Lecture 7 — The Laplace Transform

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- For an input $x(t) = e^{st}$ passed through $h(t)$, we get the Laplace transform determined as:

$$y(t) = H(s)e^{st}$$

  - With s in some form $s = \sigma + j\omega$

- $H(s)$ can be expressed as:

$$H(s) = \int_{-\infty}^{\infty} e^{-s\tau}h(\tau)\,d\tau$$

  - Taking $s = j\omega$, this is equivalent to writing:

$$H(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega\tau}h(\tau)\,d\tau$$

  - This is known as the continuous time Fourier transform; note that this is a special case of the Laplace Transform

- The Laplace Transform is defined as:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

  - Example transform:

$$x(t) = e^{-at}u(t) \longleftrightarrow X(s) = \frac{1}{s+a}$$