

Lecture 1

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- Key Features of Quantum Mechanics

1. Probabilistic outcome of measurements

- Compute probabilities exactly, and that is the most complete information possible

2. Dual wave-particle nature of matter

- Which one we observe depends on the experiment performed

3. Conjugate variables (from classical mechanics) develop “uncertainty” relations

- Wave theory relation:

$$\Delta x \Delta p \geq \hbar$$

$$\Delta E \Delta t \geq \hbar$$

- Classical mechanics is “contained” in Quantum mechanics, which includes classical electricity and magnetism

4. Every particle and object built from particles, including light, falls into one of two classes:

- Fermions (spin is any odd multiple of $\hbar/2$)
 - * Electrons are an example
- Bosons (spin is an integer multiple of \hbar , including 0)
 - * Photons are an example

5. The properties of Quantum mechanics not falling into features 1-4 are largely familiar from classical physics

- Similar but Different

- Every electron has exactly the same spin, charge, and mass
- Every photon has exactly the same spin, charge, and mass

- This indicates no inequality among particles of the same type
- Time-Evolution in Quantum Mechanics
 - Time-evolution is given by the Hamiltonian, \mathcal{H}
 - Hamilton's Equations give us:

$$\frac{d\vec{q}}{dt} = \frac{\partial \mathcal{H}}{\partial \vec{p}}$$

- We may write the Hamiltonian as:

$$\mathcal{H} = \frac{\vec{p}^2}{2m} \longrightarrow \frac{\partial \mathcal{H}}{\partial \vec{p}} = \frac{\vec{p}}{m} = \vec{v}$$

- Alternatively, we may write the force as:

$$\vec{F} = \frac{d\vec{p}}{dt} = -\frac{\partial \mathcal{H}}{\partial \vec{q}}$$

- Stern-Gerlach Experiments
 - Took place in 1922 with Otto Stern and Walther Gerlach
 - The act of observing a quantum particle affects its measurable properties in a way foreign to our experience
 - The experiment looks like this:

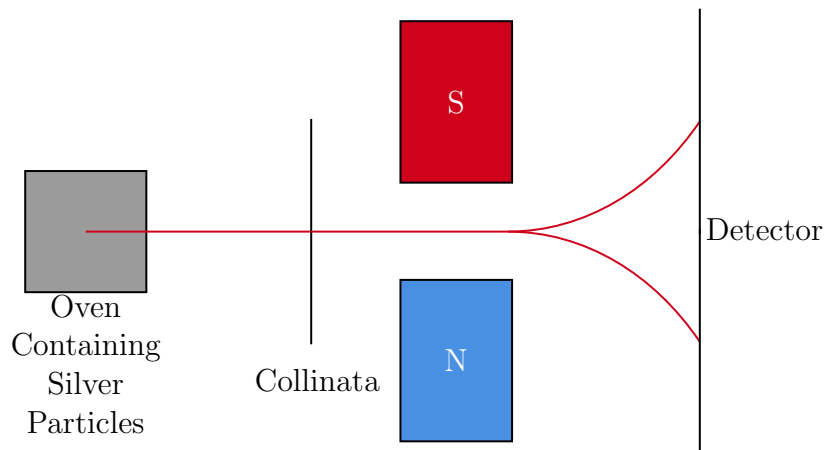


Figure 1: Set Up of Stern-Gerlach Experiment

- Assuming an atom has a monopole moment, $\vec{\mu}$, the potential energy of the interaction with a magnetic field \vec{B} is $E = \vec{\mu}\vec{B}$
- Consider a classical description of the atom's moment:

$$\mu = IA$$

- * Where I is the electrical current and A is the area of the loop

- A particle of charge q traveling at speed v in a circle of radius r gives us:

$$\mu = \frac{qvr}{2} = \frac{qL}{2m}$$

- * Where $L = mvr$ is the orbitable angular momentum

- Particles carry an intrinsic angular momentum, \vec{S} , called spin

$$\vec{\mu} = \frac{gq\vec{S}}{2m}$$

- * Where g is the gyroscopic ratio

- Noting that silver atoms were used is important, as different atoms give different results. Considering the shell filling of silver, we know that it extends to a singular atom in the $5s$ shell.

- * Since the mass of the nucleus $\geq 2000m_e$, we find a ratio of magnetic moments as:

$$\frac{\vec{\mu}_{nuc}}{\vec{\mu}_{e^-}} \ll 1$$

- * Hence, we have $\vec{\mu}_{Ag} = -g\frac{e}{2m_e}\vec{S}$, where e is the magnitude of an electron's charge

- * This produces a force of $F_z = -g\frac{e}{2m_e}S_z \cdot \frac{2B_z}{2z}$

- * Thus, the deflection of the beam in the Stern-Gerlach experiment is a measure of the projection of the intrinsic spin onto the z -axis

- * Note, the heat of the oven randomizes the direction of $\vec{\mu}$, and, classically, we have $S_z = |\vec{S}|\cos(\theta)$ and should be continuous over an S_z range given by: $-|\vec{S}| \leq S_z \leq |\vec{S}|$

- But, only two beams are observed!

- Only two S_z components are possible, since $S_z = \pm\hbar/2$

- We know:

$$\hbar \approx 1.0546 \cdot 10^{-34}[\text{J s}] = 6.5821 \cdot 10^{-16}[\text{eVs}]$$

$$\hbar = \frac{h}{2\pi} \text{ (Planck's Constant)}$$

- * This indicates a quantization of an electron's intrinsic angular momentum along the z axis
- * In the case that $S_z = +\hbar/2$, we call this spin “up”
- * In the case that $S_z = -\hbar/2$, we call this spin “down”
- * The quantity S_z itself is called an “observable”
- * The device depicted within the Stern-Gerlach experiment is called an “analyzer”, since it sorts the input into two possible outputs
- * New notation: $S_z = +\hbar/2$ can be written in Dirac notation as:

$$\left| +\frac{\hbar}{2} \right\rangle \Rightarrow |+\rangle$$

- * The negative (down) spin becomes:

$$\left| -\frac{\hbar}{2} \right\rangle \Rightarrow |-\rangle$$

- * A “ket” can be depicted as follows:

$$|\cdot\rangle$$

- Or, in its general form:

$$|\psi\rangle$$

- * Note that for up and down spin, occasionally one may encounter:

$$|+\rangle = |\uparrow\rangle$$

$$|-\rangle = |\downarrow\rangle$$

– Experiment 1:

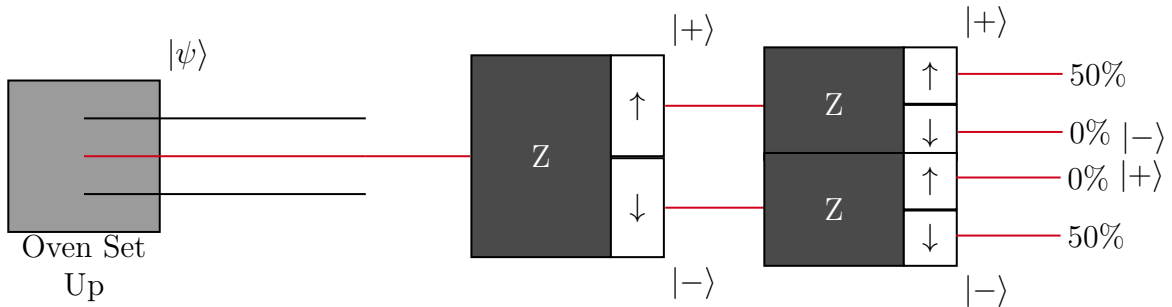


Figure 2: Experimental Setup

- * Although both Stern-Gerlach analyzers in the experiment are the same, they play different roles
 - The first analyzer prepares the beam and the second analyzer measures the beam
 - The first analyzer is often referred to as a state preparation device
- Experiment 2:

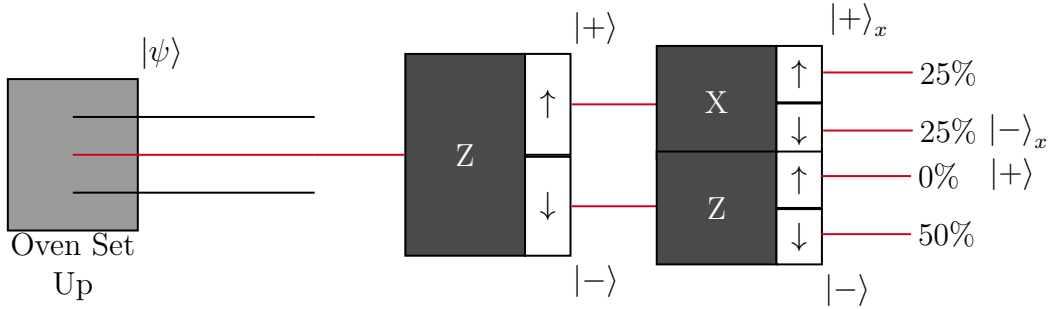


Figure 3: Experimental Setup

- * The second analyzer is rotated by 90° with respect to the first and aligned with the x -axis
 - Only two possible outputs from second analyzer
 - Results would be unchanged if we used the lower part of the first analyzer
 - One can not predict which of the second analyzer parts any particular atom will emerge from
 - These results highlight the probabilistic nature of quantum mechanics; quantum mechanics is a complete description of reality (as far as we know)
- Experiment 3:

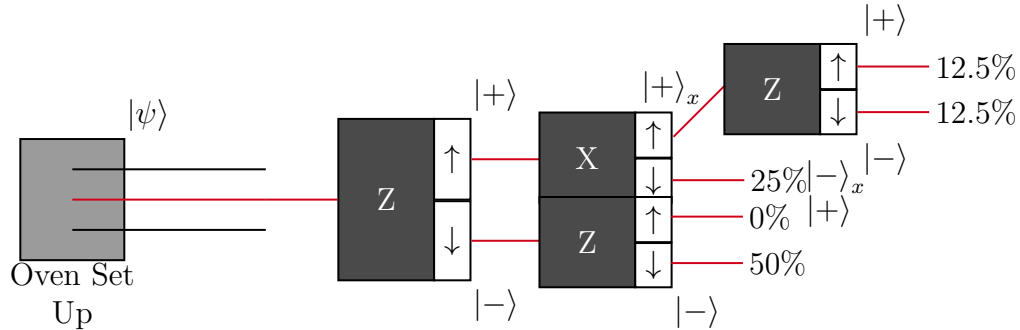


Figure 4: Experimental Setup

* This tells us that the S_x and S_z are incompatible observables, which means we can not know the values of both simultaneously

– Experiment 4:

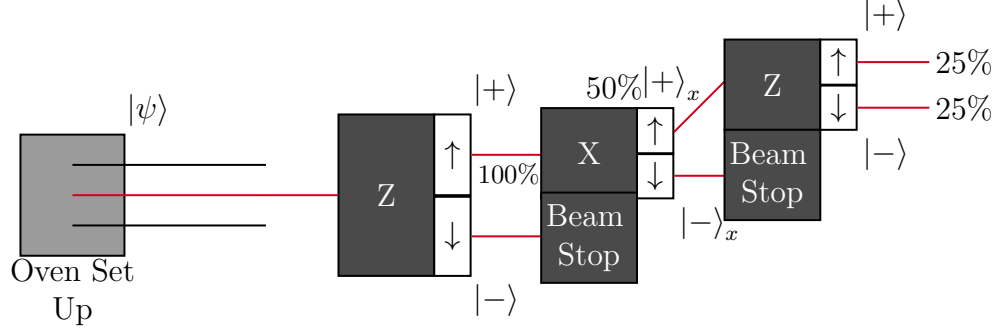


Figure 5: Experimental Setup Part One

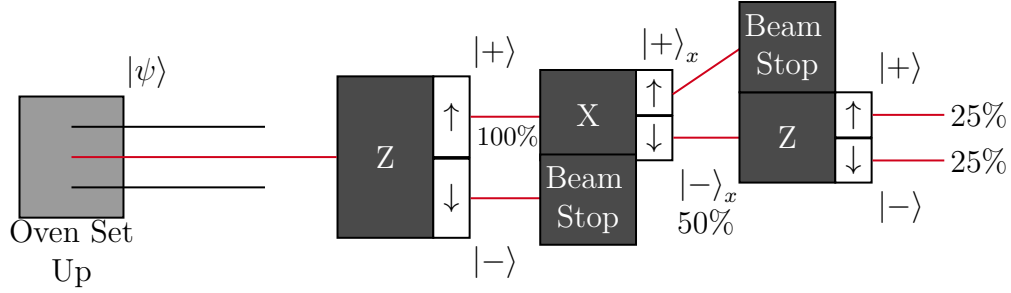


Figure 6: Experimental Setup Part Two

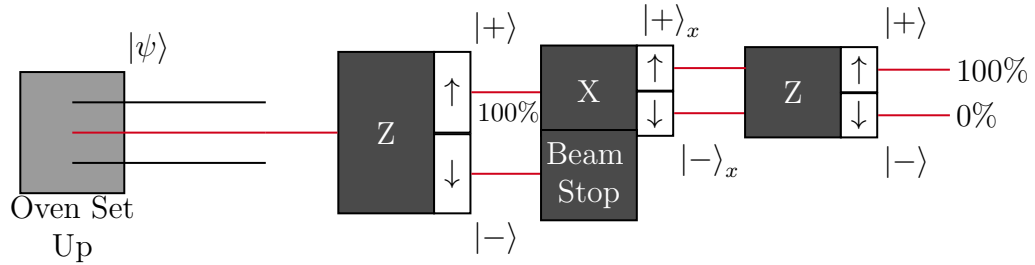


Figure 7: Experimental Setup Part Three

* Results are akin to the double slit experiment and destructive interference