Lecture 10

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- Types of Buses
  - Slack Bus (always one)
    - Known: Voltage magnitude V and the phase angle θ
    - Unknown: Net powers P and Q
  - PV (Generator) Buses (there are n_{PV} such buses)
    - Known: Voltage magnitude V and real power P
      - Net powers are:

$$P = P_G - P_D$$
$$Q = Q_G - Q_D$$

    - Unknown: Phase angle θ and reactive power Q
  - PQ (Load) Buses (there are n_{PQ} such buses)
    - Known: Real and reactive powers P and Q
      - Net powers are:

$$P = -P_D$$
$$Q = -Q_D$$

    - Unknown: Voltage magnitude V and phase angle θ
  - Note: for these buses P and Q refers to the net injected power (*e.g.* If a generator supplies P_G and power P_D is dissipated, then P refers to $P_G - P_D$)

- Working with Buses
  - Given that $\hat{S}_k = \hat{V}_k \hat{I}_k^*$ and $Y_{bus} \cdot V = I$, we may write:

$$\hat{I}_k = \sum_{m=1}^{n} Y_{bus}^*(k, m) \cdot \hat{V}_m^*$$

- This gives us:

$$\hat{S}_k = \hat{V}_k \sum_{m=1}^{n} Y^*_{bus}(k,m) \cdot \hat{V}^*_m$$

- Considering a system with n buses (where $n = 1 + n_{PV} + n_{PQ}$):
  - The total number of equations matches the total number of unknowns
  - The equations are non-linear and therefore requires solutions of nonlinear algebraic equations
  - Unknown voltages and phases may be written as:

$$X^T = [\underbrace{\theta_2, \theta_3, \cdots, \theta_n}_{n-1} \Big| \underbrace{V_{n_{PV}+2}, \cdots, V_n}_{n_{PQ}}]$$

  - For a given bus k:

$$P_k = P_G - P_D = V_k \sum V_m (G_{bus}\cos(\theta_{bus}) + B_{bus}\sin(\theta_{km}) - P_k^{sch} = 0 \quad \text{(for } PQ \text{ and } PV \text{ buses)}$$

$$Q_k = V_k \sum V_m (G_{bus}\sin(\theta_{bus}) - B_{bus}\cos(\theta_{km}) - Q_k^{sch} = 0 \quad \text{(for } PQ \text{ buses)}$$

    * We see that the solutions to these equations may be represented by a non-linear vector X, which solves X^T from above

- Solving Non-linear Algebraic Equations
  - We will use the Newton-Raphson method
  - Scalar case ($F(x)$ is a non-linear function, and x is a single variable):
    * We may begin by Taylor expanding around some value x_o such that:

$$F(x) \approx F(x_o) + \frac{\partial F}{\partial x}\Big|(x - x_o) + \overbrace{\cdots}^{\text{higher order terms we ignore}}$$

    * We let $\dfrac{\partial F}{\partial x} = F_x$ and plug in the value to get:

$$F_x(x - x_o) + F(x_o) \approx 0$$
$$x \approx x_o - F_x^{-1} F(x_o)$$

    * A k-iteration counter allows us to write:

$$x^{k+1} \approx x^k - (F_x^k)^{-1} F(x^k)$$

    * Starting with $x^1 = x_o$, continue updating x^k until $|x^{k+1} - x^k| \leq \varepsilon$ (the ε is known as the termination threshold)

* We may define the Jacobian to rewrite the above formula as:

$$\bar{x}^{k+1} = \bar{x}^k - [J(x^k)]^{-1} F(x^k)$$

- Applying this to the case of the power flow problem
  - Assume $\hat{x}^o$ and iterate until $\underbrace{||F^{k+1}||}_{\text{norm}|F_j(x)|} < \varepsilon$ (typically $10^{-4}[p.u.]$)
  - Taking the transpose, we may write:

$$[F(\bar{x})]^T = \Big[\underbrace{P_2(x) - P_2^{sch}, P_3(x) - P_3^{sch}, \cdots, P_n(x) - P_n^{sch}}_{n}$$

$$\Big| \underbrace{Q_{n_{PQ}+2}(x) - Q_{n_{PQ}+2}^{sch}, \cdots, Q_n(x) - Q_n^{sch}}_{n_{PQ}} \Big]$$

    * The Jacobian matrix will be quite sparse