

Lecture 2 — Random Variables

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- A random variable is a function that maps the outcomes of a random experiment into a set of real numbers
- The Probability Mass Function (PMF)
 - The PMF is a probability measure that gives us probabilities of the possible values for a random variable
 - The PMF may be defined as:

$$P_x(x) = \begin{cases} P(X = x), & \text{if } x \in S_x \\ 0, & \text{Otherwise} \end{cases}$$

- The probability mass function can be obtained using the probabilities of the corresponding sample space outcomes
- The Bernoulli Random Variable
 - We may write this as $X = \text{Bernoulli}(p)$
 - This can be expressed as:

$$P_x(x) = \begin{cases} 1 - p, & x = 0 \\ p, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

- Expectation Value: $E[X] = p$
 - $\text{Var}[X] = p(1 - p)$
- The Binomial Random Variable
 - We may write this as $X = \text{Binomial}(n, p)$

- This can be expressed as:

$$P_x(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$$

- Expectation Value: $E[X] = np$
- $\text{Var}[X] = np(1-p)$

- The Geometric Random Variable

- We may write this as: $X = \text{Geometric}(p)$
- This can be expressed as:

$$P_x(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, 3 \dots \\ 0, & \text{otherwise} \end{cases}$$

- Expectation Value: $E[X] = \frac{1}{p}$
- $\text{Var}[X] = \frac{1-p}{p^2}$

- The Poisson Random Variable

- We may write this as: $X = \text{Poisson}(a)$
- This can be expressed as:

$$P_K(k) = \begin{cases} \frac{a^k}{k!} e^{-a}, & k = 0, 1, 2, 3 \dots \\ 0, & \text{otherwise} \end{cases}$$

- The Poisson random variable can be described using the average arrival rate $a = \lambda T$
- Expectation Value: $E[X] = \alpha$
- $\text{Var}[X] = \alpha$

- The Pascal or Negative Binomial Random Variable

- We may write this as: $X = \text{Pascal}(k, p)$
- This can be expressed as:

$$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k}, & x = k, k+1, \dots \\ 0, & \text{otherwise} \end{cases}$$

- Expectation Value: $E[X] = \frac{k}{p}$
- $\text{Var}[X] = \frac{k(1-p)}{p^2}$

- The Discrete Uniform Random Variable

- We may write this as: $N = \text{Uniform}(k, l)$
- This can be expressed as:

$$P_N(n) = \begin{cases} \frac{1}{l-k-1}, & l-k = 2, 3, 4, \dots \\ 0, & \text{otherwise} \end{cases}$$

- Expectation Value: $E[X] = \frac{l+k}{2}$
- $\text{Var}[X] = \frac{l-k}{12}(l-k+2)$

- The Cumulative Distribution Function (CDF)

- The CDF of a discrete random variable is defined for all values of the random variable as:

$$F_x(x) = P[X \leq x] = \sum_{j: x_j \leq x} P_x(x_j)$$

- For any value x , $F_x(x)$ is the probability that the observed value of X will be at most x

- Variance

$$\text{Var}[X] = E[(X - \mu_x)^2]$$

- $\text{Var}[X_1 + X_2 + \dots + X_n] = \text{Var}[X_1] + \text{Var}[X_2] + \dots + \text{Var}[X_n]$

- The Exponential Random Variable

- We may write this as: $X = \text{exponential}(\lambda)$
- This can be expressed as:

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

- Expectation value: $\frac{1}{\lambda}$
- $\text{Var}[X] = \frac{1}{\lambda^2}$

- The Gaussian or Normal Random Variable

- $Z = \text{Normal}(\mu, \sigma)$

$$f_Z(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$