Lecture 4 — Classifications/Interconnections of Systems

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- System Representation
  - A system takes a signal as an input and transforms it into an output
  - This is written as $x(t)$ passed through transformation function $T\{\cdots\}$ makes $y(t)$
- Linear Systems and the Principle of Superposition
  - A homogenous system has zero output for zero input (if $x(t)$ transforms to $y(t)$, then $ax(t) \to ay(t)$)
  - Additive: $x_1(t)$ causes response $y_1(t)$ and $x_2(t)$ causes response $y_2(t)$, then $x_1(t) + x_2(t)$ causes $y_1(t) + y_2(t)$
  - A linear system is both homogenous and additive (the superposition principle applies)
- Linearity
  - The system with an input-output relationship $y(t) = t^2 x(t)$ is linear
  - We can prove linearity by saying:

$$x_1(t) \to y_1(t) = T\{x_1(t)\} = t^2 x_1(t) \quad \text{and} \quad x_2(t) \to y_2(t) = T\{x_2(t)\} = t^2 x_2(t)$$

  - and then proving:

$$T\{a_1 x_1(t) + a_2 x_2(t)\} = t^2(a_1 x_1(t) + a_2 x_2(t)) = a_1 t_1(t) + a_2 y_2(t)$$

  - The system with an input-output relationship $y(t) = x^2(t)$ is non-linear
- Incrementally Linear Systems
  - The system described by $y[n] = 2x[n] + 4$ is non-linear

- The system has a non-zero output for a zero input
- The system is sometimes described as incrementally linear, meaning that the difference between two output responses is a linear function of the difference between their corresponding inputs

$$y_2[n] - y_1[n] = 2(x_2[n] - x_2[n])$$

- A system described by a linear constant coefficient differential or difference equation is incrementally linear

$$\frac{d}{dt}v_c(t) + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

- The system's response can be split into two parts: the zero-input response, due to the initial conditions, and the zero-state response
- In general, a system described by a linear constant coefficient differential or difference equation is incrementally linear; it has a response equal to the sum of a zero-input response and the zero-state response

• Time Invariance

  - A system is time invariant if the behavior/parameters of the system are fixed over time; the nature of the response is not expected to change if the experiment is performed now or a few days later
  - A network initially at rest and composed of RLC components and other commonly used active components with fixed parameters is time invariant
  - A system described by a linear constant coefficient differential/difference equation is time invariant if initially at rest. If not initially at rest or if the coefficients of the differential/difference become time-dependent, the system becomes time varying

• Dynamic Versus Instantaneous System

  - Instantaneous or Memoryless System
    * The output of the system at any instants depends only on the current instant of the input; the history of the input or system response do not affect the current output
    * Resistive networks are memoryless
  - Dynamic Systems or Systems with Memory
    * A system is said to be dynamic if the response is determined by the input signal over the past interval of time and/or system response over the past interval of time

• Causal Versus Non-Causal Systems

- Causal Systems
  - The output of a causal system at any instant depends on the current and previous values of the input; a causal system is non-anticipative
  - All memoryless systems are causal
- Non-Causal Systems
  - A system is said to be non-causal if the response depends on future values of the input

- Stability
  - The input is bounded if there exists a positive finite value B_x such that:

$$|x(t)| < B_x < \infty \quad \forall t$$
$$|x[n]| < B_x < \infty \quad \forall n$$

  - A system is stable if for every bounded input, the output is also bounded; *i.e.* there exists a positive finite value, B_y such that:

$$|y(t)| < B_y < \infty \quad \forall t$$
$$|y[n]| < B_y < \infty \quad \forall n$$

  - This type of stability is referred to as bounded input/bounded output (BIBO) stability
  - Stability of physical systems usually results from a mechanism that dissipates energy, such as a resistor or losst transmission line in an electrical system, and friction in a mechanical system
  - Consider a constant applied force, $f(t) = F$, applied to a vehicle initially at rest; the vehicle will continue to accelerate until the frictional force, which is proportional to the velocity, balances the applied force ($\rho v = F$)
  - The velocity of the vehicle is therefore bounded, and the maximum velocity is given by $v = F/\rho$

- Invertibility and Inverse Systems
  - Consider a system S which processes an input signal, $x(t)/x[n]$ to produce an output $y(t)/y[n]$; if an inverse system S_i exists tha can reproduce the original input signal $x(t)/x[n]$ using the output, $y(t)/y[n]$, the system is said to be invertible
  - Cascading a system with its inverse system results in an identity system, producing an output identical to the input
  - Data transmitted over a communication channel can be distorted due to the channel's non-ideal frequency response; an inverse system for the channel, known as an equalizer, can be used to compensate for this distortion

- To demonstrate that a system is invertible, one must find the inverse system; conversely, to show that a system is non-invertible, it's necessary to identify two different inputs that result in the same output