Lab Five
Power Systems Analysis
EECE5682

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Abstract

This laboratory experiment orients the performer with the Power Education Toolbox (PET) program. Furthermore, the experiment solidifies concepts related to bus-based systems, including power flow solutions, generation of an admittance matrix, and the affects of connecting/disconnecting components.

KEYWORDS: PET, bus, power flow, admittance matrix

1 Introduction & Objectives

We begin by constructing the provided 5-bus system in the Power Education Toolbox (PET) program. The system looks as follows:

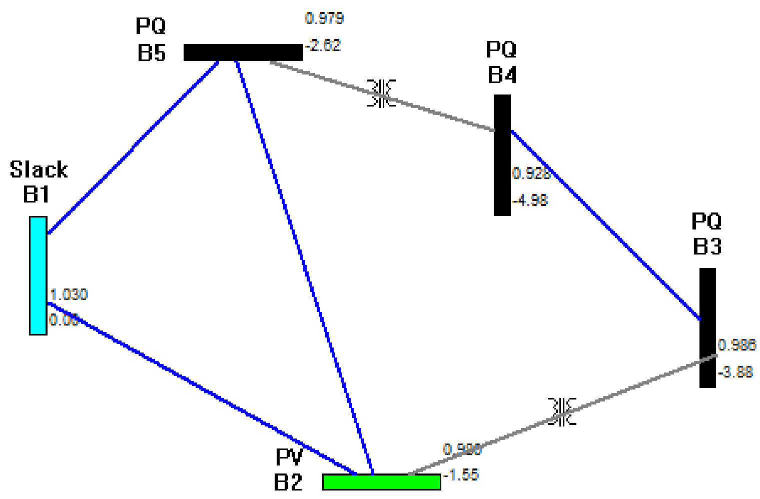

Figure 1: The Five-Bus System

2 Results

The Results sown in Parts (1-4) are obtained from the data files generated by PET.

2.1 Part 1

```
          CONVERGENCE SUMMARY
 ITER             DELP              DELQ

  0.0          0.960294
  0.5                            1.009047
  1.0          0.174055
  1.5                            0.024444
  2.0          0.011893
  2.5                            0.001464
  3.0          0.001561
  3.5                            0.000081
  4.0          0.000109

**************************** POWER FLOW REPORT ****************************

BUS    VOLTAGE    ANGLE         GENERATION               LOAD
NO.     (PU)      (DEG)      (MW)       (MVAR)       (MW)        (MVAR)


   1  1.0300     0.00   139.99496    50.81974    0.00000     0.00000
  TO-BUS     5       71.81894 MW     8.27556 MVAR
  TO-BUS     2       68.17602 MW    42.54447 MVAR

   2  0.9800    -1.60   135.00000    42.64991   75.00000    60.00000
  TO-BUS     5       40.35718 MW   -26.90932 MVAR
  TO-BUS     3       86.18706 MW    87.60418 MVAR
  TO-BUS     1      -66.53333 MW   -78.04241 MVAR

   3  1.0249    -3.86     0.00000     0.00000   60.00000    50.00000
  TO-BUS     4       26.18853 MW    30.94921 MVAR
  TO-BUS     2      -86.18706 MW   -80.94917 MVAR

   4  0.9439    -4.80     0.00000     0.00000   70.00000    55.00000
  TO-BUS     5      -45.17653 MW     0.97466 MVAR
  TO-BUS     3      -24.82372 MW   -55.97490 MVAR

   5  0.9831    -2.66     0.00000     0.00000   65.00000    45.00000
  TO-BUS     1      -70.01796 MW   -54.15830 MVAR
  TO-BUS     2      -40.15642 MW     8.44325 MVAR
  TO-BUS     4       45.17653 MW     0.71506 MVAR
```

Figure 2: Power Flow — Part 1

```
1      1  (      9.109589,     -25.508904)
2      2  (     10.882353,     -61.858901)
3      3  (      2.000000,     -25.850000)
4      4  (      2.000000,     -19.413368)
5      5  (      9.991942,     -46.638316)
5      1  (     -4.109589,      10.958904)
1      5  (     -4.109589,      10.958904)
2      1  (     -5.000000,      15.000000)
1      2  (     -5.000000,      15.000000)
5      2  (     -5.882353,      23.529412)
2      5  (     -5.882353,      23.529412)
4      3  (     -2.000000,       6.000000)
3      4  (     -2.000000,       6.000000)
5      4  (      0.000000,      13.020833)
4      5  (      0.000000,      13.020833)
3      2  (      0.000000,      21.739130)
2      3  (      0.000000,      21.739130)
```

Figure 3: Y_{Bus} — Part 1

2.2 Part 2

```
           CONVERGENCE SUMMARY
 ITER               DELP               DELQ

  0.0          0.960294
  0.5                             1.009047
  1.0          0.174055
  1.5                             0.024444
  2.0          0.011893
  2.5                             0.001464
  3.0          0.001561
  3.5                             0.026499
  4.0          0.009876
  4.5                             0.001969
  5.0          0.000252
  5.5                             0.000105
**************************** POWER FLOW REPORT ***************************

BUS   VOLTAGE   ANGLE        GENERATION            LOAD
NO.    (PU)     (DEG)      (MW)       (MVAR)     (MW)       (MVAR)


   1  1.0300     0.00   140.13101    53.89642    0.00000     0.00000
  TO-BUS     5       71.87868 MW     9.36384 MVAR
  TO-BUS     2       68.20592 MW    44.53257 MVAR

   2  0.9788    -1.58   135.00000    40.00000   75.00000    60.00000
  TO-BUS     5       40.34643 MW   -27.60321 MVAR
  TO-BUS     3       86.16390 MW    87.43051 MVAR
  TO-BUS     1      -66.51393 MW   -79.83661 MVAR

   3  1.0237    -3.85     0.00000     0.00000   60.00000    50.00000
  TO-BUS     4       26.16471 MW    30.77342 MVAR
  TO-BUS     2      -86.16390 MW   -80.77473 MVAR

   4  0.9429    -4.80     0.00000     0.00000   70.00000    55.00000
  TO-BUS     5      -45.19308 MW     0.75512 MVAR
  TO-BUS     3      -24.80669 MW   -55.75271 MVAR

   5  0.9822    -2.65     0.00000     0.00000   65.00000    45.00000
  TO-BUS     1      -70.05352 MW   -55.13915 MVAR
  TO-BUS     2      -40.14263 MW     9.18945 MVAR
  TO-BUS     4       45.19308 MW     0.93917 MVAR
```

Figure 4: Power Flow — Part 2

```
1    1  (     9.109589,    -25.508904)
2    2  (    10.882353,    -61.858901)
3    3  (     2.000000,    -25.850000)
4    4  (     2.000000,    -19.413368)
5    5  (     9.991942,    -46.638316)
5    1  (    -4.109589,     10.958904)
1    5  (    -4.109589,     10.958904)
2    1  (    -5.000000,     15.000000)
1    2  (    -5.000000,     15.000000)
5    2  (    -5.882353,     23.529412)
2    5  (    -5.882353,     23.529412)
4    3  (    -2.000000,      6.000000)
3    4  (    -2.000000,      6.000000)
5    4  (     0.000000,     13.020833)
4    5  (     0.000000,     13.020833)
3    2  (     0.000000,     21.739130)
2    3  (     0.000000,     21.739130)
```

Figure 5: Y_{Bus} — Part 2

2.3 Part 3

```
          CONVERGENCE SUMMARY
 ITER            DELP              DELQ

  0.0          0.960294
  0.5                           1.189175
  1.0          0.176448
  1.5                           0.024831
  2.0          0.011840
  2.5                           0.001362
  3.0          0.001713
  3.5                           0.000098
  4.0          0.000119

*************************** POWER FLOW REPORT **************************

BUS   VOLTAGE   ANGLE        GENERATION              LOAD
NO.    (PU)     (DEG)      (MW)       (MVAR)       (MW)        (MVAR)


  1  1.0300     0.00   140.14602    49.48200    0.00000     0.00000
  TO-BUS     5       71.65923 MW     7.03298 MVAR
  TO-BUS     2       68.48679 MW    42.45046 MVAR

  2  0.9800    -1.61   135.00000    21.33284   75.00000    60.00000
  TO-BUS     5       39.78992 MW   -29.27043 MVAR
  TO-BUS     3       87.06034 MW    68.53895 MVAR
  TO-BUS     1      -66.83836 MW   -77.93116 MVAR

  3  1.0339    -3.88     0.00000     0.00000   60.00000    50.00000
  TO-BUS     4       27.06192 MW    34.50617 MVAR
  TO-BUS     2      -87.06034 MW   -63.12906 MVAR

  4  0.9476    -4.77     0.00000     0.00000   70.00000    55.00000
  TO-BUS     5      -44.47562 MW     4.39449 MVAR
  TO-BUS     3      -25.52451 MW   -59.39468 MVAR

  5  0.9841    -2.67     0.00000     0.00000   65.00000    45.00000
  TO-BUS     1      -69.88875 MW   -53.04729 MVAR
  TO-BUS     2      -39.58479 MW    10.80168 MVAR
  TO-BUS     4       44.47562 MW    -2.75433 MVAR
```

Figure 6: Power Flow — Part 3

```
1      1  (       9.109589,      -25.508904)
2      2  (      10.882353,      -61.858901)
3      3  (       2.000000,      -25.650000)
4      4  (       2.000000,      -19.413368)
5      5  (       9.991942,      -46.638316)
5      1  (      -4.109589,       10.958904)
1      5  (      -4.109589,       10.958904)
2      1  (      -5.000000,       15.000000)
1      2  (      -5.000000,       15.000000)
5      2  (      -5.882353,       23.529412)
2      5  (      -5.882353,       23.529412)
4      3  (      -2.000000,        6.000000)
3      4  (      -2.000000,        6.000000)
5      4  (       0.000000,       13.020833)
4      5  (       0.000000,       13.020833)
3      2  (       0.000000,       21.739130)
2      3  (       0.000000,       21.739130)
```

Figure 7: Y_{Bus} — Part 3

2.4 Part 4

```
              CONVERGENCE SUMMARY
  ITER              DELP                DELQ

   0.0          0.960294
   0.5                             1.097912
   1.0          0.163774
   1.5                             0.043224
   2.0          0.020720
   2.5                             0.005107
   3.0          0.000910
   3.5                             0.000382

 **************************** POWER FLOW REPORT ***************************

 BUS   VOLTAGE   ANGLE       GENERATION            LOAD
 NO.    (PU)     (DEG)      (MW)       (MVAR)     (MW)        (MVAR)


    1  1.0300     0.00   139.31754    64.25957    0.00000     0.00000
   TO-BUS     5        74.02404 MW    20.81969 MVAR
   TO-BUS     2        65.22739 MW    43.43988 MVAR

    2  0.9800    -1.49   135.00000    31.11910   75.00000    60.00000
   TO-BUS     5        46.91624 MW    -2.89952 MVAR
   TO-BUS     3        76.71990 MW    53.11388 MVAR
   TO-BUS     1       -63.63718 MW   -79.09526 MVAR

    3  0.9340    -3.94     0.00000     0.00000   60.00000    50.00000
   TO-BUS     4        16.72826 MW    -1.58446 MVAR
   TO-BUS     2       -76.71990 MW   -48.39773 MVAR

    4  0.9068    -5.25     0.00000     0.00000   70.00000    55.00000
   TO-BUS     5       -53.51378 MW   -31.88358 MVAR
   TO-BUS     3       -16.49206 MW   -23.12670 MVAR

    5  0.9727    -2.58     0.00000     0.00000   65.00000    45.00000
   TO-BUS     1       -71.84075 MW   -65.17135 MVAR
   TO-BUS     2       -46.68237 MW   -15.22950 MVAR
   TO-BUS     4        53.51378 MW    35.36267 MVAR
```

Figure 8: Power Flow — Part 4

```
1       1  (        9.109589,       -25.508904)
2       2  (       10.882353,       -57.452787)
3       3  (        2.000000,       -25.850000)
4       4  (        2.000000,       -19.413368)
5       5  (        9.991942,       -46.638316)
5       1  (       -4.109589,        10.958904)
1       5  (       -4.109589,        10.958904)
2       1  (       -5.000000,        15.000000)
1       2  (       -5.000000,        15.000000)
5       2  (       -5.882353,        23.529412)
2       5  (       -5.882353,        23.529412)
4       3  (       -2.000000,         6.000000)
3       4  (       -2.000000,         6.000000)
5       4  (        0.000000,        13.020833)
4       5  (        0.000000,        13.020833)
3       2  (        0.000000,        19.607843)
2       3  (        0.000000,        19.607843)
```

Figure 9: Y_{Bus} — Part 4

2.5 Part 5

Experimentally, we tweak the values of the real power load to find the point at which the power flow diverges. When changing by 50[MW], we see that this occurs at 420[MW]. Changing values slightly, we observe that the power flow diverges for $P_L \geq 409$[MW]

2.6 Part 6

Now modifying to the power constraints for bus 2, as specified in Part (2), we find that the power flow diverges for $P_L \geq 388$[MW].

2.7 Part 7

For the power flow to not diverge, we need at least .34[$p.u.$] shunt capacitance. We slowly increase this value to find that, to maintain a bus voltage greater than .9[$p.u.$] at a 400[MW] real load, with power constraints from Part (2), the shunt capacitance must be at least 1.51[$p.u.$].

3 Analysis

3.1 Constructing Y_{bus}

We may begin by constructing an admittance matrix based on the provided parameters. First and foremost, we may determine the zero parameters based on disconnected buses:

$$Y_{bus} = \begin{bmatrix} - & - & 0 & 0 & - \\ - & - & - & 0 & - \\ 0 & - & - & - & 0 \\ 0 & 0 & - & - & - \\ - & - & 0 & - & - \end{bmatrix}$$

From here, we find the off-diagonal elements by taking the inverse of the impedance. This gives us:

$$y_{12} = y_{21} = -\frac{1}{.02 + .06j} = -5 + 15j$$

$$y_{15} = y_{51} = -\frac{1}{.03 + .08j} = -4.1096 + 10.9589j$$

$$y_{23} = y_{32} = -\frac{1}{(.92)(.05j)} = 21.739j$$

$$y_{25} = y_{52} = -\frac{1}{.01 + .04j} = -5.8824 + 23.5294j$$

$$y_{34} = y_{43} = -\frac{1}{.05 + .15j} = -2 + 6j$$

$$y_{45} = y_{54} = -\frac{1}{(.96)(.08j)} = 13.021j$$

Putting this into our matrix, we get:

$$Y_{bus} = \begin{bmatrix} - & -5+15j & 0 & 0 & -4.1096+10.9589j \\ -5+15j & - & 21.739j & 0 & -5.8824+23.5294j \\ 0 & 21.739j & - & -2+6j & 0 \\ 0 & 0 & -2+6j & - & 13.021j \\ -4.1096+10.9589j & -5.8824+23.5294j & 0 & 13.021j & - \end{bmatrix}$$

Finally, we determine the diagonal terms:

$$y_{11} = -y_{12} - y_{15} + \frac{(B_{12}+B_{15})j}{2} = 9.0196 - 25.5089j$$

Using a similar method, we get:

$$y_{22} = 10.882 - 61.859j$$
$$y_{33} = 2 - 25.85j$$
$$y_{44} = 2 - 19.413j$$
$$y_{55} = 9.992 - 46.638j$$

This gives us the final bus matrix (for parts 1 and 2) as:

$$\boxed{\begin{bmatrix} 9.0196-25.5089j & -5+15j & 0 & 0 & -4.1096+10.9589j \\ -5+15j & 10.882-61.859j & 21.739j & 0 & -5.8824+23.5294j \\ 0 & 21.739j & 2-25.85j & -2+6j & 0 \\ 0 & 0 & -2+6j & 2-19.413j & 13.021j \\ -4.1096+10.9589j & -5.8824+23.5294j & 0 & 13.021j & 9.992-46.638j \end{bmatrix}}$$

We may observe that this is in line with the generated buses from the results section.
With the connection of the shunt capacitor in part (3), we modify the bus to get:

$$\boxed{\begin{bmatrix} 9.0196-25.5089j & -5+15j & 0 & 0 & -4.1096+10.9589j \\ -5+15j & 10.882-61.859j & 21.739j & 0 & -5.8824+23.5294j \\ 0 & 21.739j & 2-25.65j & -2+6j & 0 \\ 0 & 0 & -2+6j & 2-19.413j & 13.021j \\ -4.1096+10.9589j & -5.8824+23.5294j & 0 & 13.021j & 9.992-46.638j \end{bmatrix}}$$

Finally, changing the tap in part (4) would result in:

$$\boxed{\begin{bmatrix} 9.0196-25.5089j & -5+15j & 0 & 0 & -4.1096+10.9589j \\ -5+15j & 10.882-57.453j & 19.608j & 0 & -5.8824+23.5294j \\ 0 & 19.608j & 2-25.65j & -2+6j & 0 \\ 0 & 0 & -2+6j & 2-19.413j & 13.021j \\ -4.1096+10.9589j & -5.8824+23.5294j & 0 & 13.021j & 9.992-46.638j \end{bmatrix}}$$

We may see that the constructed buses match those generated in PET, with small variations due to rounding.

3.2 Conclusions

First and foremost, we may see that the admittance matrix simulated is precisely as expected. From parts one to two, there is no difference since no new components are connected. For part three, the addition of a shunt capacitor causes a shift in bus 3's diagonal term. For part 4, the change in the tap value of the transform causes a change in bus 2 and bus 3's values.
For the power flow, we may observe that, transitioning from part one to two, as expected, the generator from bus 1 must pick up the slack caused by stricter limitations on bus 2's reactive power flow. Furthermore, as expected, we may see that the addition of a shunt capacitor in part 3 changes very little for the real power flow; however, the reactive power flow to bus 3 decreases, while it increases for the other buses. Conversely, the increase of the tap in part 4 results in decreased real power flow to bus 3, while the real power flow to other buses increases (with little change in reactive power flow). As such, we see that the PET program does a great job of simulating bus responses to changes in power flow set up.

3.3 Voltage Magnitude versus Real Power Load

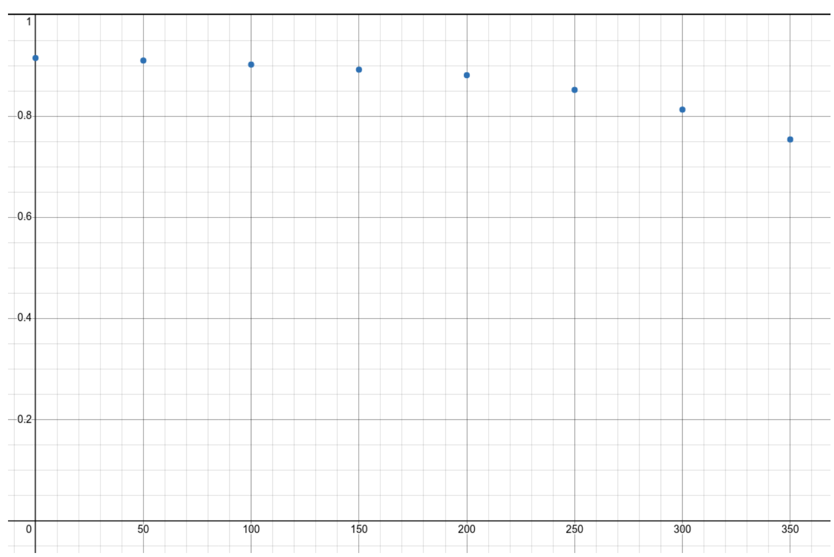

Figure 10: Plot for Part 5

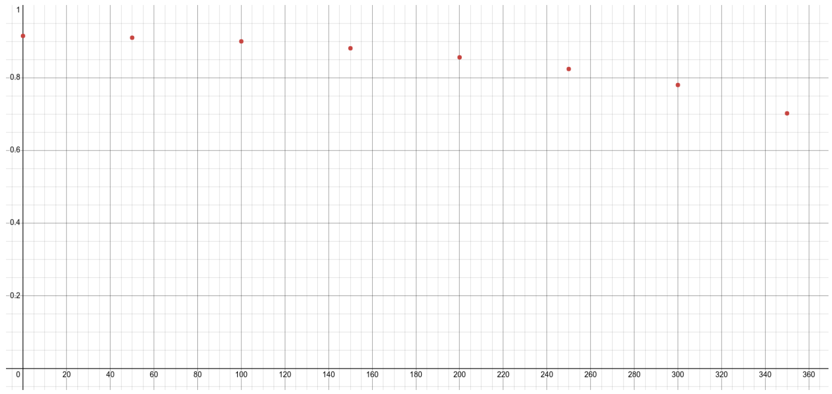

Figure 11: Plot for Part 6

Based on the PV plots, we may determine that the voltage is inversely related to the real power load, while reactive power load stays constant. This is the reason that reactive power injection is desirable, as it allows for voltage magnitude to remain more steady.

3.4 How to Determine the Necessary Capacitance for Part 7

Using the set up of the problem, we could simulate using the Newton-Raphsson method to determine the necessary capacitance value.

3.5 Converting the Capacitance to Farads

Given the base, we know that the capacitor base impedance is:

$$X_b = \frac{V^2}{Q}$$

$$X_b = \frac{(34.5 \cdot 10^3)^2}{100 \cdot 10^6 \sin(60) j}$$

$$X_b = -13.744j[\Omega]$$

We know that capacitance may be written as:

$$X_C = \frac{1}{j\omega C}$$

This means:

$$\frac{1}{\omega C} = 13.744$$

$$C = \frac{1}{13.744\omega}$$

$$C = \frac{1}{13.744(120\pi)}$$
$$C_b = 193[\mu\text{F}]$$

Now that we know the base in farads, we may simply multiply by the per-unit value to get:

$$C = C_b C_{pu}$$
$$C = (1.51)(193 \cdot 10^{-6})$$
$$\boxed{C = 291[\mu\text{F}]}$$