

Linear Algebra 2.1 Homework

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1.

$$x = -4$$

$$y = 22$$

3.

$$2x + 1 = 5$$

$$x = \frac{5-1}{2} = 2$$

$$3y - 5 = 4$$

$$y = \frac{4+5}{3} = 9$$

5. (a)

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ -2 & -1 \end{bmatrix}$$

(c)

$$2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 0 & 0 \end{bmatrix}$$

(e)

$$\begin{bmatrix} .5 & 1 \\ 1 & .5 \end{bmatrix} + \begin{bmatrix} -3 & -2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} -2.5 & -1 \\ 5 & 2.5 \end{bmatrix}$$

7. (a)

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix}$$

(c)

$$2 \begin{bmatrix} 2 & 1 & 1 \\ -1 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix}$$

(d)

$$\begin{bmatrix} 4 & 2 & 2 \\ -2 & -2 & 8 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 5 & -2 \\ 1 & -3 & 10 \end{bmatrix}$$

(e)

$$\begin{bmatrix} 1 & .5 & .5 \\ -.5 & -.5 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 4 \\ -3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2.5 & 4.5 \\ -3.5 & .5 & 0 \end{bmatrix}$$

9. (a)

$$\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

(b)

$$\begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} - \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

(c)

$$2 \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 0 & 6 \\ -2 & -8 & 0 \end{bmatrix}$$

(d)

$$2 \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} - \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

(e)

$$\frac{1}{2} \begin{bmatrix} 6 & 0 & 3 \\ -1 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 8 & -1 \\ 4 & -3 \end{bmatrix} = \text{Not Possible!}$$

11. (a) $c_{21} = 2(-3) - 3(0) = -6$

(b) $c_{13} = 2(4) - 3(-7) = 29$

13.
$$\begin{cases} 4x = 2y + 8 \\ 4y = 2z + 2x \\ 4z = -2x + 10 \\ -4 = 2 - 2x \end{cases}$$

$$-x = -3$$

$$x = 3$$

$$4z = -2(3) + 10$$

$$z = \frac{4}{4}$$

$$z = 1$$

$$4y = 2(1) + 2(3)$$

$$y = \frac{8}{4}$$

$$y = 2$$

15. (a)

$$\begin{bmatrix} 1(2) + 2(-1) & 1(-1) + 2(8) \\ 4(2) + 2(-1) & 4(-1) + 2(8) \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 6 & 12 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2(1) - (4) & 2(2) - (2) \\ -1 + 8(4) & -2 + 8(2) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 31 & 14 \end{bmatrix}$$

17. (a)

$$\begin{bmatrix} 2(0) + 4 + 3(-4) & 2(1) - 1 - 3 & 2(2) - 3 + 3(-2) \\ 5(0) - 4 - 2(-4) & 5 + 1 + 2 & 5(2) + 3 - 2(-2) \\ 2(0) + 2(-4) + 3(-4) & 2 + 2 - 3 & 2(2) + 2(3) + 3(-2) \end{bmatrix} = \begin{bmatrix} -8 & -2 & -5 \\ 4 & 8 & 17 \\ -20 & 1 & 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 0(2) + 1(5) + 2(2) & 0(-1) + 1(1) + 2(2) & 0(3) + 1(-2) + 2(3) \\ -4(2) + 5 + 3(2) & 4 + 1 + 3(2) & -4(3) - 2 + 3(3) \\ -4(2) - 5 - 2(2) & 4 - 1 - 2(2) & -4(3) + 2 - 2(3) \end{bmatrix} = \begin{bmatrix} 9 & 5 & 4 \\ 3 & 11 & -5 \\ -17 & -1 & -16 \end{bmatrix}$$

20. (a)

$$\begin{bmatrix} 3 + 2(2) + 1 & 3(2) - 2 - 2 \\ -3 + 0(2) + 4 & -3(2) + 0(-1) + 4(-2) \\ 4 - 2(2) - 4 & 4(2) + 2 - 4(-2) \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 1 & -14 \\ -4 & 18 \end{bmatrix}$$

(b) Not Possible!

21. (a)

$$\begin{bmatrix} 3(2) + 2(3) + 1(0) \end{bmatrix} = \begin{bmatrix} 12 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 2(3) & 2(2) & 2 \\ 3(3) & 3(2) & 3 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 & 2 \\ 9 & 6 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

28. (a) Not Possible!

(b)

$$\begin{bmatrix} 1 + 6(6) & 6(13) & 3 + 6(8) & -2 + 6(-17) & 4 + 6(20) \\ 4 + 2(6) & 2(13) & 4(3) + 2(8) & 4(-2) + 2(-17) & 4(4) + 2(20) \end{bmatrix} = \begin{bmatrix} 37 & 78 & 51 & -104 & 124 \\ 16 & 26 & 28 & -42 & 56 \end{bmatrix}$$

29. Defined, 3x4

31. Defined, 4x2

33. Defined, 3x2

35. Undefined, the two can not be added because they have different dimensions

37.

$$\begin{aligned}
 & \left[\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ 1 & -2 & 2 & 0 \end{array} \right] \\
 & R_1 - R_2 \rightsquigarrow R_1 \\
 & \left[\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 1 & -2 & 2 & 0 \end{array} \right] \\
 & R_2 - R_1 \rightsquigarrow R_2 \\
 & \left[\begin{array}{ccc|c} 1 & 1 & -3 & 0 \\ 0 & -3 & 5 & 0 \end{array} \right] \\
 & R_2 + \frac{1}{3}R_1 \rightsquigarrow R_2 \text{ and } \frac{1}{3}R_2 \rightsquigarrow R_2 \\
 & \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{3} & 0 \\ 0 & 1 & \frac{5}{3} & 0 \end{array} \right] \\
 & z = t \\
 & x = \frac{4}{3}t \\
 & y = -\frac{5}{3}t
 \end{aligned}$$

$$45. \quad \left[\begin{array}{ccc} 1 & -5 & 2 \\ -3 & 1 & -1 \\ 0 & -2 & 5 \end{array} \right] \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} -20 \\ 8 \\ -16 \end{array} \right]$$

$$\begin{aligned}
& \left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ -3 & 1 & -1 & 8 \\ 0 & -2 & 5 & -16 \end{array} \right] \\
& R_2 + 3R_1 \rightsquigarrow R_2 \\
& \left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ 0 & -14 & 5 & -52 \\ 0 & -2 & 5 & -16 \end{array} \right] \\
& R_2 - R_3 \rightsquigarrow R_2 \\
& \left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ 0 & -12 & 0 & -36 \\ 0 & -2 & 5 & -16 \end{array} \right] \\
& -\frac{1}{12}R_2 \rightsquigarrow R_2 \\
& \left[\begin{array}{ccc|c} 1 & -5 & 2 & -20 \\ 0 & 1 & 0 & 3 \\ 0 & -2 & 5 & -16 \end{array} \right] \\
& R_1 + 5R_2 \rightsquigarrow R_1 \text{ and } R_3 + 2R_2 \rightsquigarrow R_3 \\
& \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 5 & -10 \end{array} \right] \\
& \frac{1}{5}R_3 \rightsquigarrow R_3 \\
& \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right] \\
& R_1 - 2R_3 \rightsquigarrow R_1 \\
& \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array} \right] \\
& \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}
\end{aligned}$$

$$49. -\begin{bmatrix} 1 \\ 3 \end{bmatrix} - 4\begin{bmatrix} -1 \\ -3 \end{bmatrix} - 2\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$$51. \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 0\begin{bmatrix} -5 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$$

53.

$$\begin{aligned}
\mathbf{A} &= \left(\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \right)^{-1} \\
&\quad \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array} \right] \\
&\quad R_2 - 3R_1 \rightsquigarrow R_2 \\
&\quad \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -3 & 1 \end{array} \right] \\
&\quad R_1 + 2R_2 \rightsquigarrow R_1 \text{ and } -R_2 \rightsquigarrow R_2 \\
&\quad \left[\begin{array}{cc|cc} 1 & 0 & -5 & 2 \\ 0 & 1 & 3 & -1 \end{array} \right] \\
\mathbf{A} &= \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}
\end{aligned}$$

$$55. \quad \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{cases} a + 2c = 6 \\ b + 2d = 3 \\ 3a + 4c = 19 \\ 3b + 4d = 2 \end{cases}$$

$$\begin{aligned}
&\quad \left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 6 \\ 0 & 1 & 0 & 2 & 3 \\ 3 & 0 & 4 & 0 & 19 \\ 0 & 3 & 0 & 4 & 2 \end{array} \right] \\
&\quad R_4 - 2R_2 \rightsquigarrow R_2 \text{ and } R_3 - 2R_1 \rightsquigarrow R_1 \\
&\quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -4 \\ 3 & 0 & 4 & 0 & 19 \\ 0 & 3 & 0 & 4 & 2 \end{array} \right] \\
&\quad R_3 - 3R_1 \rightsquigarrow R_3 \text{ and } R_4 - 3R_2 \rightsquigarrow R_4 \\
&\quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 4 & 0 & -2 \\ 0 & 0 & 0 & 4 & 14 \end{array} \right] \\
&\quad .25R_3 \rightsquigarrow R_3 \text{ and } .25R_4 \rightsquigarrow R_4 \\
&\quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{7}{2} \end{array} \right] \\
S &= \left\{ \left(7, -4, -\frac{1}{2}, \frac{7}{2} \right) \right\}
\end{aligned}$$

$$57. \mathbf{A}^2 = \left(\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right)^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$59. \text{ For a diagonal matrix, } \mathbf{AB} = \mathbf{BA} = \begin{bmatrix} -10 & 0 \\ 0 & -12 \end{bmatrix}$$

61. (a)

For \mathbf{AB} and \mathbf{BA}

$$\mathbf{A} = [a_{ij}]$$

$$\mathbf{B} = [b_{ij}]$$

(b)

For \mathbf{AB}

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

For \mathbf{BA}

$$c_{ij} = \sum_{k=1}^n b_{ik} a_{kj}$$

(c)

For \mathbf{AB}

When $i \neq j$

$a_{ik} b_{kj} = 0$ because it is not on the diagonal

When $i = j$

$$a_{ik} b_{kj} \neq 0$$

For \mathbf{BA}

When $i \neq j$

$b_{ik} a_{kj} = 0$ because it is not on the diagonal

When $i = j$

$$b_{ik} a_{kj} \neq 0$$

$\mathbf{AB} = \mathbf{BA}$ if $b_{ik} a_{kj} = a_{ik} b_{ij}$

Because $i = j, k = i = j$

$$\therefore b_{kk} a_{kk} = a_{kk} b_{kk}$$

$$63. 1 - 2 + 3 = 2$$

67. (a)

$$\text{Tr}(\mathbf{A}) = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} = a_1 + d_1$$

$$\text{Tr}(\mathbf{B}) = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = a_2 + d_2$$

$$\text{Tr}(\mathbf{A}) + \text{Tr}(\mathbf{B}) = a_1 + a_2 + d_1 + d_2$$

$$\text{Tr}(\mathbf{A} + \mathbf{B}) = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix} = a_1 + a_2 + d_1 + d_2$$

$$\therefore \text{Tr}(\mathbf{A} + \mathbf{B}) = \text{Tr}(\mathbf{A}) + \text{Tr}(\mathbf{B})$$

(b)

$$\mathbf{A} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

$$\text{Tr}(c\mathbf{A}) = c(x + w)$$

$$c\text{Tr}(\mathbf{A}) = c(x + w)$$

$$\therefore \text{Tr}(c\mathbf{A}) = c\text{Tr}(\mathbf{A})$$

69. Multiplication of a similar matrix always, no matter the order, results in the same product matrix. Therefore, $x = -y$, and $w = z = c$, where c is any real constant

$$71. \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1}$$

$$\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1}$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array} \right]$$

$$R_1 - R_2 \rightsquigarrow R_1$$

$$\left[\begin{array}{cc|cc} 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 1 \end{array} \right]$$

R_1 became zero, so this matrix is not invertible