Lecture 14

Michael Brodskiy

Professor: A. Ali

October 31, 2024

- Per Unit System
  - Pick one S_{base} for the entire system (for three-phase systems, it will be $S_{3\phi}$)
  - Then, identify the transformer that separates the system into voltage zone, and label as zone $1, 2, \cdots, n$
    * Voltage or turns ratios should be provided in the problem
  - Selecting V_{base} for zone 1: line-to-line voltage for a 3-ϕ system. Then, calculate V_{base} for all other zones:

$$V_{base(j)} = \frac{n_j}{n_i} V_{base(i)}, \quad j = 2, 3, \cdots$$

  - Then:

$$z_{base(i)} = \frac{V_{base(i)}}{S_{3\phi}}$$
$$I_{base} = \frac{\sqrt{3}V_{base(i)}}{S_{3\phi}}$$

  - Powers become:

$$P_{[p.u.]} = V_{[p.u.]} I_{[p.u.]} \cdot pf$$
$$Q_{[p.u.]} = V_{[p.u.]} I_{[p.u.]} \sin(\phi) \qquad (\phi = \cos^{-1}(pf))$$

  - Using the single-phase equivalent, we obtain:

$$\hat{V}_{S[p.u.]} = \hat{I}_{[p.u.]}(jx_{t[p.u.]} + R_{[p.u.]} + jx_{[p.u.]}) + \hat{V}_{load[p.u.]}$$

    * Note: x_t is the transformer reactance, and x is the line reactance

- Y_{bus}: An $n \times n$ sparse, square matrix (where n is the number of buses)

$$Y_{bus} = G + jB$$

  - $Y_{bus}(i,j) \neq 0$ only if bus i and j are connected
  - $Y_{bus}(i,i) = \sum$ admittances of all branches connected to at one end to bus i
  - Modifying Y_{bus}:
    * Removing/adding a branch with admittance y_n connected between k and m gives us (for added branch, subtract from off-diagonals and add to diagonals, and vice versa for removal):

$$\begin{aligned} Y_{bus}^{new}(k,m) &\leftarrow Y_{bus}^{old}(k,m) \mp y_n \\ Y_{bus}^{new}(m,k) &\leftarrow Y_{bus}^{old}(m,k) \mp y_n \\ Y_{bus}^{new}(k,k) &\leftarrow Y_{bus}^{old}(k,k) \pm y_n \\ Y_{bus}^{new}(m,m) &\leftarrow Y_{bus}^{old}(m,m) \pm y_n \end{aligned}$$

    * Adding/removing a shunt with y_n at k:

$$Y_{bus}^{new}(k,k) \leftarrow Y_{bus}^{old}(k,k) \pm y_n$$

    * Kron Reduction:

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \rightarrow (Y_{11} - Y_{12}Y_{22}^{-1}Y_{21})V_1 = I_1$$

- Power Flow Problem

$$\begin{aligned} P_i(x) &= V_i \sum_{k=1}^{n} V_k \left(G_{ik}\cos(\theta_{ik}) + B_{ik}\sin(\theta_{ik}) \right) \\ Q_i(x) &= V_i \sum_{k=1}^{n} V_k \left(G_{ik}\sin(\theta_{ik}) - B_{ik}\cos(\theta_{ik}) \right) \end{aligned}$$

  - We may obtain (using the Newton-Raphsson method):

$$X^{k+1} = X^k - [J]^{-1}F(x^k)$$

$$X^T = [\underbrace{\theta_2, \theta_3, \cdots, \theta_n}_{n-1}; \overbrace{V_{n_{PV}+2}, \cdots, V_n}^{n_{PQ}}]$$

    * We check the norm: if $||F(x)|| < \varepsilon$ then stop