Lecture 1

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- Key Features of Quantum Mechanics
  1. Probabilistic outcome of measurements
     - Compute probabilities exactly, and that is the most complete information possible
  2. Dual wave-particle nature of mature
     - Which one we observe depends on the experiment performed
  3. Conjugate variables (from classical mechanics) develop "uncertainty" relations
     - Wave theory relation:

$$\Delta x \Delta p \geq \hbar$$
$$\Delta E \Delta t \geq \hbar$$

     - Classical mechanics is "contained" in Quantum mechanics, which includes classical electricity and magnetism
  4. Every particle and object built from particles, including light, falls into one of two classes:
     - Fermions (spin is any odd multiple of $\hbar/2$)
       * Electrons are an example
     - Bosons (spin is an integer multiple of $\hbar$, including 0)
       * Photons are an example
  5. The properties of Quantum mechanics not falling into features 1-4 are largely familiar from classical physics
- Similar but Different
  - Every electron has exactly the same spin, charge, and mass
  - Every photon has exactly the same spin, charge, and mass

- This indicates no inequality among particles of the same type

- Time-Evolution in Quantum Mechanics
  - Time-evolution is given by the Hamiltonian, $\mathcal{H}$
  - Hamilton's Equations give us:

$$\frac{d\vec{q}}{dt} = \frac{\partial \mathcal{H}}{\partial \vec{p}}$$

  - We may write the Hamiltonian as:

$$\mathcal{H} = \frac{\vec{p}^2}{2m} \longrightarrow \frac{\partial \mathcal{H}}{\partial \vec{p}} = \frac{\vec{p}}{m} = \vec{v}$$

  - Alternatively, we may write the force as:

$$\vec{F} = \frac{d\vec{p}}{dt} = -\frac{\partial \mathcal{H}}{\partial \vec{q}}$$

- Stern-Gerlach Experiments
  - Took place in 1922 with Otto Stern and Walther Gerlach
  - The act of observing a quantum particle affects its measurable properties in a way foreign to our experience
  - The experiment looks like this:

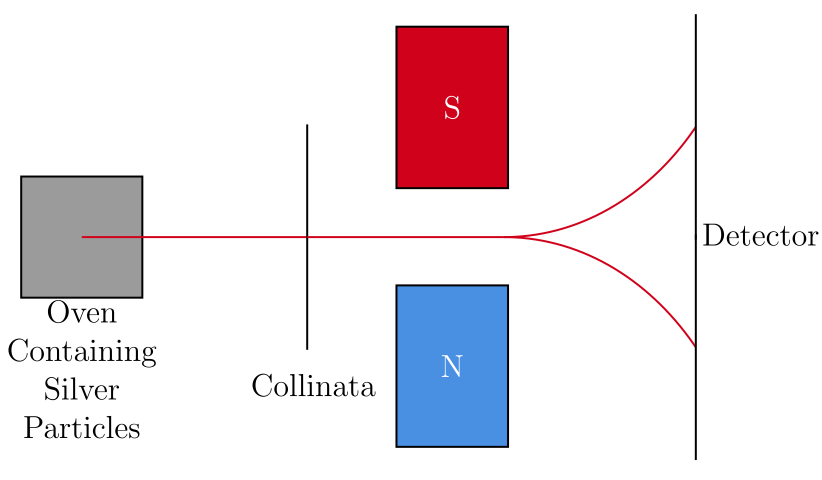

Figure 1: Set Up of Stern-Gerlach Experiment

- Assuming an atom has a monopole moment, $\vec{\mu}$, the potential energy of the interaction with a magnetic field $\vec{B}$ is $E = \vec{\mu}\vec{B}$
- Consider a classical description of the atom's moment:

$$\mu = IA$$

  * Where I is the electrical current and A is the area of the loop
- A particle of charge q traveling at speed v in a circle of radius r gives us:

$$\mu = \frac{qvr}{2} = \frac{qL}{2m}$$

  * Where $L = mvr$ is the orbitable angular momentum
- Particles carry an intrinsic angular momentum, $\vec{S}$, called spin

$$\vec{\mu} = \frac{gq\vec{S}}{2m}$$

  * Where g is the gyroscopic ratio
- Noting that silver atoms were used is important, as different atoms give different results. Considering the shell filling of silver, we know that it extends to a singular atom in the $5s$ shell.
  * Since the mass of the nucleus $\geq 2000m_e$, we find a ratio of magnetic moments as:

$$\frac{\vec{\mu}_{nuc}}{\vec{\mu}_{e^-}} << 1$$

  * Hence, we have $\vec{\mu}_{\text{Ag}} = -g\frac{e}{2m_e}\vec{S}$, where e is the magnitude of an electron's charge
  * This produces a force of $F_z = -g\frac{e}{2m_e}S_z \cdot \frac{2B_z}{2z}$
  * Thus, the deflection of the beam in the Stern-Gerlach experiment is a measure of the projection of the intrinsic spin onto the z-axis
  * Note, the heat of the oven randomizes the direction of $\vec{\mu}$, and, classicly, we have $S_z = |\vec{S}|\cos(\theta)$ and should be continuous over an S_z range given by: $-|\vec{S}| \leq S_z \leq |\vec{S}|$
    · But, only two beams are observed!
    · Only two S_z components are possible, since $S_z = \pm\hbar/2$
- We know:

$$\hbar \approx= 1.0546 \cdot 10^{-34}[\text{J s}] = 6.5821 \cdot 10^{-16}[\text{eVs}]$$

$$\hbar = \frac{h}{2\pi} \text{ (Planck's Constant)}$$