Homework 5

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1. **Estimations of de Broglie Waves**

   (a) Boltzmann Constant in $\frac{\mathrm{J}}{\mathrm{K}} = 1.381 \cdot 10^{-23}$; Room temperature in $\mathrm{K} = 293$

$$K_{avg} = \frac{3}{2} k_b T$$

$$p = \sqrt{2mK_{avg}}$$

$$\lambda_{avg} = \frac{6.626 \cdot 10^{-34}}{\sqrt{3 \cdot 28.013 \cdot 1.66 \cdot 10^{-27} \cdot 293.15 \cdot 1.38 \cdot 10^{-24}}}$$

$$\boxed{\lambda_{avg} = 2.79 \cdot 10^{-11}[\mathrm{m}] = .0279[\mathrm{nm}]}$$

   (b) $.02[\mathrm{eV}] = 3.204 \cdot 10^{-21}[\mathrm{J}]$

$$\lambda_{avg} = \frac{h}{\sqrt{2mK}}$$

$$\frac{h}{\sqrt{2mK}} = \frac{6.626 \cdot 10^{-34}}{\sqrt{2 \cdot 1.675 \cdot 10^{-27} \cdot 3.204 \cdot 10^{-21}}}$$

$$\boxed{\lambda_{avg} = 2.022 \cdot 10^{-10}[\mathrm{m}] = .2022[\mathrm{nm}]}$$

   (c) $1\left[\frac{\mathrm{m}}{\mathrm{yr}}\right] = 3.17 \cdot 10^{-8}\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$

$$\lambda_{avg} = \frac{6.626 \cdot 10^{-34}}{.001 \cdot 3.17 \cdot 10^{-8}}$$

$$\boxed{\lambda_{avg} = 2.09 \cdot 10^{-23}[\mathrm{m}] = 20.9[\mathrm{ym}]}$$

2. **de Broglie Wave of a Proton**

   (a) $L = .01[\mathrm{m}]$, so the round-trip distance for one oscillation is $2L = .02[\mathrm{m}]$. Thus:

$$2L = n\lambda$$

   Rearranging, we get:

$$\boxed{\lambda = \frac{2L}{n}}$$

   (b)

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$$\frac{h}{\sqrt{2mK}} = \frac{2L}{n}$$

$$2mK = \left(\frac{nh}{2L}\right)^2$$

$$K = \frac{\frac{n^2h^2}{4L^2}}{2m}$$

For $n = 1$

$$\boxed{K_1 = \frac{\frac{(6.626 \cdot 10^{-34})^2}{4(.01)^2}}{2(1.67 \cdot 10^{-27})} = 2.05 \cdot 10^{-18}[\mathrm{eV}]}$$

For $n = 2$

$$\boxed{K_2 = \frac{\frac{4(6.626 \cdot 10^{-34})^2}{4(.01)^2}}{2(1.67 \cdot 10^{-27})} = 8.2 \cdot 10^{-18}[\mathrm{eV}]}$$

3. e^- and e^+ Annihilation

(a)

$$\boxed{\lambda_{e^-,e^+} = \frac{6.626 \cdot 10^{-34}}{9.109 \cdot 10^{-31} \cdot 3 \cdot 10^6} = .2425[\mathrm{nm}]}$$

(b)

$$E = mc^2 + \sum K$$

$$\left(9.109 \cdot 10^{-31}\right)\left(3 \cdot 10^8\right)^2 + \frac{1}{2}\left(9.109 \cdot 10^{-31}\right)\left(3 \cdot 10^6\right)^2$$

$$\boxed{E = 8.2 \cdot 10^{-14}[\mathrm{J}] = 511,803.7[\mathrm{eV}]}$$

$$\boxed{\lambda = \frac{E}{hc} = \frac{511,803.7}{1240} = 412.75[\mathrm{nm}]}$$

$$E = pc^{1}$$

$$\boxed{p = \frac{E}{c} = 2.73 \cdot 10^{-22}\left[\frac{\mathrm{kg\,m}}{\mathrm{s}}\right] = .511\left[\frac{\mathrm{MeV}}{\mathrm{c}}\right]}$$

[1] for photons

4. Uncertainty

(a)

$$\Delta x \Delta p \approx \frac{h}{2\pi}$$

$$\lambda \Delta p \approx \frac{h}{2\pi}$$

$$\Delta p \approx \left(\frac{h}{\lambda}\right)\frac{1}{2\pi}$$

$$\boxed{\Delta p \approx \frac{p}{2\pi}}$$

(b)

$$\lambda = \frac{h}{\gamma p}$$

$$E_k = m_n c^2(\gamma - 1)$$

$$\frac{E_k}{m_n c^2} = \gamma - 1$$

$$\gamma = \frac{10}{(6.231 \cdot 10^{12}) \cdot 1.675 \cdot 10^{-27} \cdot (3 \cdot 10^8)^2} + 1$$

$$\gamma = 1.0107$$

$$v = \sqrt{c^2 - c^2\left(\frac{1}{1.0107}\right)^2}$$

$$v = .145c$$

$$\lambda = \frac{6.626 \cdot 10^{-34}}{1.0107 \cdot 1.675 \cdot 10^{-27} \cdot .145c}$$

$$\boxed{\lambda = 9 \cdot 10^{-15}[\mathrm{m}] = 9[\mathrm{fm}]}$$

9[fm] is greater than 1[fm] but less than 10[fm], so atom nuclei may be used to demonstrate the wave nature of 10[MeV] neutrons.