Linear Algebra 4.3 Homework

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1. W is a subspace of $\mathbf{V}$
   (a) W contains the origin, and is therefore not empty ✓
   (b) $W \leq \mathbf{V}$ ✓
   (c) $(x_1, x_2, x_3, 0) + (y_1, y_2, y_3, 0) = (x_1 + y_1, x_2 + y_2, x_3 + y_3, 0)$ closed under addition ✓
   (d) $c(x_1, x_2, x_3, 0) = (cx_1, cx_2, cx_3, 0)$ closed under multiplication ✓

4. W is a subspace of $\mathbf{V}$
   (a) W contains the origin, and is therefore not empty ✓
   (b) $W \leq \mathbf{V}$ ✓
   (c) $w_1 = \begin{bmatrix} a_1 & b_1 \\ a_1 - 2b_1 & 0 \\ 0 & c_1 \end{bmatrix}$ and $w_2 = \begin{bmatrix} a_2 & b_2 \\ a_2 - 2b_2 & 0 \\ 0 & c_2 \end{bmatrix}$, then
   $w_1 + w_2 = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ a_1 + a_2 - 2(b_1 + b_2) & 0 \\ 0 & c_1 + c_2 \end{bmatrix}$, where $\begin{array}{c} a = a_1 + a_2 \\ b = b_1 + b_2 \\ c = c_1 + c_2 \end{array}$
   closed under addition ✓
   (d) $cw_1 = c \begin{bmatrix} a_1 & b_1 \\ a_1 - 2b_1 & 0 \\ 0 & c_1 \end{bmatrix} = \begin{bmatrix} ca_1 & cb_1 \\ ca_1 - 2cb_1 & 0 \\ 0 & cc_1 \end{bmatrix}$, where $\begin{array}{c} a = ca_1 \\ b = cb_1 \\ c = cc_1 \end{array}$
   closed under multiplication ✓

5. W is a subspace of $\mathbf{V}$
   (a) W contains the origin, and is therefore not empty ✓
   (b) $W \leq \mathbf{V}$ ✓
   (c) $f + g$ is closed under addition ✓
   (d) cf is closed under multiplication ✓

8. It is not closed under multiplication. Given some vector $c\langle 2, x_1, x_2$, where $c \neq 1$, the value changes, so it is not a subspace

12. It is not closed under addition. Given $v_1 = x + 1$ and $v_2 = 1 - x$, the sum is 2, which is not a linear function of the form $ax + b$

15. It is not closed under multiplication. Given a value $c \neq 1$, cW is not in the vector space **V**

16. It is not closed under addition. Given another $M_{3,1}$ matrix, for example, $\begin{bmatrix} \sqrt{a} & 0 & a \end{bmatrix}$, and adding it to the original matrix generates a matrix that is not in **V**

21. No, it does not contain the origin

23. Yes, because it contains the origin, and is closed under multiplication and addition

27. Yes, because it contains the origin, and is closed under multiplication and addition

30. Yes, because it is not empty, and is closed under multiplication and addition

31. No, because it is not closed under multiplication

33. No, because it is not closed under addition

43. (a) True – This is one of the four axioms that a subspace must follow
    (b) True – If **V** and **W** are subspaces of the same vector space, then anything contained within **V** and **W** must also be a subspace
    (c) False – Although this is possible, it is not definite, as **U** could be smaller than **V**, which is smaller than **U**

44. (a) True – The origin must exist in a vector space, and a subspace can be equal to the vector space itself
    (b) True – The origin must always be contained in a subspace
    (c) True – This is one of 4 axioms that a subspace must follow
    (d) False – It is possible that something contained within a vector space is not itself a vector space

47. (a) $C(-\infty, \infty)$ is a subspace of $F(-\infty,)$
        i. $C(-\infty, \infty)$ contains the origin ✓
        ii. $C(-\infty, \infty) \leq F(-\infty, \infty)$ ✓
        iii. $f + g$ is closed under addition ✓
        iv. cf is closed under multiplication ✓

48. (a) S contains the origin ✓
    (b) $S \leq C[0, 1]$ ✓
    (c) $f + g$ is continuous because it is integrable and therefore closed under addition✓

(d) cf is still continuous, and, therefore, closed under multiplication ✓

51. W is a subspace of $\mathbf{V}$

(a) Because W is a subspace, it is closed under multiplication. Since $a\mathbf{x}$ and $b\mathbf{y}$ are in W, so is $a\mathbf{x} + b\mathbf{y}$.

(b) If $a = 1$ and $b = 0$, $a\mathbf{x}$ is in W. Therefore, if $a = 1$ and $b = 1$, $a\mathbf{x} + b\mathbf{y}$ is in W, meaning W is closed under addition and scalar multiplication.

52. W is a subspace of $\mathbf{V}$

(a) W is not empty ✓

(b) $W \leq \mathbf{V}$ ✓

(c) $\overrightarrow{\mathbf{x}}_1 = ax_1 + by_1 + cz_1$ and $\overrightarrow{\mathbf{x}}_2 = ax_2 + by_2 + cz_2$, then $\overrightarrow{\mathbf{x}}_1 + \overrightarrow{\mathbf{x}}_2 = a(x_1 + x_2) + b(y_1 + y_2) + c(z_1 + z_2)$, where $\begin{matrix} x = x_1 + x_2 \\ y = y_1 + y_2 \\ z = z_1 + z_2 \end{matrix}$ closed under addition ✓

(d) $k\overrightarrow{\mathbf{x}}_1 = kax_1 + kby_1 + kcz_1$, where $\begin{matrix} x = kx_1 \\ y = ky_1 \\ z = kz_1 \end{matrix}$ closed under multiplication ✓

54. W is a subspace $\mathbf{V}$

(a) W contains the origin $(0, 0, \ldots, 0)$ ✓

(b) $W \leq \mathbb{R}^n$ ✓

(c) $A\mathbf{x}_1 + A\mathbf{x}_2 = \mathbf{0} + \mathbf{0} = \mathbf{0}$ closed under addition ✓

(d) $c(A\mathbf{x}) = cA\mathbf{x} = c\dot{0} = 0$ closed under multiplication ✓