Homework 7

Michael Brodskiy
Professor: I. Salama

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1. First and foremost, we see that the period is:

$$T_o = 6$$

We know that the Fourier coefficient formula is given by:

$$C_n = \frac{1}{T_o}\int_{-T_o/2}^{T_o/2} x(t)e^{-2\pi jnt/T_o}\,dt$$

We substitute our known values to get:

$$C_n = \frac{1}{6}\int_{-3}^{3} 2e^{-(\pi jnt)/3}\,dt$$
$$C_n = \frac{1}{3}\int_{-1}^{1} e^{-(\pi jnt)/3}\,dt$$
$$C_n = \frac{1}{3}\left(-\frac{3}{\pi jn}e^{-(\pi jnt)/3}\right)\Big|_{-1}^{1}$$

We solve this to get:

$$C_n = -\frac{1}{\pi jn}\left[e^{-(pijn)/3} - e^{(\pi jn)/3}\right]$$
$$\boxed{C_n = \frac{2}{n\pi}\sin\left(\frac{n\pi}{3}\right)}$$

We then substitute to find the values given:

$$\boxed{\begin{cases} C_1 & = .5513 \\ C_2 & = .2757 \\ C_3 & = 0 \\ C_{-1} & = .5513 \\ C_{-2} & = .2757 \\ C_{-3} & = 0 \end{cases}}$$

We then calculate C_0 separately:

$$C_0 = \frac{1}{6}\int_{-1}^{1} 2\,dt$$

$$C_0 = \frac{1}{6}(2(1) - 2(-1))$$

$$\boxed{C_0 = \frac{2}{3}}$$

2. (a) We may use the transform table to write:

$$Y(\omega) = \frac{j\omega}{(j\omega)^2 + \omega_o^2}$$

For a transform of $\cos(t)$. Combining this with the shifting property, we may write:

$$\boxed{X(\omega) = \frac{j\omega + 3}{(j\omega + 3)^2 + 25}}$$

(b) We may write the sinusoid as:

$$Y(\omega) = \frac{\omega_o}{(j\omega)^2 + \omega_o^2}$$

We then apply the shifting property to get:

$$X^1(\omega) = \frac{2}{(j\omega + 1)^2 + 4}$$

Finally, applying the t term, we get:

$$X(\omega) = j\frac{d}{d\omega}\left[\frac{2}{(j\omega + 1)^2 + 4}\right]$$

$$X(\omega) = j\left[\frac{2(-2j)}{[(j\omega + 1)^2 + 4]^2}\right]$$

$$\boxed{X(\omega) = \frac{4}{[(j\omega + 1)^2 + 4]^2}}$$

3. (a) We first list the relevant properties:

$$x(t-t_o) \rightarrow e^{-j\omega t_o}X(j\omega)$$
$$x(-t) \rightarrow X(-j\omega)$$

Deconstructing the expression, we may write:

$$X_1(\omega) = e^{-j\omega(4)}X(-j\omega) + e^{-j\omega(-4)}X(-j\omega)$$
$$X_1(\omega) = X(-j\omega)[e^{-4j\omega} + e^{4j\omega}]$$

We then use the property that:

$$e^{-j\omega} + e^{j\omega} = 2\cos(\omega)$$

To get:

$$\boxed{X_1(\omega) = 2\cos(4\omega)X(-j\omega)}$$

(b) We first list the relevant properties:

$$x(at) \rightarrow \frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$$
$$x(t-t_o) \rightarrow e^{-j\omega t_o}X(j\omega)$$
$$tx(t) \rightarrow j\frac{d}{d\omega}[X(j\omega)]$$

The expression given to us is of the form:

$$x(t) = tx(at-t_o)$$

We rewrite the expression as:

$$x_2(t) = tx(a[t-t_o/a])$$

Which lets us determine $a = 3$ and $T_o = -2$. As such, we obtain:

$$X_2(\omega) = j\frac{d}{d\omega}\left[\frac{e^{\frac{2j\omega}{3}}}{3}X\left(\frac{j\omega}{3}\right)\right]$$
$$\boxed{X_2(\omega) = \frac{j}{3}\frac{d}{d\omega}\left[e^{\frac{2j\omega}{3}}X\left(\frac{j\omega}{3}\right)\right]}$$

(c)

4. (a)

(b)

(c)

(d)

(e)

5. (a)

 (b)

 (c)

 (d)

6. (a)

 (b)

 (c)

7. (a)

 (b)

8. (a)

 (b)