Homework 1

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1. Binomial Expansion Exercise $\rightarrow (1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2$

   (a) $\left(\sqrt{1-\frac{u^2}{c^2}}\right)^{-1} = \left(1-\frac{u^2}{c^2}\right)^{-\frac{1}{2}} \longrightarrow b = -\frac{u^2}{c^2}; n = -\frac{1}{2} \longrightarrow \boxed{\gamma = 1 + \frac{u^2}{2c^2} + \frac{3u^4}{8c^4}}$

   (b) $\sqrt{1-\frac{u^2}{c^2}} = \left(1-\frac{u^2}{c^2}\right)^{\frac{1}{2}} \longrightarrow b = -\frac{u^2}{c^2}; n = \frac{1}{2} \longrightarrow \boxed{\frac{1}{\gamma} = 1 - \frac{u^2}{2c^2} - \frac{u^4}{8c^4}}$

2. Inertial Reference Frames

   (a) The two are not necessarily equal; the results depend on the motion of each respective frame of reference, mostly due to the effect of length contraction

   (b) Again, this value would not necessarily be the same in each frame; here, the mass of the proton is dependent on the relative speed of the proton with respect to each frame

   (c) Einstein's second postulate means that this is equal in both reference frames

   (d) This would not necessarily be equal in different frames, as, due to the effect of time dilation, if movement with respect to one reference frame is much closer to the speed of light than the other, the faster moving object will experience time in a slower manner

   (e) Because Newton's first law depends on relative motion of a frame of reference, results would be different depending on which frame this is experienced in

   (f) This would be the same, as the order of periodic elements is not influenced by motion, and, thus would remain the same regardless of reference frames

   (g) The charge of an electron is a fundamental constant, and most definitely unaffected by motion, and, thus, it would remain the same in both frames

3. Michelson-Morley Experiment

   (a) • Horizontal distance to travel: L to the right and L to the left; Horizontal velocity: $c - v$ to the right, and $c + v$ to the left. Because time is distance over velocity, we obtain:

   $$t_{\parallel} = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2Lc}{c^2 - v^2} = \frac{2L}{c}\frac{1}{1-\frac{v^2}{c^2}}$$

   • Vertical distance to travel L up and L down; Velocity vector: c, horizontal velocity $u \longrightarrow$ vertical velocity : $\sqrt{c^2 - v^2}$

   $$t_{\perp} = \frac{2L}{\sqrt{c^2-v^2}} = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

   $$\boxed{\Delta t = \frac{2L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)}$$

(b)

$$f = \frac{v}{\lambda}; f = \frac{N}{\Delta t}; N = \frac{v}{\lambda}\Delta t$$

$$\Delta t = \frac{4L}{c}\left(\frac{1}{1-\frac{v^2}{c^2}} - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\Delta t = \frac{4(11)}{30000}\left(\frac{1}{1-\frac{(30000)^2}{(3\cdot 10^8)^2}} - \frac{1}{\sqrt{1-\frac{(30000)^2}{(3\cdot 10^8)^2}}}\right)$$

$$= 7.333[\text{ps}]$$

$$(7.333\cdot 10^{-12})\cdot\frac{30000}{500\cdot 10^{-9}} = (7.333)\frac{30}{500} = .44$$

- $\boxed{\text{There was a fringe shift of .44}}$