Homework 4

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1. (a) We begin by taking the Laplace transform to get:

$$H(s) = -\frac{1}{s-1} \quad \text{and} \quad X(s) = \frac{2}{s}\left[e^{-s} - e^{-2s}\right]$$

We then multiply the two to get:

$$Y(s) = X(s)H(s)$$
$$Y(s) = -\frac{2}{s(s-1)}\left[e^{-s} - e^{-2s}\right]$$

Using partial fraction decomposition, we may write the equivalent such that:

$$-\frac{2}{s(s-1)} = \frac{A}{s-1} + \frac{B}{s} = As + B(s-1)$$

We use $s = 0, 1$ to get:

$$A = -2,\, B = 2 \rightarrow -\frac{2}{s-1} + \frac{2}{s}$$

We now distribute this in the above case to get:

$$Y(s) = -\frac{2}{s-1}\left[e^{-s} - e^{-2s}\right] + \frac{2}{s}\left[e^{-s} - e^{-2s}\right]$$
$$Y(s) = -\frac{2e^{-s}}{s-1} + \frac{2e^{-2s}}{s-1} + \frac{2e^{-s}}{s} - \frac{2e^{-2s}}{s}$$

Finally, we take the inverse transform to get:

$$\boxed{y(t) = -2u(1-t) + 2e^{t-1}u(1-t) + 2u(2-t) - 2e^{t-2}u(2-t)}$$

(b) Differentiating one of the inputs is the same as differentiating the output. Thus, we may say:

$$g(t) = \frac{d}{dt}[y(t)]$$

$$\boxed{g(t) = 2e^{t-1}u(1-t) - 2e^{t-2}u(2-t)}$$

(c) As stated in (b) $\underline{g(t) = (d/dt)[y(t)]}$

(d) $z(t)$ is the same as $g(t)$. Since taking the differential is a linear operation, it does not matter if this is done to the impulse response or to $x(t)$. Therefore, we get:

(e) By linearity of the transform, we may say:

$$y_1(t) = 2y(t-1)$$

Therefore, we may obtain:

$$\boxed{y_1(t) = -4u(2-t) + 4e^{t-2}u(2-t) + 4u(3-t) - 4e^{t-3}u(3-t)}$$

2. (a)
   (b)
   (c)
   (d)
3. (a)
   (b)
   (c)
4. (a)
   (b)
   (c)
   (d)
5. (a)
   (b)
   (c)
   (d)
6. (a)
   (b)
7. (a)
   (b)
8.