Homework 1

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September 10, 2024

1. Express each of the following complex numbers in polar form and plot them

   (a) 8

$$r = \sqrt{8^2 + 0^2} = 8$$
$$\theta = 0$$
$$z(r, \theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(8, 0) = 8(\cos(0) + j\sin(0))$$
$$\therefore \text{ In polar: } \boxed{z = 8}$$

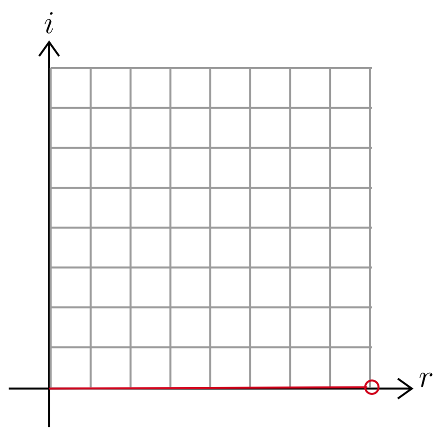


Figure 1: $z = 8$ Plotted on the Imaginary Plane

(b) -5

$$r = \sqrt{(-5)^2 + 0^2} = 5$$
$$\theta = \pi$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(5,\pi) = 5(\cos(\pi) + j\sin(\pi))$$
$$\therefore \text{ In polar: } \boxed{z = -5 = 5e^{\pi j}}$$

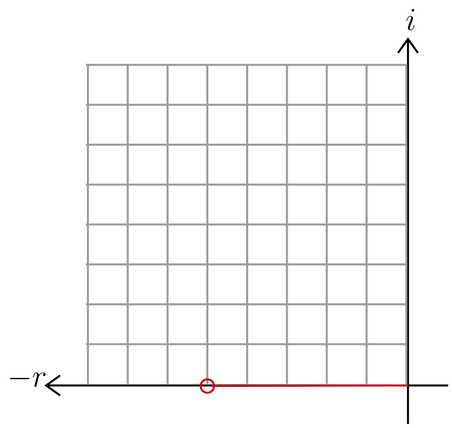


Figure 2: $z = -5$ Plotted on the Imaginary Plane

(c) $2j$

$$r = \sqrt{0^2 + (2)^2} = 2$$
$$\theta = \frac{\pi}{2}$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(2, .5\pi) = 2j$$
$$\therefore \text{ In polar: } \boxed{z = 2j = 2e^{.5\pi j}}$$

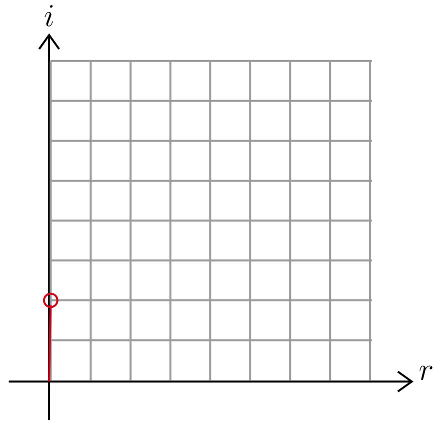


Figure 3: $z = 2j$ Plotted on the Imaginary Plane

(d) $\frac{1}{4}(1-j)^5$

$$.25(1-j)^2(1-j)^3$$
$$.25(-2j)(1-j)(1-j)^2$$
$$.25(-2-2j)(-2j)$$
$$z = j - 1$$
$$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$
$$\theta = \frac{3\pi}{4}$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(\sqrt{2}, .75\pi) = j - 1$$
$$\therefore \text{ In polar: } \boxed{z = j - 1 = \sqrt{2}e^{.75\pi j}}$$

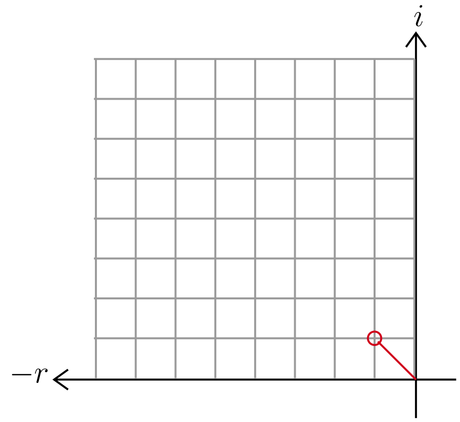


Figure 4: $z = \frac{1}{4}(1-j)^5$ Plotted on the Imaginary Axis

(e) $\frac{(1+j)}{j}e^{\frac{j\pi}{3}}$

$$\frac{(1+j)}{j} \cdot \frac{-j}{-j} = 1 - j$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{b}{a}$$

$$\frac{b}{a} = \sqrt{3}$$

$$b = a\sqrt{3}$$

$$\sqrt{(a\sqrt{3})^2 + a^2} = 1$$

$$4a^2 = \pm 1$$

$$a = \frac{1}{2}$$

$$b = \frac{\sqrt{3}}{2}$$

$$\frac{1}{2}(1-j)(1+\sqrt{3}j) \rightarrow \frac{1}{2}((\sqrt{3}+1) + (\sqrt{3}-1)j)$$

$$r = \frac{1}{2}\sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = \sqrt{(4+2\sqrt{3}) + (4-2\sqrt{3})}$$

$$r = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = .26179$$

$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$

$$z(\sqrt{2}, .26179) = \frac{1}{2}(\sqrt{3}+1) + (\sqrt{3}-1)j$$

$$\therefore \text{ In polar: } \boxed{z = \frac{1}{2}(\sqrt{3}+1) + (\sqrt{3}-1)j = \sqrt{2}e^{.26179j}}$$

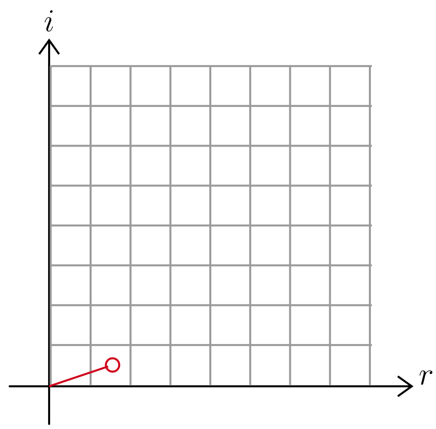


Figure 5: $z = \frac{(1+j)}{j}e^{\frac{j\pi}{3}}$ Plotted on the Imaginary Axis

(f) $(\sqrt{3} - j^5)(1 + j)$

$$j^5 = j \to (\sqrt{3} - j)(1 + j) = (\sqrt{3} + (\sqrt{3} - 1)j + 1)$$
$$(\sqrt{3} + 1) + (\sqrt{3} - 1)j$$
$$r = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = \sqrt{(4 + 2\sqrt{3}) + (4 - 2\sqrt{3})}$$
$$r = \sqrt{8} = 2\sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = .26179$$
$$z(r, \theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(2\sqrt{2}, .26179) = (\sqrt{3} + 1) + (\sqrt{3} - 1)j$$
$$\therefore \text{ In polar: } \boxed{z = (\sqrt{3}+1) + (\sqrt{3}-1)j = 2\sqrt{2}e^{.26179j}}$$

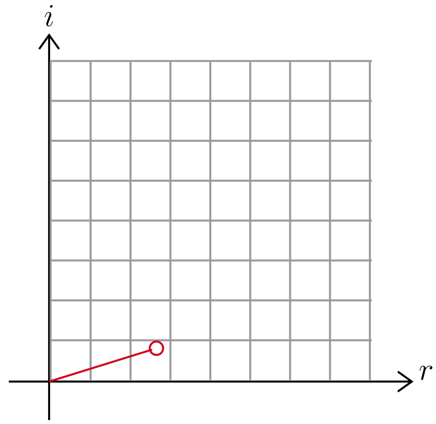


Figure 6: $z = (\sqrt{3} - j^5)(1 + j)$ Plotted on the Imaginary Axis

(g) $\frac{2(\sqrt{3}-j)}{1+j\sqrt{3}}$

$$\frac{2\sqrt{3} - 2j}{1 + j\sqrt{3}} \cdot \frac{1 - j\sqrt{3}}{1 - j\sqrt{3}} = -2j$$

$$r = \sqrt{0^2 + (-2)^2} = 2$$

$$\theta = \frac{3\pi}{2}$$

$$z(r, \theta) = r(\cos(\theta) + j\sin(\theta))$$

$$z(2, 1.5\pi) = -2j$$

$$\therefore \text{ In polar: } \boxed{z = -2j = 2e^{1.5\pi j}}$$

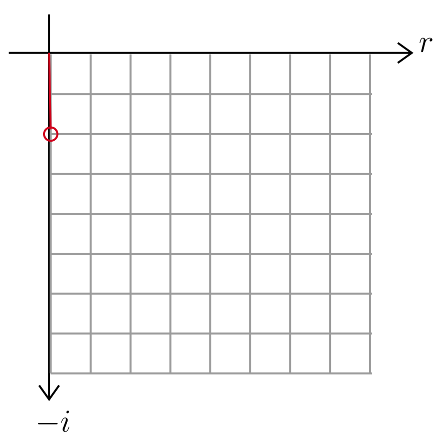


Figure 7: $z = \frac{2(\sqrt{3}-j)}{1+j\sqrt{3}}$ Plotted on the Imaginary Axis

2. Determine the value of E_∞ and P_∞ for each of the following signals and indicate whether the signal is a power or energy signal or neither.

(a) $x_1(t) = \begin{cases} 5e^{j(4t+\pi/3)}, & t \geq 2 \\ 0, & \text{Otherwise} \end{cases}$

$$E_\infty = \int_2^\infty 25\left[\cos\left(4t+\frac{\pi}{3}\right) + j\sin\left(4t+\frac{\pi}{3}\right)\right]^2 dt$$

Period of the sinusoids is $\pi/2$

$$E_\infty = \frac{50}{\pi}\int_2^{2+\frac{\pi}{2}} \left[\cos\left(4t+\frac{\pi}{3}\right) + j\sin\left(4t+\frac{\pi}{3}\right)\right]^2 dt$$

(b) $x_2(t) = \begin{cases} 2+2\cos(t), & 0 < t < 2\pi \\ 0, & \text{Otherwise} \end{cases}$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (2+2\cos(t))^2\, dt$$

$$P_\infty = 6$$

$$\therefore \text{ Power is finite}$$

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} (2+2\cos(t))^2\, dt$$

$$P_\infty = \infty$$

$$\therefore \text{ Energy is infinite}$$

$\boxed{\text{Since power is finite and energy is infinite, this is a power signal}}$

(c) $x_3[n] = \begin{cases} (.5)^n, & n \geq 0 \\ 0, & \text{Otherwise} \end{cases}$

$$E_\infty = \lim_{N\to\infty} \sum_{n=0}^{N} (.25)^n$$

A geometric series must be finite:

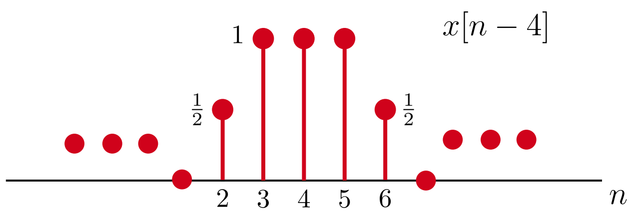
$$E_\infty \approx \left(\frac{1}{1-.25}\right) \approx \frac{4}{3}$$

$$P_\infty = \lim_{N\to\infty} \frac{4/3}{2N+1} \approx 0$$

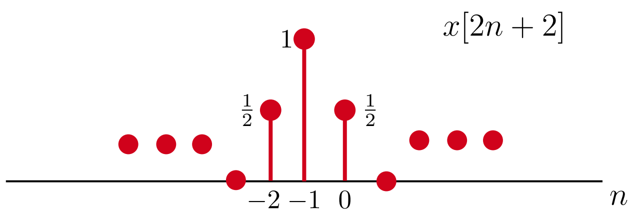
$\boxed{\text{As such, because energy is finite and average power is 0, this is an energy signal}}$

3. For the discrete time signal shown in Figure P1.3, sketch, and carefully label each of the following.

    (a) $x[n-4]$



    (b) $x[2n+2]$



4. For the continuous time signal shown in Figure P1.4, sketch, and carefully label each of the following.

    (a) $x(t+3)$

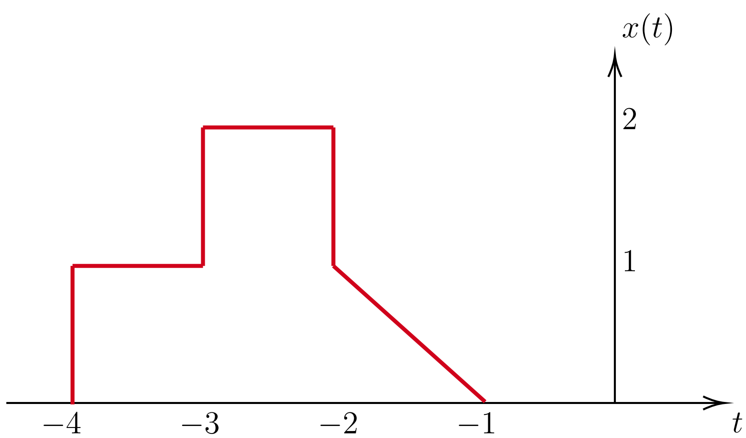


Figure 8: Figure Showing Transformation $x(t) \to x(t+3)$

(b) $x\left(3-\frac{2}{3}t\right)$

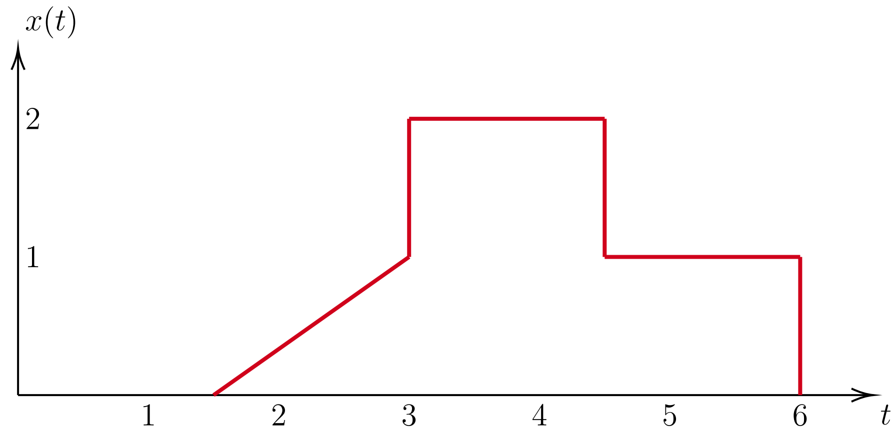


Figure 9: Figure Showing Transformation $x(t) \to x\left(3-\frac{2}{3}t\right)$

5. Determine and sketch the even and odd parts of the signals depicted in Figure P1.5. Label your sketches carefully.

   (a)

   (b)

6. Determine and sketch the even and odd parts of the signal depicted in Figure P1.6. Label your sketches carefully.

7. Express the real part of each of the following signals in the form $Ae^{-at}\cos(\omega t+\phi)$ where A, a, ω and ϕ are real numbers with $A>0$ and $-\pi<\phi\leq\pi$.

   (a) $x_1(t) = 4e^{-2t}\sin\left(10t+\frac{3\pi}{4}\right)\cos\left(10t+\frac{3\pi}{4}\right)$

Per trig identities, we can rewrite this as:

$$2e^{-2t}\sin\left(20t+\frac{3\pi}{2}\right)$$

Per another identity, we can convert sin $\to$ cos:

$$2e^{-2t}\cos\left(\frac{\pi}{2}-20t-\frac{3\pi}{2}\right)$$

$$x_1(t) = 2e^{-2t}\cos\left(-20t-\pi\right)$$

Since $\cos(x)=\cos(-x)$, we finally write:

$$\boxed{x_1(t) = 2e^{-2t}\cos\left(20t+\pi\right)}$$

(b) $x_2(t) = j(1-j)e^{(-5+j\pi)t}$

8. Determine whether each of the following continuous time signals is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x(t) = 5\cos\left(400\pi t + \frac{\pi}{4}\right)$

   (b) $x(t) = 20e^{j(\pi t - 2)}$

   (c) $x(t) = 2\left[\sin\left(50\pi t - \frac{\pi}{3}\right)\right]^2$

   (d) $x(t) = \begin{cases} 2\sin(5\pi t), & t \geq 0 \\ -2\sin(-5\pi t), & t < 0 \end{cases}$

9. Determine whether each of the following discrete time signals is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x[n] = 2\cos\left(\frac{7}{11}n + \frac{\pi}{2}\right)$

   (b) $x[n] = \cos(\pi n) + 4\sin\left(\frac{\pi}{4}n^2\right)$

   (c) $x[n] = 3\sin\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right) - 3\cos\left(\frac{\pi}{6}n + \frac{\pi}{3}\right)$