Homework 8

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1. (a)

   (b)

2. (a) The signal $x(t)$ may be expressed in several parts: a step up by 1 at $t = -1$ to $t = 1$, a slope of 1 between -1 and 1, and a step up of 2 for $t > 1$. Thus, we get:

$$\boxed{x(t) = u(t+1) + r(t+1) + u(t-1) - r(t-1)}$$

   This lets us find:

$$\boxed{\frac{dx}{dt} = \delta(t+1) + u(t+1) + \delta(t-1) - u(t-1)}$$

   And finally:

$$\boxed{\frac{d^2x}{dt^2} = \delta(t+1) - \delta(t-1)}$$

   (b)

   (c)

4. (a) Per the theorem, we may write:

$$\int_{-\infty}^{\infty} |x(t)|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

$$2\pi\int_{-\infty}^{\infty} |x(t)|^2 = \int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

   This allows us to write:

$$A = 4\pi\int_{0}^{\infty} e^{-4t}\,dt$$

$$-\pi\left(e^{-4t}\right)\Big|_0^\infty$$
$$-\pi\left(e^{-4\infty}-1\right)$$

Thus, we obtain:

$$\boxed{A=\pi}$$

(b) Per our Fourier transform properties, we may write:

$$tx(t)\rightarrow j\frac{d}{d\omega}X(\omega)$$

For $y(t)=te^{-2|t|}$ this gives us:

$$Y(j\omega)=j\frac{d}{d\omega}\left[\frac{4}{(4+\omega^2)}\right]$$

Differentiating gives us the final answer as:

$$\boxed{Y(j\omega)=-\frac{8j\omega}{(4+\omega^2)^2}}$$

(c) By the duality property, we know that if $x(t)\leftrightarrow X(j\omega)$, then:

$$x(t)\leftrightarrow 2\pi X(-j\omega)$$

As such, we may write:

$$-\frac{8jt}{(4+t^2)^2}\leftrightarrow 2\pi(-\omega)e^{-2|\omega|}$$
$$\frac{t}{(4+t^2)^2}\leftrightarrow -j\pi\omega e^{-2|\omega|}$$

Thus, we see that:

$$\boxed{\mathcal{F}\left\{\frac{4t}{(4+t^2)^2}\right\}=-j\pi\omega e^{-2|\omega|}}$$

5. We may compute the response of the system using the convolution; the convolution may be more easily computed using the Fourier transform such that:

$$y(t)=h(t)*x(t)\leftrightarrow Y(j\omega)=H(j\omega)X(j\omega)$$

We can see that the given response may be written as:

$$H(j\omega)=\begin{cases} e^{-3j\omega}, & |\omega|\leq 4\\ 0, & \text{otherwise}\end{cases}$$

(a) We may see that the transform becomes:

$$X_1(j\omega) = \pi e^{N\pi j\omega}\left[\delta(\omega-10)+\delta(\omega+10)\right]$$

We may observe that the response and signal do not have common values for which they are non-zero. This gives us:

$$Y_1(j\omega) = 0$$

This ultimately means:

$$\boxed{y_1(t) = 0}$$

(b) We may see that the transform becomes:

$$X_2(j\omega) = 5\pi\left[\delta(\omega-2)+\delta(\omega+2)\right]$$

We multiply the two together to get:

$$Y_2(j\omega) = 5\pi e^{-3j\omega}\left[\delta(\omega-2)+\delta(\omega+2)\right]$$

We see this introduces a delay of three units, which gives us:

$$\boxed{y_2(t) = 5\cos(2(t-3))}$$

(c) We may see that the transform becomes:

$$X_3(j\omega) = \begin{cases} 1, & |\omega| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Multiplying the two together creates a delayed sink function, but with the boundaries of the input:

$$Y_3(j\omega) = \begin{cases} e^{-3j\omega}, & |\omega| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Transforming back, we get:

$$\boxed{y_3(t) = \frac{\sin(t-3)}{\pi(t-3)}}$$

(d) We may observe that the input signals within the passband of the filter simply introduce a delay at the output, thus, we may conclude:

$$\boxed{y_4(t) = \left(\frac{\sin(t-3)}{\pi(t-3)}\right)^2}$$

6. (a)
   (b)
8. (a)
   (b)
   (c)
   (d)
9.
10.