Homework 1

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Chapter 2:

4. First, we find the angle difference, ϕ:

$$\phi_1 = \cos^{-1}(.707)$$
$$\phi_1 \approx 45^\circ$$

From here, we find the initial load current, based on the provided load voltage and power draw:

$$V_L = 440\,[\text{V}]$$
$$\hat{I}_L = \frac{200 \cdot 10^3}{\sqrt{3}(44)(.707)} \angle - \phi_1$$
$$\hat{I}_L = 371.19 \angle - 45^\circ$$

We know that power may be expressed as:

$$S = P + jQ$$

We find the reactive power of the load:

$$Q = P\frac{\sin(\phi_1)}{\cos(\phi_1)} = P\tan(\phi_1)$$
$$Q = 200[\text{kVAr}]$$

Adding the capacitors in parallel allows us to sum the net positive VArs:

$$Q_{net} = 200 - 50 = 150[\text{kVAr}]$$

The new power factor becomes:

$$\phi_2 = \tan^{-1}\left(\frac{150[\text{kVAr}]}{200[\text{kW}]}\right) = \tan^{-1}(.75) = 36.87^\circ$$

$$\boxed{pf_2 = \cos(36.87) = .8 \text{ lagging}}$$

The current then becomes:

$$I_{L2} = \frac{200 \cdot 10^3}{\sqrt{3}(440)(.8)} = 328.04$$

$$\boxed{\hat{I}_{L2} = 328.04\angle - 36.87^\circ}$$

5. (a) We begin by finding the reactive component of the load:

$$Q = P\tan\big(\cos^{-1}(pf)\big) = 10\tan\big(\cos^{-1}(.9)\big)$$
$$Q = 4.843[\text{kVAr}]$$

Putting the components together, we get:

$$S = P + jQ$$

$$\boxed{S = 10 + 4.843j[\text{V A}]}$$

(b) To find the magnitude of the current, we may use the following relation:

$$P = IV(pf)$$

We insert known values:

$$10^4 = (I)(416)(.9)$$

This gives us:

$$|I| = \frac{10^4}{416(.9)}$$

$$\boxed{|I| = 26.709[\text{A}]}$$

(c) Assuming $\angle I = 0$, we can say:

$$\hat{I} = 26.709\angle 0^\circ[\text{A}]$$

And from this we say:

$$\hat{V} = 416\angle\cos^{-1}(.9)$$

$$\hat{V} = 416\angle 25.84°[\mathrm{V}]$$

We multiply the two together to find the power:

$$p(t) = \hat{I}\hat{V} = 26.709\angle 0° \cdot 416\angle 25.84°$$
$$p(t) = 11.111\angle 25.84°[\mathrm{kW}]$$

11. The first step has us converting Δ connections to 'Y' connections:

$$Z_Y = -\frac{Z_\Delta}{3} = -\frac{1}{3}j[\Omega$$
$$\hat{V}_{a''n} = \frac{\sqrt{3}}{3}\hat{V}_{ab''}\angle - 30° = \frac{\sqrt{3}}{3}\angle - 30°[\mathrm{V}]$$

We may then find values for all of the voltages (only the angles change):

$$\begin{bmatrix} V_{a''n} \\ V_{b''n} \\ V_{c''n} \end{bmatrix} = \frac{\sqrt{3}}{3}\angle \begin{bmatrix} -30 \\ -150 \\ 90 \end{bmatrix}^{\circ} [\mathrm{V}]$$

Using equivalence, we can develop a single-phase circuit as follows:

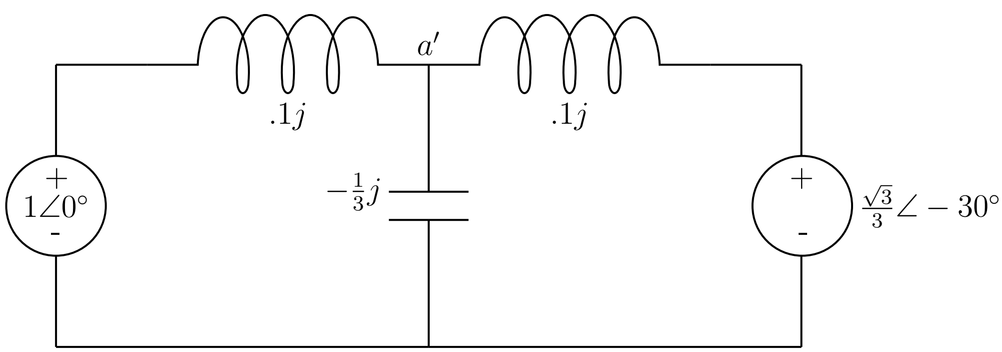

Figure 1: Single Phase Equivalent Circuit

We can write equations using the voltages at a':

$$\frac{V_{an} - V_{a'n}}{.1j} + \frac{V_{a''n} - V_{a'n}}{.1j} + \frac{3V_{a'n}}{j} = 0$$

We move $V_{a'n}$ to one side:

$$\frac{17V_{a'n}}{j} = \frac{10V_{an}}{j} + \frac{10V_{a''n}}{j}$$

$$V_{a'n} = \frac{10(V_{an} + V_{a''n})}{17}$$
$$V_{a'n} = \frac{10(1 + .5 - .2887j)}{17}$$
$$V_{a'n} = \frac{15 - 2.887j}{17}$$
$$V_{a'n} = .8824 - .1698j$$

Converting back to phasor form, we find:

$$\hat{V}_{a'n} = .8986\angle - 10.9°[\text{V}]$$

Given that the rest of the voltages are offset by 120°, we may write:

$$\boxed{\begin{bmatrix} V_{a'n} \\ V_{b'n} \\ V_{c'n} \end{bmatrix} = .8986\angle \begin{bmatrix} -10.9 \\ -130.9 \\ 109.1 \end{bmatrix}^{\circ}} [\text{V}]$$

From here, we finally find $V_{a'b'}$ since we know this is the difference between $V_{a'n}$ and $V_{b'n}$:

$$V_{a'b'} = V_{a'n} - V_{b'n}$$
$$V_{a'b'} = .8986\left(\angle - 10.9° - \angle - 130.9°\right)$$
$$V_{a'b'} = .8986\left[(.982 - .1891j) - (-.6547 - .7559j)\right]$$
$$V_{a'b'} = .8986\left[1.6367 + .5668j\right]$$
$$V_{a'b'} = 1.4707 + .5093j$$

And finally we convert to phasor form:

$$\boxed{\hat{V}_{a'b'} = 1.5564\angle 19.1°[\text{V}]}$$

12. We begin by, as always, converting to a 'Y' configuration:

$$Z_C \rightarrow \frac{Z_C}{3} = -\frac{10}{3}j[\Omega]$$

This leads to a parallel combination, the equivalent impedance of which we may calculate:

$$Z_{eq} = \frac{-(10/3)(10)}{10 - (10/3)}j = -5j[\Omega]$$

The circuit can then be phase-simplified by drawing:

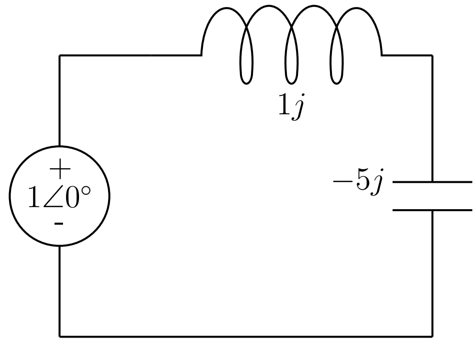

Figure 2: Single Phase Equivalent Circuit

Summing impedances, the total becomes:

$$Z_t = -4j[\Omega]$$

The line current can then be calculated as:

$$I_a = \frac{1}{-4j}$$
$$\hat{I}_a = \frac{(1\angle 0°)j}{4}$$
$$\boxed{\hat{I}_a = .25\angle 90°[\mathrm{A}]}$$

To find the current through a capacitor, we must find the voltage across one of load phase components:

$$V_{pha} = (-5j)(.25\angle 90°)$$
$$V_{pha} = (-5j)(.25j)$$
$$\hat{V}_{pha} = 1.25\angle 0°[\mathrm{V}]$$

Because of the parallel nature of the capacitor-inductor network, this voltage flows into both branches. To find the current across one capacitor, we find:

$$I_{cap} = \frac{(\sqrt{3})1.25\angle 0°}{-10j}$$
$$I_{cap} = \frac{(\sqrt{3})1.25j}{10}$$
$$I_{cap} = .2165j$$

$$\boxed{I_{cap} = .2165\angle 90°[\mathrm{A}]}$$

The complex power of the load may be calculated by using the product of the voltage and current phasors. This gives us:

$$S = 3|\hat{V}||\hat{I}|$$

$$S = 3\,(1.25)\,(.25)$$

$$\boxed{S = .9375[\mathrm{V\,A}]}$$