Linear Algebra 5.2 Homework

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<u>Problems 1, 8, 9, 11, 13, 15, 17, 23, 24, 29, 33, 36, 39, 41, 43, 45, 47, 49, 50, 51, 65, 67, 75, 79, 85</u>

1. This does define an inner product
   (a) $(\overrightarrow{u}, \overrightarrow{v}) = 3u_1v_1 + u_2v_2 = 3v_1u_1 + v_2u_2 = (\overrightarrow{v}, \overrightarrow{u})$ ✓
   (b) $(\overrightarrow{u}, \overrightarrow{v} + \overrightarrow{w}) = 3u_1(v_1 + w_1) + u_2(v_2 + w_2) = 3u_1v_1 + 3u_1w_1 + u_2v_2 + u_2w_2 = (\overrightarrow{u}, \overrightarrow{v}) + (\overrightarrow{u}, \overrightarrow{w})$ ✓
   (c) $c(\overrightarrow{u}, \overrightarrow{v}) = c(3u_1v_1 + u_2v_2) = 3cu_1v_1 + cu_2v_2 = (c\overrightarrow{u}, \overrightarrow{v})$ ✓
   (d) $(\overrightarrow{v}, \overrightarrow{v}) = 3(v_1)^2 + (v_2)^2$ ✓

8. This does define an inner product
   (a) $(\overrightarrow{u}, \overrightarrow{v}) = \frac{1}{2}u_1v_1 + \frac{1}{4}u_2v_2 + \frac{1}{2}u_3v_3 = \frac{1}{2}v_1u_1 + \frac{1}{4}v_2u_2 + \frac{1}{2}v_3u_3 = (\overrightarrow{v}, \overrightarrow{u})$ ✓
   (b) $(\overrightarrow{u}, \overrightarrow{v} + \overrightarrow{w}) = \frac{1}{2}u_1(v_1 + w_1) + \frac{1}{4}u_2(v_2 + w_2) + \frac{1}{2}u_3(v_3 + w_3) = \frac{1}{2}u_1v_1 + \frac{1}{2}u_1w_1 + \frac{1}{4}u_2v_2 + \frac{1}{4}u_2w_2 + \frac{1}{2}u_3v_3 + \frac{1}{2}u_3w_3 = (\overrightarrow{u}, \overrightarrow{v}) + (\overrightarrow{u}, \overrightarrow{w})$ ✓
   (c) $c(\overrightarrow{u}, \overrightarrow{v}) = c(\frac{1}{2}u_1v_1 + \frac{1}{4}u_2v_2 + \frac{1}{2}u_3v_3) = \frac{1}{2}cu_1v_1 + \frac{1}{4}cu_2v_2 + \frac{1}{2}cu_3v_3 = (c\overrightarrow{u}, \overrightarrow{v})$ ✓
   (d) $(\overrightarrow{v}, \overrightarrow{v}) = \frac{1}{2}(v_1)^2 + \frac{1}{4}(v_2)^2 + \frac{1}{2}(v_3)^2$ ✓

9. This does <u>not</u> define an inner product because it fails axiom 4, which states that the inner product of $\overrightarrow{v}$ with itself only equals zero if $\overrightarrow{v}$ itself is zero. This is not true, as, for $\overrightarrow{v} = \langle 0, c\rangle$, the function fails the axiom.

11. This does <u>not</u> define an inner product, as it fails axiom 4. This is because, for any vector $\overrightarrow{v}$ where $v_1 = v_2$, the vector equals zero, which fails axiom 4.

13. This does <u>not</u> define an inner product, as it fails axiom 1. This is because, for any $\overrightarrow{v} = \langle 0, 0, c\rangle$, this fails, as $(\overrightarrow{u}, \overrightarrow{v}) = -u_1u_2u_3$, but $(\overrightarrow{v}, \overrightarrow{u}) = 0$

15. This does <u>not</u> define an inner product, as it fails axiom 3. This is because $c(\overrightarrow{u}, \overrightarrow{v}) = c((u_1v_1)^2 + (u_2v_2)^2 + (u_3v_3)^2)$, but $(c\overrightarrow{u}, \overrightarrow{v}) = c^2u_1^2v_1^2 + c^2u_2^2v_2^2 + c^2u_3^2v_3^2$

17. (a) $3(5)+4(-12)=-33$
    (b) $\sqrt{3^2+4^2}=5$
    (c) $\sqrt{5^2+(-12)^2}=13$
    (d) $\sqrt{(-2)^2+(16)^2}=2\sqrt{65}$

23. (a) $2(8)(8)+3(0)(3)+(-8)(16)=0$
    (b) $\sqrt{2(8)(8)+(-8)(-8)}=8\sqrt{3}$
    (c) $\sqrt{2(8)^2+3(3)^2+(16)^2}=\sqrt{411}$
    (d) $||\overrightarrow{u}-\overrightarrow{v}||=\langle 0,-3,-24\rangle \Rightarrow \sqrt{3(-3)^2+(-24)^2}=3\sqrt{67}$

24. (a) $(1)(2)+2(1)(5)+(1)(2)=14$
    (b) $\sqrt{1^2+2(1)^2+1^2}=2$
    (c) $\sqrt{2^2+2(5)^2+2^2}=\sqrt{58}$
    (d) $\langle -1,-4,-1\rangle \Rightarrow \sqrt{(-1)^2+2(-4)^2+(-1)^2}=\sqrt{34}$

29. (a) $2(2)(-2)+(-4)(1)+(-3)(1)+2(1)(0)=-15$
    (b) $\sqrt{2(2)^2+(-4)^2+(-3)^2+2(1)^2}=\sqrt{35}$
    (c) $\sqrt{2(-2)^2+(1)^2+(1)^2}=\sqrt{10}$
    (d) $\begin{bmatrix} 4 & -5 \\ -4 & 1 \end{bmatrix} \Rightarrow \sqrt{2(4)^2+(-5)^2+(-4)^2+2(1)^2}=\sqrt{75}=5\sqrt{3}$

33. This is an inner product for P_2

    (a) $\langle p,q\rangle = a_0b_0+2a_1b_1+a_2b_2=b_0a_0+2b_1a_1+b_2a_2=\langle q,p\rangle$ ✓
    (b) $\langle p,q+r\rangle = a_0(b_0+c_0)+2a_1(b_1+c_1)+a_2(b_2+c_2)=a_0b_0+a_0c_0+2a_1b_1+2a_1c_1+a_2b_2+a_2c_2=\langle p,q\rangle+\langle p,r\rangle$ ✓
    (c) $c\langle p,q\rangle = c(a_0b_0+2a_1b_1+a_2b_2)=ca_0b_0+2ca_1b_1+ca_2b_2=\langle cp,q\rangle$ ✓
    (d) $\langle p,p\rangle=(a_0)^2+2(a_1)^2+(a_2)^2\geq 0$ ✓

36. (a) $(1)(1)+(1)(0)+\frac{1}{2}(2)=2$
    (b) $\sqrt{1^2+1^2+\left(\frac{1}{2}\right)^2}=\frac{3}{2}$
    (c) $\sqrt{(1)^2+(2)^2}=\sqrt{5}$
    (d) $x-\frac{3}{2}x^2 \Rightarrow \sqrt{1^2+\left(-\frac{3}{2}\right)^2}=\frac{\sqrt{13}}{2}$

39. (a) $\int_{-1}^{1} 4x^2-1\,dx=\left(\frac{4}{3}x^3-x\right)\Big|_{-1}^{1}=\frac{2}{3}$
    (b) $\sqrt{\int_{-1}^{1} 1\,dx}=\sqrt{(x)\Big|_{-1}^{1}}=\sqrt{2}$
    (c) $\sqrt{\int_{-1}^{1}\left(4x^2-1\right)^2\,dx}=\sqrt{2\int_0^1 16x^4-8x^2+1\,dx}=\sqrt{\left(\frac{32}{5}x^5-\frac{16}{3}x^3+2x\right)\Big|_0^1}=\sqrt{\frac{46}{15}}$

(d) $2-4x^2 \Rightarrow \sqrt{2\int_0^1 \left(2-4x^2\right)^2\,dx} = \sqrt{2\int_0^1 4-16x^2+16x^4\,dx} =$
$\sqrt{\left(8x-\frac{32}{3}x^3+\frac{32}{5}x^5\right)\Big|_0^1} = \sqrt{\frac{56}{15}} = \frac{2\sqrt{14}}{\sqrt{15}}$

41. (a) $\int_{-1}^1 xe^x\,dx = \left(xe^x-e^x\right)\Big|_{-1}^1 = \frac{2}{e}$

(b) $\sqrt{2\int_0^1 x^2\,dx} = \sqrt{\left(\frac{2}{3}x^3\right)\Big|_0^1} = \frac{\sqrt{6}}{3}$

(c) $\sqrt{\int_{-1}^1 e^{2x}\,dx} = \sqrt{\left(\frac{1}{2}e^{2x}\right)\Big|_{-1}^1} = \sqrt{\frac{e^2}{2}-\frac{1}{2e^2}}$

(d) $x-e^x \Rightarrow \sqrt{\int_{-1}^1 \left(x-e^x\right)^2\,dx} = \sqrt{\int_{-1}^1 x^2-2xe^x+e^{2x}\,dx} =$
$\sqrt{\left(\frac{1}{3}x^3-2xe^x+2e^x+\frac{1}{2}e^{2x}\right)\Big|_{-1}^1} = \sqrt{\frac{2}{3}-\frac{4}{e}+\frac{e^2}{2}-\frac{1}{2e^2}}$

43. $\cos^{-1}\left(\frac{-33}{5(13)}\right) = 120.5°$

45. $\cos^{-1}\left(\frac{15}{(\sqrt{3(-4)^2+(3)^2})(\sqrt{5^2})}\right) = 66.6°$

47. $\cos^{-1}(0) = 90°$

49. $\cos^{-1}\left(\frac{1}{3}\right) = 70.5°$

50. $\cos^{-1}(0) = 90°$

51. $\int_{-1}^1 x^3\,dx = 0 \Rightarrow \cos^{-1}(0) = 90°$

65. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(x)\cos(x)\,dx = \int_0^{\frac{\pi}{2}} \sin(2x)\,dx =$
$\left(-\frac{1}{2}\cos(2x)\right)\Big|_0^{\frac{\pi}{2}} = 0$, so they are orthogonal

67. $\frac{1}{2}\int_{-1}^1 5x^4-3x^2\,dx \Rightarrow \int_0^1 5x^4-3x^2\,dx = \left(x^5-x^3\right)\Big|0^1 = 0$, so they are orthogonal

75. (a) $\frac{\langle \overrightarrow{u},\overrightarrow{v}\rangle}{\langle \overrightarrow{v},\overrightarrow{v}\rangle}\overrightarrow{v} = \frac{1(1)+3(2)-6(2)}{(-1)^2+1^2+2^2+2^2} = \langle \frac{1}{2},-\frac{1}{2},-1,-1\rangle$

(b) $\frac{\langle \overrightarrow{u},\overrightarrow{v}\rangle}{\langle \overrightarrow{u},\overrightarrow{u}\rangle}\overrightarrow{u} = \frac{1(1)+3(2)-6(2)}{1^2+3^2+(-6)^2} = \langle 0,-\frac{5}{46},-\frac{15}{46},\frac{15}{23}\rangle$

79. $\int_0^1 xe^x\,dx = \left(xe^x-e^x\right)\Big|_0^1 = 1,\ \int_0^1 e^{2x}\,dx = \left(\frac{1}{2}e^{2x}\right)0^1 = \frac{e^2}{2}-\frac{1}{2} \Rightarrow \frac{2}{e^2-1}g = \frac{2}{e^2-1}e^x$

85. (a) False. The dot product is the only euclidean product, but others may be defined.

(b) False. The magnitude of $\overrightarrow{v}$ can only equal zero if $\overrightarrow{v} = 0$