The Rutherford-Bohr Model of the Atom

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Contents

1 Basic Properties of Atoms

- Around 1900, knowledge about atoms was:
  - Size: Small, 1[Å]/.1[nm]
  - Stable: Forces balance
  - Atoms contain electrons (e^-) and maintain neutral charge
  - Atoms can emit and absorb electromagnetic radiation

2 Scattering Experiments and Thomson Model

- An early atom model: J.J. Thomson (1904):

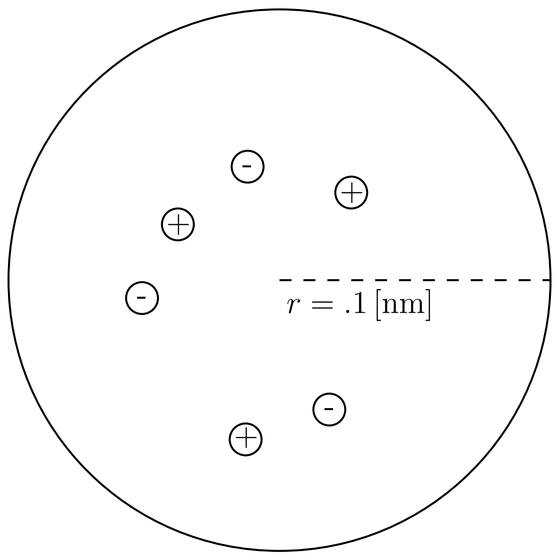

Figure 1: The Jelium Model ("Plum-pudding Model")

3 The Rutherford Nuclear Atom

- Rutherford discovered two rays, α and β-rays, which he used to experiment with atoms
  - α rays are essentially He^{2+} atoms
    - The positive charge would mean that the atoms should deflect α rays
- The Geiger-Marsden Observation

– Their observation found:

$$p(\text{backscattering}) \approx 10^{-4}$$

– This is much larger than expected

– Rutherford proposed that the charge and mass of atoms are concentrated in a region called the nucleus

- The Bohr Model
  - Proposed by Niels Bohr (1913)
  - Atoms resembled a miniature planetary system

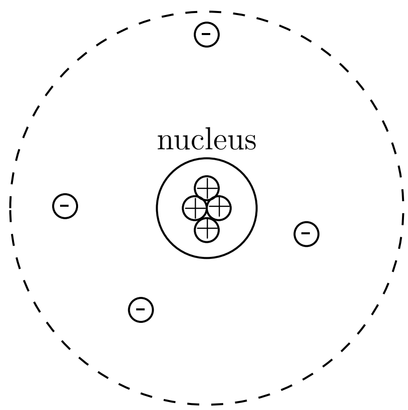

Figure 2: The Bohr Model

  - From this, the Coulomb interaction (force) was determined:

$$\boxed{F = \frac{1}{4\pi\varepsilon_o}\frac{|q_1||q_2|}{r^2}}$$

  - The kinetic energy was determined as:

$$\boxed{K = \frac{1}{8\pi\varepsilon_o}\frac{e^2}{r}}$$

  - If the electron is radiating, it slows down and moves toward nucleus to collapse?
    * Niels Bohr hypothesized that electrons may exist in “stationary states” without radiating electromagnetic energy
  - $L = rp = rmv = n\hbar \to v = \frac{n\hbar}{mr}$
    * Where L is the angular momentum, r is the radius, n is a quantized number, and m is the mass

- Substituting this into kinetic energy, we get the permitted radius, r:

$$\boxed{r_n = \frac{4\pi\varepsilon_o\hbar^2}{me^2}n^2}$$

- Using electron information, we get:

$$\frac{4\pi\varepsilon_o\hbar^2}{me^2} = .0529[\text{nm}]$$

- This value is known as the Bohr radius

- Hydrogen Atom and Bohr Model:
  - Radius: $r_n = a_o n^2, \qquad n = 1, 2, 3$
  - Energy: $E_n = \frac{-13.6[\text{eV}]}{n^2}$
  - $\Delta E = E_n - E_1$ is the excitation energy, where E_n is the nth excited state, and E_1 is the ground state
    - $|E_n|$ is the binding energy of e^- in state n (ionization energy)
  - Optical transitions[1] result in absorption or emission of a photon
    - In stationary state, there is no electromagnetic energy radiation
    - e^- can emit radiation when moving from n_1 to n_2

4 Line Spectra

- The absorption or emission from atoms may be used to create an emission spectra
- A general equation was generated:

$$\boxed{\lambda = \lambda_{\text{limit}}\frac{n^2}{n^2 - n_o^2}, \qquad n = n_o + 1, n_o + 2, \cdots}$$

- For the Balmer series, $n_o = 2$
- For the Lyman series, $n_o = 1$
- The Ritz combination principle:

$$f_1 + f_2 = f_3$$

  - This principle shows that the sum of any two emission frequencies results in a frequency that is also in the spectrum

[1] Transitions using a photon

- The wavelength of the transition becomes:

$$\lambda = \frac{c}{f} = \frac{64\pi^3\varepsilon_o^2\hbar^3 c}{me^4}\frac{n_1^2 n_2^2}{n_1^2 - n_2^2} = \frac{1}{R_\infty}\frac{n_1^2 n_2^2}{n_1^2 - n_2^2}$$

- Where R_∞ is the Rydberg constant, equal to $1.097 \cdot 10^7[\mathrm{m}^{-1}]$
- Lyman series are only observed in the absorption spectrum
- The following summarizes emission spectra
  - Isolated atoms are in the ground state most of the time
  - Excited state lives fro a short time (picoseconds to femtoseconds)
  - The absorption spectrum only occurs from the ground state
  - Balmer series are not found in the absorption spectrum

5 The Bohr Model

- For an atom with $z > 1$
  - For a nucleus of charge ze, the Coulomb force is:

$$F = \frac{1}{4\pi\varepsilon_o}\frac{|q_1||q_2|}{r^2} = \frac{ze^2}{4\pi\varepsilon_o r^2}$$

- Analyzing a Hydrogen Atom in Quantum Mechanics
  - The potential energy, derived from the Coulomb force, is:

$$U(r) = -\frac{e^2}{4\pi\varepsilon_o r}$$

  - This may be converted to be in terms of x, and plugged into the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} - \frac{e^2}{4\pi\varepsilon_o x}\psi(x) = E\psi(x)$$

  - To keep this finite, the following two conditions need to be met:

$$\begin{cases} x \to 0, & \psi(x) = 0 \\ x \to \infty, & \psi(x) = 0 \end{cases}$$

  - Using $\psi(x) = Axe^{-bx}$ with the Schrödinger equation, solving would obtain:

$$\boxed{b = \frac{me^2}{4\pi\varepsilon_o\hbar^2} = \frac{1}{\alpha_o} \Rightarrow \text{Bohr Radius}}$$

- Using the normalization condition, and $\psi(x) = Axe^{-bx} = Axe^{\frac{x}{\alpha_o}}$, we get:

$$A = \frac{2}{\alpha_o^{\frac{3}{2}}}$$

- Conclusions:
  * For the ground state, there is an uncertainty in the location of e^-
  * The most probable region to find e^- is near $x = \alpha_o$ (consistent with Bohr model)
  * But, it's possible to find e^- anywhere (which is very difference from Bohr)

- Using Classical Mechanics
  - The angular momentum in a planetary system is constant, and vector $\vec{L}(L_x, L_y, L_z)$ has three components
    * l is the angular momentum quantum number; it determines the length of the vector
    * m_l is the magnetic number; it determines one of the components of the vector
    * We have:

$$|\vec{L}| = \sqrt{l(l+1)}\hbar, \qquad l = 0, 1, 2 \cdots$$