Homework 2

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1 Twins

1. (a)
   - Total distance for Mary-Kate: $2L_0 = 32$[light-years]
   - Total time for Mary-Kate: $\Delta t_0 = 20$[yr]

$$v\Delta t_0 = 2L$$
$$v\Delta t_0 = 2L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$v = \frac{2L_0\sqrt{1-\frac{v^2}{c^2}}}{\Delta t_0}$$
$$v^2 = \frac{(32)^2c^2\left(1-\frac{v^2}{c^2}\right)}{20^2}$$
$$v^2 = 2.56c^2 - 2.56v^2$$
$$3.56v^2 = 2.56c^2$$
$$\boxed{v = .848c}$$

   (b) According to the result from (a), the speed at which Mary-Kate traveled is $.848c$. Applying the time dilation formula using this knowledge yields:

$$\Delta t = \frac{20}{\sqrt{1-(.848)^2}}$$
$$\Delta t = 37.7\text{[yr]}$$

   So Ashley is $37.7-20 = 17.7$ years older than Mary-Kate when Mary-Kate returns

2 Spherical Waves

2. The Lorentz Transformation is given as:

$$\begin{cases} x' = \dfrac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t-\frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}} \end{cases}$$

Because it is stated that the pulse begins at time $t = 0$, it can be assumed that, at this time, the pulse is not moving. As such, the transformations reduce to:

$$\begin{cases} x' = \dfrac{x-(0)t}{\sqrt{1-\frac{0^2}{c^2}}} = x \\ y' = y \\ z' = z \\ t' = \dfrac{t-\frac{0}{c^2}x}{\sqrt{1-\frac{0^2}{c^2}}} = t \end{cases}$$

In this manner, we substitute each transformation into the original formula, which yields:

$$\boxed{x'^2 + y'^2 + z'^2 = (ct')^2 = 0}$$

3 Pole Vaulting

3. (a) Using the length contraction formula, the proper length (L_0) as 20[m], and the observed length (L) as 10[m], we obtain:

$$10 = 20\sqrt{1-\frac{u^2}{c^2}}$$

$$.25 = 1 - \frac{u^2}{c^2}$$

$$.25c^2 = c^2 - u^2$$

$$u^2 = .75c^2$$

$$\boxed{u = .866c}$$

As Ming's speed in reference frame of observer O

(b) As with the phenomenon of clock desynchronization, because Ming is moving towards the garage, it appears to him that the farther door closes first. In such a manner, the time difference between the state alternation of the doors (closed to open) can be expressed as the following:

$$\Delta t' = \frac{\frac{uL}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Delta t' = \frac{\frac{(10)(.866)}{c}}{\sqrt{1-(.866)^2}}$$

$$\Delta t' = 5.7328 \cdot 10^{-8}[\mathrm{s}]$$

In this time, Ming is able to travel the following distance:

$$(5.7328)(3)(.866) = 14.894[\mathrm{m}]$$

Because the garage is actually 10 meters long, and Ming travels 14.894[m], he is able to enter and exit the garage prior to it closing. In this manner, he is 4.894[m] away from the garage by the time the door closes. As such, Ming is able to safely enter and exit the 10[m] garage despite having a 20[m] pole.

4 Meson Decay

4. π Meson speed: $v_x = \pm.815c$; K Meson speed: $u = .453c$

   This would mean, using the Lorentz velocity transformation, the first π meson particle would have a speed of:

$$v'_{x1} = \frac{.453 - .815}{1 - (.815)(.453)}c$$

$$\boxed{v'_{x1} = -.574c}$$

   And the other, using $v_x = -.815c$, would have a speed of:

$$v'_{x2} = \frac{.453 + .815}{1 + (.815)(.453)}c$$

$$\boxed{v'_{x2} = .926c}$$

5 Meter Stick

5. Because the motion is parallel, only the x component of the meter stick experiences length contraction. Thus, to find the components, we would perform the following:

$$L_y = (1[\mathrm{m}])\sin(30) = .5[\mathrm{m}]$$
$$L_x = (1[\mathrm{m}])\cos(30) = .866[\mathrm{m}]$$

   Following contraction, L_x becomes:

$$L'_x = .866(\sqrt{1 - .81}) = .378[\mathrm{m}]$$

   Thus, the new length of the meter stick becomes:

$$L = \sqrt{(.378)^2 + (.5)^2}$$

$$\boxed{L = .627[\mathrm{m}]}$$