Homework 3

Michael Brodskiy
Professor: A. Abur

October 22, 2024

<u>Chapter 9</u>:

1.

2. We can expand the given table to write:

From-To	Reactance $[p.u.]$	Admittance $[pu]$
1-2	$.04j$	$-25j$
1-6	$.06j$	$-16.667j$
2-4	$.03j$	$-33.333j$
2-3	$.02j$	$-50j$
3-4	$.08j$	$-12.5j$
4-5	$.06j$	$-16.667j$
5-6	$.05j$	$-20j$

This allows us to write the corresponding connections into the matrix:

$$Y_{bus} = \begin{bmatrix} & 25 & & & & 16.667 \\ 25 & & 50 & 33.333 & & \\ & 50 & & 12.5 & & \\ & 33.333 & 12.5 & & 16.667 & \\ & & & 16.667 & & 20 \\ 16.667 & & & & 20 & \end{bmatrix} j$$

By observation from the diagram, we may write all of the following admittances as zero:

From	To
1	3,4,5
2	5,6
3	1,5,6
4	1,6
5	1,2,3
6	2,3,4

This gives us:

$$Y_{bus} = \begin{bmatrix} & 25 & 0 & 0 & 0 & 16.667 \\ 25 & & 50 & 33.333 & 0 & 0 \\ 0 & 50 & & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & & 16.667 & 0 \\ 0 & 0 & 0 & 16.667 & & 20 \\ 16.667 & 0 & 0 & 0 & 20 & \end{bmatrix} j$$

We see that now only the diagonal is left. Thus, we write KVL equations whilst imagining a current element I_n for each bus n. This allows us to write:

$$\begin{aligned} I_1 &= (V_1 - V_2)(-25j) + (V_1 - V_6)(-16.667j) \\ I_2 &= (V_2 - V_1)(-25j) + (V_2 - V_3)(-50j) + (V_2 - V_4)(-33.333j) \\ I_3 &= (V_3 - V_2)(-50j) + (V_3 - V_4)(-12.5j) \\ I_4 &= (V_4 - V_2)(-33.333j) + (V_4 - V_3)(-12.5j) + (V_4 - V_5)(-16.667j) \\ I_5 &= (V_5 - V_4)(-16.667j) + (V_5 - V_6)(-20j) \\ I_6 &= (V_6 - V_1)(-16.667j) + (V_6 - V_5)(-20j) \end{aligned}$$

Simplifying the equations, we obtain

$$\begin{aligned} I_1 &= j[-41.667V_1 + 25V_2 + 16.667V_6] \\ I_2 &= j[-108.33V_2 + 25V_1 + 50V_3 + 33.333V_4] \\ I_3 &= j[-62.5V_3 + 50V_2 + 12.5V_4] \\ I_4 &= j[-62.5V_4 + 33.333V_2 + 12.5V_3 + 16.667V_5] \\ I_5 &= j[-36.667V_5 + 16.667V_4 + 20V_6] \\ I_6 &= j[-36.667V_6 + 16.667V_1 + 20V_5] \end{aligned}$$

Given that we know:

$$I_n = Y_{bus} V_n$$

We look at the coefficients from the currents to obtain the diagonals:

$$\boxed{Y_{bus} = \begin{bmatrix} -41.667 & 25 & 0 & 0 & 0 & 16.667 \\ 25 & -108.33 & 50 & 33.333 & 0 & 0 \\ 0 & 50 & -62.5 & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & -62.5 & 16.667 & 0 \\ 0 & 0 & 0 & 16.667 & -36.667 & 20 \\ 16.667 & 0 & 0 & 0 & 20 & -36.667 \end{bmatrix} j[p.u.]}$$

3. Adding another connection between buses 1 and 5 would mean that only the nm-th elements are modified, with nm being any combination of 1 and 5. Thus, we see that, with this $-10j$ admittance, we get two new terms at 15 and 51:

$$Y_{bus} = \begin{bmatrix} ? & 25 & 0 & 0 & 10 & 16.667 \\ 25 & -108.33 & 50 & 33.333 & 0 & 0 \\ 0 & 50 & -62.5 & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & -62.5 & 16.667 & 0 \\ 10 & 0 & 0 & 16.667 & ? & 20 \\ 16.667 & 0 & 0 & 0 & 20 & -36.667 \end{bmatrix} j$$

We then re-check the current equations for buses 1 and 5:

$$I_1 = j[-51.667V_1 + 25V_2 + 10V_5 + 16.667V_6]$$
$$I_5 = j[-46.667V_5 + 10V_1 + 16.667V_4 + 20V_6]$$

Thus, we get:

$$\boxed{Y_{bus} = \begin{bmatrix} -51.667 & 25 & 0 & 0 & 10 & 16.667 \\ 25 & -108.33 & 50 & 33.333 & 0 & 0 \\ 0 & 50 & -62.5 & 12.5 & 0 & 0 \\ 0 & 33.333 & 12.5 & -62.5 & 16.667 & 0 \\ 10 & 0 & 0 & 16.667 & -46.667 & 20 \\ 16.667 & 0 & 0 & 0 & 20 & -36.667 \end{bmatrix} j[p.u.]}$$

4. We may begin finding diagonal entries by summing all of the admittances incident at each bus:

$$Y_{11} = \frac{1}{.2j} + \frac{1}{.25j} + \frac{.24j}{2} + \frac{.3j}{2}$$
$$Y_{22} = \frac{1}{.2j} + \frac{1}{.1j} + \frac{.24j}{2} + \frac{.16j}{2} + .3j$$
$$Y_{33} = \frac{1}{.25j} + \frac{1}{.1j} + \frac{.16j}{2} + \frac{.3j}{2} - .6j$$

Solving, we get:

$$Y_{11} = -5j - 4j + .12j + .15j$$
$$Y_{22} = -5j - 10j + .12j + .08j + .3j$$
$$Y_{33} = -4j - 10j + .08j + .15j - .6j$$

And finally:

$$Y_{11} = -8.73j[p.u.]$$
$$Y_{22} = -14.5j[p.u.]$$
$$Y_{33} = -14.37j[p.u.]$$

(a) The off-diagonal elements will simply be the admittances of the connections, which gives us:

$$\boxed{Y_{bus} = \begin{bmatrix} -8.73 & 5 & 4 \\ 5 & -14.5 & 10 \\ 4 & 10 & -14.37 \end{bmatrix} j}$$

(b) With bus 2 gone, we may write:

$$Y_{11} = -4j + \frac{.3j}{2}$$
$$Y_{33} = -4j + \frac{.3j}{2} - .6j$$

And then:

$$Y_{11} = -3.85j$$
$$Y_{33} = -4.45j$$

The off-diagonal terms are then simply $4j$:

$$\boxed{Y_{bus} = \begin{bmatrix} -3.85 & 4 \\ 4 & -4.45 \end{bmatrix} j}$$

5. First, we get the current flow at bus 1:

$$I_1 = \frac{.9}{1.15j} = -.7826j$$

We now write KCL equations which integrate the new voltage source:

$$-.7826j = V_1(-.8696j) + (V_1 - V_2)(.252j) + (V_1 - V_3)(.3243j)$$
$$0 = (V_2)(.3j) + (V_2 - V_1)(.252j) + (V_2 - V_3)(.1626j)$$
$$0 = (V_3)(-.6j) + (V_3 - V_1)(.3423j) + (V_3 - V_2)(.1626j)$$

This gives us:

$$-.2933V_1 - .252V_2 - .3243V_3 = -.7826$$

$$[.7146V_2 - .252V_1 - .1626V_3]j = 0$$

$$[-.0951V_3 - .3423V_1 - .1626V_2]j = 0$$

From which we may build:

$$Z_{bus} = \left(\begin{bmatrix} -.2933 & -.252 & -.3243 \\ -.252 & .7146 & -.1626 \\ -.3423 & -.1626 & -.0951 \end{bmatrix} j \right)^{-1}$$

Taking the inverse, we get:

$$\boxed{Y_{bus} = \begin{bmatrix} -1.2946 & .3945 & 3.7403 \\ .4347 & -1.1399 & .4668 \\ 3.9167 & .529 & -3.7454 \end{bmatrix} j}$$

Note that, due to rounding of decimals, the matrix produced is asymmetric, even though the admittance matrix should exhibit symmetry.