Determinants & Elementary Row Operations

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February 24, 2021

- Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices and $\mathbf{B}$ be a matrix resulting from $\mathbf{A}$ by:

  1. interchanging two rows of $\mathbf{A} \Rightarrow \det(\mathbf{B}) = -\det(\mathbf{A})$
  2. Multiplying row of $\mathbf{A}$ by a non-zero constant $\Rightarrow \det(\mathbf{B}) = c\det(\mathbf{A})$, where c is a scalar
  3. Add multiple of row of $\mathbf{A}$ to another row $\Rightarrow \det(\mathbf{B}) = \det(\mathbf{A})$ (no change)
  4. ex. Given $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6$

     (a) $\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = 6$

     (b) $\begin{vmatrix} 3a & 3b & 3c \\ 2d & 2e & 2f \\ -g & -h & -i \end{vmatrix} = -6(-6) = 36$

     (c) $\begin{vmatrix} a-d & b-e & c-f \\ 2d & 2e & 2f \\ g & h & i \end{vmatrix} = 2(-6) = -12$

- If a matrix has a row of zeros, then the determinant is zero

- If one row or column is a scalar multiple of another, then the determinant is