Homework 4

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November 15, 2024

Chapter 11:

8. First, we need to find the incremental costs. This gives us:

$$IC_i = \frac{dC_i}{dP_{Gi}}$$

Which results in:

$$\begin{cases} IC_1 & = 8 + .003P_{G1} \\ IC_2 & = 8 + .001P_{G2} \\ IC_3 & = 7.5 + .002P_{G3} \end{cases}$$

We want to find a point at which:

$$IC_1 = IC_2 = IC_3$$

This gives us:

$$8 + .003P_{G1} = 8 + .001P_{G2}$$
$$3P_{G1} = P_{G2}$$

And:

$$7.5 + .002P_{G3} = 8 + .001P_{G2}$$
$$P_{G3} = \frac{500 + P_{G2}}{2}$$

We can then apply these equations to the three given cases:

(a) At 500[MW]:

$$P_{G1} + P_{G2} + P_{G3} = 500$$

We substitute to get:

$$\frac{1}{3}P_{G2} + P_{G2} + \frac{500 + P_{G2}}{2} = 500$$
$$\frac{1}{3}P_{G2} + P_{G2} + \frac{1}{2}P_{G2} = 250$$
$$\frac{11}{6}P_{G2} = 250$$
$$\boxed{P_{G2} = 136.36[\mathrm{MW}]}$$

From our formulas, we may solve to get:

$$P_{G1} = \frac{1}{3}P_{G2}$$
$$\boxed{P_{G1} = 45.455[\mathrm{MW}]}$$

And finally, we find the last value:

$$P_{G3} = 500 - 136.36 - 45.455$$
$$\boxed{P_{G3} = 318.19[\mathrm{MW}]}$$

This gives an optimal dispatch of:

$$\boxed{\left\{\begin{array}{ll} P_{G1} & = 45.455 \\ P_{G2} & = 136.36 \\ P_{G3} & = 318.19 \end{array}\right. [\mathrm{MW}]}$$

And a cost of:

$$C_t = 1450 + 8(45.455) + .0015(45.455)^2 + 8(136.36) + .0005(136.36)^2 +$$
$$7.5(318.19) + .001(318.19)^2$$
$$\boxed{C_t = 5,404.6 \left[\frac{\$}{\mathrm{hr}}\right]}$$

(b) At 1000[MW]:

$$P_{G1} + P_{G2} + P_{G3} = 500$$

We substitute to get:

$$\frac{1}{3}P_{G2} + P_{G2} + \frac{500 + P_{G2}}{2} = 1000$$

$$P_{G2} = \frac{6}{11}(750)$$

$$\boxed{P_{G2} = 409.09[\text{MW}]}$$

Then we find P_{G1}:

$$P_{G1} = \frac{1}{3}P_{G2}$$

$$\boxed{P_{G1} = 136.36[\text{MW}]}$$

And finally, we find the last value:

$$P_{G3} = 1000 - 409.09 - 136.36$$

$$\boxed{P_{G3} = 454.55[\text{MW}]}$$

This gives an optimal dispatch of:

$$\boxed{\begin{cases} P_{G1} & = 136.36 \\ P_{G2} & = 409.09 \ [\text{MW}] \\ P_{G3} & = 454.55 \end{cases}}$$

And a cost of:

$$C_t = 1450 + 8(136.36) + .0015(136.36)^2 + 8(409.09) + .0005(409.09)^2 + \\ 7.5(454.55) + .001(454.55)^2$$

$$\boxed{C_t = 9,540.9 \left[\frac{\$}{\text{hr}}\right]}$$

(c) At 2000[MW]:

$$P_{G1} + P_{G2} + P_{G3} = 500$$

We substitute to get:

$$\frac{1}{3}P_{G2} + P_{G2} + \frac{500 + P_{G2}}{2} = 2000$$

$$P_{G2} = \frac{6}{11}(1750)$$

$$\boxed{P_{G2} = 954.55[\text{MW}]}$$

Then we find P_{G1}:

$$P_{G1} = \frac{1}{3}P_{G2}$$

$$\boxed{P_{G1} = 318.18[\text{MW}]}$$

And finally, we find the last value:

$$P_{G3} = 2000 - 954.55 - 318.18$$

$$\boxed{P_{G3} = 727.27[\text{MW}]}$$

This gives an optimal dispatch of:

$$\boxed{\begin{cases} P_{G1} & = 318.18 \\ P_{G2} & = 954.55 \ [\text{MW}] \\ P_{G3} & = 727.27 \end{cases}}$$

And a cost of:

$$C_t = 1450 + 8(318.18) + .0015(318.18)^2 + 8(954.55) + .0005(954.55)^2 + \\ 7.5(727.27) + .001(727.27)^2$$

$$\boxed{C_t = 18,223 \left[\frac{\$}{\text{hr}}\right]}$$

9. When the loads are shared evenly, we know that:

$$P_{G1} = P_{G2} = P_{G3} = \frac{P_D}{3}$$

(a) At 500[MW]:

$$P_{G1} = P_{G2} = P_{G3} = 166.67[\text{MW}]$$

This gives a cost of:

$$C_t = 1450 + 23.5(166.67) + .003(318.18)^2$$

$$\boxed{C_t = 5,450 \left[\frac{\$}{\text{hr}}\right]}$$

This gives a cost increase of:

$$\Delta C = 5450 - 5404.6$$

$$\boxed{\Delta C = 45.4 \left[\frac{\$}{\text{hr}}\right]}$$

(b) At 1000[MW]:

$$P_{G1} = P_{G2} = P_{G3} = 333.33[\text{MW}]$$

This gives a cost of:

$$C_t = 1450 + 23.5(333.33) + .003(333.33)^2$$
$$\boxed{C_t = 9,616.6 \left[\frac{\$}{\text{hr}}\right]}$$

This gives a cost increase of:

$$\Delta C = 9616.6 - 9540.9$$
$$\boxed{\Delta C = 75.682 \left[\frac{\$}{\text{hr}}\right]}$$

(c) At 2000[MW]:

$$P_{G1} = P_{G2} = P_{G3} = 666.67[\text{MW}]$$

This gives a cost of:

$$C_t = 1450 + 23.5(666.67) + .003(666.67)^2$$
$$\boxed{C_t = 18,450 \left[\frac{\$}{\text{hr}}\right]}$$

This gives a cost increase of:

$$\Delta C = 18,450 - 18,223$$
$$\boxed{\Delta C = 227.09 \left[\frac{\$}{\text{hr}}\right]}$$

10. (a) Since the first generator must be producing at least 50[MW], we set it to its minimum value and continue to find P_{G2} and P_{G3}:

$$P_{G2} + P_{G3} = 450$$

This lets us find:

$$P_{G2} = \frac{2}{3}(200)$$
$$P_{G2} = 133.33[\text{MW}]$$

We then find:

$$P_{G3} = 133.33(.5) + 250$$

$$\boxed{P_{G3} = 316.67[\text{MW}]}$$

This gives an optimal dispatch of:

$$\boxed{\begin{cases} P_{G1} & = 50 \\ P_{G2} & = 133.33 \;[\text{MW}] \\ P_{G3} & = 316.67 \end{cases}}$$

And a cost of:

$$C_t = 1450 + 8(50) + .0015(50)^2 + 8(133.33) + .0005(133.33)^2 + \\ 7.5(316.67) + .001(316.67)^2$$

$$\boxed{C_t = 5,404.6 \left[\frac{\$}{\text{hr}}\right]}$$

We may observe that the cost remains the same.

(b) At 1000[MW]:

We may observe that, because all of the generation is already within limits, the dispatch would remain the same as in problem 8:

$$\boxed{\begin{cases} P_{G1} & = 136.36 \\ P_{G2} & = 409.09 \;[\text{MW}] \\ P_{G3} & = 454.55 \end{cases}}$$

And a cost of:

$$\boxed{C_t = 9,540.9 \left[\frac{\$}{\text{hr}}\right]}$$

(c) We may observe that the second generator is producing beyond its limitations. This makes us cap the value at 800, which gives us:

$$P_{G1} + P_{G3} = 1200$$

$$\frac{5}{2} P_{G1} = 950$$

$$\boxed{P_{G1} = 380[\text{MW}]}$$

We then find:

$$P_{G3} = 250 + 1.5(380)$$

$$\boxed{P_{G3} = 820[\text{MW}]}$$

The optimal dispatch thus becomes:

$$\boxed{\begin{cases} P_{G1} & = 380 \\ P_{G2} & = 800 \;\; [\text{MW}] \\ P_{G3} & = 820 \end{cases}}$$

And a cost of:

$$C_t = 1450 + 8(380) + .0015(380)^2 + 8(800) + .0005(800)^2 + \\ 7.5(820) + .001(820)^2$$

$$\boxed{C_t = 18,249 \left[\frac{\$}{\text{hr}}\right]}$$