Linear Algebra 4.5 Homework

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Problems 1-6, 10, 13, 15, 18, 22, 23, 25, 33, 37, 41, 45, 47, 67, 73, 78, 81

1. $\{(1,0,0,0,0,0),(0,1,0,0,0,0),(0,0,1,0,0,0),(0,0,0,1,0,0),(0,0,0,0,1,0),(0,0,0,0,0,1)\}$
2. $\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$
3. $$\left\{\begin{matrix} \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, & \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}, & \begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, \\ \begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix}, & \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}, & \begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}, \\ \begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}, & \begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}, & \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix} \end{matrix}\right\}$$
4. $\left\{\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$
5. $\{1,x,x^2,x^3,x^4\}$
6. $\{1,x,x^2\}$

10. The set does not span $\mathbb{R}^2$ so it is not a basis
13. It is not basis because it is linearly dependent and does not span $\mathbb{R}^2$
15. The set is linearly dependent, and does not span $\mathbb{R}^3$, so it is not a basis for it
18. The set does not span $\mathbb{R}^3$

22. The set is linearly dependent

23. The set is linearly dependent

25. The set does not span P_2

33. The set is linearly independent

37. Not a basis because it is linearly dependent

41. It is a basis for $\mathbb{R}^3$

    (a) S is linearly independent ✓
    (b) S spans $\mathbb{R}^3$ ✓

45. It is a basis for $\mathbb{R}^4$

    (a) S is linearly independent ✓
    (b) S spans $\mathbb{R}^4$ ✓

47. It is a basis for P_3

    (a) $\begin{vmatrix} 1 & -4 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix} \neq 0$ so it is linearly independent ✓
    (b) S spans P_3 ✓

67. $\{(0,1),(1,0)\}, \{(1,1),(0,1)\}, \{(1,1),(1,0)\}$

73. (a) W forms a line
    (b) There is only one term, so it is a basis of itself: $\{(2,1,-1)\}$
    (c) One term, so dimension $= 1$

78. (a) Basis: $\{(1,0,1,2)\}, \{(4,1,0,-1)\}$
    (b) Two terms, so dimension $= 2$

81.

$$\begin{aligned} \text{If } S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}, \text{ then there exists a solution to:} \\ \mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_n = 0 \\ \text{The same can be said for } S = \{c\mathbf{v}_1, c\mathbf{v}_2, \ldots, c\mathbf{v}_n\} \\ c\mathbf{v}_1 + c\mathbf{v}_2 + \cdots + c\mathbf{v}_n = 0 \\ \text{Multiplying both sides by } \frac{1}{c} \text{ the following is obtained:} \\ \mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_n = 0 \\ \therefore \text{ both are basis for } V \end{aligned}$$