# Lecture 3 — The Unit Step and Unit Impulse Functions

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September 16, 2024

- The Unit Step Function
  - The step function is discontinuous at $t = 0$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

- The Dirac Delta (Unit Impulse) Function
  - As $\epsilon$ reduces to zero, the derivative to the unit step function reduces to a Dirac delta function

$$\frac{d}{dt}[u(t)] = \lim_{\epsilon \to 0} \begin{cases} \frac{1}{\epsilon}, & -\frac{\epsilon}{2} \leq t < \frac{\epsilon}{2} \\ 0, & \text{otherwise} \end{cases}$$

  - The Dirac delta function has an infinite amplitude and unit area, and a scaled impulse $k\delta(t)$ has an area equal to $k$:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

  - Substitute $x = \tau - t \to u(t)$:

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = \int_{0}^{\infty} \delta(t - x)\, dx$$

  - Useful Properties:

$$\int_{-\infty}^{\infty} \delta(t - t_o)\, dt = 1$$

$$\int_{-\infty}^{\infty} A\delta(t - t_o)\, dt = A$$

$$g(t)\delta(t-t_o) = g(t_o)\delta(t-t_o)$$

$$\int_{-\infty}^{\infty} g(t)\delta(t-t_o)\,dt = g(t_o) \text{ (sifting property)}$$

* The derivative property ($x(t)$ is continuous and $t_1 < t_o < t_2$):

$$\int_{t_1}^{t_2} x(t)\dot{\delta}(t-t_o)\,dt = -\dot{x}(t_o)$$

$$\int_{t_1}^{t_2} x(t)\delta^n(t-t_o)\,dt = (-1)^n x^n(t_o)$$

  · Where $\delta^n(t-t_o)$ is the n-th derivative of $\delta(t-t_o)$ and $x^n$ is the n-th derivative of $x(t)$

- The Unit Ramp Function

$$r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = \begin{cases} t, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$