Continuous Fourier Transform Properties and Common Functions

$f(t)$	$F(j\omega)$
$x(t)$	$X(j\omega)$
$ax_1(t)+bx_2(t)$	$aX_1(j\omega)+bX_2(j\omega)$
$x(t-t_o)$	$e^{-j\omega t_o}X(j\omega)$
$e^{j\omega_o t}x(t)$	$X(j(\omega-\omega_o))$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
$x(-t)$	$X(-j\omega)$
$x^*(t)$	$X^*(-j\omega)$
$x_1(t)*x_2(t)$	$X_1(j\omega)X_2(j\omega)$
$x(t)y(t)$	$\frac{1}{2\pi}[X(j\omega)*Y(j\omega)]$
$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
$\int_{-\infty}^{t}x(t)\,dt$	$\frac{X(j\omega)}{j\omega}+\pi X(0)\delta(\omega)$
real and even	real and even
real and odd	imaginary and odd
$Even\left\{x(t)\right\}$	$Re\left\{X(j\omega)\right\}$
$Odd\left\{x(t)\right\}$	$jIm\left\{X(j\omega)\right\}$
$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2\,d\omega$

$f(t)$	$F(j\omega)$
$\delta(t)$	1
$\delta(t-t_o)$	$e^{-j\omega t_o}$
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$e^{-at}u(t)\ (Re\left\{a\right\}>0)$	$\frac{1}{a+j\omega}$
$te^{-at}u(t)\ (Re\left\{a\right\}>0)$	$\frac{1}{(a+j\omega)^2}$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(a+j\omega)^n}$
$e^{j\omega_o t}$	$2\pi\delta(\omega-\omega_o)$
$\cos(\omega_o t)$	$\pi\left[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)\right]$
$\sin(\omega_o t)$	$\frac{\pi}{j}\left[\delta(\omega-\omega_o)-\delta(\omega+\omega_o)\right]$
1	$2\pi\delta(\omega)$
$\text{rect}(t)$	$\frac{2}{\omega}\sin(\omega t)$
$\frac{1}{\pi t}\sin(\omega t)$	$\text{rect}(\omega)$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$
$\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_o t}$	$2\pi\sum_{k=-\infty}^{\infty}a_k\delta(\omega-k\omega_o)$

Discrete Fourier Transform Properties and Common Functions

$f[n]$	$F(e^{j\Omega})$
$x_k[n]=\begin{cases}x\left[\frac{n}{k}\right]\\0\end{cases}$	$X(e^{jk\Omega})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_0^{2\pi}X(e^{j\theta})Y(e^{j(\Omega-\theta)})$
$x[n]-x[n-1]$	$(1-e^{-j\Omega})X(e^{j\Omega})$
$\sum_{-\infty}^{n}x[n]$	$\frac{X(e^{j\Omega})}{1-e^{-j\Omega}}+\pi X(1)\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$
$\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2$	$\frac{1}{2\pi}\int_0^{2\pi}\lvert X(e^{j\Omega})\rvert^2\,d\Omega$
$a^n u[n]$	$\frac{1}{1-ae^{-j\Omega}}$

$f[n]$	$F(e^{j\Omega})$
$\frac{(n+r-1)!}{n!(r-1)!}a^n u[n]$	$\frac{1}{(1-ae^{-j\Omega})^n}$
$u[n]$	$\frac{1}{1-e^{-j\Omega}}+\pi\sum_{k=-\infty}^{\infty}\delta(\Omega-2\pi k)$
$\text{rect}(n)$	$\sin\left(\Omega\left(n+\frac{1}{2}\right)\right)\sin^{-1}\left(\frac{\Omega}{2}\right)$
$\frac{\sin(\Omega n)}{n\pi}$	$\text{rect}(\Omega)$ (repeats every 2π)
$\cos(\Omega_o n)$	$\pi\left[\sum_{k=-\infty}^{\infty}\delta(\Omega-\Omega_o-2\pi k)\right.$
	$\left.+\,\delta(\Omega+\Omega_o-2\pi k)\right]$

Note, in the first property of discrete time, k is a multiple of n
Discrete time maintains properties of continuous time, except that:

$$\boxed{\begin{cases} j\omega \to & e^{j\Omega} \\ \omega \to & \Omega \\ t \to & n \end{cases}}$$

Laplace Transform Properties and Common Functions

$f(t)$	$F(s)$	R.O.C
$x(t)$	$X(s)$	
$x(t-t_o)$	$e^{-st_o}X(j\omega)$	R
$e^{s_o t}x(t)$	$X(s-s_o)$	$R + Re\{s_o\}$
$x^*(t)$	$X^*(s^*)$	R
$-tx(t)$	$\frac{d}{ds}X(s)$	
$\frac{d}{dt}x(t)$	$sX(s)$	
$\int_{-\infty}^{t} x(t)\,dt$	$\frac{X(s)}{s}$	$> R \cap Re\{s\} > 0$
$u(t)$	$\frac{1}{s}$	$Re\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$Re\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{(s)^n}$	$Re\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{(s)^n}$	$Re\{s\} < 0$
$\underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	s^{-n}	All s

$f(t)$	$F(s)$	R.O.C
$e^{-at}u(t)$	$\frac{1}{s+a}$	$Re\{s\} > -a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$Re\{s\} < -a$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$Re\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$Re\{s\} < 0$
$e^{-j\omega_o t}u(t)$	$\frac{1}{s+j\omega_o}$	$Re\{s\} > 0$
$\cos(\omega_o t)u(t)$	$\frac{s}{s^2+\omega_o^2}$	$Re\{s\} > 0$
$\sin(\omega_o t)u(t)$	$\frac{\omega_o}{s^2+\omega_o^2}$	$Re\{s\} > 0$
$e^{-at}\cos(\omega_o t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_o^2}$	$Re\{s\} > -a$
$e^{-at}\sin(\omega_o t)u(t)$	$\frac{\omega_o}{(s+a)^2+\omega_o^2}$	$Re\{s\} > -a$
$u_n(t) = \frac{d^n}{dt^n}\delta(t)$	s^n	All s
$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	$> R_1 \cap R_2$

Energy and Power:

$$E_\infty = \lim_{T\to\infty}\int_{-T}^{T} \underbrace{|x(t)|^2}_{p(t)}\,dt \qquad E_\infty = \lim_{N\to\infty}\sum_{n=-N}^{N} \underbrace{|x[n]|^2}_{p[n]}$$

$$P_\infty = \frac{1}{2T}\lim_{T\to\infty}\int_{-T}^{T} \underbrace{|x(t)|^2}_{p(t)}\,dt \qquad P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N} \underbrace{|x[n]|^2}_{p[n]}$$

Convolution: $\quad y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[n] \qquad y(t) = \int_0^t x(t-\tau)h(t)\,d\tau$

Geometric Series Properties: $\quad \sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r} \qquad \sum_{n_1}^{n_2} r^k = \frac{r^{n_1}-r^{n_2+1}}{1-r} \qquad \sum_{k=n}^{\infty} = \frac{r^n}{1-r}$

System Properties:

Linear:

$$ax_1 + bx_2 = ay_1 + by_2$$

Memory:

Depends only on current n or t

Stability:

$$\int_{-\infty}^{\infty} x(t)\,dt < \infty \text{ or } \sum_{n=-\infty}^{\infty} x[n] < \infty$$

Time-Invariant:

$$x(t - t_o) = y(t - t_o)$$

$$x[n - n_o] = y[n - n_o]$$

Causal:

Depends on current or past n or t

Non-invertible:

Two inputs produce same output