Exam 2 Practice Problems

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1 Conceptual Questions

1. Infinite Wells and de Broglie Waves

   Because the wavelength may be defined as $\lambda = \dfrac{2L}{n}$, the situation is similar to the oscillation of a standing wave, like a string, fixed at two ends. As such, a particle moving in a similar manner would have standing de Broglie waves as solutions to the Schrödinger equation.

2. Wave Normalization

   Purely mathematically, an un-normalized wave may be a solution to the Schrödinger equation; however, when applying this concept to physical quantities, it is necessary for it to be normalized. This is because the integral over the entire probability function must be equal to 1 or 100%, and, thus, the function needs to be normalized to meet this requirement.

3. The Physical Meaning of $\displaystyle\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = 1$

   $\psi(x)$ represents the wave function, which describes the probability of a particle being in a given position. Taking the magnitude of the wave function and squaring it, or $|\psi(x)|^2$ generates a probability density distribution. By integrating over the whole function, a probability of 1, or 100% would be calculated, as it is definite that the particle is somewhere within the entirety of the wave.

4. Harmonic Oscillators

   The ground state energy occurs when $n = 0$; given the formula $E_n = \left(n + \frac{1}{2}\right)\hbar\omega_0$, this means the ground state energy is $E_0 = \frac{1}{2}\hbar\omega_0$. The difference between an energy level and the ground state energy may be expressed as $E_n - E_0 = n\hbar\omega_0$. This means that the smallest energy difference would be when $n = 1$, and the second smallest would occur when $n = 2$, yielding a value of $E_2 - E_0 = 2\hbar\omega_0$

2 Problems

1. Using Normalization Conditions

   Setting up the normalization equation, we get:

$$\int_{-\frac{L}{2}}^{0} \left(C\left(\frac{2x}{L} + 1\right)\right)^2 dx + \int_{0}^{\frac{L}{2}} \left(C\left(\frac{-2x}{L} + 1\right)\right)^2 dx = 1$$

   To simplify integration we can do the following:

$$\int_{-\frac{L}{2}}^{0} \left(\frac{2xC}{L} + C\right)^2 dx + \int_{0}^{\frac{L}{2}} \left(\frac{-2xC}{L} + C\right)^2 dx$$

$$C^2\int_{-\frac{L}{2}}^{0}\left(\frac{4x^2}{L^2}+\frac{4x}{L}+1\right)dx + C^2\int_{0}^{\frac{L}{2}}\left(\frac{4x^2}{L^2}-\frac{4x}{L}+1\right)dx$$

$$C^2\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{8x^2}{L^2}+2\right)dx$$

Finally, we must solve:

$$C^2\left(\frac{8x^3}{3L^2}+2x\right)\Big|_{-\frac{L}{2}}^{\frac{L}{2}}$$

$$C^2\left(\left(\frac{L}{3}+L\right)-\left(-\frac{L}{3}-L\right)\right)$$

Returning the "= 1", we get:

$$C^2\left(\frac{L}{3}\right)=1$$

$$C^2=\frac{3}{L}$$

Finally, we get:

$$\boxed{C=\pm\sqrt{\frac{3}{L}}}$$

2. Applying Boundary Conditions to Find Constants

   First and foremost, because e^x diverges, we know that $\boxed{C=0}$. This leaves A, B, and D. We know the functions must be continuous at boundary, meaning that, at $x=0$, the functions need to equal each other:

$$A\sin(k_0(0))+B\cos(k_0(0))=De^{-k_1(0)}$$

$$B=D$$

   Next, we know that the first order derivatives of the function must be continuous as well. The derivatives are:

$$\begin{cases} Ak_0\cos(k_0x)-Bk_0\sin(k_0x), & x<0 \\ -Dk_1e^{-k_1x}, & x>0 \end{cases}$$

   Similarly to the first step, we must plug in $x=0$, or the boundary where this changes at. This generates:

$$Ak_0=-Dk_1$$

Rearranging in terms of A, we get:

$$\boxed{B = D = -\frac{k_0 A}{k_1}}$$