# Homework 2

Michael Brodskiy
Professor: A. Abur

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Chapter 5:

15. We need to establish a per-unit base. We may begin by choosing $S_{b1} = 30[\text{MV A}]$ and $V_{b1} = 13.8[\text{V}]$. The impedance in zone 1 will remain the same:

$$X_{s1}^{new} = X_{s1}^{old}\left(\frac{30}{30}\right)\left(\frac{13.8}{13.8}\right)^2$$
$$X_{s1}^{new} = .1pu$$

We adapt the other impedances accordingly:

$$X_{T1}^{new} = .1\left(\frac{30}{20}\right)\left(\frac{132}{138}\right)^2$$
$$X_{T1}^{new} = .1372pu$$

$$X_{T2}^{new} = .12\left(\frac{30}{15}\right)\left(\frac{138}{138}\right)^2$$
$$X_{T2}^{new} = .24pu$$

$$X_{s2}^{new} = .08\left(\frac{30}{20}\right)\left(\frac{138}{138}\right)^2$$
$$X_{s2}^{new} = .12pu$$

$$Z_L^{new} = (20 + j100)\left(\frac{30}{138^2}\right)$$
$$Z_L^{new} = (.0315 + .15753j)pu$$

The power of the motor can also be found as:

$$V_{s2} = \frac{13.8}{13.8}$$
$$V_{s2} = 1pu$$
$$S_{s2} = \frac{20}{30}$$
$$S_{s2} = .667pu$$

Using the obtained values, we may generate the following diagram:

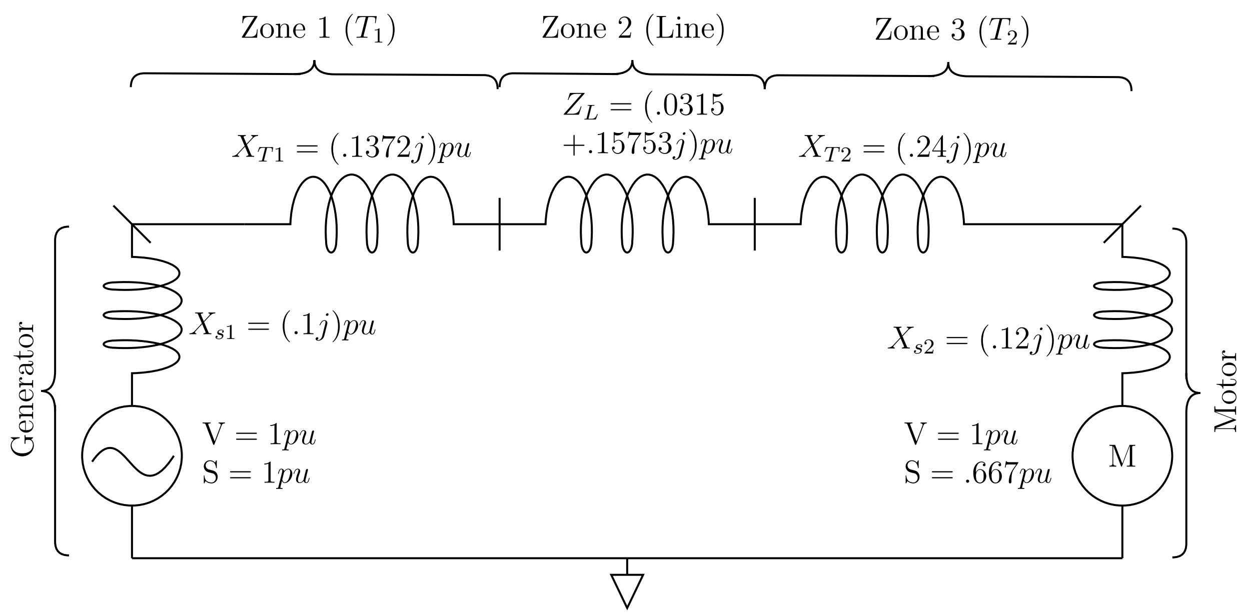


Figure 1: Impedance Diagram for Problem 1

16. (a) We need to begin by calculating the base current. This can be done by doing the following:

$$I_b = \frac{S_b}{V_b\sqrt{3}}$$
$$I_b = \frac{30}{13.8\sqrt{3}}$$
$$I_b = 1.2551pu$$

We then need to find the current at the motor. We start with finding the phase angle:

$$\cos^{-1}(.85) = 31.788°$$

Since the power factor is leading, we get:

$$\phi = -31.788^\circ$$

The power at the motor becomes:

$$S = \frac{15}{.85}\angle\phi$$
$$S = 17.647\angle - 31.788^\circ$$
$$S = 15 - 9.296j$$

Converting to per-unit, we get:

$$S = (0.5 - .31j)pu$$

The motor voltage is:

$$V_m = \frac{13.2}{13.8}$$
$$V_m = .9565pu$$

Thus, we can calculate the current as:

$$I_m = \frac{S^*}{V}$$
$$I_m = \frac{.5 + .31j}{.9565}$$
$$\boxed{I_m = (.5227 + .3241j)pu}$$

Since the line is fully serial, the current throughout the line is the same, giving us:

$$\boxed{I_G = (.5227 + .3241j)pu}$$
$$\boxed{I_L = (.5227 + .3241j)pu}$$

The voltage at the generator terminals can be calculated once we find the overall impedance of the line. This gives:

$$X_t = .0315 + .15753j + .1372j + .24j + .1j + .12j$$
$$\boxed{X_t = (.0315 + .7547j)pu}$$

The voltage at the terminals is then:

$$V_G = V_m + IX_t$$

$$V_G = .9565 + (.5227 + .3241j)(.0315 + .7547j)$$
$$\boxed{V_G = (0.7284 + 0.4047j)pu}$$

We can find the sending-end line voltage by taking the above value and subtracting the voltage drop across $X_g$ and $X_{T1}$:

$$V_L = V_G - I(X_{s1} + X_{T1})$$
$$V_L = .7284 + .4047j - (.5227 + .3241j)[(.1j) + (.1372j)]$$
$$\boxed{V_L = (.8052 + .2807j)pu}$$

The complex power supplied by the generator may be found by multiplying the voltage and current values together:

$$S_G = I^* V_G$$
$$S_G = (.5227 - .3241j)(.7284 + .4047j)$$
$$\boxed{S_G = (.5119 - .02454j)pu}$$

In summary, we obtained the following in per-unit:

$$\boxed{\begin{cases} I_m &= .5227 + .3241j \\ I_G &= .5227 + .3241j \\ I_L &= .5227 + .3241j \\ V_G &= .7284 + .4047j \\ V_L &= .8052 + .2807j \\ S_G &= .5119 - .02454j \end{cases}}$$

(b) We may use the per-unit values obtained in (a) to easily convert back to real values:

$$I = (.5227 + .3241j)\left(\frac{30}{13.8}\right)$$
$$\boxed{I = 1.1363 + 0.7046j[\text{kA}]}$$

$$V_G = (.7284 + .4047j)(13.8)$$
$$\boxed{V_G = 10.0519 + 5.5849j[\text{kV}]}$$

$$V_L = (.8052 + .2807j)(13.8)$$
$$\boxed{V_L = 11.1118 + 3.8737j[\text{kV}]}$$

$$S_G = (.5119 - .02454j)(30)$$

$$\boxed{S_G = 15.357 - .7362j[\text{MV A}]}$$

In summary, we obtained the following:

$$\boxed{\begin{cases} I_m &= 1.1363 + .7046j[\text{kA}] \\ I_G &= 1.1363 + .7046j[\text{kA}] \\ I_L &= 1.1363 + .7046j[\text{kA}] \\ V_G &= 10.0519 + 5.5849j[\text{kV}] \\ V_L &= 11.1118 + 3.8737j[\text{kV}] \\ S_G &= 15.357 - .7362j[\text{MV A}] \end{cases}}$$

17. The load impedance may be represented as:

$$Z_L = 14.14 + 14.14j[\Omega]$$

In per-unit, we get:

$$Z_L = \left(\frac{30}{13.8^2}\right)(14.14 + 14.14j)$$
$$Z_L = (2.2275 + 2.2275j)pu$$

We then calculate the line current:

$$I = \frac{V_G}{X_{T1} + X_{T2} + Z_L + Z_{L2}}$$
$$I = \frac{.9565}{.1372j + .24j + .0315 + .15753j + 2.2275 + 2.2275j}$$
$$\boxed{I = (.1697 - .2075j)pu}$$

Using this, we find the load voltage:

$$V_L = IZ_L$$
$$V_L = (.1697 - .2075j)(2.2275 + 2.2275j)$$
$$\boxed{V_L = (.8402 - .0842j)pu}$$

The voltage then becomes:

$$V_L = (.840199 - .084205)(13.8)$$
$$\boxed{V_L = 11.6 - 1.162j[\text{kV}]}$$

And finally, the current is:

$$I_L = \frac{V_L}{Z_L\sqrt{3}}$$

$$I_L = \frac{(11.5947 - 1.162j)}{(14.14 + 14.14j)\sqrt{3}}$$

$$\boxed{I_L = .213 - .2604j[\text{kA}]}$$

In summary, we obtained the following values:

$$\boxed{\left\{ \begin{array}{lclcl} I_L & = & (.1697 - .2075j)pu & = & .213 - .2604j[\text{kA}] \\ V_L & = & (.8402 - .0842j)pu & = & 11.6 - 1.162j[\text{kV}] \end{array} \right.}$$