Homework 2

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1 Twins

1. (a)
   - Total distance for Mary-Kate: $2L_0 = 32$[light-years]
   - Total time for Mary-Kate: $\Delta t_0 = 20$[yr]

$$v\Delta t_0 = 2L$$
$$v\Delta t_0 = 2L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$v = \frac{2L_0\sqrt{1-\frac{v^2}{c^2}}}{\Delta t_0}$$
$$v^2 = \frac{(32)^2c^2\left(1-\frac{v^2}{c^2}\right)}{20^2}$$
$$v^2 = 2.56c^2 - 2.56v^2$$
$$3.56v^2 = 2.56c^2$$
$$\boxed{v = .848c}$$

   (b) According to the result from (a), the speed at which Mary-Kate traveled is $.848c$. Applying the time dilation formula using this knowledge yields:

$$\Delta t = \frac{20}{\sqrt{1-(.848)^2}}$$
$$\Delta t = 37.7[\text{yr}]$$

   So Ashley is $37.7-20 = 17.7$ years older than Mary-Kate when Mary-Kate returns

2 Spherical Waves

2. The Lorentz Transformation is given as:

$$\begin{cases} x' = \dfrac{x-ut}{\sqrt{1-\frac{u^2}{c^2}}} \\ y' = y \\ z' = z \\ t' = \dfrac{t-\frac{u}{c^2}x}{\sqrt{1-\frac{u^2}{c^2}}} \end{cases}$$

Because it is stated that the pulse begins at time $t = 0$, it can be assumed that, at this time, the pulse is not moving. As such, the transformations reduce to:

$$\begin{cases} x' = \dfrac{x-(0)t}{\sqrt{1-\frac{0^2}{c^2}}} = x \\ y' = y \\ z' = z \\ t' = \dfrac{t-\frac{0}{c^2}x}{\sqrt{1-\frac{0^2}{c^2}}} = t \end{cases}$$

In this manner, we substitute each transformation into the original formula, which yields:

$$\boxed{x'^2 + y'^2 + z'^2 = (ct')^2 = 0}$$

3 Pole Vaulting

3. (a) Using the length contraction formula, the proper length (L_0) as 20[m], and the observed length (L) as 10[m], we obtain:

$$10 = 20\sqrt{1-\frac{u^2}{c^2}}$$

$$.25 = 1 - \frac{u^2}{c^2}$$

$$.25c^2 = c^2 - u^2$$

$$u^2 = .75c^2$$

$$\boxed{u = .866c}$$

As Ming's speed in reference frame of observer O

(b) As with the phenomenon of clock desynchronization, because Ming is moving towards the garage, it appears to him that the farther door closes first. In such a manner, the time difference between the state alternation of the doors (closed to open) can be expressed as the following:

$$\Delta t' = \frac{\frac{uL}{c^2}}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$\Delta t' = \frac{\frac{(5)(.866)}{c}}{\sqrt{1-(.866)^2}}$$

$$\Delta t' = 2.8864 \cdot 10^{-8}[\mathrm{s}]$$

In this time, Ming is able to travel the following distance:

$$(2.8864)(3)(.866) = 7.4469[\text{m}]$$

Because the garage is actually 10 meters long, Ming travels 7.45[m], during which the first door is still open. It then closes, and the second door opens, which allows Ming to pass through without stopping or damaging the garage. As such, Ming is able to safely enter the 10[m] garage despite having a 20[m] pole.

4 Meson Decay

4. π Meson speed: $v_x = \pm.815c$; K Meson speed: $u = .453c$

   This would mean, using the Lorentz velocity transformation, the first π meson particle would have a speed of:

$$v'_{x1} = \frac{.453 - .815}{1 - (.815)(.453)}c$$
$$\boxed{v'_{x1} = -.574c}$$

   And the other, using $v_x = -.815c$, would have a speed of:

$$v'_{x2} = \frac{.453 + .815}{1 + (.815)(.453)}c$$
$$\boxed{v'_{x2} = .926c}$$

5 Meter Stick

5. Because the motion is parallel, only the x component of the meter stick experiences length contraction. Thus, to find the components, we would perform the following:

$$L_y = (1[\text{m}])\sin(30) = .5[\text{m}]$$
$$L_x = (1[\text{m}])\cos(30) = .866[\text{m}]$$

   Following contraction, L_x becomes:

$$L'_x = .866(\sqrt{1 - .81}) = .378[\text{m}]$$

   Thus, the new length of the meter stick becomes:

$$L = \sqrt{(.378)^2 + (.5)^2}$$
$$\boxed{L = .627[\text{m}]}$$