Introduction to Modern Physics

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Contents

1 Modern Physics

- Modern physics is a set of developments that emerged around 1900
- This led to the development of the Theory of Relativity and Quantum Theory
- Some theories of classical physics which helped develop modern physics, include:
  - Newton's law of mechanics, which describes interactions among microscopic particles
  - Maxwell's equations, which unify electricity and magnetism
  - The laws of thermodynamics
- In the early 20$^{\text{th}}$ century, two theories emerged:
  - Special Theory of Relativity (1905) — Einstein
  - Quantum Theory (1900) — Planck
- Classical Relativity
  - A theory of relativity provides a mathematical basis for expressing physical laws in different frames of reference
  - The mathematical basis is called a transformation
  - Ex. Two observers, O, who is still, and O', who is moving, are at rest in their own frames of reference (FOR). Relative velocity is defined as $\overrightarrow{u}$. For this course, an inertial FOR will be used, meaning Newton's law holds, where $v = 0$, or constant, unless $\overrightarrow{F} \neq 0$. O and O' observe the same event.
    * Four quantities describe this event for O: x, y, z, t
    * For O', these quantities are: x', y', z', t'
    * Assuming postulate: $t = t'$
      · Also, at $t = 0$, the two origins coincide
    * To find x' from x, this would become $x' = x - ut$
    * y' and z' remain equal to y and z, respectively
    * This is defined as a Galilean Transformation
    * As velocity is the first derivative, this yields $\begin{cases} v_x = \dfrac{dx}{dt} \\ v_y = \dfrac{dy}{dt} \\ v_z = \dfrac{dz}{dt} \end{cases}$ and $\begin{cases} v_{x'} = v_x - u \\ v_{y'} = v_y \\ v_{z'} = v_z \end{cases}$ for O and O', respectively
    * This means the acceleration components are all equal
- Consequences of classical relativity

- From Maxwell's equations, it is concluded that light is an electromagnetic wave
  * Light travels in some medium, at speed $\boxed{c = \dfrac{1}{\sqrt{\mu_0 \epsilon_0}}} \approx 3 \times 10^8 \left[\frac{\text{m}}{\text{s}}\right]$
  * A postulate from Maxwell is that there is a preferred frame of reference with "ether" at rest, in which the speed of light is precisely c
  * Ether — An invisible, massless medium

- Michelson-Morley Experiment (1887)

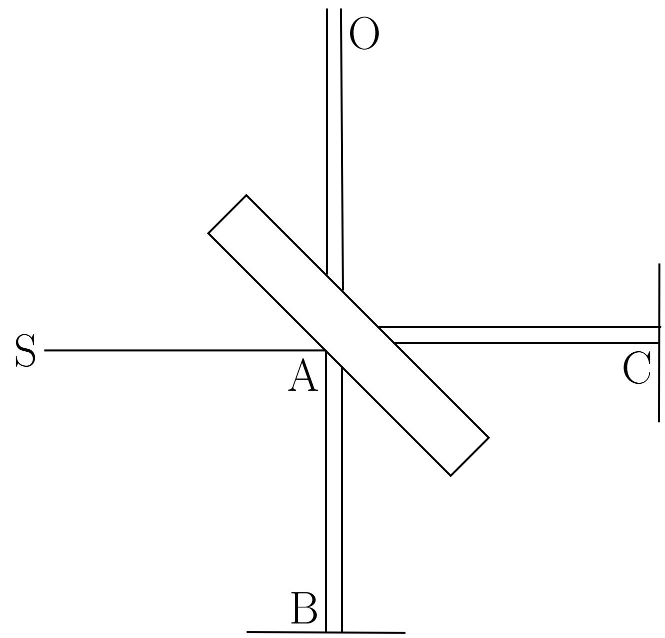

Figure 1: The Michelson-Morley Setup

  - S is the source, O is an observer, and A, B, and C, are points along the path of light
  - Generated a "fringe" pattern using light and mirrors
  - Interference or "fringe" appears due to phase difference of light
    * Path difference: $2|AB - AC|$
    * Light travels faster through a cross-stream pattern
  - With the same setup shown, they then rotated the device 90°
    * 2^{nd} contribution then changes sign
    * Thus, phase difference changes
    * Number of fringes was measured

      * The result: There was no observable change of fringe pattern — the movement of ether was mapped out to be a speed of $u < 5 \left[\frac{\text{km}}{\text{s}}\right]$
      * This experiment was redone over the course of many years, most recently Herman at al. (2009), with $u < 10^{-8} \left[\frac{\text{cm}}{\text{s}}\right]$
  - This indicates that c is a constant, in any inertial reference frame

- Einstein's postulates for inertial relativity

  1. The principle of relativity — The physical laws are the same in all inertial reference frames
  2. The principle of the constancy of the speed of light — The speed of light in free space has the same value c in all inertial reference frames

  - The second postulate requires observers in all inertial reference frames to measure the same speed of c for the light beam
  - This explains the failure of Michelson & Morley
  - Now we can "dispose" of the ether hypothesis

  1. 1^{st} postulate doesn't allow a preferred frame of reference where ether stays at rest
  2. 2^{nd} postulate doesn't allow only a single frame of reference with light moving at speed c

2 The Relativity of Time

- Time is relative

  - The time for light to hit a mirror and bounce back would be calculated by $\boxed{\Delta t_0 = \frac{2L_0}{c}}$
  - If an observer were to watch a mirror moving at speed $\overrightarrow{u}$, as shown in figure 2, the light would appear to have a triangular path
  - This would mean that the time difference is scaled by $\left(\sqrt{1 - \frac{u^2}{c^2}}\right)^{-1}$, which means $\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{u^2}{c^2}}}}$
  - This phenomenon is known as time dilation, which means that time moves slower for an observer moving faster than another observer

    O measures a longer time than O' — this is a general result of special relativity — even the growth and aging of living systems is affected

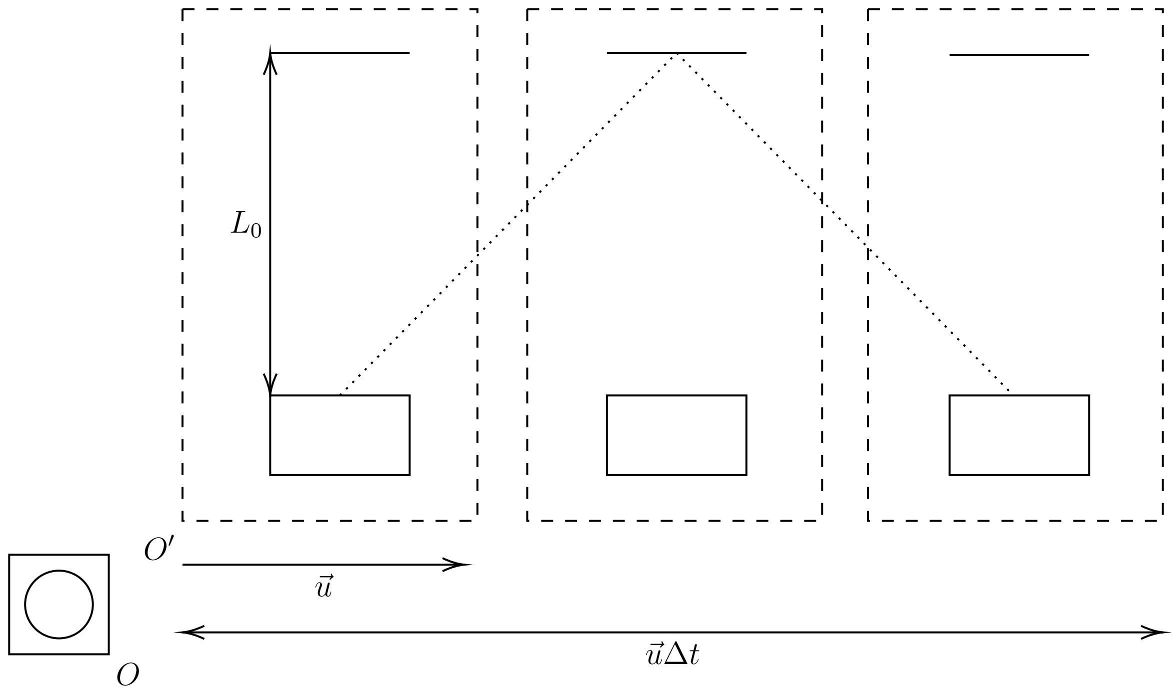

Figure 2: O observes the movement of O'

- Δt_0 is known as the "proper time", which is the time measured in the same reference frame as the motion
- Δt is always longer than Δt_0, no matter what $\overrightarrow{u}$ is
- This experiment is verified by $\begin{cases} \text{decaying elemental particles} \\ \text{atomic clocks} \end{cases}$
- Example: muon → Muon is the combination of air and cosmic rays; it decays with $t_0 = 2.2[\mu\text{s}]$
- The muon should decay significantly faster than it is able to reach Earth, and, thus, it shouldn't be measurable from the Earth's surface — but it still is; this is because the muon experiences time more slowly, slowing its decay in our frame of reference from Earth
  - Muons can not actually travel at the speed of light; the speed is closer to $0.999978c$

3 The Relativity of Length

– Another consequence is that length is relative; the moving device is now timed sideways

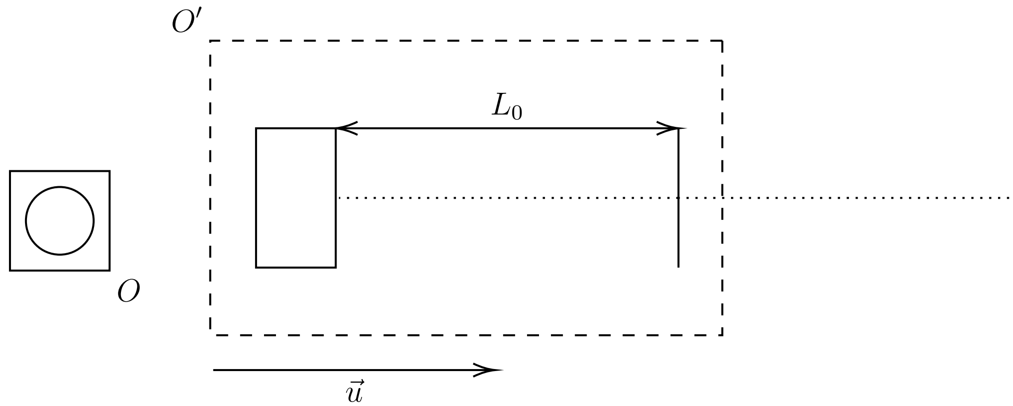

Figure 3: Length Becomes Relative

– The light is emitted when O' is at its starting position, and reaches the mirror at time Δt_1; it travels back to the emitter in interval Δt_2

– This results in a series of calculations:

$$c\Delta t_1 = L + u\Delta t_1 \Rightarrow \Delta t_1 = \frac{L}{c-u}$$
$$c\Delta t_2 = L - u\Delta t_2 \Rightarrow \Delta t_2 = \frac{L}{c+u}$$
$$\Delta t_{\text{total}} = \frac{L}{c-u} + \frac{L}{c+u} =$$
$$\frac{2Lc}{c^2-u^2} \Rightarrow \frac{2L}{c}\frac{c^2}{c^2-u^2}$$

– Finally, this yields

$$L = L_0\sqrt{1-\frac{u^2}{c^2}}$$

– This effect is called "length contraction"

– O' measures the proper length, L_0, because it is at rest with respect to the object

– Conclusion: An object in motion is measured to have a shorter length than at when it is at rest

– In the case of the muon, an observer on Earth experiences time dilation, while an observer following the muon experiences length contraction

    * A time dilation in one reference frame (say, O on Earth), is equivalent to a length contraction in another reference frame (say, O' traveling with the muon)

4 The Doppler Effect

- Classical Doppler Effect
  - An observer (O) moving relative to a source (S) of a (sound) wave detects a frequency (f') different from that emitted by the source (f)
  - The difference experienced is given by the formula below, where v is the speed of the wave in a given medium, v_s is the speed of S relative to the medium, and v_o is the speed of the observer:

$$\boxed{f' = f\frac{v \pm v_o}{v \mp v_s}}$$

  - The first option (addition in numerator and subtraction in denominator) occurs when O and S are moving toward each other; the second option (subtraction in the numerator and addition in the denominator) occurs when O and S are moving away from each other
  - This means that the speed of O and S with respect to the medium determines the Doppler Effect; however, for light, no medium is necessary, meaning a theory for light is necessary, where only the relative motion between S and O matters
    * This led to the development of the Theory of Relativity
  - Consider S at rest in the frame of reference of observer O. Observer O' moves relative to S at speed u. O observes S to emit N waves at frequency f in a time interval given by:

$$\boxed{\Delta t_o = \frac{N}{f}}$$

  - In the reference frame of O', the time interval is $\Delta t'$ due to time dilation, and the wavelength becomes

$$\boxed{\lambda' = \frac{c\Delta t' + u\Delta t'}{N} = \frac{(c+u)\Delta t'}{f\Delta t_o}}$$

$$\boxed{f' = \frac{c}{\lambda'} = \frac{f\Delta t_o}{\Delta t'}\frac{c}{c+u}}$$

- Applying the formula for time dilation, the frequency in the reference frame of O' becomes:

$$f' = f\frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}} = f\frac{\sqrt{1+\frac{u}{c}}\sqrt{1-\frac{u}{c}}}{\sqrt{1+\frac{u}{c}}}$$
$$= \boxed{f\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}}$$

- This is known as the relativistic Doppler Effect[1]

5 Lorentz Transformation

- Galilean transformation is not consistent with Einstein's postulates
- A new set of transformations that is capable of predicting the relativistic effects is necessary
- This transformation relates the measurements of O (x, y, z, t) to those of O' (x', y', z', t')
- Some necessary properties are:
  1. Linear equations (1^{st} power of space and time)
  2. Consistent with Einstein's postulates
  3. Reduces to Galilean transformation when $u << c$, or $\dfrac{u}{c} << 1$
- This new transformation is known as the Lorentz transformation. This generates the following:

$$\boxed{\begin{cases} x' &= \frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t-\frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}} \end{cases}}$$

- When $u << c$, this generates the Galilean transformation:

$$\boxed{\begin{cases} x' &= x - ut \\ y' &= y \\ z' &= z \\ t' &= t \end{cases}}$$

[1] The sign of u changes if S and O' are moving toward each other

- Velocity Transformation
  - If O observes a particle traveling with velocity v (v_x, v_y, v_z), what velocity, v', does O' observe for the particle?
  - Based on the Lorentz velocity transformation:

$$\boxed{\begin{cases} v'_x &= \dfrac{v_x - u}{1 - \frac{v_x u}{c^2}} \\ v'_y &= \dfrac{v_y\sqrt{1 - \frac{u^2}{c^2}}}{1 - \frac{v_x u}{c^2}} \\ v'_z &= \dfrac{v_z\sqrt{1 - \frac{u^2}{c^2}}}{1 - \frac{v_x u}{c^2}} \end{cases}}$$

  - A strange result here: $v'_y, v'_z \neq v_y, v_z$, even if $y', z' = y, z$. This is because $t \neq t'$

$$dt' = \frac{dt - \frac{u}{c^2}dx}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$v'_y = \frac{dy'}{dt'} = dy\frac{\sqrt{1 - \frac{u^2}{c^2}}}{1 - \frac{v_x u}{c^2}} = \frac{v_y\sqrt{1 - \frac{u^2}{c^2}}}{1 - \frac{v_x u}{c^2}}$$

  - The same method is used to find v'_z
- Simultaneity and Clock Synchronization
  - If two events happen at the same time in one inertial reference frame, do they still happen simultaneously (in another moving inertial reference frame)? No
  - If the distance L, between the clocks was 0, then they still happen simultaneously

6 The Twin Paradox

- A pair of twins on Earth are named Bob and Alice. While Bob stands on Earth, Alice joins NASA and travels to a distant planet and back. Bob knows that Alice's clock will move slower because of time dilation, and she should be younger than Bob when she returns. For two observers in relative motion, each thinks the other's clock is running slow. Alice experiences Bob and Earth making a round-trip journey; for her, it seems that Bob should also be younger. This is known as the twin paradox.
- Each twin expects the other to be younger. How can this be explained?

- For Alice to make the journey to the planet, she needs to accelerate quickly to escape Earth's atmosphere, and only then can she move in an inertial reference frame. To turn around she would then have to decelerate to turn around. This is inconsistent with Special Relativity, as multiple frames of reference, both inertial and not, are experience.
- Conceptually, Bob and Alice are NOT equivalent for any observer in any inertial reference frame
- If the situation is mirrored, and Bob and Alice travel the same distance, L, in opposite directions, then, upon returning, their age would be the same
- In principle, though, no Special Relativity laws should be applied within the regions experiencing acceleration

* It is actually Alice who is in motion, and her clock is running slow, so she will be the younger one
  - Bob stays in one inertial reference frame (Earth)
  - Alice has to decelerate to return to Earth
  - This asymmetry is the basic cause

* Suppose the planet is at rest relative to the Earth and the distance is 6 light-years, while Alice is traveling at a speed of $.6c$
  - According to Bob, it will take 10 years for Alice (6 / .6) to reach the planet and 10 years to return, so 20 years have passed for Bob
  - Due to length contraction, Alice experiences the distance as $L = L_0\sqrt{1-\frac{u^2}{c^2}}$, or $L = 6(\sqrt{1-.36}) = 4.8$ light years, which means it takes her 4.8 / .6 = 8 years one way, or 16 years for a round-trip
  - This means she will be effectively 4 years younger than Bob, or Bob will be 4 years older

* Another way to understand the twin paradox
  - Bob sends Alice a light signal each year on his birthday
  - These sort of light signals can be viewed as waves

$$\boxed{\text{Outward:} \quad f' = f\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}}$$

  - This means that, for Alice, the frequency outward is:

$$f' = (1)(.5) = .5\left[\frac{1}{\text{yr}}\right]$$

- And the frequency inward is:

$$f' = (1)(2) = 2\left[\frac{1}{\text{yr}}\right]$$

- Which means, for her, the total is $8(.5) + 8(2) = 20$ signals
- Thus, the total for Alice is 16 years, while it is 20 years for Bob
- As such, we obtain the same result

- Spacetime Diagrams
  - Switch space and time axes
  - A line representing motion is called a "worldline"
  - $\tan(\theta)$ = velocity (because it becomes distance per time)

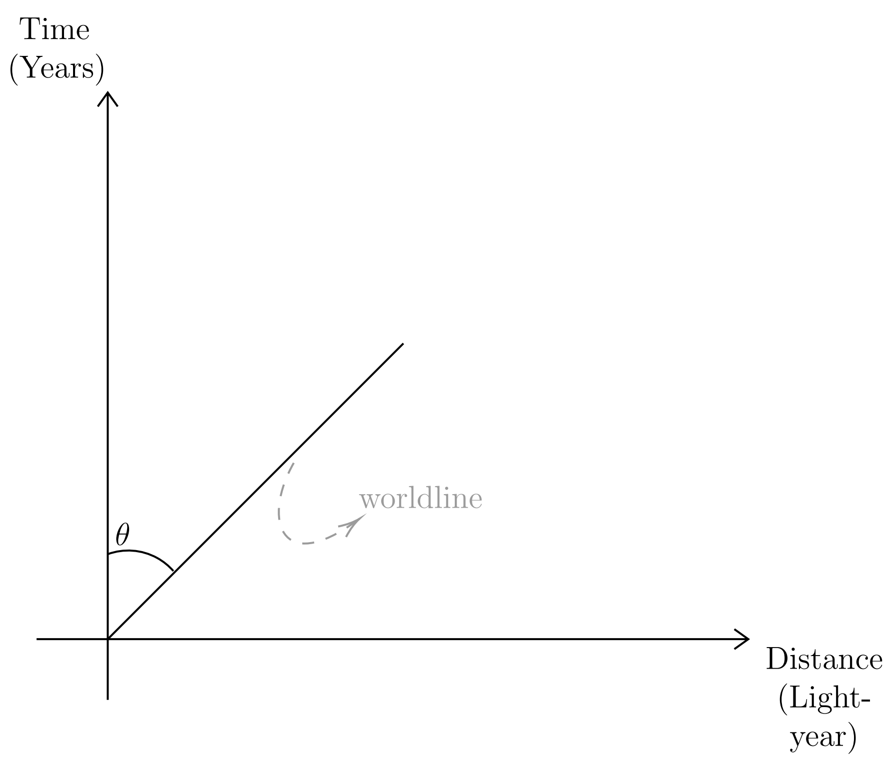

Figure 4: A Spacetime Diagram Example

  - The confining angles are $-45° \leq \theta \leq 45°$ because the line for traveling light would form a line at the two endpoint values
  - "Light cones" are possible value ranges formed by mirrored 45° lines
  - Bob and Alice's worldlines, respectively, would look as follows:

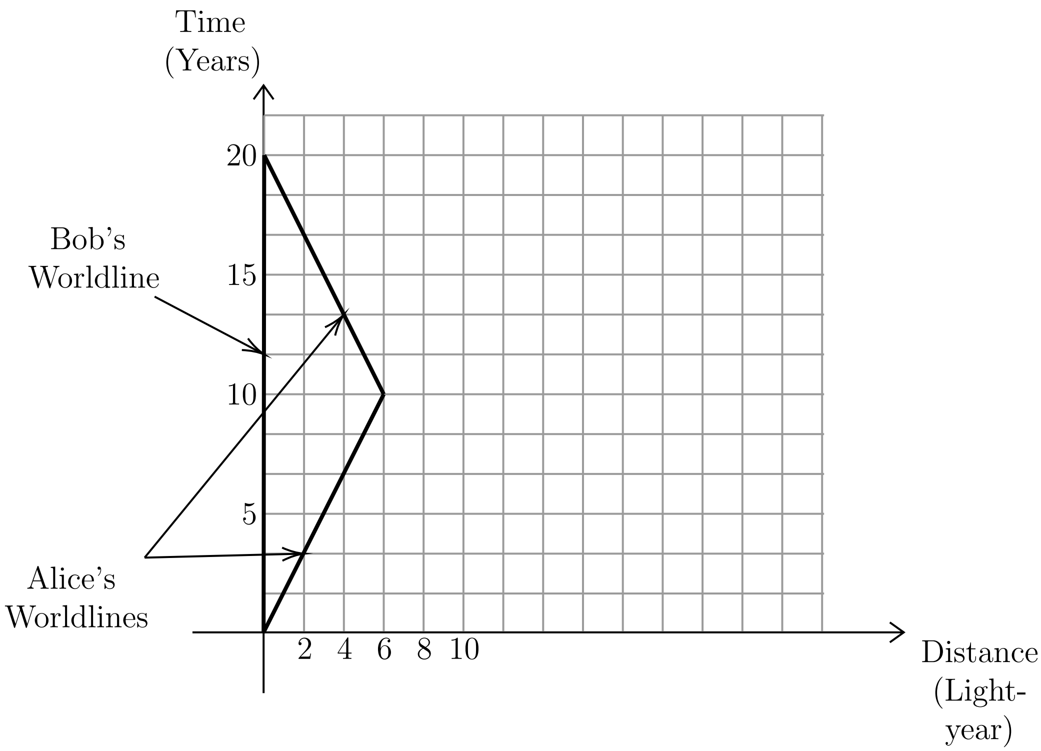

Figure 5: Bob and Alice's Spacetime Diagrams

  - Bob's worldline is vertical, signifying no motion
  - Alice has two straight lines (two inertial reference frames), as she moves to and then away from the distance object
  - Light signals travel from Bob to Alice along the 45° direction
  - Light signals travel from Alice to bob along the −45° direction

- Lorentz transformation prevents relative speed from reaching or exceeding c

7 Relativistic Dynamics

- Applies to the energies and momentums of objects

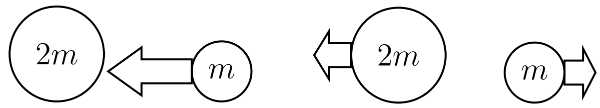

Figure 6: Momentum Collision

- Assuming the $2m$ mass is initially at rest, and m is moving at $-.75c$, how would the final result be derived?
- Using conservation of momentum, there should be $-.75mc$ total momentum at the end; however, a Lorentz transformation must be used when working with relativistic dynamics
- So is momentum conserved in relativistic dynamics?
  - No, $p_i \neq p_f$
- Thus, it was necessary to modify the mass in relativistic dynamics so that conservation of momentum holds true
  - The new mass had to be a function of the velocity
  - The following function was hypothesized, with v as the speed of an object in an inertial reference frame:

$$\boxed{m(v) = \frac{m}{\sqrt{1 - \dfrac{v^2}{c^2}}}}$$

  - At $v = 0$, the mass would be as normal, also known as the rest mass
  - But, as v approaches c, the mass approaches infinity!
  - This would mean relativistic momentum is defined as:

$$\overline{p} = m(v)\overline{v} \Rightarrow \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} m\overline{v}$$

  - The relativistic momentum conservation formula can then be applied to relativistic kinetic energy conservation
- Kinetic Energy in Relativity Theory
  - In classical physics, the work-energy theorem is:

$$K = W = \int_0^x F\,dx \Rightarrow F = \frac{d\overline{p}}{dt} \Rightarrow \int_0^{\overline{p}} \frac{dx}{dt}\,dp$$

  - $\dfrac{dx}{dt} = v$, so we have velocity involved

- Substituting and using integration by parts, we get:

$$\int_0^p v\,dp = pv - \int p\,dv$$

- This means that the kinetic energy may be defined as:

$$K = pv - \int_0^v p\,dv$$

$$K = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} - \int_0^v \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}\,dv$$

$$K = \frac{mv^2}{\sqrt{1-\frac{v^2}{c^2}}} + mc^2\sqrt{1-\frac{v^2}{c^2}} - mc^2$$

$$\boxed{K = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2}$$

- Sometimes, this formula is simplified as $K = \gamma mc^2 - mc^2$
  * γmc^2 is referred to as the relativistic total energy (E)
  * mc^2 is referred to as the rest energy (E_0)

$$E = K + E_0 = \gamma mc^2$$

- This is the relativistic kinetic energy
- When v approaches c, kinetic energy approaches
- This means it will take infinite energy to accelerate to the speed of light

• Bertozz et al Experiment (1964)
  - Accelerated an electron using an electric field
  - Found that electrons could not be accelerated to the speed of light
  - Verified the relativistic energy theorem

• The Energy-Momentum Relationship can be defined as:

$$E^2 = (mc^2)^2 + (pc)^2$$

  - When $v \approx c$, $E \approx pc$

- This means, for massless particles (*e.g.* photons):

$$\boxed{E = pc}$$

- This means the rest energy for an electron is $E_0 = .511[\text{MeV}]$
- The momentum of an electron (at $v = .75c$) would be:

$$p = \frac{m_e v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{.511 \cdot .75}{\sqrt{1 - .75^2}}$$

$$p = .58\left[\frac{\text{MeV}}{c}\right]$$

- The following are some important units to understand:
  - Energy: MeV
  - Momentum: MeV/c
  - Mass: MeV/c^2