Introduction to Modern Physics

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Contents

1 Modern Physics

- Modern physics is a set of developments that emerged around 1900
- This led to the development of the Theory of Relativity and Quantum Theory
- Some theories of classical physics which helped develop modern physics, include:
  - Newton's law of mechanics, which describes interactions among microscopic particles
  - Maxwell's equations, which unify electricity and magnetism
  - The laws of thermodynamics
- In the early 20$^{\text{th}}$ century, two theories emerged:
  - Special Theory of Relativity (1905) — Einstein
  - Quantum Theory (1900) — Planck
- Classical Relativity
  - A theory of relativity provides a mathematical basis for expressing physical laws in different frames of reference
  - The mathematical basis is called a transformation
  - Ex. Two observers, O, who is still, and O', who is moving, are at rest in their own frames of reference (FOR). Relative velocity is defined as $\vec{u}$. For this course, an inertial FOR will be used, meaning Newton's law holds, where $v = 0$, or constant, unless $\vec{F} \neq 0$. O and O' observe the same event.
    * Four quantities describe this event for O: x, y, z, t
    * For O', these quantities are: x', y', z', t'
    * Assuming postulate: $t = t'$
      · Also, at $t = 0$, the two origins coincide
    * To find x' from x, this would become $x' = x - ut$
    * y' and z' remain equal to y and z, respectively
    * This is defined as a Galilean Transformation
    * As velocity is the first derivative, this yields $\begin{cases} v_x = \dfrac{dx}{dt} \\ v_y = \dfrac{dy}{dt} \\ v_z = \dfrac{dz}{dt} \end{cases}$ and $\begin{cases} v_{x'} = v_x - u \\ v_{y'} = v_y \\ v_{z'} = v_z \end{cases}$ for O and O', respectively
    * This means the acceleration components are all equal
- Consequences of classical relativity

– From Maxwell's equations, it is concluded that light is an electromagnetic wave

  * Light travels in some medium, at speed $\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} \approx 3 \times 10^8 \left[\frac{\text{m}}{\text{s}}\right]$
  * A postulate from Maxwell is that there is a preferred frame of reference with "ether" at rest, in which the speed of light is precisely c
  * Ether — An invisible, massless medium

- Michelson-Morley Experiment (1887)

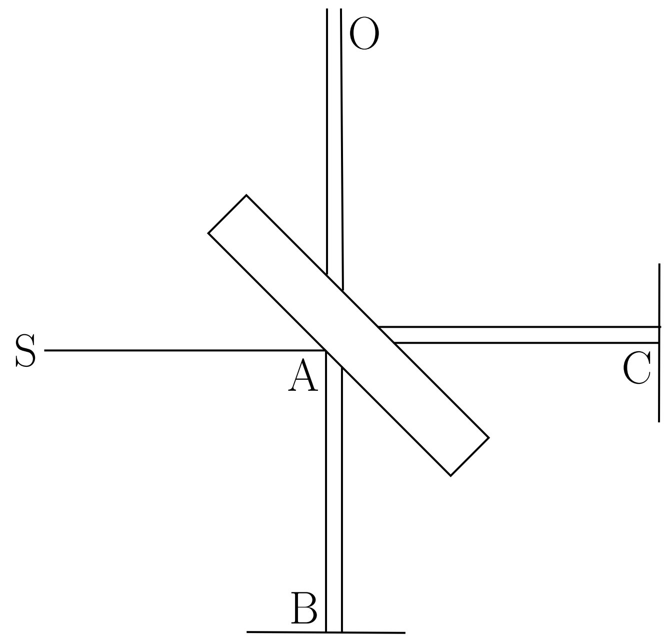

Figure 1: The Michelson-Morley Setup

– S is the source, O is an observer, and A, B, and C, are points along the path of light

– Generated a "fringe" pattern using light and mirrors

– Interference or "fringe" appears due to phase difference of light

  * Path difference: $2|AB - AC|$
  * Light travels faster through a cross-stream pattern

– With the same setup shown, they then rotated the device 90°

  * 2^{nd} contribution then changes sign
  * Thus, phase difference changes
  * Number of fringes was measured

    * The result: There was no observable change of fringe pattern — the movement of ether was mapped out to be a speed of $u < 5 \left[\frac{\text{km}}{\text{s}}\right]$
    * This experiment was redone over the course of many years, most recently Herman at al. (2009), with $u < 10^{-8} \left[\frac{\text{cm}}{\text{s}}\right]$
  - This indicates that c is a constant, in any inertial reference frame

- Einstein's postulates for inertial relativity

  1. The principle of relativity — The physical laws are the same in all inertial reference frames
  2. The principle of the constancy of the speed of light — The speed of light in free space has the same value c in all inertial reference frames

  - The second postulate requires observers in all inertial reference frames to measure the same speed of c for the light beam
  - This explains the failure of Michelson & Morley
  - Now we can "dispose" of the ether hypothesis

  1. 1^{st} postulate doesn't allow a preferred frame of reference where ether stays at rest
  2. 2^{nd} postulate doesn't allow only a single frame of reference with light moving at speed c

2 The Relativity of Time

- Time is relative

  - The time for light to hit a mirror and bounce back would be calculated by $\boxed{\Delta t_0 = \dfrac{2L_0}{c}}$
  - If an observer were to watch a mirror moving at speed $\overrightarrow{u}$, as shown in figure 2, the light would appear to have a triangular path
  - This would mean that the time difference is scaled by $\left(\sqrt{1-\dfrac{u^2}{c^2}}\right)^{-1}$, which means $\boxed{\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\dfrac{u^2}{c^2}}}}$
  - This phenomenon is known as time dilation, which means that time moves slower for an observer moving faster than another observer

    O measures a longer time than O' — this is a general result of special relativity — even the growth and aging of living systems is affected

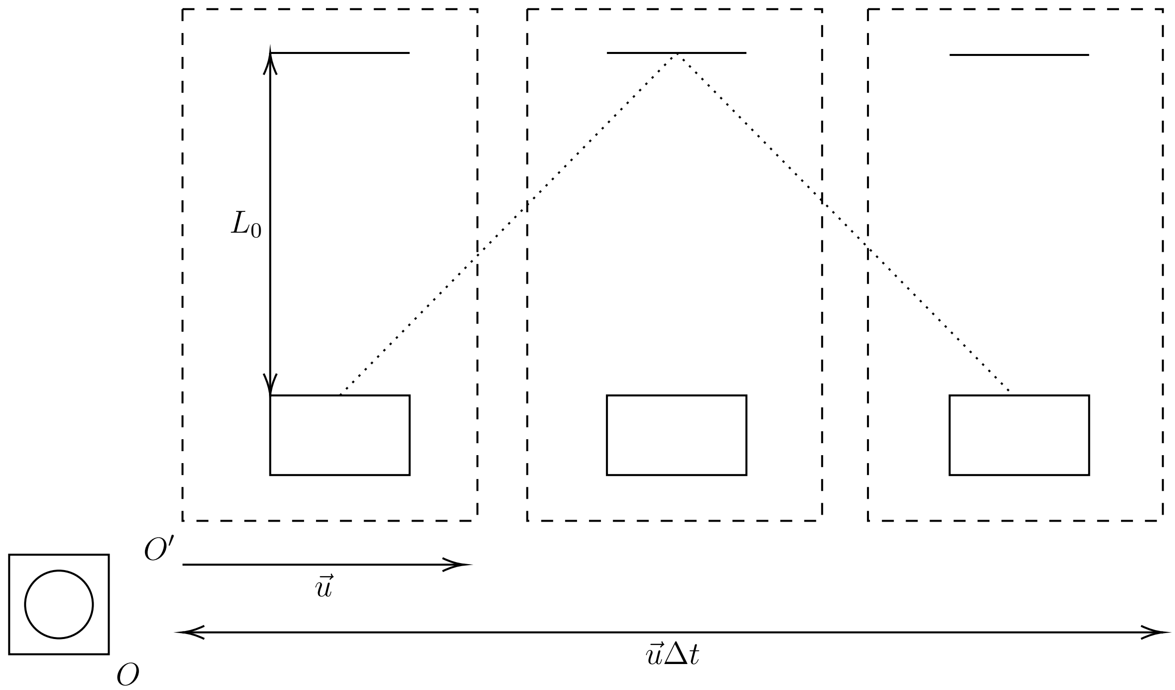

Figure 2: O observes the movement of O'

- Δt_0 is known as the "proper time", which is the time measured in the same reference frame as the motion
- Δt is always longer than Δt_0, no matter what $\overrightarrow{u}$ is
- This experiment is verified by $\begin{cases} \text{decaying elemental particles} \\ \text{atomic clocks} \end{cases}$
- Example: muon → Muon is the combination of air and cosmic rays; it decays with $t_0 = 2.2[\mu\text{s}]$
- The muon should decay significantly faster than it is able to reach Earth, and, thus, it shouldn't be measurable from the Earth's surface — but it still is; this is because the muon experiences time more slowly, slowing its decay in our frame of reference from Earth
  - Muons can not actually travel at the speed of light; the speed is closer to $0.999978c$

3 The Relativity of Length

– Another consequence is that length is relative; the moving device is now timed sideways

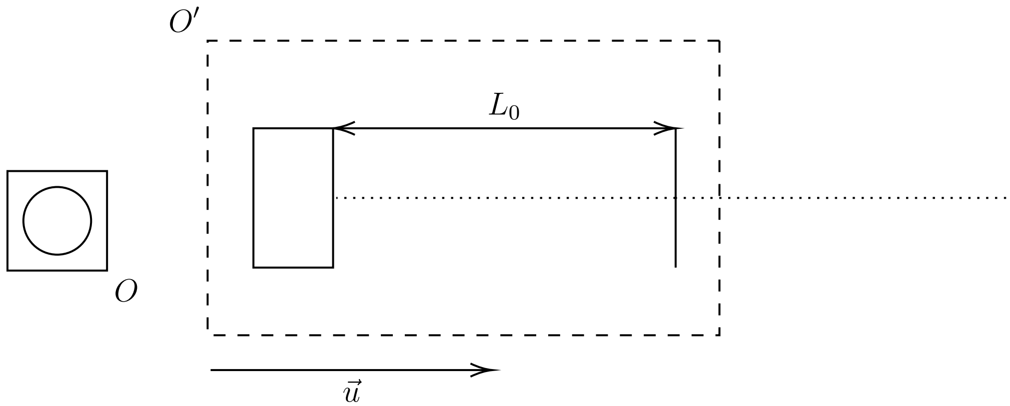

Figure 3: Length Becomes Relative

– The light is emitted when O' is at its starting position, and reaches the mirror at time Δt_1; it travels back to the emitter in interval Δt_2

– This results in a series of calculations:

$$c\Delta t_1 = L + u\Delta t_1 \Rightarrow \Delta t_1 = \frac{L}{c-u}$$
$$c\Delta t_2 = L - u\Delta t_2 \Rightarrow \Delta t_2 = \frac{L}{c+u}$$
$$\Delta t_{\text{total}} = \frac{L}{c-u} + \frac{L}{c+u} =$$
$$\frac{2Lc}{c^2-u^2} \Rightarrow \frac{2L}{c}\frac{c^2}{c^2-u^2}$$

– Finally, this yields

$$L = L_0\sqrt{1-\frac{u^2}{c^2}}$$

– This effect is called "length contraction"

– O' measures the proper length, L_0, because it is at rest with respect to the object

– Conclusion: An object in motion is measured to have a shorter length than at when it is at rest

– In the case of the muon, an observer on Earth experiences time dilation, while an observer following the muon experiences length contraction

    * A time dilation in one reference frame (say, O on Earth), is equivalent to a length contraction in another reference frame (say, O' traveling with the muon)

4 The Doppler Effect

- Classical Doppler Effect
  - An observer (O) moving relative to a source (S) of a (sound) wave detects a frequency (f') different from that emitted by the source (f)
  - The difference experienced is given by the formula below, where v is the speed of the wave in a given medium, v_s is the speed of S relative to the medium, and v_o is the speed of the observer:

$$\boxed{f' = f\frac{v \pm v_o}{v \mp v_s}}$$

  - The first option (addition in numerator and subtraction in denominator) occurs when O and S are moving toward each other; the second option (subtraction in the numerator and addition in the denominator) occurs when O and S are moving away from each other
  - This means that the speed of O and S with respect to the medium determines the Doppler Effect; however, for light, no medium is necessary, meaning a theory for light is necessary, where only the relative motion between S and O matters
    * This led to the development of the Theory of Relativity
  - Consider S at rest in the frame of reference of observer O. Observer O' moves relative to S at speed u. O observes S to emit N waves at frequency f in a time interval given by:

$$\boxed{\Delta t_o = \frac{N}{f}}$$

  - In the reference frame of O', the time interval is $\Delta t'$ due to time dilation, and the wavelength becomes

$$\boxed{\lambda' = \frac{c\Delta t' + u\Delta t'}{N} = \frac{(c+u)\Delta t'}{f\Delta t_o}}$$

$$\boxed{f' = \frac{c}{\lambda'} = \frac{f\Delta t_o}{\Delta t'}\frac{c}{c+u}}$$

- Applying the formula for time dilation, the frequency in the reference frame of O' becomes:

$$f' = f\frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}} = f\frac{\sqrt{1+\frac{u}{c}}\sqrt{1-\frac{u}{c}}}{\sqrt{1+\frac{u}{c}}}$$
$$= \boxed{f\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}}$$

- This is known as the relativistic Doppler Effect[1]

5 Lorentz Transformation

- Galilean transformation is not consistent with Einstein's postulates
- A new set of transformations that is capable of predicting the relativistic effects is necessary
- This transformation relates the measurements of O (x, y, z, t) to those of O' (x', y', z', t')
- Some necessary properties are:
  1. Linear equations (1^{st} power of space and time)
  2. Consistent with Einstein's postulates
  3. Reduces to Galilean transformation when $u << c$, or $\dfrac{u}{c} << 1$
- This new transformation is known as the Lorentz transformation. This generates the following:

$$\boxed{\begin{cases} x' &= \frac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t-\frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}} \end{cases}}$$

- When $u << c$, this generates the Galilean transformation:

$$\boxed{\begin{cases} x' &= x-ut \\ y' &= y \\ z' &= z \\ t' &= t \end{cases}}$$

[1] The sign of u changes if S and O' are moving toward each other

- Velocity Transformation
  - If O observes a particle traveling with velocity v (v_x, v_y, v_z), what velocity, v', does O' observe for the particle?
  - Based on the Lorentz velocity transformation:

$$\boxed{\begin{cases} v'_x &= \dfrac{v_x - u}{1 - \frac{v_x u}{c^2}} \\ v'_y &= \dfrac{v_y\sqrt{1-\frac{u^2}{c^2}}}{1 - \frac{v_x u}{c^2}} \\ v'_z &= \dfrac{v_z\sqrt{1-\frac{u^2}{c^2}}}{1 - \frac{v_x u}{c^2}} \end{cases}}$$

  - A strange result here: $v'_y, v'_z \neq v_y, v_z$, even if $y', z' = y, z$. This is because $t \neq t'$

$$dt' = \frac{dt - \frac{u}{c^2}dx}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$v'_y = \frac{dy'}{dt'} = dy\frac{\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{v_x u}{c^2}} = \frac{v_y\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{v_x u}{c^2}}$$

  - The same method is used to find v'_z
- Simultaneity and Clock Synchronization
  - If two events happen at the same time in one inertial reference frame, do they still happen simultaneously (in another moving inertial reference frame)? No
  - If the distance L, between the clocks was 0, then they still happen simultaneously