Lab One
Power Systems Analysis
EECE5682

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Abstract

This laboratory experiment explores three-phase circuit modeling via SimuLink integration into MATLAB. The experiment simulates a provided circuit design and demonstrates the differences in output power of balanced and unbalanced three-phase conditions.

KEYWORDS: three-phase, modeling, SimuLink, MATLAB, power, balanced, unbalanced

1 Introduction & Objectives

This experiment begins by integrating the following diagram into MATLAB's SimuLink environment:

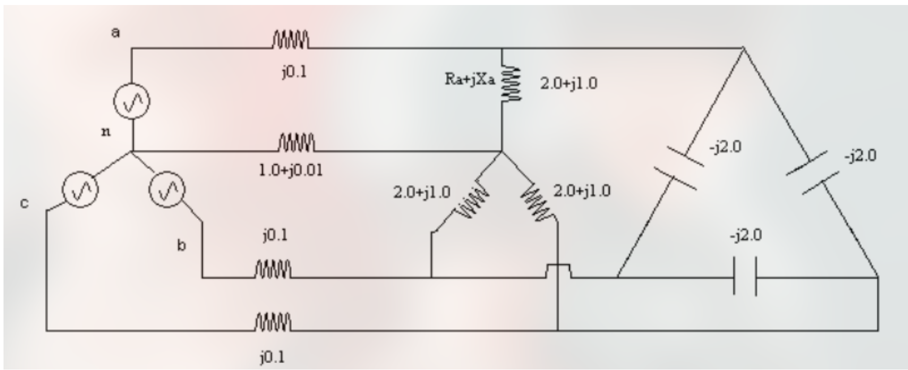

Figure 1: Simulated Circuit

After simulating the circuit, data related to the instantaneous power, voltage, and current, is taken to calculate expected values. The experiment is then repeated for an unbalanced circuit with $R_a = 20[\Omega]$. For the balanced case, we assume $|V_{in}| = 1[\text{V}]$, with phase angles $0°$, $120°$, and $-120°$.

2 Results & Analysis

2.1 Balanced Case ($R_a = 2[\Omega]$)

The first step of the experiment is to plot the per-phase instantaneous power versus time:

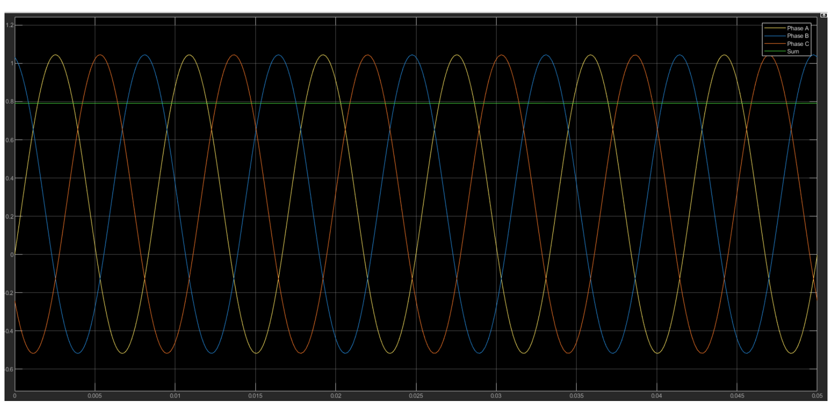

Figure 2: Instantaneous Power of the Balanced Circuit Shown in Figure 1

As expected for a balanced circuit, each power is equivalent in magnitude, but with an offset of phase proportional to 120°. From here, we use the Fourier transform blocks to obtain the phasors for V_{in} and I_{ni}, respectively. The blocks are shown in Figures 3-5 below:

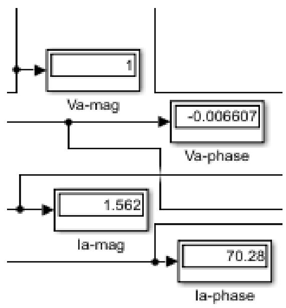

Figure 3: Voltage and Current Phasor Blocks, Phase a

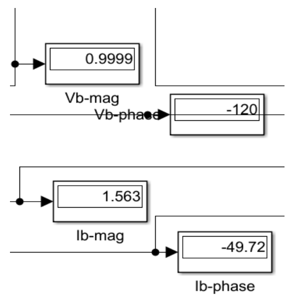

Figure 4: Voltage and Current Phasor Blocks, Phase b

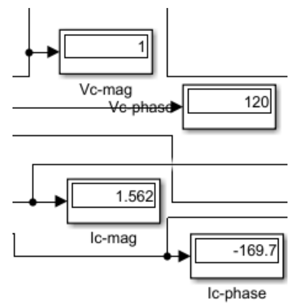

Figure 5: Voltage and Current Phasor Blocks, Phase c

From the blocks, the phasors for voltage and current may be observed as:

$$\hat{V}_{[a,b,c]n} = 1\angle \begin{bmatrix} 0 \\ -120 \\ 120 \end{bmatrix}^{\circ} \text{ [V]}$$

$$\hat{I}_{n[a,b,c]} = 1.562\angle \begin{bmatrix} 70.28 \\ -49.72 \\ -169.72 \end{bmatrix}^{\circ} \text{ [A]}$$

From here, we may solve for the apparent power using the formula:

$$\hat{S} = \frac{1}{2}\left[|\hat{V}||\hat{I}|\cos\left(\phi\right) + j|\hat{V}||\hat{I}|\sin\left(\phi\right)\right]$$

This gives us:

$$\hat{S}_{[a,b,c]} = \frac{1}{2}\left[|1||1.562|\cos(-70.28) + j|1||1.562|\sin(-70.28)\right]$$

$$\hat{S}_{[a,b,c]} = .2635 - .7352j[\text{V A}]$$

Thus, we see each source delivers .2635[W] of real power and −.7352[VAr] of reactive power.

Looking at Figure 2, we see that the sum power (the green line) is just below .8[W]. This makes sense, as, summing the power values of all of the phases gives us a corresponding (DC power) value:

$$3(.2635) = .7905[\text{W}] \approx P_{tot}$$

The instantaneous three-phase power may be seen in Figure 11 below:

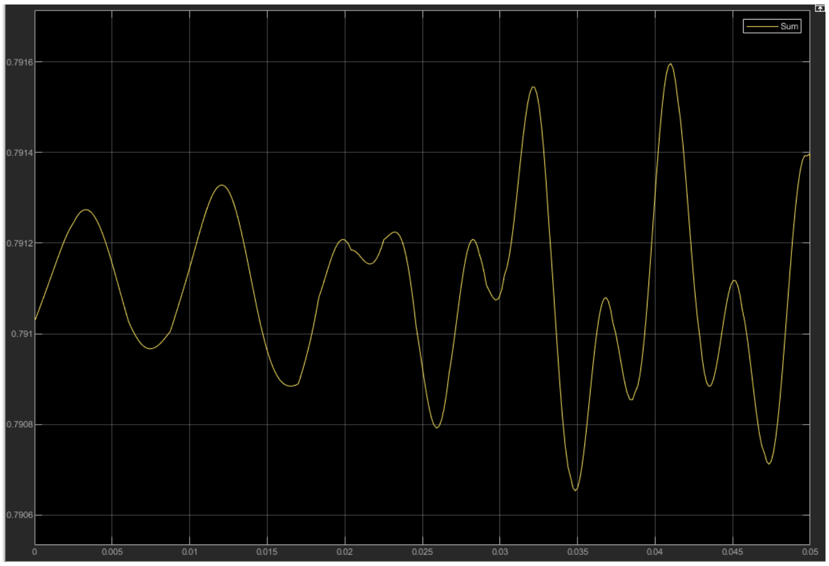

Figure 6: Instantaneous Power

Although it appears that the power fluctuates, further inspection of the waveform shows that it, more or less stays around the expected value of .7905[W, with fluctuations of no more than $\pm 10^{-2}$, most likely a result of the simulation. Thus, the green line in Figure 1 provides the best approximation, as a balanced three-phase circuit would generate constant instantaneous total power. We now proceed to analyzing an unbalanced circuit.

2.2 Unbalanced Case ($R_a = 20[\Omega]$)

The first step of the experiment is to plot the per-phase instantaneous power versus time:

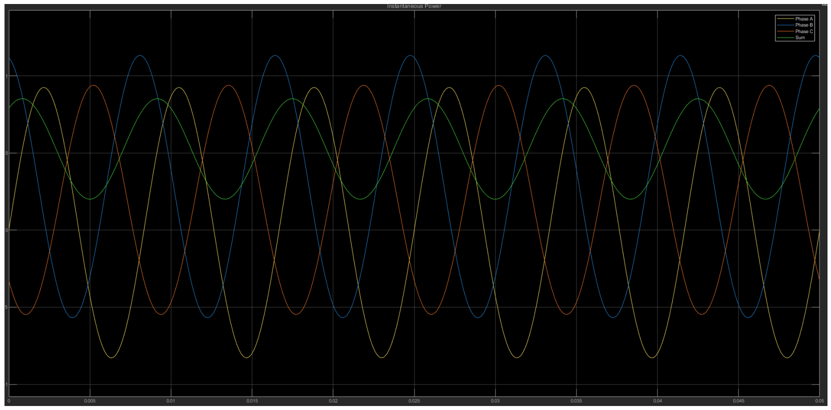

Figure 7: Instantaneous Power of an Unbalanced Circuit Shown in Figure 1

Unlike the balanced case, we see that the powers vary in magnitude, as well as phase. This is expected, as the change of one resistor value would cause a different power delivery in phase a, and the same power delivery in phases b and c. From here, we use the Fourier transform blocks to obtain the phasors for V_{in} and I_{ni}, respectively. The blocks are shown in Figures 8-10 below:

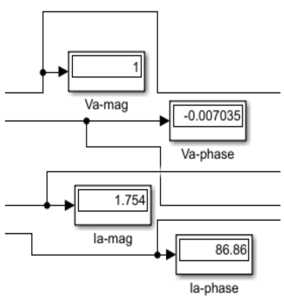

Figure 8: Voltage and Current Phasor Blocks, Phase a

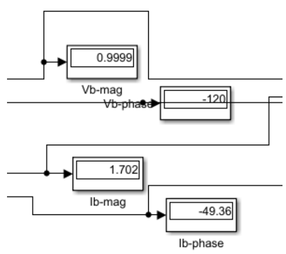

Figure 9: Voltage and Current Phasor Blocks, Phase b

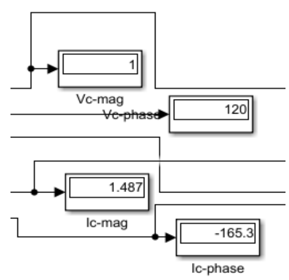

Figure 10: Voltage and Current Phasor Blocks, Phase c

From the blocks, the phasors for voltage and current may be observed as:

$$\hat{V}_{[a,b,c]n} = 1\angle \begin{bmatrix} 0 \\ -120 \\ 120 \end{bmatrix}^{\circ} \text{ [V]}$$

$$\hat{I}_{n[a,b,c]} = \begin{bmatrix} 1.754 \\ 1.702 \\ 1.487 \end{bmatrix} \angle \begin{bmatrix} 86.86 \\ -49.36 \\ -165.3 \end{bmatrix}^{\circ} \text{ [A]}$$

We once again solve for the apparent power. This gives us:

$$\hat{S}_{[a]} = \frac{1}{2}\left[|1||1.754|\cos(-86.86) + j|1||1.754|\sin(-86.86)\right]$$

$$\hat{S}_{[b]} = \frac{1}{2}\left[|1||1.702|\cos(-70.64) + j|1||1.702|\sin(-70.64)\right]$$

$$\hat{S}_{[c]} = \frac{1}{2}\left[|1||1.487|\cos(-74.27) + j|1||1.487|\sin(-74.27)\right]$$

$$\hat{S}_{[a,b,c]} = \begin{bmatrix} .048038 \\ .2821 \\ .1962 \end{bmatrix} + j\begin{bmatrix} -.875683 \\ -.8029 \\ -.7171 \end{bmatrix} [V\,A]$$

Thus, we see each source delivers a differing amount of real and reactive power, which is to be expected in an unbalanced circuit. The real and reactive powers, respectively, are shown below:

$$P = \begin{bmatrix} .048038 \\ .2821 \\ .1962 \end{bmatrix} [\mathrm{W}] \quad \text{and} \quad Q = \begin{bmatrix} -.875683 \\ -.8029 \\ -.7171 \end{bmatrix} [\mathrm{VAr}]$$

Looking at Figure 7, we see that the sum power (the green line), due to the unbalanced conditions, generates a sinusoid. We may also see that the sinusoid is, on average, just above .5[W]. Once again, if we sum these powers, we find the average total power (DC power):

$$.048038 + .2821 + .1962 = .5263[\mathrm{W}] \approx P_{avg}$$

The instantaneous three-phase power may be seen in Figure 11 below:

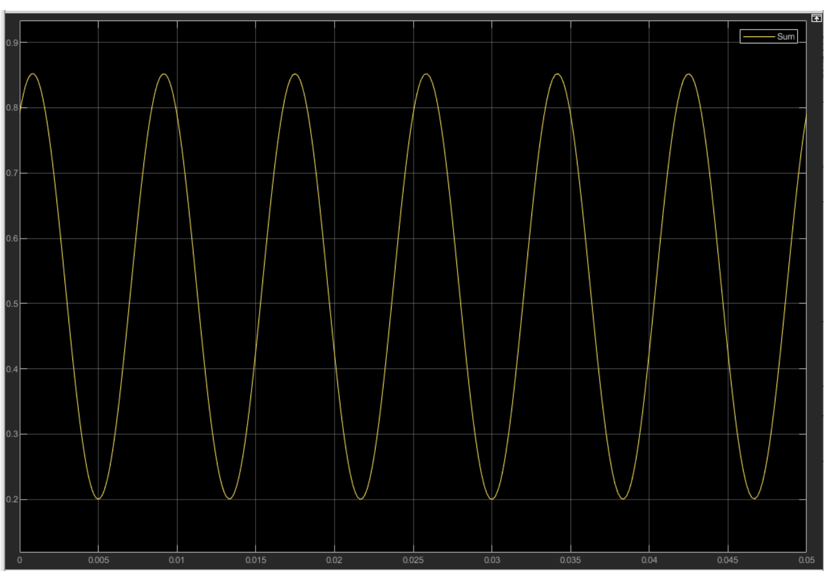

Figure 11: Instantaneous Power

As expected, we see that the instantaneous power of an unbalanced three-phase circuit behaves in a manner similar to a single phase circuit (that is, it forms a sinusoid with frequency twice that of the voltage/current).

3 Conclusion

a. With the balanced circuit, the frequency of each phase's power doubles (that is, it goes from the 60[Hz] of the voltage and current to 120[Hz]). The derivation for the instantaneous power may be found below:

   In a given circuit, assume the voltage and current take the form: $v(t) = V_{max}\cos(\omega t + \theta_v)$ and $i(t) = I_{max}\cos(\omega t + \theta_i)$. The instantaneous power is then:

$$p(t) = V_{max}I_{max}\cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

   Using trigonometric identities and $\phi = \theta_v - \theta_i$, we may write:

$$p(t) = V_{max}I_{max}[\cos(\phi) + \cos(2\omega t + \theta_v + \theta_i)]$$

   Eliminating the average power (DC offset), we see:

$$\boxed{p(t) - p_{avg} = V_{max}I_{max}\cos(2\omega t + \theta_v + \theta_i)}$$

   Note that in the sinusoid, ignoring the phase offset, the frequency is doubled ($\omega \to 2\omega$). For this reason, the power experiences 120[Hz] instead of 60[Hz].

b. For a balanced circuit, the frequency of the instantaneous power is 0[Hz], as the power should not fluctuate I*i.e.* it is constant). For the unbalanced circuit, the frequency is twice that of the voltage/current.