Linear Algebra 2.3 Homework

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1.

$$\mathbf{AB} = \begin{bmatrix} 2(3)-5 & 2(-1)+1(2) \\ 5(3)+3(-5) & -5+3(2) \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5.

$$\mathbf{AB} = \begin{bmatrix} -2(-\frac{4}{3})+2(-\frac{4}{3})+1 & -\frac{2(-5)}{3}+\frac{2(-8)}{3}+2 & -2(1)+2(1) \\ -\frac{4}{3}+\frac{4}{3} & -\frac{5}{3}+\frac{8}{3} & 1-1 \\ -\frac{4}{3}+\frac{4}{3} & -\frac{8}{3}+4\left(\frac{2}{3}\right) & 1(1) \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

9.

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right]$$

$$R_2 - 3R_1 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$R_1 - 2R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$\mathbf{A}^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

13.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - R_2 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 - R_2 \widetilde{\rightarrow} R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 3 & 6 & 5 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - 3R_1 - 5R_2 \widetilde{\rightarrow} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -3 & 2 & -1 \end{array}\right]$$

$$R_2 - R_3 \widetilde{\rightarrow} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \\ 0 & 1 & 0 & -3 & 2 & -1 \end{array}\right]$$

$$R_2 \widetilde{\leftrightarrow} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -3 & 2 & -1 \\ 0 & 0 & 1 & 3 & -3 & 2 \end{array}\right]$$

$$\mathbf{A}^{-1} = \left[\begin{array}{ccc} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{array}\right]$$

17.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ -2 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - R_1 \widetilde{\to} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ -3 & -1 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$R_3 + R_2 \widetilde{\to} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$R_1 - 2R_3 \widetilde{\to} R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 3 & -2 & -2 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$R_2 - 3R_1 \widetilde{\to} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 3 & -2 & -2 \\ 0 & -2 & 0 & -9 & 7 & 6 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$-\frac{1}{2} R_2 \widetilde{\to} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 3 & -2 & -2 \\ 0 & 1 & 0 & \frac{9}{2} & -\frac{7}{2} & -3 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$R_1 - R_2 \widetilde{\to} R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{3}{2} & \frac{3}{2} & 1 \\ 0 & 1 & 0 & \frac{9}{2} & -\frac{7}{2} & 3 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$\mathbf{A}^{-1} = \begin{bmatrix} -\frac{3}{2} & \frac{3}{2} & 1 \\ \frac{9}{2} & -\frac{7}{2} & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

25. The matrix is noninvertible because it has a zero row

35.

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$a=\frac{7}{2},\ b=-\frac{3}{4},\ c=\frac{1}{5},\ d=\frac{4}{5}$$
$$\frac{1}{\left(\frac{7}{2}\right)\left(\frac{4}{5}\right)-\left(-\frac{3}{4}\right)\left(\frac{1}{5}\right)}=\frac{20}{59}$$
$$\frac{20}{59}\begin{bmatrix} \frac{4}{5} & \frac{3}{4} \\ -\frac{1}{5} & \frac{7}{2} \end{bmatrix}=\begin{bmatrix} \frac{16}{59} & \frac{15}{59} \\ -\frac{4}{59} & \frac{70}{59} \end{bmatrix}$$

37. (a)

$$\left(\mathbf{A}^2\right)^{-1}$$
$$\mathbf{A}^2=\begin{bmatrix} 2 & 0(2)-2(3) \\ -1(0)-3 & 2+3(3) \end{bmatrix}$$
$$\left(\begin{bmatrix} 2 & -6 \\ -3 & 11 \end{bmatrix}\right)^{-1}=\frac{1}{22-18}\begin{bmatrix} 11 & 6 \\ 3 & 2 \end{bmatrix}$$
$$=\begin{bmatrix} \frac{11}{4} & \frac{3}{2} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix}$$

(b)

$$\left(\mathbf{A}^{-1}\right)^2$$
$$\mathbf{A}^{-1}=\frac{1}{0-2}\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$
$$\left(\begin{bmatrix} -\frac{3}{2} & -1 \\ -\frac{1}{2} & 0 \end{bmatrix}\right)^2=\begin{bmatrix} \left(\frac{3}{2}\right)^2+\frac{1}{2} & \frac{-3}{2}(-1) \\ -\frac{1}{2}\left(-\frac{3}{2}\right) & -1\left(-\frac{1}{2}\right) \end{bmatrix}$$
$$\begin{bmatrix} \frac{11}{4} & \frac{3}{2} \\ \frac{3}{4} & \frac{1}{2} \end{bmatrix}$$

39. (a)

$$\left(\mathbf{A}^2\right)^{-1}$$
$$\text{Diagonal matrix, square each term: }\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$
$$\left(\begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right)^{-1}=\text{ Diagonal matrix, inverse terms}$$
$$\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{9} \end{bmatrix}$$

(b)

$$\left(\mathbf{A}^{-1}\right)^2$$

$$\text{Diagonal matrix, inverse each term: } \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

$$\left(\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}\right)^2 = \text{ Diagonal matrix, square terms}$$

$$\begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{9} \end{bmatrix}$$

43. (a)

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

$$\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{bmatrix} 2+6+\frac{5}{8} & -1+2+\frac{5}{2} & \frac{3}{2}-8+\frac{5}{4} \\ -\frac{3}{4}+3+\frac{1}{16} & \frac{3}{8}+1+\frac{1}{4} & -\frac{9}{16}-4+\frac{1}{8} \\ \frac{1}{4}+\frac{3}{4}+\frac{1}{2} & -\frac{1}{8}+\frac{1}{4}+2 & \frac{3}{16}-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{69}{8} & \frac{7}{2} & -\frac{21}{4} \\ \frac{37}{16} & \frac{13}{8} & -\frac{71}{16} \\ \frac{3}{2} & \frac{17}{8} & \frac{3}{16} \end{bmatrix}$$

(b)

$$(\mathbf{A}^\intercal)^{-1} = (\mathbf{A}^{-1})^\intercal$$

$$(\mathbf{A}^{-1})^\intercal = \begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} & 1 \\ \frac{3}{4} & -2 & \frac{1}{2} \end{bmatrix}$$

(c)

$$(2\mathbf{A})^{-1} = \frac{1}{2}\mathbf{A}^{-1}$$

$$\frac{1}{2}\mathbf{A}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} & \frac{3}{8} \\ \frac{3}{4} & \frac{1}{4} & -1 \\ \frac{1}{8} & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

45. (a)

$$\mathbf{A}^{-1} = \frac{1}{1(-2)-2(1)}\begin{bmatrix} -2 & -2 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$x = 1,\ y = -1$$

(b)

$$\mathbf{A}^{-1} = \frac{1}{1(-2)-2(1)} \begin{bmatrix} -2 & -2 \\ -1 & 1 \end{bmatrix}$$
$$\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} 10 \\ -6 \end{bmatrix} =$$
$$\begin{bmatrix} 2 \\ 4 \end{bmatrix}$$
$$x = 2,\ y = 4$$

47. (a)

$$\mathbf{A}^{-1} = \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right]$$
$$R_1 - R_3 \widetilde{\rightarrow} R_2 \text{ and } R_1 - R_2 \widetilde{\rightarrow} R_3$$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 4 & 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & 1 & -1 & 0 \end{array}\right]$$
$$\frac{1}{2}R_3 \widetilde{\rightarrow} R_3 \text{ and } \frac{1}{4}R_2 \widetilde{\rightarrow} R_2$$
$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right]$$
$$R_1 - 2R_2 - R_3 \widetilde{\rightarrow} R_1$$
$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \end{array}\right]$$
$$\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & 0 & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} =$$
$$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$
$$x = 1,\ y = 1,\ z = -1$$

53.

$$\mathbf{A}^{-1} = \frac{1}{-9+2x} \begin{bmatrix} -3 & -x \\ 2 & 3 \end{bmatrix}$$
$$\frac{1}{-9+2x} = 1$$
$$2x - 9 = -1$$
$$x = 4$$

55.

System needs to be parallel:

$$-2R_2 \to \begin{bmatrix} 4 & 6 \end{bmatrix}$$
$$x = 6$$

57.

$$(2\mathbf{A})^{-1} = \frac{1}{2}\mathbf{A}^{-1}$$
$$\mathbf{A}^{-1} = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$
$$\frac{1}{16-24}\begin{bmatrix} 8 & -4 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} -1 & \frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} \end{bmatrix}$$

67. (a)

$$\mathbf{A} = \mathbf{A}^\intercal$$

(b)

$\mathbf{A}^{-1}$ exists

(c)

This means:

$$\mathbf{A}^{-1} = (\mathbf{A}^\intercal)^{-1}$$
$$(\mathbf{A}^\intercal)^{-1} = (\mathbf{A}^{-1})^\intercal$$
$$\therefore \mathbf{A}^{-1} = (\mathbf{A}^{-1})^\intercal$$

68.

$$\mathbf{ABC} = \mathbf{I}$$
$$\mathbf{AB} = \mathbf{C}^{-1}$$
$$\mathbf{A} = \mathbf{C}^{-1}\mathbf{B}^{-1}$$
$$\mathbf{B}^{-1} = \mathbf{CA}$$

70. (a)

$\mathbf{A}$ is singular or non-singular

(b)

If $\mathbf{A}$ is singular, this is already true

(c)

$$
\begin{aligned}
\text{If } \mathbf{A} \text{ is non-singular, then:}\\
\mathbf{A}^2 &= \mathbf{A}\\
\mathbf{A}(\mathbf{A}-\mathbf{I}) &= \mathbf{O}\\
\mathbf{A} \text{ is either } \mathbf{O} \text{ (singular) or } \mathbf{I}
\end{aligned}
$$

75. (a)

$$\left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right)^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

(b)

$$\left(\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}\right)^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$