Systems of Linear Equations

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- Linear Equation:
  1. One Unknowns: $3x = 5$
  2. Two Unknowns: $ax + by = 5$
  3. Three Unknowns: $ax + by + cz = 5$
  4. n Unknowns: $a_0 + a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0$
- Solution Set:
  1. With two unknowns, given $a \neq 0$ and $b \neq 0$: $S = \{(x, y)|ax + by = c\}$
  2. Empty Set: $a = 0$, $b = 0$, $c \neq 0$
  3. Whole xy plane: $a = 0$, $b = 0$, $c = 0$
- Systems of Linear Equations:
  1. Example of a system: $\begin{array}{ll} x - 2y = 7 & (E_1) \\ 3x + y = 7 & (E_2) \end{array}$
  2. Solved with elimination or back substitution
  3. For the given example: $S = \{(3, -2)\}$
- Operations on Equations:
  1. Two Equations are Interchanged
  2. Multiply and Equation by a non-zero Constant
  3. A Constant Multiple of one Equation can be Added to Another
- Types of Solutions:
  1. Consistent System $-$ One Unique (!) Solution
  2. Inconsistent System $-$ No Solutions (ø)

3. Dependent System − Infinitely Many Solutions

- Example: $\begin{array}{ll} 3x + y = 7 & (E_1) \\ 2y = 14 - 6x & (E_2) \end{array}$

1. Dependent System − Set Up A Parameter
2. Let $x = t$, then $y = 7 - 3t$. This means the set is $S = \{(t, 7 - 3t) \,|\, t \in \mathbb{R}\}$

- Systems of Linear Equations with 3 Unknowns

1. $a_1x_1 + a_2x_2 + a_3x_3 = d$ (plane)
2. Types of Solutions:
   (a) Ordered Triple (Point) − Consistent
   (b) Empty Set (ø) − Inconsistent
   (c) Same Plane − Dependent
   (d) Line of Intersection − Consistent
3. Example: $\begin{array}{ll} 5x + 4y - 2z = 9 & (E_1) \\ 5x + 5y + z = 16 & (E_2) \\ 5x + 4y - z = 10 & (E_3) \end{array}$

$$\begin{aligned} E_1 - E_2 \Longrightarrow -y - 3z = -7\,(E_4) \\ E_1 - E_3 \Longrightarrow z = 1 \\ E_4(y, 1) \Longrightarrow y = 4 \\ E_1(x, 4, 1) \Longrightarrow 5x + 16 - 2 = 9 \\ 5x = -5 \Longrightarrow x = -1 \\ S = \{(-1, 4, 1)\} \end{aligned}$$