Lecture 4

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September 19, 2024

- Transformers
  - A device which has two coils, a primary and secondary, both of which are physically wrapped around a magnetic rectangular donut-like shape
  - The magnetic material traps the flux generated by pushing a current through the primary coil
  - Some flux will "leak" into the empty central portion of the shape from both coils, called the "leakage flux"
  - Given N_1 turns on the primary coil and N_2 turns on the secondary coil, the total flux becomes:

$$N_1\phi = \lambda_1$$
$$N_2\phi = \lambda_2$$

$$N_1\frac{d\phi}{dt} = V_1$$
$$N_2\frac{d\phi}{dt} = V_2$$

    * With ϕ as the flux through one turn, and λ_n as the total flux through the coil
  - This gives us a voltage ratio:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

  - Thus, transformers are used to step the voltage up/down
  - In the case that there is no leakage flux (an ideal transformer), the powers are equal: $V_1 i_1 = V_2 i_2$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{i_2}{i_1}$$

- Expressing in matrix notation, we may write:

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix} \begin{bmatrix} V_2 \\ i_2 \end{bmatrix}$$

  - With $n = \dfrac{N_1}{N_2}$

- To support the model, a "magnetizing inductor" (which is non-existent) may be introduced to derive the actual load current
  - i_ϕ is the excitation current, which is typically 1-3% of the load current

- The Per-Unit System
  - Per-unit value of a quantity (could be a voltage, a current, impedance, power, etc.) is the actual value divided by the base value
  - Step 1
    - Choose the base value for 2 out of many variables
  - Step 2
    - Determine the base values for all remaining variables
  - For a change of base in a transformer, we may write:

$$x_{pu2} = x_{pu1} \left(\frac{V_{b1}}{V_{b2}} \right)^2 \left(\frac{S_{b2}}{S_{b1}} \right)$$