Quiz 2

Michael Brodskiy
Professor: I. Salama

October 17, 2024

I.

- Linearity: **Linear**
  We may test for linearity by checking that:
  $$ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$$
  First, we find:
  $$ax_1(t) + bx_2(t) = ae^{2t}x_1(t-1)\cos\left(\frac{\pi}{4}t\right) + be^{2t}x_2(t-1)\cos\left(\frac{\pi}{4}t\right)$$
  And then we find:
  $$ay_1(t) + by_2(t) = ae^{2t}x_1(t-1)\cos\left(\frac{\pi}{4}t\right) + be^{2t}x_2(t-1)\cos\left(\frac{\pi}{4}t\right)$$
  Therefore, because the condition from above is met, the system is linear
- Time-Variance: **Time-Varying**
  We may text for time-variance by checking:
  $$x(t-t_o) = y(t-t_o)$$
  First, we find:
  $$x(t-t_o) = e^{2t}x(t-t_o-1)\cos\left(\frac{\pi}{4}t\right)$$
  Then, we find:
  $$y(t-t_o) = e^{2(t-t_o)}x(t-t_o-1)\cos\left(\frac{\pi}{4}(t-t_o)\right)$$
  Therefore, because $x(t-t_o) \neq y(t-t_o)$, the system is time-varying
- Causality: **Causal** — By observation, we may see that the function depends on only past or present values of t, and, therefore, it is causal

- Stability: **Unstable** — We may find whether the function is stable by integrating over its entire range and determining whether the value is finite. By observation, the positive exponential, e^{2t}, would diverge, and, therefore, the integrate would evaluate to infinity. Therefore, the system is unstable
- Invertibility: **Non-invertible** — We know that a system is non-invertible if two values of t produce the same response. Since the sinusoid oscillates, we may observe that there are many repeated values. Most evidently, we may see that $y(t)$ would be zero for every $t = 4n + 2$. Therefore, the system is not invertible

II. A. We may find $y_1[n]$ by taking $y_1[n] = x_1[n] * h[n]$. Given that $x_1[n]$ consists only of a delta function, we may use the sifting property to evaluate:

$$x[n] * \delta[n - n_o] = x[n - n_o]$$

We may write our case as:

$$y_1[n] = h[n] * (5\delta[n-2])$$
$$y_1[n] = 5h[n-2]$$

Since we know the system is linear and time invariant, we may write:

$$\boxed{y_1[n] = 5\left(\frac{1}{4}\right)^{n-2} u[n-2]}$$

B. To simplify analysis, we may define $x_2[n] = 4\left(\delta[n+1] + \delta[n] + \delta[n-1]\right)$. Using the same sifting property as part (A), we write:

$$y_2[n] = h[n] * x_2[n]$$
$$y_2[n] = 4h[n] * (\delta[n+1] + \delta[n] + \delta[n-1])$$
$$y_2[n] = 4h[n+1] + 4h[n] + 4h[n-1]$$

We may write this as:

$$y_2[n] = 4\left(\frac{1}{4}\right)^{n+1} u[n+1] + 4\left(\frac{1}{4}\right)^{n} u[n] + 4\left(\frac{1}{4}\right)^{n-1} u[n-1]$$

Given that $4 = \frac{1}{4}^{-1}$, we may simplify this as:

$$\boxed{y_2[n] = \left(\frac{1}{4}\right)^{n} u[n+1] + \left(\frac{1}{4}\right)^{n-1} u[n] + \left(\frac{1}{4}\right)^{n-2} u[n-1]}$$