The Wave-Like Properties of Particles

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Contents

1 De Broglie's Hypothesis

- After Einstein's theory, it was determined that light has dual particle-wave nature
- In 1924, Louis de Broglie proposes a hypothesis:
  - Any object moving with a momentum p is associated with a wave of wavelength λ, where:

$$\boxed{\lambda = \frac{h}{p}}$$

  - λ refers to the "De Broglie" wavelength, h is the Planck constant, and p is the momentum
  - For experimental measurement of the wave–like behavior of particles, the double and single-slit experiments were performed

2 Experimental Evidence for De Broglie Waves

- Particle Diffraction Experiment
  - For light of wavelength λ incident on a slit of width a, the diffraction pattern has a minimum at angles:

$$\boxed{a\sin(\theta) = n\lambda, \qquad n = 1, 2, 3, \cdots}$$

  - Each of the atoms acts as a scatter
  - The scattered electron waves interfere
  - The crystal serves as a diffraction grating
  - The maxima occurs at angle:

$$\boxed{d\sin(\phi) = n\lambda}$$

  - Where λ is the de Broglie wavelength

2.1 Double-Slit Experiment

- Question: Through which slit does the particle pass?
- Result: No diffraction pattern on the screen
- if we check which slit the particle passes through:
  - Particle behavior is measured

- We can not observe its wave nature simultaneously! (Principle of complementarity)

- Conclusion:
  - The electron will behave as a wave or a particle

3 Heisenberg Uncertainty Relationships

- Applying the uncertainty relationship to de Broglie waves:

$$p = \frac{h}{\lambda} \Rightarrow dp = -\frac{h}{\lambda^2}\, d\lambda \Rightarrow \Delta p = \frac{h}{\lambda^2}\Delta\lambda$$

- Finally, this yields:

$$\Delta x \Delta p \approx \varepsilon h$$

- From quantum mechanics:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\varepsilon = \frac{1}{4\pi}$$

$$\Delta x \Delta p \geq \frac{1}{2}\hbar$$

- Where $\hbar = \dfrac{h}{2\pi}$