Homework 6

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Permitted Wave Functions

(a) One reason why this function is not permitted is because it violates the normalization condition, $\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = 1$; more specifically, solving for the boundary conditions makes it violate this:

$$A\cos(kx) = B\sin(kx)$$
$$A\cos(0) = B\sin(0)$$
$$A = 0$$

Differentiating to find B:

$$0 = Bk\cos(kx)$$
$$B = 0$$

Because both constants are zero, the integral over the entire boundary does not equal 1.

(b) $\psi(x) = \dfrac{Ae^{-kx}}{x}$ can not be a solution because it is discontinuous; at the point $x = 0$, the function has a discontinuity.

(c) $A\sin^{-1}(kx)$ can not be a solution because it is discontinuous. $\sin^{-1}$ is only valid for values in the range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, and, thus, it must have a discontinuity somewhere in its domain, unless k were to have the value of zero; in such a case, the function would violate the normalization condition, as it would be zero over its whole domain.

(d) $A\tan(kx)$ can not be a solution because it is discontinuous every $n\pi$ intervals.

The Schrödinger Equation

For a particle with $\psi(x) = Cxe^{-bx}$, plugging into the Schrödinger equation would yield:

$$\left(-\frac{\hbar^2}{2m}\right)(b^2Cxe^{-bx} - 2bCe^{-bx}) + U(x)(Cxe^{-bx}) = E(Cxe^{-bx})$$
$$E = \left(-\frac{\hbar^2}{2m}\right)\left(b^2 - \frac{2b}{x}\right) + U(x)$$

The x terms balance and cancel out because E is constant:

$$E = -\frac{\hbar^2 b^2}{2m}$$

This makes $U(x)$ equal to the final term left when E cancels out:

$$U(x) = -\frac{b\hbar^2}{mx}$$

Expectation Values

(a) In ground state $(n = 1)$

$$\psi(x) = \sqrt{\frac{2}{L}} sin\left(\frac{n\pi x}{L}\right)$$

$$\int_0^L \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right) x\,dx = \frac{L}{2}$$

(b) In first excited state $(n = 2)$

$$\int_0^L \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right) x\,dx = \frac{L}{2}$$

It appears that the expected position value of any such particle in a unidimensional well would be $\frac{L}{2}$

A Particle in a 3D Box

	$n(x, y, z)$	Degeneracies
$22E_o$	$(3, 3, 2)$	$(3, 2, 3)$, $(2, 3, 3)$
$21E_o$	$(4, 2, 1)$	$(1, 2, 4)$, $(1, 4, 2)$, $(4, 1, 2)$, $(2, 1, 4)$, $(2, 4, 1)$
$19E_o$	$(3, 3, 1)$	$(3, 1, 3)$, $(1, 3, 3)$
$18E_o$	$(4, 1, 1)$	$(1, 4, 1)$, $(1, 1, 4)$
$17E_o$	$(3, 2, 2)$	$(2, 3, 2)$, $(2, 2, 3)$
$14E_o$	$(3, 2, 1)$	$(1, 2, 3)$, $(1, 3, 2)$, $(3, 1, 2)$, $(2, 1, 3)$, $(2, 3, 1)$
$12E_o$	$(2, 2, 2)$	None
$11E_o$	$(3, 1, 1)$	$(1, 3, 1)$, $(1, 1, 3)$
$9E_o$	$(2, 2, 1)$	$(1, 2, 2)$, $(2,\ 1,\ 2)$
$6E_o$	$(2,\ 1,\ 1)$	$(1, 2, 1)$, $(1,\ 1,\ 2)$
$3E_o$	$(1, 1, 1)$	None

Figure 1: Energy Levels of $E_n(n_x, n_y, n_z) = E_o(n_x^2 + n_y^2 + n_z^2)$

Quantum Simple Harmonic Oscillator

(a)

(b)