# Homework 3

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October 2, 2024

1. Classifying systems as memory-less, time-invariant, linear, causal, and/or stable:

   (a) $y(t) = 5e^{4t}x(t-1)$

   - Memory: **not** memory-less; the $x(t-1)$ term means the system relies on values other than the present value; therefore, it is not memory-less
   - Time-Invariant: **not** time-invariant; we may see that, $y(t-t_o)$ changes the $t$ value in the exponential and $x(t)$ statement, while $x(t-t_o)$ changes only the $x(t)$ statement; thus, it is not time-invariant, since $x(t-t_o) \neq y(t-t_o)$
   - Linear: the system **is** linear (see below) because $ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$

   $$ax_1(t) + bx_2(t) \to a5e^{4t}x_1(t-1) + b5e^{4t}x_2(t-1)$$
   $$ay_1(t) + by_2(t) \to a5e^{4t}x_1(t-1) + b5e^{4t}x_2(t-1)$$

   - Causal: the system **is** causal , because it only depends on past or present values (ex. $t = 0 \to y(t) = 5e^{4(0)}x(-1)$)
   - Stable: Given that the system depends on an exponential $e^{4t}$, its maximum value is unbounded and, therefore, it is **unstable**

   (b) $y(t) = \displaystyle\int_{-\infty}^{\frac{t}{2}} x(\tau)\,d\tau$

   - Memory: **not** memory-less; the system depends on a shift of the $t$ parameter $(t/2)$, and, therefore, does not always depend on the current value of time
   - Time-Invariant: **not** time-invariant; $y(t-t_o) \neq x(t-t_o)$ (see below)

   $$x(t-t_o) \to \int_{-\infty}^{\frac{t}{2}} x(\tau - t_o)\,d\tau$$
   $$y(t-t_o) \to \int_{-\infty}^{\frac{(t-t_o)}{2}} x(\tau - t_o)\,d\tau$$
   $$\therefore x(t-t_o) \neq y(t-t_o)$$

- Linear: the system **is** linear; it follows both the superposition and homogeneity principles (see below)

$$ay_1(t) + by_2(t) \rightarrow a \int_{-\infty}^{\frac{t}{2}} x_1(\tau)\, d\tau + b \int_{-\infty}^{\frac{t}{2}} x_2(\tau)\, d\tau$$
$$ax_1(t) + bx_2(t) \rightarrow a \int_{-\infty}^{\frac{t}{2}} x_1(\tau)\, d\tau + b \int_{-\infty}^{\frac{t}{2}} x_2(\tau)\, d\tau$$
$$\therefore ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$$

- Causal: the system **is not** causal; integration depends on future values when $t < 0$
- Stable: the system **is not** stable (see below)

$$y(t) = \int_{-\infty}^{\frac{t}{2}} x(\tau)\, d\tau$$
$$h(t) = \int_{-\infty}^{\frac{t}{2}} \delta(\tau)\, d\tau = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$
$$h(t) \rightarrow u(t)$$
$$\int_{-\infty}^{\infty} h(t)\, dt = \infty$$

(c) $y(t) = 4 + 5\frac{d^2}{dt^2}x(t)$

- Memory: the system is **not** memory-less; the use of a differential implies that the system depends on past values
- Time-Invariant: the system **is** time-invariant (see below)

$$x(t - t_o) \rightarrow 4 + 5\frac{d^2}{dt^2}x(t - t_o)$$
$$y(t - t_o) \rightarrow 4 + 5\frac{d^2}{dt^2}x(t - t_o)$$
$$\therefore x(t - t_o) = y(t - t_o)$$

- Linear: the system is **not** linear (see below)

$$ax_1(t) + bx_2(t) = \left(4 + 5a\frac{d^2}{dt^2}x_1(t)\right) + \left(4 + 5b\frac{d^2}{dt^2}x_2(t)\right)$$
$$ay_1(t) + by_2(t) = a\left(4 + 5\frac{d^2}{dt^2}x_1(t)\right) + b\left(4 + 5\frac{d^2}{dt^2}x_2(t)\right)$$
$$\therefore ax_1(t) + bx_2(t) \neq ay_1(t) + by_2(t)$$

- Causal: the system **is** causal because it only depends on past or present values
- Stable: the system is **unstable** because it is unbounded

(d) $y(t) = \begin{cases} 0, & t < 0 \\ x(t-2) + 2x(t), & t \geq 0 \end{cases}$

- Memory: the system is **not** memory-less, since the $x(t-2)$ term depends on a past value
- Time-Invariant: the system is **not** time-invariant (see below)

$$x(t - t_o) \rightarrow \begin{cases} 0, & t < 0 \\ x(t - 2 - t_o) + 2x(t - t_o), & t \geq 0 \end{cases}$$

$$y(t - t_o) \rightarrow \begin{cases} 0, & t < 2 \\ x(t - 2 - t_o) + 2x(t - t_o), & t \geq 2 \end{cases}$$

$$\therefore x(t - t_o) \neq y(t - t_o)$$

- Linear: the system **is** linear (see below)

$$ax_1(t) + bx_2(t) \rightarrow \begin{cases} 0, & t < 0 \\ ax_1(t-2) + 2ax_1(t) + bx_2(t-2) + 2bx_2(t), & t \geq 0 \end{cases}$$

$$ay_1(t) + by_2(t) \rightarrow \begin{cases} 0, & t < 0 \\ ax_1(t-2) + 2ax_1(t) + bx_2(t-2) + 2bx_2(t), & t \geq 0 \end{cases}$$

$$\therefore ax_1(t) + bx_2(t) = ay_1(t) + by_2(t)$$

- Causal: the system **is** causal because it only depends on past or present values
- Stable: the system **is** stable, because it does not tend to diverge

Problem 1 can be tabulated as follows:

| System | a | b | c | d |
|---|---|---|---|---|
| Memory-Less | no | no | no | no |
| Time-Invariant | no | no | yes | no |
| Linear | yes | yes | no | yes |
| Causal | yes | no | yes | yes |
| Stable | no | no | no | yes |

2. Classifying systems as memory-less, time-invariant, linear, causal, and/or stable:

   (a) $y[n] = x[n+1] - 2x[n-4]$

   (b) $y[n] = \text{Even}\{x[n-1]\}$

   (c) $y[n] = 5x[3n+1]$

   (d) $y[n] = \begin{cases} 0, & n = 2 \\ x[n], & \text{otherwise} \end{cases}$

3. (a)

   (b)

   (c)

(d)

4.

5.

6. (a)

   (b)

   (c)

7.

8. (a)

   (b)

9. (a)

   (b)