

The Particle-Like Properties of Electromagnetic Waves

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1 Review of Electromagnetic Waves

- Reviewing the nature of light (or electromagnetic waves)

- A plane wave

- * A plane wave traveling in the positive x direction:

$$\begin{cases} \vec{E} = \vec{E}_o \sin(kx - \omega t) \\ \vec{B} = \vec{B}_o \sin(kx - \omega t) \end{cases}$$

- * The direction of energy transport would be $\vec{E} \times \vec{B}$
 - * $|\vec{E}|$ and $|\vec{B}|$ are constant at a given t
 - * The power, P , of the wave:

$$P = \frac{E}{\Delta t} = \frac{1}{\mu_o c} E_o^2 A \sin^2(kx - \omega t)$$

- * Two important features:

1. Intensity (average power per unit area) is proportional to E_o^2

$$P_{avg} = \int_0^T P(t) dt$$

2. The intensity of the system fluctuates with time

$$\frac{P_{avg}}{A}$$

- A spherical wave

- * Spreads out uniformly along the three axis

2 The Photoelectric Effect

- Experiment performed by Heinrich Hertz (1887)
- When a metal surface is illuminated, light electrons can be emitted from the surface
- The Experiment:
 - Connect emitter and collector to an external circuit
 - Apply a negative potential to the circuit collector
 - Increase the potential difference $((-V) - (+V))$ to be more negative
 - At some point, even the most energetic electrons do not have enough kinetic energy to reach the collector

- The maximum kinetic energy to reach the collector with the stopping voltage, V_s , is:

$$K_{max} = eV_s$$

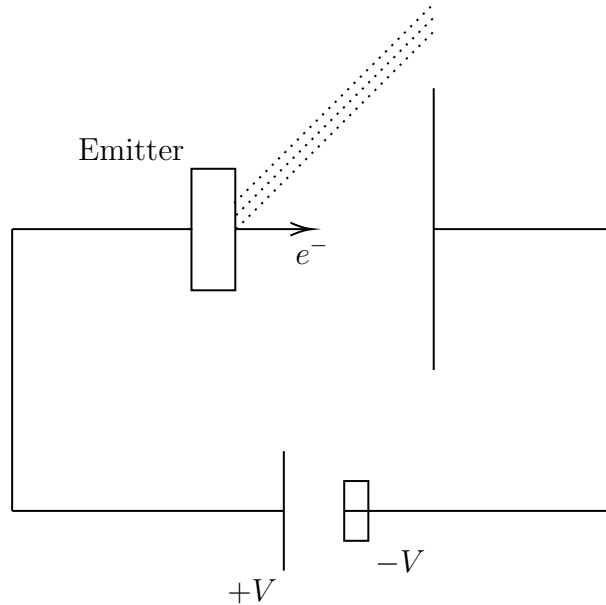


Figure 1: Set up of the Photoelectric Effect Experiment

- The classical picture: The energy of light with intensity I is absorbed by electrons, $E_{light} > E_{binding}$, e is released
- What does the classical wave theory predict?
 1. The maximum kinetic energy of the electrons, K_{max} , is proportional to the intensity of light
 2. The effect occurs for light with any frequency or wavelength
 3. e^- are released after a finite Δt
- Experimental Results
 1. For a fixed f or λ , K_{max} is independent of the intensity of light
 2. The effect occurs only if $f > f_{cutoff}$
 3. The first electrons are emitted almost instantaneously ($< 10^{-9}[s]$)
- This means everything that classical wave theory predicted was essentially incorrect

3 Thermal Radiation

- The Quantum Theory of the Photoelectric Effect
 - Developed by Albert Einstein (in 1905), based on Max Planck’s idea explaining thermal radiation
 - Assumptions:
 - * The energy of electromagnetic waves is not continuously distributed
 - * The energy is concentrated in localized bands or “quanta”
 - * This quanta is called “photon”
 - The energy of a photon is $E = hf$, where h is Planck’s constant, and f is the frequency

$$f = \frac{c}{\lambda} \Rightarrow E = \frac{hc}{\lambda}$$

- Photons travel at speed c , and are technically massless, so:

$$p = \frac{E}{c} = \frac{h}{\lambda}$$

- If $E = hf > \phi$, then photoelectrons are released; E is the photon energy, and ϕ is the work function
- The kinetic energy of the electron is:

$$K_{max} = hf - \phi$$

- Evidently, the intensity is not relevant; a larger intensity would mean more photons in a unit area, which means more electrons released; this means there is more current.
- In 1915, Robert Millikan performed an experiment (won 1923 Nobel Prize)
 - Determined Planck’s constant ($h = 6.57 \cdot 10^{-34} \text{ J s}$)
 - Fairly accurate, modern calculations found $h = 6.626 \cdot 10^{-34} \text{ J s}$
- The Photoelectric Effect Formula:

$$\lambda_c = \frac{hc}{\phi}$$

- Where λ_c is the cutoff wavelength, h is Planck’s constant, and ϕ is the work function

4 Compton Effect

- From classical physics (optics), $\lambda_1 = \lambda_2$

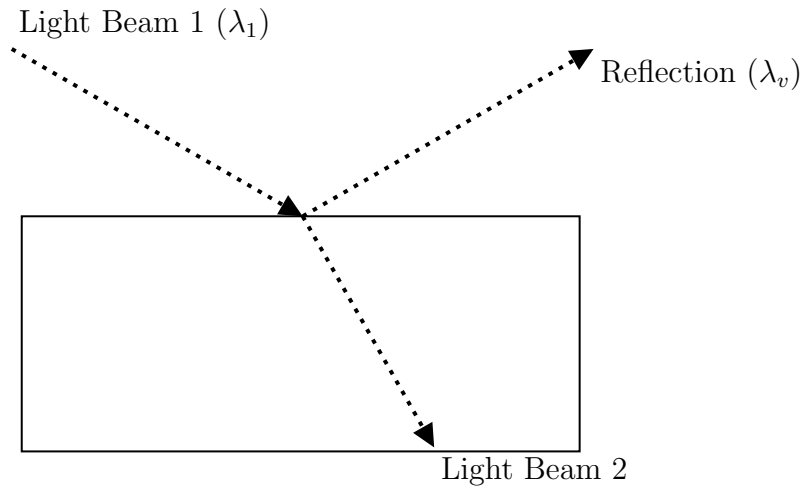


Figure 2: Classical Reflection of a Light Beam

- There will be a scattering process (Figure 3)
 - An incident photon (E, p) collides with a stationary electron
 - The photon (E', p') is scattered with angle θ
 - The electron (E_e, p_e) is scattered with angle ϕ

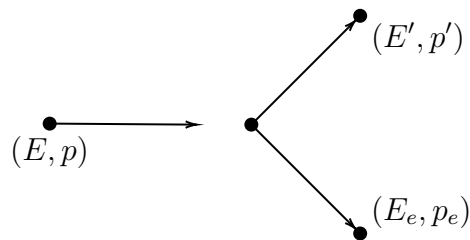


Figure 3: Collision of Photon and Electron

- Conservation of Energy and Momentum

- $E_i = E_f \Rightarrow E + m_e c^2 = E' + E_e$

- Momentum

- * $p_{xi} = p_{xf} \Rightarrow p = p' \cos(\theta) + p_e \cos(\phi)$
- * $p_{yi} = p_{yf} \Rightarrow 0 = p' \sin(\theta) + p_e \sin(\phi)$

– Conservation Laws:

$$\boxed{E - E' = E_e - m_e c^2 = K_e}$$

– After a lot of algebra and formula manipulations, we get:

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos(\theta))}$$

- λ' is the wavelength of the scattered photon, λ is the wavelength of the incident photon, and $\frac{h}{m_e c}$ is the Compton wavelength (approximately .002426[nm])
- Thus, after scattering, we know $\lambda' \geq \lambda$

$$\lambda' - \lambda = \begin{cases} 0, & \text{when } \theta = 0 \\ \frac{2h}{m_e c}, & \text{when } \theta = 180 \end{cases}$$

- Electron has maximum energy when θ is closer to 180, and minimum energy when θ is closer to 0
- The experiment was done by Arthur Compton in 1923
 - * A beam of x-rays of wavelength λ is incident on carbon (“nearly free” electrons); measured the scattered x-ray intensity as a function of θ