Lecture 6 — Properties of Linear Time Invariant Systems

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October 7, 2024

- Changing the order of cascaded LTI systems does not change the overall response (commutative)
- You can determine the overall response by first applying the input to the first system, computing its output, and then using that output as the input to the second system; alternatively, you can find the impulse response of the equivalent system, $h_{eq} = h_1 * h_2$, and use it to find the overall response, $y = x * h_{eq}$ (associative)
- Two systems in parallel with a single input can be added together to find the output, $y = x * (h_1 + h_2)$ (distributive)
- If an invertible system is cascaded with its inverse system, the output will be the same as the input; the system formed by cascading an invertible system with its inverse is referred to as the identity system $(y(t)/y[n] = x(t) * h(t) * h_i(t)/x[n] * h[n] * h_i[n] \to y(t)/y[n] = x(t)/x[n])$
  - Known as the identity LTI system
- An 0N$^{\text{N¡0 th}}$ order LCCDE differential equation is an implicit description of the output in terms of the input

$$a_N \frac{d^N y(t)}{dt^N} + a_{N-1} \frac{d^{N-1} y(t)}{dt^{N-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_M \frac{d^M x(t)}{dt^M} + \cdots + b_o x(t)$$

- Since the derivative operation is irreversible, to solve for $y(t)$, further information about the conditions of $y(t)$ and its derivatives at the start of the interval is needed. These conditions are referred to as the initial our auxiliary conditions
- The term linear in LCC does not imply that the system is linear, but rather indicates that the equation is represented by linear combinations of the derivative of both input and output

- The complete solution may be written as:

$$y(t) = y_p(t) + y_h(t)$$

  - Where the p component is the particular solution
    - Also known as the forced solution
    - Has the same form of the applied input
    - Represents the steady-state response of the system
  - And the h component is the homogenous solution

- The homogenous solution may be found using:

$$\sum_{n=0}^{N} a_n \frac{d^n y(t)}{dt^n} = 0$$

- The initial rest conditions imply that the system is causal, which means that if the input $x(t) = 0$ for $t < t_o$, then the output can not exist before $t = t_o$. This also enforces linearity and time invariance