Lecture 5 — Linear Time Invariant Systems

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- The Impulse Response
  - Linear algebra concepts apply to LTI system analysis; if the response to basis input signals are known, any general input can be expressed as a linear combination of these basis signals, allowing the output to be determined
  - The system is fully characterized by its responses to the basis signals
  - In the time domain representation, we use delayed unit impulse functions as basis signals
  - The impulse response, $h(t)/h[n]$, of a linear time invariant system is defined as the response of the system to a unit impulse input, $x(t) = \delta(t)/x[n] = \delta[n]$
  - The response of a linear time invariant system with impulse response $h[n]$ to a general input $x[n]$ can be obtained using the convolution sum; the convolution sum sifts through the sequence values, $x[k]$. respresented by an impulse located at $n = k$, and finds the corresponding response given by $h[n-k]$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \Longrightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- The Convolution Sum
  - The convolution sum may also be expressed as:

$$y[n] = x[n] * h[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

- Geometric Series

– Sums of geometric series are given by:

$$\sum_{k=0}^{n} r^k = \frac{(1 - r^{n+1})}{1 - r}$$

$$\sum_{k=n_1}^{n_2} r^k = \frac{(r^{n_1} - r^{n_2+1})}{1 - r}$$

$$\sum_{k=n_1}^{\infty} r^k = \frac{r^{n_1}}{1 - r}$$

  * For $|r| < 1$