The Particle-Like Properties of Electromagnetic Waves

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Contents

1 Review of Electromagnetic Waves

- Reviewing the nature of light (or electromagnetic waves)
  - A plane wave
    - A plane wave traveling in the positive x direction:

$$\begin{cases} \vec{E} = \vec{E}_o \sin(kx - \omega t) \\ \vec{B} = \vec{B}_o \sin(kx - \omega t) \end{cases}$$

    - The direction of energy transport would be $\vec{E} \times \vec{B}$
    - $|\vec{E}|$ and $|\vec{B}|$ are constant at a given t
    - The power, P, of the wave:

$$P = \frac{E}{\Delta t} = \frac{1}{\mu_o c} E_o^2 A \sin^2(kx - \omega t)$$

    - Two important features:
      1. Intensity (average power per unit area) is proportional to E_o^2

$$P_{avg} = \int_0^T P(t)\, dt$$

      2. The intensity of the system fluctuates with time

$$\frac{P_{avg}}{A}$$

  - A spherical wave
    - Spreads out uniformly along the three axis

2 The Photoelectric Effect

- Experiment performed by Heinrich Hertz (1887)
- When a metal surface is illuminated, light electrons can be emitted from the surface
- The Experiment:
  - Connect emitter and collector to an external circuit
  - Apply a negative potential to the circuit collector
  - Increase the potential difference $((-V) - (+V))$ to be more negative
  - At some point, even the most energetic electrons do not have enough kinetic energy to reach the collector

– The maximum kinetic energy to reach the collector with the stopping voltage, V_s, is:

$$K_{max} = eV_s$$

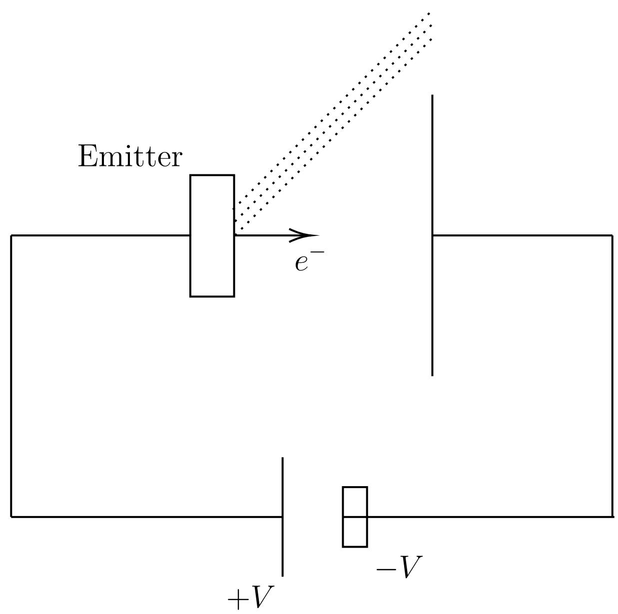

Figure 1: Set up of the Photoelectric Effect Experiment

– The classical picture: The energy of light with intensity I is absorbed by electrons, $E_{light} > E_{binding}$, e is released

– What does the classical wave theory predict?

1. The maximum kinetic energy of the electrons, K_{max}, is proportional to the intensity of light
2. The effect occurs for light with any frequency or wavelength
3. e^- are released after a finite Δt

- Experimental Results

1. For a fixed f or λ, K_{max} is independent of the intensity of light
2. The effect occurs only if $f > f_{\text{cutoff}}$
3. The first electrons are emitted almost instantaneously ($< 10^{-9}$[s])

– This means everything that classical wave theory predicted was essentially incorrect

- The Quantum Theory of the Photoelectric Effect
  - Developed by Albert Einstein (in 1905), based on Max Planck's idea explaining thermal radiation
  - Assumptions:
    * The energy of electromagnetic waves is not continuously distributed
    * The energy is concentrated in localized bands or "quanta"
    * This quanta is called "photon"
  - The energy of a photon is $\boxed{E = hf}$, where h is Planck's constant, and f is the frequency

$$\boxed{f = \frac{c}{\lambda} \Rightarrow E = \frac{hc}{\lambda}}$$

  - Photons travel at speed c, and are technically massless, so:

$$\boxed{p = \frac{E}{c} = \frac{h}{\lambda}}$$

  - If $E = hf > \phi$, then photoelectrons are released; E is the photon energy, and ϕ is the work function
  - The kinetic energy of the electron is:

$$\boxed{K_{max} = hf - \phi}$$

  - Evidently, the intensity is not relevant; a larger intensity would mean more photons in a unit area, which means more electrons released; this means there is more current.

- In 1915, Robert Millikan performed an experiment (won 1923 Nobel Prize)
  - Determined Planck's constant ($h = 6.57 \cdot 10^{-34}\,\mathrm{J\,s}$)
  - Fairly accurate, modern calculations found $h = 6.626 \cdot 10^{-34}\,\mathrm{J\,s}$

- The Photoelectric Effect Formula:

$$\boxed{\lambda_c = \frac{hc}{\phi}}$$

  - Where λ_c is the cutoff wavelength, h is Planck's constant, and ϕ is the work function

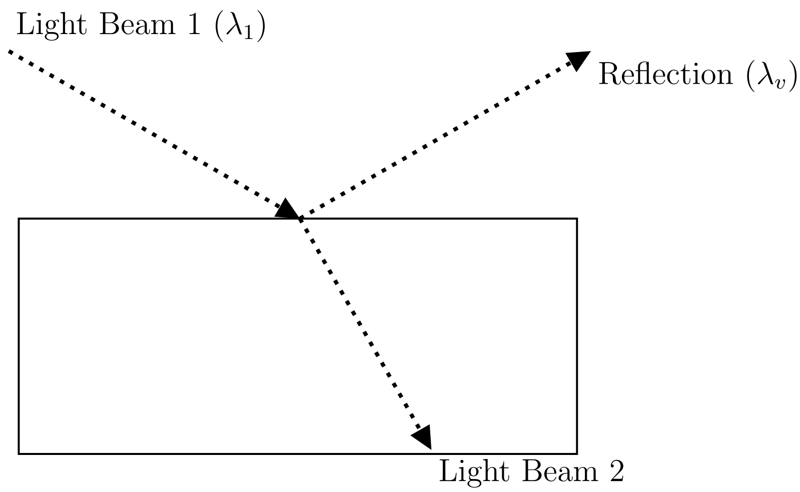

Figure 2: Classical Reflection of a Light Beam

3 Compton Effect

- From classical physics (optics), $\lambda_1 = \lambda_2$
- There will be a scattering process (Figure 3)
  - An incident photon (E, p) collides with a stationary electron
  - The photon (E', p') is scattered with angle θ
  - The electron (E_e, p_e) is scattered with angle ϕ

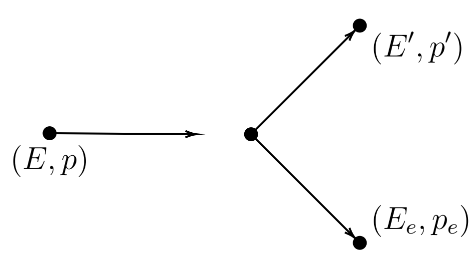

Figure 3: Collision of Photon and Electron

- Conservation of Energy and Momentum
  - $E_i = E_f \Rightarrow E + m_e c^2 = E' + E_e$
  - Momentum
    - $p_{xi} = p_{xf} \Rightarrow p = p' \cos(\theta) + p_e \cos(\phi)$

    * $p_{yi} = p_{yf} \Rightarrow 0 = p' \sin(\theta) + p_e \sin(\phi)$
  - Conservation Laws:

$$\boxed{E - E' = E_e - m_e c^2 = K_e}$$

  - After a lot of algebra and formula manipulations, we get:

$$\boxed{\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos(\theta))}$$

  - λ' is the wavelength of the scattered photon, λ is the wavelength of the incident photon, and $\dfrac{h}{m_e c}$ is the Compton wavelength (approximately .002426[nm])
  - Thus, after scattering, we know $\lambda' \geq \lambda$

$$\lambda' - \lambda = \begin{cases} 0, & \text{when } \theta = 0 \\ \dfrac{2h}{m_e c}, & \text{when } \theta = 180 \end{cases}$$

  - Electron has maximum energy when θ is closer to 180, and minimum energy when θ is closer to 0
  - The experiment was done by Arthur Compton in 1923
    * A beam of x-rays of wavelength λ is incident on carbon ("nearly free" electrons); measured the scattered x-ray intensity as a function of θ

4 Thermal Radiation

- The total intensity $\left(\int I(\lambda)\, d\lambda\right)$ increases as T is increased
- Stefan's Law: $I = \sigma T^4$
- λ_{max} decreases as T increases
- Wien's Displacement: $\lambda_{\text{max}} T = 2.9 \cdot 10^{-3}[\text{m K}]$
- Classical theory outline of $I(\lambda)$ vs. λ:
  - A box is filled with an EM standing wave (black body)
  - The number of standing waves with a wavelength between λ and $\lambda + d\lambda$ is:

$$N(\lambda)\, d\lambda = \frac{8\pi V}{\lambda^4}\, d\lambda$$

– Each standing wave carries E_{avg}

$$E_{avg} = k_B T$$

– Energy Density (# of standing waves per volume):

$$u(\lambda)\, d\lambda = \frac{8\pi}{\lambda^4} k_B T\, d\lambda$$

$$I(\lambda) = \frac{c}{4} u(\lambda)$$

$$\boxed{I(\lambda) = \frac{2\pi c}{\lambda^4} k_B T}$$

– This is known as the Rayleigh-Jones formula

  * It approaches the correct intensity at large wavelengths
  * Fails at small wavelengths
  * Known as the ultraviolet catastrophe

- Quantum Theory of Thermal Radiation

  – The theory was proposed by Max Planck in 1900

  – What was needed? $u(\lambda) \to 0$ when $\lambda \to 0$

  – An oscillating atom can absorb or emit energy only in discrete bundles

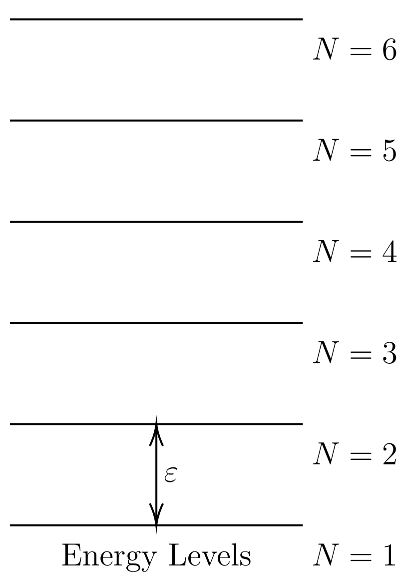

Figure 4: Oscillator with Multiple Energy Levels

  – Energy level movement can occur only in integer multiples of N

- This becomes:

$$\boxed{E_n = n\varepsilon}$$

- Where E_n is discrete, and n represents the # of quanta ($n = 1, 2, 3, \cdots$)
- The difference is E is not a continuous variable anymore
- Classically, $E_{avg} = k_B T$
- The quantum version of E_n for N oscillators is:

$$\boxed{N_n = N\left(1 - e^{-\dfrac{\varepsilon}{k_B T}}\right) e^{-\dfrac{n\varepsilon}{k_B T}}}$$

- Summing all values of N_n would yield:

$$\sum_{n=0}^{\infty} N_n = N$$

- This means E_{avg} becomes:

$$\boxed{E_{avg} = \frac{\left(\displaystyle\sum_{n=0}^{\infty} N_n E_n\right)}{\left(\displaystyle\sum_{n=0}^{\infty} N_n\right)}}$$

- The numerator is the total energy for all oscillators
- The equation can be simplified to:

$$\boxed{E_{avg} = \frac{\varepsilon}{e^{\left(\dfrac{\varepsilon}{k_B T}\right)} - 1}}$$

or

$$\boxed{E_{avg} = \frac{\frac{hc}{\lambda}}{e^{\left(\dfrac{hc}{\lambda k_B T}\right)} - 1}}$$

- Using the quantum formula from above, the formula for intensity becomes:

$$\boxed{I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\left(\dfrac{hc}{\lambda k_B T}\right)} - 1}}$$

- A relation between the Stefan-Boltzmann constant and the Planck constant

$$\boxed{\sigma = \frac{2\pi^5 k_B^4}{15c^2h^3}}$$

- Summary
  - Photoelectric Effect: Absorption of electromagnetic radiation
  - Thermal Radiation: Emission of electromagnetic waves
  - Both concluded that we have to use quanta to understand light and radiation