Homework 3

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Chapter 9:

1. Per the connection/disconnection rules of Y_{bus}, we know that adding a connection between 2 and 4 will result only in changes to Y_{22}, Y_{24}, Y_{42}, and Y_{44}. Thus, we may rewrite all other values from the matrix obtained in Example 9.4:

$$Y_{bus} = \begin{bmatrix} 1.4 - j18.66 & -1.4 + j18.66 & 0 & 0 & 0 \\ -1.4 + j18.66 & ??? & -.318 + j3.98 & ??? & 0 \\ 0 & -.318 + j3.98 & .636 - j7.73 & -.318 + j3.98 & 0 \\ 0 & ??? & -.318 + j3.98 & ??? & -.932 + j12.43 \\ 0 & 0 & 0 & -.932 + j12.43 & .932 - j12.43 \end{bmatrix}$$

Per our connection rules, connecting a line from terminal 2 to terminal 4 results in addition of the new admittance to diagonal entries:

$$Y'_{22} = Y_{22} + (.01 + .15j)^{-1} + .11j$$
$$Y'_{44} = Y_{44} + (.01 + .15j)^{-1} + .11j$$

This gives us:

$$Y'_{22} = (2.16 - 29.104j) + (.01 + .15j)^{-1} + .11j$$
$$Y'_{44} = (1.692 - 22.877j) + (.01 + .15j)^{-1} + .11j$$

And finally:

$$Y'_{22} = 2.6025 - 35.6312j[p.u.]$$
$$Y'_{44} = 2.1345 - 29.4042j[p.u.]$$

We then find the equivalent admittance by taking the inverse of the impedance:

$$Z_{eq} = \frac{(.01+.15j)^2}{2(.01+.15j)}$$
$$Y_{eq} = \frac{2(.01+.15j)}{(.01+.15j)^2}$$
$$Y_{eq} = \frac{2}{(.01+.15j)}$$
$$Y_{24} = Y_{42} = -Y_{eq} = -.885 + 13.2743j[p.u.]$$

This allows us to construct the modified matrix as:

$$\boxed{Y_{bus} = \begin{bmatrix} 1.4 - j18.66 & -1.4 + j18.66 & 0 & 0 & 0 \\ -1.4 + j18.66 & 2.6025 - 35.6312j & -.318 + j3.98 & -.885 + 13.2743j & 0 \\ 0 & -.318 + j3.98 & .636 - j7.73 & -.318 + j3.98 & 0 \\ 0 & -.885 + 13.2743j & -.318 + j3.98 & 2.1345 - 29.4042j & -.932 + j12.43 \\ 0 & 0 & 0 & -.932 + j12.43 & .932 - j12.43 \end{bmatrix}}$$

2. We can expand the given table to write:

From-To	Reactance $[p.u.]$	Admittance $[pu]$
1-2	$.04j$	$-25j$
1-6	$.06j$	$-16.667j$
2-4	$.03j$	$-33.333j$
2-3	$.02j$	$-50j$
3-4	$.08j$	$-12.5j$
4-5	$.06j$	$-16.667j$
5-6	$.05j$	$-20j$

This allows us to write the corresponding connections into the matrix:

$$Y_{bus} = \begin{bmatrix} & 25 & & & & 16.667 \\ 25 & & 50 & 33.333 & & \\ & 50 & & 12.5 & & \\ & 33.333 & 12.5 & & 16.667 & \\ & & & 16.667 & & 20 \\ 16.667 & & & & 20 & \end{bmatrix} j$$

By observation from the diagram, we may write all of the following admittances as zero:

From	To
1	3,4,5
2	5,6
3	1,5,6
4	1,6
5	1,2,3
6	2,3,4

This gives us:

$$Y_{bus} = \begin{bmatrix} & 25 & 0 & 0 & 0 & 16.667 \\ 25 & & 50 & 33.333 & 0 & 0 \\ 0 & 50 & & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & & 16.667 & 0 \\ 0 & 0 & 0 & 16.667 & & 20 \\ 16.667 & 0 & 0 & 0 & 20 & \end{bmatrix} j$$

We see that now only the diagonal is left. Thus, we write KVL equations whilst imagining a current element I_n for each bus n. This allows us to write:

$$I_1 = (V_1 - V_2)(-25j) + (V_1 - V_6)(-16.667j)$$
$$I_2 = (V_2 - V_1)(-25j) + (V_2 - V_3)(-50j) + (V_2 - V_4)(-33.333j)$$
$$I_3 = (V_3 - V_2)(-50j) + (V_3 - V_4)(-12.5j)$$
$$I_4 = (V_4 - V_2)(-33.333j) + (V_4 - V_3)(-12.5j) + (V_4 - V_5)(-16.667j)$$
$$I_5 = (V_5 - V_4)(-16.667j) + (V_5 - V_6)(-20j)$$
$$I_6 = (V_6 - V_1)(-16.667j) + (V_6 - V_5)(-20j)$$

Simplifying the equations, we obtain

$$I_1 = j[-41.667V_1 + 25V_2 + 16.667V_6]$$
$$I_2 = j[-108.33V_2 + 25V_1 + 50V_3 + 33.333V_4]$$
$$I_3 = j[-62.5V_3 + 50V_2 + 12.5V_4]$$
$$I_4 = j[-62.5V_4 + 33.333V_2 + 12.5V_3 + 16.667V_5]$$
$$I_5 = j[-36.667V_5 + 16.667V_4 + 20V_6]$$
$$I_6 = j[-36.667V_6 + 16.667V_1 + 20V_5]$$

Given that we know:

$$I_n = Y_{bus}V_n$$

We look at the coefficients from the currents to obtain the diagonals:

$$\boxed{Y_{bus} = \begin{bmatrix} -41.667 & 25 & 0 & 0 & 0 & 16.667 \\ 25 & -108.33 & 50 & 33.333 & 0 & 0 \\ 0 & 50 & -62.5 & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & -62.5 & 16.667 & 0 \\ 0 & 0 & 0 & 16.667 & -36.667 & 20 \\ 16.667 & 0 & 0 & 0 & 20 & -36.667 \end{bmatrix} j[p.u.]}$$

3. Adding another connection between buses 1 and 5 would mean that only the nm-th elements are modified, with nm being any combination of 1 and 5. Thus, we see that, with this $-10j$ admittance, we get two new terms at 15 and 51:

$$Y_{bus} = \begin{bmatrix} ? & 25 & 0 & 0 & 10 & 16.667 \\ 25 & -108.33 & 50 & 33.333 & 0 & 0 \\ 0 & 50 & -62.5 & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & -62.5 & 16.667 & 0 \\ 10 & 0 & 0 & 16.667 & ? & 20 \\ 16.667 & 0 & 0 & 0 & 20 & -36.667 \end{bmatrix} j$$

We then re-check the current equations for buses 1 and 5:

$$I_1 = j[-51.667V_1 + 25V_2 + 10V_5 + 16.667V_6]$$
$$I_5 = j[-46.667V_5 + 10V_1 + 16.667V_4 + 20V_6]$$

Thus, we get:

$$\boxed{Y_{bus} = \begin{bmatrix} -51.667 & 25 & 0 & 0 & 10 & 16.667 \\ 25 & -108.33 & 50 & 33.333 & 0 & 0 \\ 0 & 50 & -62.5 & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & -62.5 & 16.667 & 0 \\ 10 & 0 & 0 & 16.667 & -46.667 & 20 \\ 16.667 & 0 & 0 & 0 & 20 & -36.667 \end{bmatrix} j[p.u.]}$$

4. We may begin finding diagonal entries by summing all of the admittances incident at each bus:

$$Y_{11} = \frac{1}{.2j} + \frac{1}{.25j} + \frac{.24j}{2} + \frac{.3j}{2}$$
$$Y_{22} = \frac{1}{.2j} + \frac{1}{.1j} + \frac{.24j}{2} + \frac{.16j}{2} + .3j$$
$$Y_{33} = \frac{1}{.25j} + \frac{1}{.1j} + \frac{.16j}{2} + \frac{.3j}{2} - .6j$$

Solving, we get:

$$Y_{11} = -5j - 4j + .12j + .15j$$
$$Y_{22} = -5j - 10j + .12j + .08j + .3j$$
$$Y_{33} = -4j - 10j + .08j + .15j - .6j$$

And finally:

$$Y_{11} = -8.73j[p.u.]$$
$$Y_{22} = -14.5j[p.u.]$$
$$Y_{33} = -14.37j[p.u.]$$

(a) The off-diagonal elements will simply be the admittances of the connections, which gives us:

$$\boxed{Y_{bus} = \begin{bmatrix} -8.73 & 5 & 4 \\ 5 & -14.5 & 10 \\ 4 & 10 & -14.37 \end{bmatrix} j}$$

(b) With bus 2 gone, we may write:

$$Y_{11} = -4j + \frac{.3j}{2}$$
$$Y_{33} = -4j + \frac{.3j}{2} - .6j$$

And then:

$$Y_{11} = -3.85j$$
$$Y_{33} = -4.45j$$

The off-diagonal terms are then simply $4j$:

$$\boxed{Y_{bus} = \begin{bmatrix} -3.85 & 4 \\ 4 & -4.45 \end{bmatrix} j}$$

5. We write KCL equations which integrate the new voltage source (note the currents I_2 and I_3 are zero, but this is not reflected below to simplify further analysis):

$$I_1 = V_1(-.8696j) + (V_1 - V_2)(-5j) + .12V_1j + (V_1 - V_3)(-4j) + .15V_1j$$
$$I_2 = (V_2)(.3j) + (V_2 - V_1)(-5j) + .12V_2j + (V_2 - V_3)(-10j) + .08V_2j$$
$$I_3 = (V_3)(-.6j) + (V_3 - V_1)(-4j) + .15V_3j + (V_3 - V_2)(-10j) + .08V_3j$$

This gives us:

$$[-9.5396V_1 + 5V_2 + 4V_3]j = I_1$$
$$[-14.5V_2 + 5V_1 + 10V_3]j = I_2$$
$$[-14.37V_3 + 4V_1 + 10V_2]j = I_3$$

From which we may build:

$$\boxed{Y_{bus} = \left(\begin{bmatrix} -9.5396 & 5 & 4 \\ 5 & -14.5 & 10 \\ 4 & 10 & -14.37 \end{bmatrix} j \right) [p.u.]}$$