Homework 1

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1. Express each of the following complex numbers in polar form and plot them

   (a) 8

$$r = \sqrt{8^2 + 0^2} = 8$$
$$\theta = 0$$
$$z(r, \theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(8, 0) = 8(\cos(0) + j\sin(0))$$
$$\therefore \text{ In polar: } \boxed{z = 8}$$

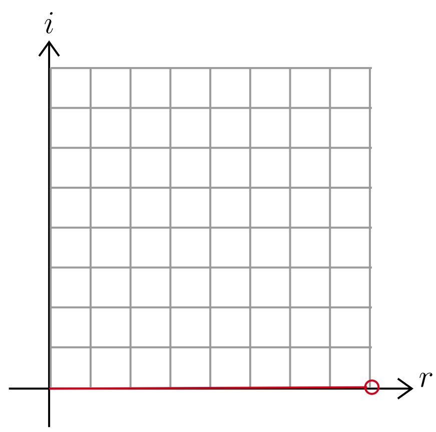

Figure 1: $z = 8$ Plotted on the Imaginary Plane

(b) -5

$$r = \sqrt{(-5)^2 + 0^2} = 5$$
$$\theta = \pi$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(5,\pi) = 5(\cos(\pi) + j\sin(\pi))$$
$$\therefore \text{ In polar: } \boxed{z = -5 = 5e^{\pi j}}$$

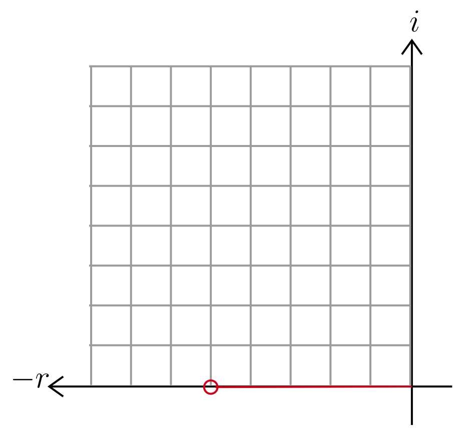

Figure 2: $z = -5$ Plotted on the Imaginary Plane

(c) $2j$

$$r = \sqrt{0^2 + (2)^2} = 2$$
$$\theta = \frac{\pi}{2}$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(2, .5\pi) = 2j$$
$$\therefore \text{ In polar: } \boxed{z = 2j = 2e^{.5\pi j}}$$

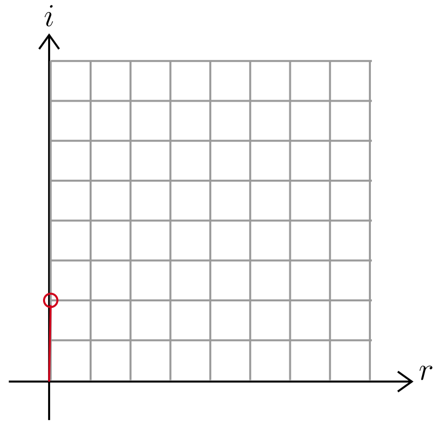

Figure 3: $z = 2j$ Plotted on the Imaginary Plane

(d) $\frac{1}{4}(1-j)^5$

$$.25(1-j)^2(1-j)^3$$
$$.25(-2j)(1-j)(1-j)^2$$
$$.25(-2-2j)(-2j)$$
$$z = j - 1$$
$$r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$
$$\theta = \frac{3\pi}{4}$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(\sqrt{2}, .75\pi) = j - 1$$
$$\therefore \text{ In polar: } \boxed{z = j - 1 = \sqrt{2}e^{.75\pi j}}$$

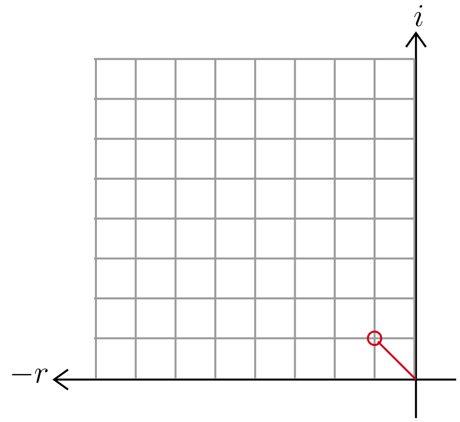

Figure 4: $z = \frac{1}{4}(1-j)^5$ Plotted on the Imaginary Axis

(e) $\frac{(1+j)}{j}e^{\frac{j\pi}{3}}$

$$\frac{(1+j)}{j} \cdot \frac{-j}{-j} = 1 - j$$
$$\tan\left(\frac{\pi}{3}\right) = \frac{b}{a}$$
$$\frac{b}{a} = \sqrt{3}$$
$$b = a\sqrt{3}$$
$$\sqrt{(a\sqrt{3})^2 + a^2} = 1$$
$$4a^2 = \pm 1$$
$$a = \frac{1}{2}$$
$$b = \frac{\sqrt{3}}{2}$$
$$\frac{1}{2}(1-j)(1+\sqrt{3}j) \rightarrow \frac{1}{2}((\sqrt{3}+1) + (\sqrt{3}-1)j)$$
$$r = \frac{1}{2}\sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = \sqrt{(4+2\sqrt{3}) + (4-2\sqrt{3})}$$
$$r = \sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = .26179$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(\sqrt{2}, .26179) = \frac{1}{2}(\sqrt{3}+1) + (\sqrt{3}-1)j$$

$$\therefore \text{ In polar: } \boxed{z = \frac{1}{2}(\sqrt{3}+1) + (\sqrt{3}-1)j = \sqrt{2}e^{.26179j}}$$

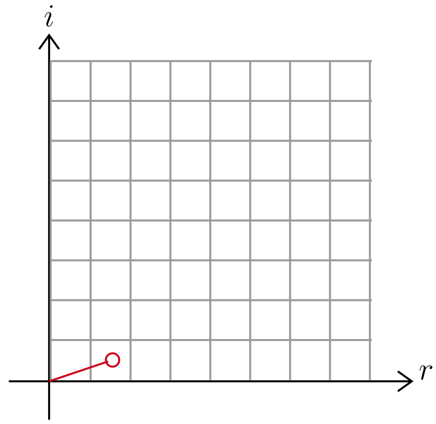

Figure 5: $z = \frac{(1+j)}{j}e^{\frac{j\pi}{3}}$ Plotted on the Imaginary Axis

(f) $(\sqrt{3} - j^5)(1+j)$

$$j^5 = j \rightarrow (\sqrt{3} - j)(1+j) = (\sqrt{3} + (\sqrt{3}-1)j + 1)$$
$$(\sqrt{3}+1) + (\sqrt{3}-1)j$$
$$r = \sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = \sqrt{(4+2\sqrt{3}) + (4-2\sqrt{3})}$$
$$r = \sqrt{8} = 2\sqrt{2}$$
$$\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = .26179$$
$$z(r,\theta) = r(\cos(\theta) + j\sin(\theta))$$
$$z(2\sqrt{2}, .26179) = (\sqrt{3}+1) + (\sqrt{3}-1)j$$
$$\therefore \text{ In polar: } \boxed{z = (\sqrt{3}+1) + (\sqrt{3}-1)j = 2\sqrt{2}e^{.26179j}}$$

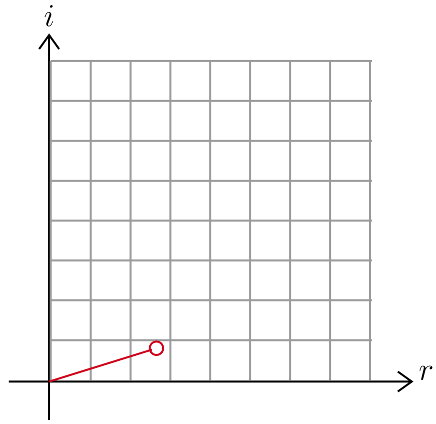

Figure 6: $z = (\sqrt{3} - j^5)(1 + j)$ Plotted on the Imaginary Axis

(g) $\frac{2(\sqrt{3}-j)}{1+j\sqrt{3}}$

$$\frac{2\sqrt{3} - 2j}{1 + j\sqrt{3}} \cdot \frac{1 - j\sqrt{3}}{1 - j\sqrt{3}} = -2j$$

$$r = \sqrt{0^2 + (-2)^2} = 2$$

$$\theta = \frac{3\pi}{2}$$

$$z(r, \theta) = r(\cos(\theta) + j\sin(\theta))$$

$$z(2, 1.5\pi) = -2j$$

$$\therefore \text{ In polar: } \boxed{z = -2j = 2e^{1.5\pi j}}$$

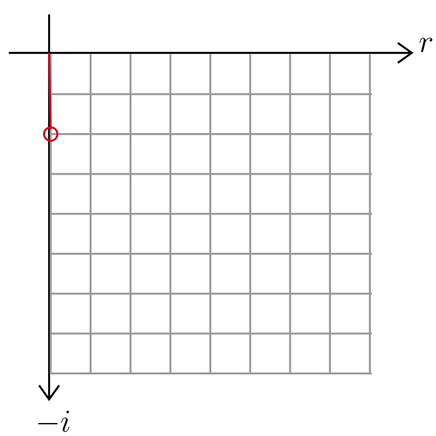

Figure 7: $z = \frac{2(\sqrt{3}-j)}{1+j\sqrt{3}}$ Plotted on the Imaginary Axis

2. Determine the value of E_∞ and P_∞ for each of the following signals and indicate whether the signal is a power or energy signal or neither.

   (a) $x_1(t) = \begin{cases} 5e^{j(4t+\pi/3)}, & t \geq 2 \\ 0, & \text{Otherwise} \end{cases}$

$$E_\infty = \int_2^{2+\frac{\pi}{4}} 25e^{j(8t+2\pi/3)}\, dt$$

$$E_\infty = \frac{25}{8}\left[\sin\left(\frac{24t+2\pi}{3}\right) - i\cos\left(\frac{24t+2\pi}{3}\right)\right]\Big|_2^{2+\frac{\pi}{4}}$$

$$E_\infty = 2.2755 + 76.43285i + 2.142 + 2.2755i$$

$$\therefore \text{ Energy if finite}$$

   (b) $x_2(t) = \begin{cases} 2+2\cos(t), & 0 < t < 2\pi \\ 0, & \text{Otherwise} \end{cases}$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (2+2\cos(t))^2\, dt$$

$$P_\infty = 6$$

$$\therefore \text{ Power is finite}$$

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} (2+2\cos(t))^2\, dt$$

$$P_\infty = \infty$$

$$\therefore \text{ Energy is infinite}$$

$$\boxed{\text{Since power is finite and energy is infinite, this is a power signal}}$$

   (c) $x_3[n] = \begin{cases} (.5)^n, & n \geq 0 \\ 0, & \text{Otherwise} \end{cases}$

3. For the discrete time signal shown in Figure P1.3, sketch, and carefully label each of the following.

   (a) $x[n-4]$
   (b) $x[2n+2]$

4. For the continuous time signal shown in Figure P1.4, sketch, and carefully label each of the following.

   (a) $x(t+3)$
   (b) $x\left(3 - \frac{2}{3}t\right)$

5. Determine and sketch the even and odd parts of the signals depicted in Figure P1.5. Label your sketches carefully.

   (a)

   (b)

6. Determine and sketch the even and odd parts of the signal depicted in Figure P1.6. Label your sketches carefully.

7. Express the real part of each of the following signals in the form $Ae^{-at}\cos(\omega t + \phi)$ where A, a, ω and ϕ are real numbers with $A > 0$ and $-\pi < \phi \leq \pi$.

   (a) $x_1(t) = 4e^{-2t}\sin\left(10t + \frac{3\pi}{4}\right)\cos\left(10t + \frac{3\pi}{4}\right)$

   (b) $x_2(t) = j(1-j)e^{(-5+j\pi)t}$

8. Determine whether each of the following continuous time signals is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x(t) = 5\cos\left(400\pi t + \frac{\pi}{4}\right)$

   (b) $x(t) = 20e^{j(\pi t - 2)}$

   (c) $x(t) = 2\left[\sin\left(50\pi t - \frac{\pi}{3}\right)\right]^2$

   (d) $x(t) = \begin{cases} 2\sin(5\pi t), & t \geq 0 \\ -2\sin(-5\pi t), & t < 0 \end{cases}$

9. Determine whether each of the following discrete time signals is periodic. If the signal is periodic, determine its fundamental period.

   (a) $x[n] = 2\cos\left(\frac{7}{11}n + \frac{\pi}{2}\right)$

   (b) $x[n] = \cos(\pi n) + 4\sin\left(\frac{\pi}{4}n^2\right)$

   (c) $x[n] = 3\sin\left(\frac{\pi}{3}n\right) + \cos\left(\frac{\pi}{4}n\right) - 3\cos\left(\frac{\pi}{6}n + \frac{\pi}{3}\right)$