Quiz 3

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A. Taking the Laplace transform of the transfer function, we get:

$$s^2Y(s) + 2sY(s) - 8Y(s) = sX(s) + 3X(s)$$

Given that the transfer function may be expressed as:

$$H(s) = \frac{Y(s)}{X(s)}$$

We rearrange to get:

$$[s^2 + 2s - 8]Y(s) = [s + 3]X(s)$$

This gives us:

$$\boxed{H(s) = \frac{s+3}{s^2 + 2s - 8}}$$

B. Given that the function is stable, we know that the ROC must contain the $j\omega$ axis. Since we see that the poles are at $s = 2, -4$, we know that the ROC can be expressed as the real values in between the two poles (double-sided):

$$\boxed{\text{R.O.C:} \; -4 < \sigma < 2}$$

C. We see that there are poles at $\sigma = 2, -4$ and a zero at $\sigma = -3$. This gives us:

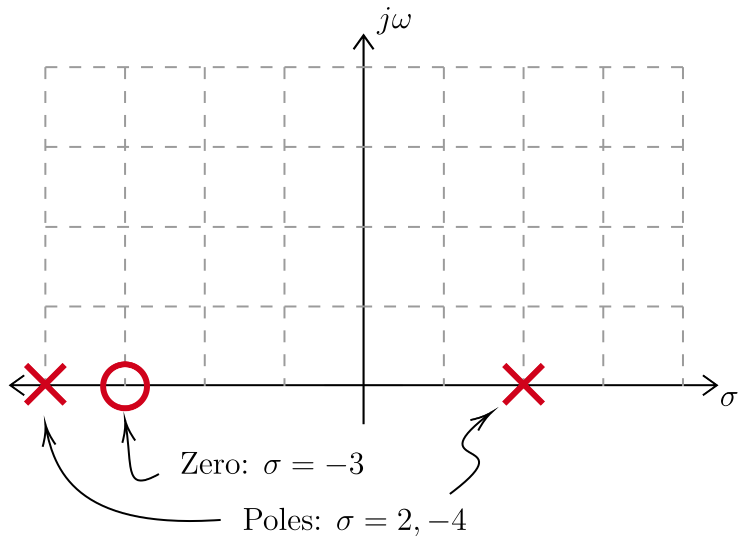

Figure 1: Pole-Zero Plot of $H(s)$

D. The system has one zero located at $\sigma = -3$.

E. Since the signal is double-sided, we know it <u>is not</u> causal.

F. We may begin by using partial fractions to obtain an equivalent $H(s)$. This gives us:

$$\frac{s+3}{(s+4)(s-2)} = \frac{A}{s+4} + \frac{B}{s-2}$$

Using partial fraction decomposition, we get $A = 1/6$ and $B = 5/6$:

$$H(s) = \frac{1/6}{s+4} + \frac{5/6}{s-2}$$

Taking the inverse transform, we get:

$$\boxed{h(t) = \frac{5}{6}e^{2t}u(-t) + \frac{1}{6}e^{-4t}u(t)}$$

G. I. Using our transform tables, the transform of this input is:

$$\boxed{X(s) = \frac{6}{s+3}}$$

Since the signal is right-handed, the R.O.C is:

$$\boxed{\text{R.O.C: } \sigma > -3}$$

II. We determine the output by multiplying together the response and input. This gives us:

$$Y(s) = \left(\frac{s+3}{(s-2)(s+4)}\right)\left(\frac{6}{s+3}\right)$$

$$\boxed{Y(s) = \frac{6}{(s-2)(s+4)}}$$

We combine the two regions of convergence, which means the R.O.C is:

$$\boxed{\text{R.O.C:} \; -3 < \sigma < 2}$$

III. We use partial fractions to get:

$$Y(s) = \frac{1}{s+4} + \frac{3}{s-2}$$

Taking the inverse transform, we get:

$$\boxed{y(t) = e^{-4t}u(t) + 3e^{2t}u(-t)}$$