Lecture 15

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- Economic Dispatch (ED) Problem
  - Given NG committed generators, $P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max}$, for $i = 1, 2, \cdots, NG$, with S_{Di} at all system, buses
  - Find P_{Gi} for $i = 1, 2, \cdots, NG$ that will minimize the total generation
  - Cost:

$$C_T = \sum_{i=1}^{NG} C_i\left(P_{Gi}\right)$$

    * Subject to:

$$\sum_{i=1}^{NG} P_{Gi} = \sum_{i=i}^{N} P_{Di} + P_{buses}$$

    * Where N is the number of buses
    * Line transmission capacity is subject to:

$$|P_{fk}| \leq P_{fk}^{max}, \quad k = 1, 2, \cdots, L$$

    * Where L refers to the number of lines
  - The generation cost can thus be expressed as:

$$C_i(P_{Gi}) = \alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2, \quad i = 1, 2, \cdots, NG$$

    * We can minimize the cost (optimize) by using the Lagrangian method
    * We may form a Lagrangian as:

$$\mathcal{L}(x, \lambda) = f(x) - \lambda g(x)$$

      · The λ term is referred to as the Lagrange multiplier

- We may then express a solution such that:

$$\begin{cases} \dfrac{\partial \mathcal{L}(x,\lambda)}{\partial x} &= 0 \\ \dfrac{\partial \mathcal{L}(x,\lambda)}{\partial \lambda} &= 0 \end{cases}$$

- Using the Lagrangian
  - We apply the Lagrangian as follows:

$$\mathcal{L}(P_{Gi},\lambda) = \sum_{i=1}^{NG} C_i(P_{Gi}) - \lambda \sum_{i=1}^{N} P_{Gi} - P_D$$

  - Taking the partial differentials, we obtain:

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \beta_i + 2\gamma_i P_{Gi} - \lambda$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{N} P_{Gi} - P_D$$

  - We set both of the above equal to zero, which lets us form:

$$\begin{bmatrix} 2\gamma_1 & \cdots & \cdots & \cdots & -1 \\ \vdots & 2\gamma_2 & & & -1 \\ \vdots & & \ddots & & -1 \\ \vdots & & & 2\gamma_{NG} & -1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ P_{GNG} \\ \lambda \end{bmatrix} = \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ \vdots \\ -\beta_{NG} \\ P_D \end{bmatrix}$$

  - The physical interpretation of λ:

$$C_T = C_T^* + \Delta C_T = \sum_{i=1}^{NG} \left\{ C_i(P_{Gi}^*) + \frac{dC_i(P_{Gi})}{dP_{Gi}} \Delta P_{Gi} \right\}$$

$$\Delta C_T = \lambda \sum_{i=1}^{NG} \Delta P_{Gi}$$

  - This cost will be expressed in dollars per mega-watt
  - The penalty factor may be expressed as:

$$L_i \triangleq \frac{1}{1 - \dfrac{\partial P_L}{\partial P_{Gi}}}$$

- If all penalty factors are assumed to be equal to one, then the economic dispatch rule is assumed to be lossless

- Computing the Loss Penalty Factors
  - First, we write:

$$P_L = \sum_{i=1}^{N} P_i$$

$$P_i = P_{Gi} - P_{Di}$$

    * Where N is the number of buses
  - We can then write θ as:

$$\theta = \begin{bmatrix} \theta_2 \\ \vdots \\ \vdots \\ \theta_N \end{bmatrix}, \quad \theta_1 = 0°$$

  - This gives us:

$$P_L(\theta) = P_i + \sum_{i=2}^{N} P_i$$

$$dP_L(\theta) = dP_i + \sum_{i=2}^{N} dP_i$$

  - We then let:

$$dP = \begin{bmatrix} dP_2 \\ \vdots \\ \vdots \\ dP_N \end{bmatrix}$$

$$\frac{\partial P_1}{\partial \theta} = \begin{bmatrix} \partial P_1/\partial \theta_2 \\ \vdots \\ \vdots \\ \partial P_1/\partial \theta_N \end{bmatrix}$$

$$d\theta = \begin{bmatrix} d\theta_2 \\ \vdots \\ \vdots \\ d\theta_N \end{bmatrix}$$

– We then know that $\dfrac{\partial P}{\partial \theta}$ is an $(N-1)\times(N-1)$ jacobian. This lets us write:

$$dP = \left[\frac{\partial P}{\partial \theta}\right]^T d\theta$$

– We can then write:

$$dP_L = \left[\frac{\partial P_1}{\partial \theta}\right]^T \left[\frac{\partial P}{\partial \theta}\right]^{-1} dP = \beta\, dP$$

$$dP_L = \beta dP + \sum_{i=2}^{N} dP_i = (\beta_2+1)dP_2 + (\beta_3+1)dP_3 + \cdots + (\beta_N+2)dP_N$$

– Ultimately, this lets us write:

$$L_i = -\frac{1}{\beta_i}$$

- Steps of ED Calculation Accounting for Losses

  1. Choose an initial dispatch $P_{G2}, P_{G3}, \cdots, P_{GNG}$
  2. Solve the power flow problem
     – $P_{G1} \rightarrow \theta$
  3. Calculate the loss penalty factors:

$$\beta = \left[\frac{\partial P_1}{\partial \theta}\right]^T \left[\frac{\partial P}{\partial \theta}\right]^{-1}$$

$$L_i = -\frac{1}{\beta_i}$$

  4. Check $L_j IC_j = IC_1 = \lambda$
     – If $L_j IC_j > \lambda$, then $P_{Gi} \leftarrow P_{Gj} - \delta P$ — If $P_{Gj} < P_{Gj}^{min}$ set $P_{Gj} = P_{Gj}^{min}$
     – If $L_j IC_j < \lambda$, then $P_{Gi} \leftarrow P_{Gj} - \delta P$ — If $P_{Gj} < P_{Gj}^{max}$ set $P_{Gj} = P_{Gj}^{max}$
  5. Go back to step 2