Vector Spaces

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March 3, 2021

- $\mathbb{R}^2$, $\mathbb{R}^3$, ..., $\mathbb{R}^n$ are all vector spaces with certain properties.
- Properties of Vector Spaces, where $\mathbf{V}$ is a set on which vector addition and scalar multiplication are defined, and $\overrightarrow{\mathbf{u}}$, $\overrightarrow{\mathbf{v}}$, and $\overrightarrow{\mathbf{w}} \in \mathbf{V}$ and c and d are scalars, then $\mathbf{V}$ is a vector space if:

  1. $\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}} \in \mathbf{V}$
  2. $\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}} = \overrightarrow{\mathbf{v}} + \overrightarrow{\mathbf{u}}$
  3. $\overrightarrow{\mathbf{u}} + (\overrightarrow{\mathbf{v}} + \overrightarrow{\mathbf{w}}) = (\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}}) + \overrightarrow{\mathbf{w}}$
  4. $\mathbf{V}$ has a zero vector for every $\overrightarrow{\mathbf{u}}$ such that $\overrightarrow{\mathbf{u}} + \mathbf{0} = \overrightarrow{\mathbf{u}}$
  5. For every $\overrightarrow{\mathbf{u}}$ in $\mathbf{V}$ there is a vector such that $\overrightarrow{\mathbf{u}} + (-\overrightarrow{\mathbf{u}} = 0$
  6. $c\overrightarrow{\mathbf{u}} \in \mathbf{V}$
  7. $c(\overrightarrow{\mathbf{u}} + \overrightarrow{\mathbf{v}}) = c\overrightarrow{\mathbf{u}} + c\overrightarrow{\mathbf{v}}$
  8. $(c+d)\overrightarrow{\mathbf{u}} = c\overrightarrow{\mathbf{u}} + d\overrightarrow{\mathbf{u}}$
  9. $c(d\overrightarrow{\mathbf{u}}) = (cd)\overrightarrow{\mathbf{u}}$
  10. $1(\overrightarrow{\mathbf{u}}) = \overrightarrow{\mathbf{u}}$

- Polynomials of Degree n

  1. $P_1(x) = \left\{ax + b \middle| a, b \in \mathbb{R}\right\}$ Like in $\mathbb{R}^2$
  2. $P_2(x) = \left\{ax^2 + bx + c \middle| a, b, c \in \mathbb{R}\right\}$ Like in $\mathbb{R}^3$
  3. $P_n(x) = \left\{a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \middle| a_i \in \mathbb{R}\right\}$ Like in $\mathbb{R}^n$
  4. $P_0(x) = \left\{a \middle| a \in \mathbb{R}\right\}$ Like in $\mathbb{R}^1$

- Standard Vector Spaces

  1. $\mathbb{R}$ = set of all real numbers
  2. $\mathbb{R}^2$ = set of all ordered pairs
  3. $\mathbb{R}^3$ = set of all ordered triples
  4. $\mathbb{R}^n$ = set of all n-tuples
  5. $C(-\infty, \infty)$ = set of all continuous functions defined on the real number line
  6. $C[a, b]$ = set of all continuous functions defined on a closed interval $[a, b]$, where $a \neq b$
  7. P = set of all polynomials
  8. P_n = set of all polynomials of degree $\leq n$ (together with the zero polynomial)
  9. $M_{m,n}$ = set of all $m \times n$ matrices
  10. $M_{n,n}$ = set of all $n \times n$ matrices

- Let $\overrightarrow{\mathbf{v}}$ be any element of a vector space $\mathbf{V}$, and let c be any scalar. Then the properties below are true.

  1. $0\overrightarrow{\mathbf{v}} = \mathbf{0}$
  2. $c\mathbf{0} = \mathbf{0}$
  3. If $c\overrightarrow{\mathbf{v}} = 0$, then $c = 0$ or $\overrightarrow{\mathbf{v}} = 0$
  4. $(-1)\overrightarrow{\mathbf{v}} = -\overrightarrow{\mathbf{v}}$