Lecture 7

Michael Brodskiy
Professor: A. Ali

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- Generator Model
  - Synchronous Generator
    - Consists of a rotor (rotating piece)
    - And a stator
    - Rotating part consists of a coil receiving D.C. current, with a magnetic orientation
    - The rotation results in a flux, which passes through the stator
    - From this, we can obtain (with $\lambda = N\phi$):

$$\frac{d\lambda}{dt} = e = \omega k \sin(\omega t)$$
$$|\hat{E}_f| = 4.44 f N \phi_{max} \propto I_{dc}, \omega$$
$$\lambda = k \cos(\omega t)$$

    - The terminal voltage may be written as:

$$\hat{V}_t = \hat{E}_f - (r + jX_s)\hat{I}_a$$

    - We see that terminal voltage may be controlled using $\hat{E}_f$
    - The net injected current at different buses (substations) may be calculated using:

$$\hat{I}_i = \frac{(P_{Gi} + jQ_{Gi})^*}{\hat{V}_i^*}$$

    - For buses without a generator, there is no source injection or load, so there is no net current injection (zero-injection bus)
      - Such substations may be used as 'switching' substations

- Transmission Lines

- For a transmission line with resistance r and impedance jx, we add two capacitors, one on each side of the impedance, with value jb
- With this model, we may write:

$$jb = \frac{1}{2} jb_{lc}$$

  - Where b_{lc} is the total line charging susceptance of the line
- At a bus with a load, we may find the current as:

$$\hat{I}_2 = \frac{(\hat{V}_2 - \hat{V}_1)}{r_{12} + jx_{12}} + \hat{V}_2 jb_{12} + \frac{(\hat{V}_2 - \hat{V}_3)}{r_{23} + jx_{23}} + \hat{V}_2 jb_{23} + \frac{(\hat{V}_2 - \hat{V}_4)}{r_{24} + jx_{24}} + \hat{V}_2 jb_{24} +$$

- Bus Admittance Matrix
  - We may build a bus admittance matrix (Y_{bux}) may be built by inspection:

$$Y_{bus}(i,i) = \text{ sum of admittances of all branches incident to bus } i$$
$$Y_{bus}(i,j) = \text{ negative of the admittance of the branch connecting bus } i \text{ and } j$$
$$Y_{bus}(i,j) = Y_{bus}(j,i)$$

  - Properties of Y_{bus}:
    - Symmetric square matrix
    - Has complex entries
    - Super sparse (*i.e.* majority of the entries are zero)
    - Y_{bus} is non-singular only if there exists at least one branch connecting a bus to ground
    - $(Y_{bus})^{-1} = Z_{bus}$ — the bus impedance matrix!
  - Properties of Z_{bus}:
    - It is a full matrix
    - It is a symmetric matrix
    - Diagonal entries are equal to the Thévenin equivalent impedances looking into the network at that bus
      - This means Thévenin equivalents may be formulated by finding Z_{bus} and finding the corresponding diagonal entry for a bus