Lecture 2 — Introduction to Signals and Systems

Michael Brodskiy

Professor: I. Salama

September 5, 2024

- Signal Power and Energy
  - Definition
    - Consider signal $x(t)$ representing the voltage or current in a unit resistance. The signal power is defined as $p(t) = |x(t)|^2$
    - It is a common terminology to refer to $|x(t)|^2$ or $|x[n]|^2$ as the signal power even if the signal does not represent voltage or current
  - Total energy in a finite duration interval
    - The total energy in an interval $T = t_2 - t_1$ is given by:

$$\text{Continuous Time } \rightarrow E = \int_{t_1}^{t_2} \underbrace{|x(t)|^2}_{p(t)} \, dt$$

$$\text{Discrete Time } \rightarrow E = \Delta T \sum_{n=n_1}^{n_2} \underbrace{|x[n]|^2}_{p(t)} \text{ where } T = (n_2 - n_1 + 1)\Delta T$$

  - The average power in a finite duration interval

$$P_{avg} = \frac{E}{t_2 - t_1} = \frac{1}{t_1 - t_2} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

or

$$P_{avg} = \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

- Power and Energy over an infinite time interval

– Energy

$$\text{Continuous Time } \rightarrow E_\infty = \lim_{T\to\infty} \int_{-T}^{T} \underbrace{|x(t)|^2}_{p(t)}\, dt$$

$$\text{Discrete Time } \rightarrow E_\infty = \lim_{N\to\infty} \cancel{\Delta T} \sum_{n=-N}^{N} \underbrace{|x[n]|^2}_{p(t)}$$

– Power

$$P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \underbrace{|x(t)|^2}_{p(t)}\, dt$$

or

$$P_\infty = \lim_{N\to\infty} \frac{E_\infty}{2N+1} = \lim_{T\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \underbrace{|x[n]|^2}_{p(t)}$$

- Energy Signals versus Power Signals
  – The energy or power of a signal quantifies the magnitude of the signal. For this measure to be meaningful, it must be finite. This requirement leads to the following classification of signals:
    * Energy
      · Signals with finite total energy ($E_\infty < \infty$)
      · They have zero average power

$$P_\infty = \lim_{T\to\infty} \frac{E_\infty}{2T} = 0$$

$$P_\infty = \lim_{N\to\infty} \frac{E_\infty}{2N+1} = 0$$

    * Power
      · Signals with finite average power ($P_\infty < \infty$)
      · They have infinite energy

$$E_\infty = \lim_{T\to\infty} 2T(P_\infty) \rightarrow \infty$$

$$E_\infty = \lim_{N\to\infty} (2N+1)(P_\infty) \rightarrow \infty$$

    * Any finite signal is automatically an energy signal (think: some value in range, 0 otherwise)

- Periodic Signals

  - Periodic signals are classified as power signals because they possess an infinite amount of energy
  - The average power of a periodic signal can be determined by averaging its power over one period:

$$P_\infty = P_{avg} = \frac{1}{T_o} \int_{-T_o/2}^{T_o/2} |x(t)|^2 \, dt$$

- Signals with neither finite power nor energy
  - Some signals have neither finite power nor energy
  - An example is a ramp signal, where $x(t) = t$, $t \geq 0$
  - Neither the energy nor the power can be defined for such signals