Lecture 18

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- For single-phase systems, power oscillates at $2\pi f$ for voltage and current at f
- For a balanced 3-phase system, the power is constant
- For power factor correction, we add a shunt capacitor at the load
- Power provided at the load may be written as (with θ_{12} as the angle difference between the two points):

$$P_l = \frac{V_1 V_2}{x} \sin(\theta_{12})$$

- We know:

$$Y_{bus}\hat{V}_{bus} = \hat{I}_{bus}$$

  - With:

$$\hat{I}_{bus} = \hat{I}_{G_k} + \hat{I}_{D_k}$$

  - The admittance matrix, $Y_{bus} = G + jB$, is:
    * Sparse
    * Square ($n \times n$), with n as the number of buses
    * Symmetric (without phase shifters)
    * Complex
  - The impedance matrix is:

$$Z_{bus} = Y_{bus}^{-1}$$

    * Full
    * Square
    * Complex

- The incidence matrix A bus to branch $n \times l$

$$A_{ij} = \begin{cases} 1, & i \text{ is sending end of } j \\ -1, & i \text{ is receiving end of } j \\ 0, & \text{Otherwise} \end{cases}$$

  - From this, we may get:

$$Y_{bus} = AY_pA^T$$

  - Y_p is the primitive admittance matrix, consisting only of diagonal terms with z_i being the impedance of branch i

- Y_{bus} by inspection
  - $Y_{bus}(i,i)$ is the sum of the admittances of all branches incident to i, including the branches connecting to ground
  - $Y_{bus}(i,j)$ is the admittance of the branch connecting i to j

- Complex Power
  - We know that:

$$I_{bus} = V_{bus}Y_{bus}$$

  - This gives us:

$$S_k = \hat{V}_{bus}(k)\left[\hat{I}_{bus}(k)\right]^*$$

  - With the current expressed as:

$$\hat{I}_{bus}(k)^* = \sum_{m=1}^{n} Y_{bus}^*(k,m)\hat{V}_m^*$$

  - Expanding Y_{bus}, we may obtain the power components as:

$$P_k = V_k \sum_{m=1}^{n} V_m \left(G_{km}\cos(\theta_{km}) + B_{km}\sin(\theta_{km})\right)$$
$$Q_k = V_k \sum_{m=1}^{n} V_m \left(G_{km}\sin(\theta_{km}) - B_{km}\cos(\theta_{km})\right)$$

- Bus Types
  - Slack Bus
    - Known: V, θ

* Unknown: P, Q
    * Quantity: 1
  - PV Bus
    * Known: V, P
    * Unknown: Q, θ
    * Quantity: n_{PV}
  - PQ Bus
    * Known: P, Q
    * Unknown: V, θ
    * Quantity: n_{PQ}

- Mismatch Vector
  - We can form a vector as:

$$X = [\underbrace{\theta_2, \theta_3, \cdots, \theta_n}_{n-1} | \overbrace{V_{n_{PV}+2}, \cdots, V_n}^{n_{PQ}}]$$

,

  - Initialize all of angles as zero and the voltages as one
  - From here, we can make the mismatch vector as:

$$F(X) = \left[\begin{array}{c} P_2(X) - (P_{G_2} - P_{D_2}) \\ P_3(X) - (P_{G_3} - P_{D_3}) \\ \vdots \\ P_n(X) - (P_{G_n} - P_{D_n}) \\ \hline Q_{n_{PV}+2}(X) - (Q_G - Q_D) \\ \vdots \\ Q_n(X) - (Q_{G_n} - Q_{D_n}) \end{array}\right]$$

  - Using the Newton-Raphsson method, we solve using:

$$X^{k+1} = X^k - [J]^{-1} F(x^k)$$

  - Equations 10.40 and 10.41 from the book help us generate the Jacobian matrix
  - Line loss may be calculated as: $P_{G_i} - P_{D_i}$
  - Line loss from k to m may be obtained using: $P_{km} + P_{mk}$
  - Slack bus by default is bus 1, but what happens if we change the slack bus to k?

    * $\theta_j^{new} = \theta_j^{old} - \theta_k^{old}$
    * For a PV bus: $P_{G_i} = P_i^{old}$, $|V_i| = V_i^{old}$, and $V_k = V_k^{old}$
        - Note that only the angle changes

- Synchronous Condenser
  - A voltage source with $\hat{E}_f = E_f \angle \delta = \hat{V}_t + jX_s \hat{I}_a$
  - We assume $\hat{V}_t = V_t \angle 0$
- Primary Control
  - If frequency drops, power increases and vice versa
- Secondary Control (Area Control Error — ACE)
  - ACE $= \beta_i \Delta P_{tie} + k \Delta \omega_i$
  - Want to minimize ACE
- Economic Dispatch
  - $C_i(P_{G_i}) = \alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2$
  - We may write (for a lossless system):

$$\sum_{i=1}^{NG} P_{G_i} - P_D = 0$$

  - This gives us a solution as:

$$\begin{bmatrix} 2\gamma_1 & \cdots & \cdots & \cdots & -1 \\ \vdots & 2\gamma_2 & 0 & 0 & -1 \\ \vdots & 0 & \ddots & 0 & -1 \\ \vdots & 0 & 0 & 2\gamma_{NG} & -1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{G_1} \\ P_{G_2} \\ \vdots \\ P_{G_{NG}} \\ \lambda \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ \vdots \\ -\beta_{NG} \\ P_D \end{bmatrix}$$

  - With power limits:
    * If $P_{G_i} > P_{G_i}^{max} \rightarrow P_{G_i} = P_{G_i}^{max}$
    * If $P_{G_i} < P_{G_i}^{min} \rightarrow P_{G_i} = P_{G_i}^{min}$
    * Then, remove the row corresponding to i
    * Re-solve using $P_D = P_D - P_{G_i}$
    * System λ versus $IC_i < \lambda$ if P_{G_i} is at $P_{G_i}^{max}$, and $IC_i > \lambda$ if P_{G_i} is at $P_{G_i}^{min}$