Lecture 8 — The Fourier Transform

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- A periodic signal, $x(t)$, with period T_o can be expressed as a sum of complex exponentials at the fundamental frequency and its harmonics. The analysis equation may be written as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

- The synthesis equation may be written as:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

- The Fourier Transform exists only if the $j\omega$ axis lies within the ROC of the Laplace Transform
- Fourier Transforms are governed by the Dirichlet conditions:
  1. $x(t)$ is absolutely integrable
  2. $x(t)$ has a finite number of maxima and minima over any finite interval
  3. $x(t)$ has a finite number of finite discontinuities over any finite interval
  4. Note: Periodic signals do not satisfy these conditions but are considered to have Fourier transforms if impulse functions are included in the Fourier representation