

# Lecture 9

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- Modifying an Existing  $Y_{bus}$ :
  - Connecting/disconnecting a line from terminals  $k$  and  $m$  results in addition/subtraction of  $y$  from diagonal entries and subtraction/addition from  $km$  and  $mk$  entries
  - Connecting/disconnecting a line from a single point  $k$  results in addition/subtraction of  $y$  from the corresponding diagonal entry
- Synchronous Generators
  - A field (excitation) voltage,  $E_f$ , is connecting to a reactance  $jX_s$
  - Taking  $\hat{V}_t = V_t \angle 0^\circ$ , and  $\hat{E}_f = \hat{V}_t + jX_s \hat{I}_a$ , we may obtain:

$$\hat{E}_f = E_f \angle \delta \quad \text{and} \quad \hat{I}_a = I_a \angle \phi$$

$$S_g = \hat{V}_t \cdot (\hat{I}_a)^*, \quad \hat{I}_a = \frac{\hat{E}_f - \hat{V}_t}{jX_s}$$

$$S_g = \frac{j}{X_s} (V_t E_f \cos(\delta) - j V_t E_f \sin(\delta) - V_t^2)$$

- This gives us:

$$P_g = \frac{V_t E_f}{X_s} \sin(\delta) = V_t I_a \cos(\phi)$$

$$Q_g = \frac{V_t E_f}{X_s} \cos(\delta) - \frac{V_t^2}{X_s}$$

- We may make some observations:

$$P_g = 0 \rightarrow \delta = 0$$

$$Q_g > 0 \rightarrow \text{acts like a capacitor (synchronous condenser)}$$

$$Q_g < 0 \rightarrow \text{acts like a reactor}$$

- Types of Buses:
  - Load/ $PQ$  bus
    - \* Supplies some real and reactive power
  - Generator/ $PV$  Bus
    - \* Supplies some real power and voltage (magnitude), with unknown reactive power
  - Slack/Swing Bus
    - \* Know the voltage and phase angle, with unknown real and reactive powers
  - Total buses in a system:

$$n = n_{PV} + n_{PQ} + 1$$

$$\left[ \underbrace{1}_{\text{slack}}, \underbrace{2, 3, \dots, (n_{PV} + 1)}_{PV \text{ buses}}, \underbrace{(n_{PV} + 2), \dots, n}_{PQ \text{ buses}} \right]$$

- Thus, we may write:

$$\underbrace{Y_{bus}}_{n \times n} \underbrace{\hat{V}}_{n \times 1} = \underbrace{\hat{I}}_{n \times 1}$$

$$Y_{bus} = G_{bus} + jB_{bus}$$

$$S_k = P_k + jQ_k = \hat{V}_k \hat{I}_k^*$$

$$\hat{I}_k = \sum_{j=1}^k Y_{bus}(k, j) \cdot \hat{V}_j$$

- Finally, we can get:

$$P_k = \sum_{j=1}^n V_k V_j (G_{kj} \cos(\theta_{kj}) + B_{kj} \sin(\theta_{kj}))$$

$$P_k = \sum_{j=1}^n V_k V_j (G_{kj} \sin(\theta_{kj}) - B_{kj} \cos(\theta_{kj}))$$