

The Wave-Like Properties of Particles

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March 1, 2023

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1 De Broglie's Hypothesis

- After Einstein's theory, it was determined that light has dual particle-wave nature
- In 1924, Louis de Broglie proposes a hypothesis:
 - Any object moving with a momentum p is associated with a wave of wavelength λ , where:

$$\lambda = \frac{h}{p}$$

- λ refers to the “De Broglie” wavelength, h is the Planck constant, and p is the momentum
- For experimental measurement of the wave-like behavior of particles, the double and single-slit experiments were performed

2 Experimental Evidence for De Broglie Waves

- Particle Diffraction Experiment
 - For light of wavelength λ incident on a slit of width a , the diffraction pattern has a minimum at angles:

$$a \sin(\theta) = n\lambda, \quad n = 1, 2, 3, \dots$$

- Each of the atoms acts as a scatter
- The scattered electron waves interfere
- The crystal serves as a diffraction grating
- The maxima occurs at angle:

$$d \sin(\phi) = n\lambda$$

- Where λ is the de Broglie wavelength

2.1 Double-Slit Experiment

- Question: Through which slit does the particle pass?
- Result: No diffraction pattern on the screen
- if we check which slit the particle passes through:
 - Particle behavior is measured

- We can not observe its wave nature simultaneously! (Principle of complementarity)
- Conclusion:
 - The electron will behave as a wave or a particle

3 Heisenberg Uncertainty Relationships

- Applying the uncertainty relationship to de Broglie waves:

$$p = \frac{h}{\lambda} \Rightarrow dp = -\frac{h}{\lambda^2} d\lambda \Rightarrow \Delta p = \frac{h}{\lambda^2} \Delta \lambda$$

- Finally, this yields:

$$\Delta x \Delta p \approx \varepsilon h$$

- From quantum mechanics:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\varepsilon = \frac{1}{4\pi}$$

$$\Delta x \Delta p \geq \frac{1}{2} \hbar$$

- Where $\hbar = \frac{h}{2\pi}$
- When a coin is flipped, or a dice is rolled:
 - No way to predict a single flip/roll
 - But, we can predict the distribution of the results from a large # of flips or rolls
 - Quantum Theory allows for the same behavior
- Wave Function
 - What is the amplitude of the de Broglie wave?
 - Checking classical waves:
 - * Waves in the ocean: Height of water level
 - * Sound wave: Volume density of molecules

- * Light waves: \vec{E}, \vec{B} field
- * de Broglie waves: The probability of finding a particle at a given (x, t)
 - This is known as ψ , the wave function
 - In n -dimensional space, it becomes $\psi(x_1, x_2, \dots, x_n, t)$
 - In classical physics, the intensity (I) of any wave is proportional to $|A|$
 - For quantum mechanics, we have the probability of final particle $P \propto |\psi|^2$
- * The requirement for wave function ψ is that $|\psi|^2 \geq 0$
- * Any physical measurement is related to $P \propto |\psi|^2$
- * ψ are generally complex #'s
- * Properties of Complex Numbers
 - $\psi = Re(\psi) + iIm(\psi)$
 - The complex conjugate is: $\psi^* = Re(\psi) - iIm(\psi)$
- In the complex plane:
 - The phase factor is $\boxed{z = |z|e^{i\theta}}$
 - The wave function of a free particle is $Ae^{i(kx - \omega t)}$ or $A[\cos(kx - \omega t) + i\sin(kx - \omega t)]$
- Behavior of a wave function:
 - Reflection and transmission at a boundary
 - Penetration of the reflected wave
 - Continuity at the boundary
- The mathematical solution for wave functions
 - The wave function itself must be continuous
 - The slope of the wave function must be continuous (“boundary condition”)
 - By confining a particle by 2 boundaries, we’ve learned:
 - * It may be anywhere in space
 - * A definite/continuous valued λ, p, E
- In the left/right region:

$$U = qV = (-e)(-V_o) = eV_o$$

- A wave confined in a well with infinite-height barriers is known as a standing wave
- The two nodes ($A = 0$) are at two boundaries

- With this standing wave:

$$\lambda_n = \frac{2L}{n}, \quad n = 1, 2, 3 \dots$$

- Where n is the number of nodes, and L is the width of the well
- From de Broglie theory, the energy becomes:

$$E_n = n^2 \left(\frac{h^2}{8mL^2} \right)$$

- Where the expression in the parenthesis is E_o
- The n term makes it so that energy is quantized
- This is the general nature of quantum particles
- A particle confined in space \rightarrow energy is quantized
- Schrödinger's Equation:
 - The behavior of the wave function is controlled by a differential equation called "Schrödinger's equation"
 - The role is similar to Newton's 2nd law
 - It is a second order differential equation