Introduction to Modern Physics

Michael Brodskiy
Professor: Q. Yan

January 12, 2023

Contents

1 Modern Physics

- Modern physics is a set of developments that emerged around 1900
- This led to the development of the Theory of Relativity and Quantum Theory
- Some theories of classical physics which helped develop modern physics, include:
  - Newton's law of mechanics, which describes interactions among microscopic particles
  - Maxwell's equations, which unify electricity and magnetism
  - The laws of thermodynamics
- In the early 20^{th} century, two theories emerged:
  - Special Theory of Relativity (1905) — Einstein
  - Quantum Theory (1900) — Planck
- Classical Relativity
  - A theory of relativity provides a mathematical basis for expressing physical laws in different frames of reference
  - The mathematical basis is called a transformation
  - Ex. Two observers, O, who is still, and O', who is moving, are at rest in their own frames of reference (FOR). Relative velocity is defined as $\overrightarrow{u}$. For this course, an inertial FOR will be used, meaning Newton's law holds, where $v = 0$, or constant, unless $\overrightarrow{F} \neq 0$. O and O' observe the same event.
    - Four quantities describe this event for O: x, y, z, t
    - For O', these quantities are: x', y', z', t'
    - Assuming postulate: $t = t'$
      - Also, at $t = 0$, the two origins coincide
    - To find x' from x, this would become $x' = x - ut$
    - y' and z' remain equal to y and z, respectively
    - This is defined as a Galilean Transformation
    - As velocity is the first derivative, this yields $\begin{cases} v_x = \dfrac{dx}{dt} \\ v_y = \dfrac{dy}{dt} \\ v_z = \dfrac{dz}{dt} \end{cases}$ and $\begin{cases} v_{x'} = v_x - u \\ v_{y'} = v_y \\ v_{z'} = v_z \end{cases}$ for O and O', respectively
    - This means the acceleration components are all equal
- Consequences of classical relativity

- From Maxwell's equations, it is concluded that light is an electromagnetic wave
  - Light travels in some medium, at speed $\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}} \approx 3 \times 10^8 \left[\frac{\text{m}}{\text{s}}\right]$
  - A postulate from Maxwell is that there is a preferred frame of reference with "ether" at rest, in which the speed of light is precisely c
  - Ether — An invisible, massless medium

- Michelson-Morley Experiment (1887)

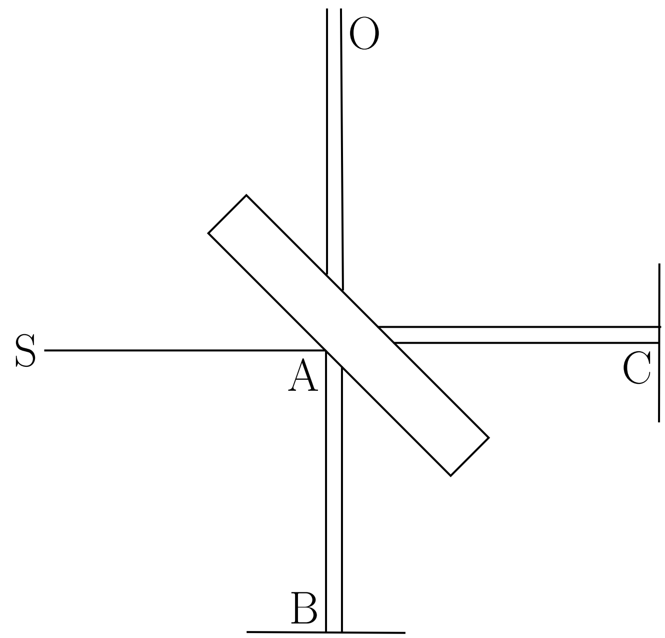

Figure 1: The Michelson-Morley Setup

- S is the source, O is an observer, and A, B, and C, are points along the path of light
- Generated a "fringe" pattern using light and mirrors
- Interference or "fringe" appears due to phase difference of light
  - Path difference: $2|AB - AC|$
  - Light travels faster through a cross-stream pattern
- With the same setup shown, they then rotated the device 90°
  - 2^{nd} contribution then changes sign
  - Thus, phase difference changes
  - Number of fringes was measured

    * The result: There was no observable change of fringe pattern — the movement of ether was mapped out to be a speed of $u < 5 \left[\frac{\text{km}}{\text{s}}\right]$
    * This experiment was redone over the course of many years, most recently Herman at al. (2009), with $u < 10^{-8} \left[\frac{\text{cm}}{\text{s}}\right]$
  - This indicates that c is a constant, in any inertial reference frame

- Einstein's postulates for inertial relativity

  1. The principle of relativity — The physical laws are the same in all inertial reference frames
  2. The principle of the constancy of the speed of light — The speed of light in free space has the same value c in all inertial reference frames

  - The second postulate requires observers in all inertial reference frames to measure the same speed of c for the light beam
  - This explains the failure of Michelson & Morley
  - Now we can "dispose" of the ether hypothesis

  1. 1^{st} postulate doesn't allow a preferred frame of reference where ether stays at rest
  2. 2^{nd} postulate doesn't allow only a single frame of reference with light moving at speed c

2 The Relativity of Time

- Time is relative

  - The time for light to hit a mirror and bounce back would be calculated by $\boxed{\Delta t_0 = \dfrac{2L_0}{c}}$
  - If an observer were to watch a mirror moving at speed $\vec{u}$, as shown in figure 2, the light would appear to have a triangular path
  - This would mean that the time difference is scaled by $\left(\sqrt{1 - \dfrac{u^2}{c^2}}\right)^{-1}$, which means $\boxed{\Delta t = \dfrac{\Delta t_0}{\sqrt{1 - \dfrac{u^2}{c^2}}}}$
  - This phenomenon is known as time dilation, which means that time moves slower for an observer moving faster than another observer

    O measures a longer time than O' — this is a general result of special relativity — even the growth and aging of living systems is affected

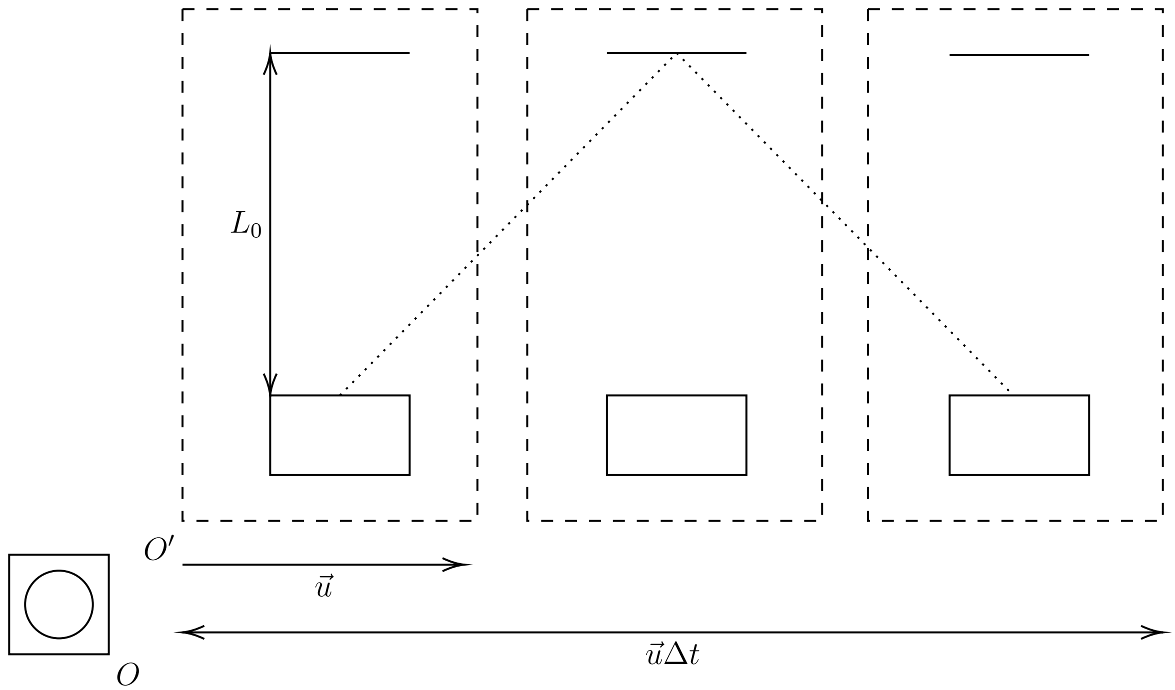

Figure 2: O observes the movement of O'

- Δt_0 is known as the "proper time", which is the time measured in the same reference frame as the motion
- Δt is always longer than Δt_0, no matter what $\overrightarrow{u}$ is
- This experiment is verified by $\begin{cases} \text{decaying elemental particles} \\ \text{atomic clocks} \end{cases}$
- Example: muon → Muon is the combination of air and cosmic rays; it decays with $t_0 = 2.2[\mu s]$
- The muon should decay significantly faster than it is able to reach Earth, and, thus, it shouldn't be measurable from the Earth's surface — but it still is; this is because the muon experiences time more slowly, slowing its decay in our frame of reference from Earth
  * Muons can not actually travel at the speed of light; the speed is closer to $0.999978c$

– Another consequence is that length is relative; the moving device is now timed sideways

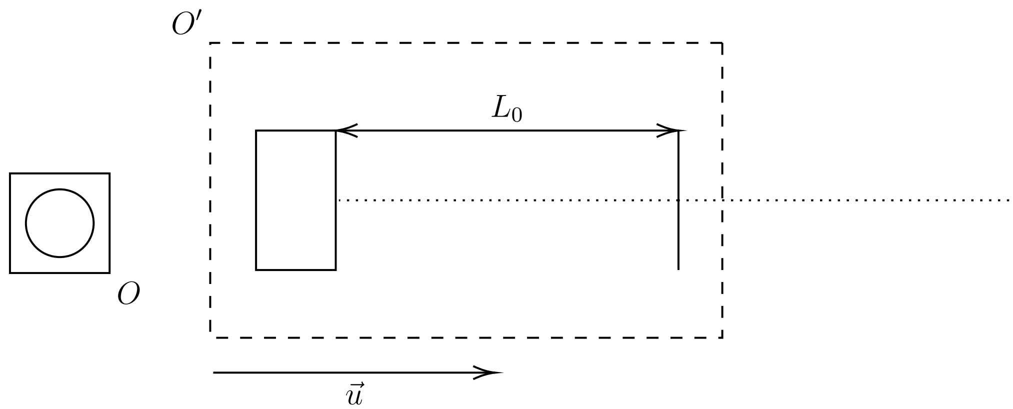

Figure 3: Length Becomes Relative

– The light is emitted when O' is at its starting position, and reaches the mirror at time Δt_1; it travels back to the emitter in interval Δt_2

– This results in a series of calculations:

$$c\Delta t_1 = L + u\Delta t_1 \Rightarrow \Delta t_1 = \frac{L}{c-u}$$

$$c\Delta t_2 = L - u\Delta t_2 \Rightarrow \Delta t_2 = \frac{L}{c+u}$$

$$\Delta t_{\text{total}} = \frac{L}{c-u} + \frac{L}{c+u} =$$

$$\frac{2Lc}{c^2-u^2} \Rightarrow \frac{2L}{c}\frac{c^2}{c^2-u^2}$$

– Finally, this yields

$$L = L_0\sqrt{1-\frac{u^2}{c^2}}$$

– This effect is called "length contraction"

– O' measures the proper length, L_0, because it is at rest with respect to the object

– Conclusion: An object in motion is measured to have a shorter length than at when it is at rest