Linear Algebra 5.2 Homework

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Problems 1, 8, 9, 11, 13, 15, 17, 23, 24, 29, 33, 36, 39, 41, 43, 45, 47, 49, 50, 51, 65, 67, 75, 79, 85

1. This does define an inner product
   (a) $(\overrightarrow{u}, \overrightarrow{v}) = 3u_1v_1 + u_2v_2 = 3v_1u_1 + v_2u_2 = (\overrightarrow{v}, \overrightarrow{u})$ ✓
   (b) $(\overrightarrow{u}, \overrightarrow{v} + \overrightarrow{w}) = 3u_1(v_1 + w_1) + u_2(v_2 + w_2) = 3u_1v_1 + 3u_1w_1 + u_2v_2 + u_2w_2 = (\overrightarrow{u}, \overrightarrow{v}) + (\overrightarrow{u}, \overrightarrow{w})$ ✓
   (c) $c(\overrightarrow{u}, \overrightarrow{v}) = c(3u_1v_1 + u_2v_2) = 3cu_1v_1 + cu_2v_2 = (c\overrightarrow{u}, \overrightarrow{v})$ ✓
   (d) $(\overrightarrow{v}, \overrightarrow{v}) = 3(v_1)^2 + (v_2)^2$ ✓

8. This does define an inner product
   (a) $(\overrightarrow{u}, \overrightarrow{v}) = \frac{1}{2}u_1v_1 + \frac{1}{4}u_2v_2 + \frac{1}{2}u_3v_3 = \frac{1}{2}v_1u_1 + \frac{1}{4}v_2u_2 + \frac{1}{2}v_3u_3 = (\overrightarrow{v}, \overrightarrow{u})$ ✓
   (b) $(\overrightarrow{u}, \overrightarrow{v} + \overrightarrow{w}) = \frac{1}{2}u_1(v_1 + w_1) + \frac{1}{4}u_2(v_2 + w_2) + \frac{1}{2}u_3(v_3 + w_3) = \frac{1}{2}u_1v_1 + \frac{1}{2}u_1w_1 + \frac{1}{4}u_2v_2 + \frac{1}{4}u_2w_2 + \frac{1}{2}u_3v_3 + \frac{1}{2}u_3w_3 = (\overrightarrow{u}, \overrightarrow{v}) + (\overrightarrow{u}, \overrightarrow{w})$ ✓
   (c) $c(\overrightarrow{u}, \overrightarrow{v}) = c(\frac{1}{2}u_1v_1 + \frac{1}{4}u_2v_2 + \frac{1}{2}u_3v_3) = \frac{1}{2}cu_1v_1 + \frac{1}{4}cu_2v_2 + \frac{1}{2}cu_3v_3 = (c\overrightarrow{u}, \overrightarrow{v})$ ✓
   (d) $(\overrightarrow{v}, \overrightarrow{v}) = \frac{1}{2}(v_1)^2 + \frac{1}{4}(v_2)^2 + \frac{1}{2}(v_3)^2$ ✓

9. This does not define an inner product because it fails axiom 4, which states that the inner product of $\overrightarrow{v}$ with itself only equals zero if $\overrightarrow{v}$ itself is zero. This is not true, as, for $\overrightarrow{v} = \langle 0, c\rangle$, the function fails the axiom.

11. This does not define an inner product, as it fails axiom 4. This is because, for any vector $\overrightarrow{v}$ where $v_1 = v_2$, the vector equals zero, which fails axiom 4.

13. This does not define an inner product, as it fails axiom 1. This is because, for any $\overrightarrow{v} = \langle 0, 0, c\rangle$, this fails, as $(\overrightarrow{u}, \overrightarrow{v}) = -u_1u_2u_3$, but $(\overrightarrow{v}, \overrightarrow{u}) = 0$

15. This does not define an inner product, as it fails axiom 3. This is because $c(\overrightarrow{u}, \overrightarrow{v}) = c((u_1v_1)^2 + (u_2v_2)^2 + (u_3v_3)^2)$, but $(c\overrightarrow{u}, \overrightarrow{v}) = c^2u_1^2v_1^2 + c^2u_2^2v_2^2 + c^2u_3^2v_3^2$

17. (a) $3(5) + 4(-12) = -33$
    (b) $\sqrt{3^2 + 4^2} = 5$
    (c) $\sqrt{5^2 + (-12)^2} = 13$
    (d) $\sqrt{(-2)^2 + (16)^2} = 2\sqrt{65}$

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