Elementary Matrices and Row Equivalence

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- To interchange R_1 and R_2, one could use the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. This is called an elementary matrix, which is obtained with exactly one operation on $\mathbf{I}$
- To multiply R_1 by 2, one could use the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
- To make R_2 equal to $R_2 - 2R_1$, one could use the matrix $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
- The inverse of an elementary matrix reverses its operation (for example, if $\mathbf{E}_1$ subtracts R_2 from R_1, then $\mathbf{E}_1^{-1}$ adds R_2 to R_1)
- Fundamental Theorem for Matrices:

  1. If $\mathbf{A}$ is a square matrix of order n, then all of the following conditions are equivalent:

     (a) $\mathbf{A}$ is invertible

     (b) $\mathbf{A}x = \mathbf{B}$ has a unique solution for any n by one column matrix $\mathbf{B}$

     (c) Only solution of $\mathbf{A}x = 0$ is the trivial solution $x = 0$

     (d) $\mathbf{A}\widetilde{R}\mathbf{I}$

     (e) $\mathbf{A}$ can be written as the product of elementary matrices