Linear Algebra 3.1 Homework

Michael Brodskiy
Instructor: Prof. Knight

3. $2(4) - 1(3) = 5$

5. $5(3) + 12 = 27$

11. $(\lambda - 3)(\lambda - 1) - 4(2) = \lambda^2 - 4\lambda - 5$

13. (a) $M_{11} = 4$
    (b) $M_{12} = 3$
    (c) $M_{21} = 2$
    (d) $M_{22} = 1$

    (a) $C_{11} = 4$
    (b) $C_{12} = -3$
    (c) $C_{21} = -2$
    (d) $C_{22} = 1$

15. (a) $M_{11} = 23$
    (b) $M_{12} = -8$
    (c) $M_{13} = -22$
    (d) $M_{21} = 5$
    (e) $M_{22} = -5$
    (f) $M_{23} = 5$
    (g) $M_{31} = 7$
    (h) $M_{32} = -22$
    (i) $M_{33} = -23$

    (a) $C_{11} = 23$
    (b) $C_{12} = 8$
    (c) $C_{13} = -22$
    (d) $C_{21} = -5$
    (e) $C_{22} = -5$
    (f) $C_{23} = -5$
    (g) $C_{31} = 7$
    (h) $C_{32} = 22$
    (i) $C_{33} = -23$

17. (a) $4(-5) + 5(-5) + 6(-5) = -75$
    (b) $2(8) + 5(-5) - 3(22) = -75$

19. About Row 2: $3[-1(3(4) - 4(-2))] + 2(1) = -58$

25. About Row 2: $3[-1(y + 1)] + 2(x + 1) = -3y + 2x - 1$

27. About Column 1: $5[6(2) + 12(-1)] + 4[3(2) + 6(-1)] = 0$

29. About Row 1:

    (a) $w\{-15[32(17)] - 24[-840 - 396] + 30[32(46)]\}$
    (b) $-x\{21[32(17)] - 24[350 + 40(18)] + 30[-32(50)]\}$
    (c) $y\{21[-840 - 396] + 15[350 + 40(18)] + 30[-220 + 40(24)]\}$
    (d) $-z\{21[32(46)] + 15[-32(50)] + 24[-220 + 24(40)]\}$

$$= 65,664w + 62,256x + 12,294 - 24,672z$$

41. About Column 1: $5[0(-2) - 6(0(2) + 0(1)) + 0(2)] = 0$

43. (a) False:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

    (b) True. In such a case, the only possible way to find a determinant is if it equals the first (and only) entry.
    (c) False. That is the definition of a minor. A cofactor could either be equal to the statement, or the negative version of the statement.

44. (a) False. One needs to form the prodcut of the diagonal entries, not the sum.
    (b) True. Generally, it is better to expand on a row or column with the most zeros, but any row or column would work.
    (c) True. Because the formula involves multiplying by the entry at the ijth point, multiplying by zero would result in zero, so this is true.

45. $(x + 3)(x + 2) - 2 = 0 \to x^2 + 5x + 4 = 0 \to (x + 1)(x + 4) = 0 \to x = -1, -4$

51. $(\lambda)\left((\lambda^2 + \lambda) - 2\right) = 0 \to \lambda\left(\lambda^2 + \lambda - 2\right) = 0 \to \lambda(\lambda - 1)(\lambda + 2) = 0 \to \lambda = 0, 1, -2$

63. $wz - xy = -(xy - wz)$ True

64. $cwz - cxy = c(wz - xy)$ True

65. $wz - xy = w(z + cy) - y(x + cw) \to wz + \cancel{cyw} - xy \cancel{- cyw}$ True

67.

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$R_2 - R_3 \widetilde{\rightarrow} R_2 \text{ and } R_1 - R_3 \widetilde{\rightarrow} R_1$$

$$\begin{vmatrix} 0 & x-z & x^2-z^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix} = (x-z)(y^2-z^2) - (y-z)(x^2-z^2)$$

$$(x-z)(y^2-z^2) - (y-z)(x^2-z^2) = (x-z)(z+y)(y-z) - (y-z)(x+z)(x-z)$$

$$(x-z)(z+y)(y-z) - (y-z)(x+z)(x-z) = (x-z)(y-z)\,(z+y-x-z)$$

$$= (x-z)(y-z)\,(z+y-x-z) = (x-z)(y-z)(y-x)$$

$$(x-z)(y-z)(y-x) = (z-x)(z-y)(y-x)$$