

Homework 3

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1. (a) We want to find the probability that both units failed. This can be expressed as event A AND event B , indicating failure of the processors. Per our formulas, we know:

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

Per the given information, we know that $P[A|B] = .3$ and $P[B] = .02$. Thus, we may calculate to get:

$$P[A \cap B] = .006$$

- (b) We may calculate the probability of no good units in five systems by simply raising part (a) to the power of 5 to get:

$$P_5[A \cap B] = 7.776 \cdot 10^{-12}$$

- (c) The probability that there is a working unit in a single computer can be given by the complement of no working units. Thus, we see:

$$P[(A \cap B)^c] = 1 - .006 = .994$$

We can then sum the possible combinations for the other two computers (at least one working or none working) to get:

$$P_3[(A \cap B)^c] = .994(.994^2 + (.994)(.006) + .006^2)$$

$$P_3[(A \cap B)^c] = .9881$$

2. (a) The probability that the system does not fail may be expressed as:

$$P[F^c] = (1 - p)^{20}$$

We can thus take the complement and say that failure will occur for:

$$P[F] = 1 - (1 - p)^{20}$$

From the problem, the system failure rate is 20%, which gives us:

$$1 - (1 - p)^{20} = .2$$

From here, we solve:

$$(1 - p)^{20} = .8$$

$$1 - p = \sqrt[20]{.8}$$

$$p = 1 - \sqrt[20]{.8}$$

$$\boxed{p = .011095}$$

(b) Similar to part (a), we now solve for a failure rate of .1:

$$1 - (1 - p)^{20} = .1$$

$$(1 - p)^{20} = .9$$

$$p = 1 - \sqrt[20]{.9}$$

$$\boxed{p = .0052542}$$

4. (a) We can calculate this by multiplying the success rates together:

$$P_3 = (.9)^2(.8)$$

$$\boxed{P_3 = .648}$$

(b) We want to calculate the probability of at least one of the components succeeding. This may be expressed as:

$$P_{1+} = \underbrace{2[(.9)(.1)(.2)]}_{\text{One router}} + \overbrace{[(.1)^2(.8)]}^{\text{Only switch}} + \underbrace{[(.9)^2(.2)]}_{\text{Both routers}} + \overbrace{2[(.9)(.8)(.1)]}^{\text{One router/one switch}} + .648$$

$$\boxed{P_{1+} = .998}$$

(c) To find the odds of exactly one of the components passing, we simply subtract cases from above in which multiple devices pass. This gives us:

$$P_1 = .998 - 2[(.9)(.8)(.1)] - [(.9)^2(.2)] - .648$$

$$\boxed{P_1 = .044}$$

- (d) Since we know the probability that no components fail, we simply take the complement to find the probability that a component does fail:

$$P_3^c = 1 - .648$$

$$\boxed{P_3^c = .352}$$

- (e) Per our formulas, we can find $P[\text{all}|\text{one}]$ as:

$$P[\text{all}|\text{one}] = \frac{.648}{.998}$$

$$\boxed{P[\text{all}|\text{one}] = .6493}$$

5. (a) Given that n can be the range of $[2, 5]$, we know that the PMF must contain:

$$\left\{ \frac{c}{4}, \frac{c}{8}, \frac{c}{16}, \frac{c}{32} \right\}$$

We know that the probabilities within a PMF must sum to 1, so we get:

$$\frac{c}{4} + \frac{c}{8} + \frac{c}{16} + \frac{c}{32} = 1$$

We simplify to find:

$$8c + 4c + 2c + c = 32$$

$$\boxed{c = \frac{32}{17}}$$

- (b) We can take the complement of the probability that two packets are transmitted to find:

$$P[3+] = 1 - \frac{1}{4} \left(\frac{32}{17} \right)$$

$$\boxed{P[3+] = \frac{7}{17} = .4118}$$

- (c) We can find the probability of an odd packet to be:

$$P[3+ \cap \text{Odd}] = \left(\frac{1}{8} + \frac{1}{32} \right) \frac{32}{17}$$

$$P[3+ \cap \text{Odd}] = \frac{5}{17}$$

We then divide by the probability of three or more to find:

$$\boxed{P[\text{Odd}|3+] = \frac{5}{7} = .7143}$$

(d) We can express the probability for a given n as:

$$P[C] = \sum_{n=2}^5 P_N(n)(.95)^n$$

This gives us the following values:

n	$P_n[C]$
2	.4247
3	.2017
4	.095824
5	.045517
Total	.7677

6. (a) Based on the given information, we may express the probability of 2 or 3 servers as c . This gives us:

$$\left\{ \frac{c}{4}, \frac{c}{2}, c, c, \frac{c}{4} \right\}$$

We know the values need to sum to 1, which gives us:

$$\frac{2c}{4} + \frac{c}{2} + 2c = 1$$

$$\frac{c}{2} + \frac{c}{2} + 2c = 1$$

$$3c = 1$$

$$\boxed{c = \frac{1}{3}}$$

Thus, we get:

$$\left\{ \frac{1}{12}, \frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{12} \right\}$$

- (b) Based on the PMF above, we sum to get:

$$P[2+] = 1 - \frac{1}{12} - \frac{1}{6}$$

$$\boxed{P[2+] = .75}$$

7. We may express the PMF of A as a binomial distribution, since the pulled resistor is either A or not. This gives us $n = 10$ and a probability of $p = 1/3$, since each resistor is equally likely. This gives us the terms of the PMF as:

$$\binom{10}{n} (1/3)^p (2/3)^{p-n}$$

We iterate to get:

$$P_n(x) = \{.017342, .086708, .19509, .26012, .22761, .13656, .056902, \\ .016258, .0030483, .00033870, .000016935\}$$

From here, we find the probability of 2 or more A resistors as:

$$P[2+] = 1 - .017342 - .086708$$

$$\boxed{P[2+] = .896}$$

8. We begin by finding the probability that a generation-ending pair is combined. We may iterate through possible values of
 9. (a)
 - (b)
 - (c)
 - (d)
10. (a)
- (b)
- (c)