w}_{n-1} \rangle}{\langle \overrightarrow{w}_{n-1}, \overrightarrow{w}_{n-1} \rangle} \overrightarrow{w}_{n-1}$