Lecture 8 — The Fourier Transform

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- A periodic signal, $x(t)$, with period T_o can be expressed as a sum of complex exponentials at the fundamental frequency and its harmonics. The analysis equation may be written as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

- The synthesis equation may be written as:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega$$

- The Fourier Transform exists only if the $j\omega$ axis lies within the ROC of the Laplace Transform
- Fourier Transforms are governed by the Dirichlet conditions:
    1. $x(t)$ is absolutely integrable
    2. $x(t)$ has a finite number of maxima and minima over any finite interval
    3. $x(t)$ has a finite number of finite discontinuities over any finite interval
    4. Note: Periodic signals do not satisfy these conditions but are considered to have Fourier transforms if impulse functions are included in the Fourier representation
- Properties of Fourier Transforms
    - Let $X(\omega) = A(\omega) + jB(\omega)$
    - $X(-\omega) = A(-\omega) + jB(-\omega) = X^*(\omega) = A(\omega) - jB(\omega)$
    - $A(-\omega) = A(\omega)$, real part is an even function
    - $B(-\omega) = -B(\omega)$, imaginary part is an odd function
    - $|X(\omega)| = \sqrt{A^2(\omega) + B^2(\omega)}$ is an even function
    - $\angle X(\omega) = \tan^{-1}\left(\frac{B(\omega)}{A(\omega)}\right)$ is an odd function
    - For a real signal: $x(t) = x^*(t) \rightarrow X(\omega) = X^*(-\omega)$ and $X(-\omega) = X^*(\omega)$