Homework 3

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January 28, 2025

1. (a) We want to find the probability that both units failed. This can be expressed as event A AND event B, indicating failure of the processors. Per our formulas, we know:

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

Per the given information, we know that $P[A|B] = .3$ and $P[B] = .02$. Thus, we may calculate to get:

$$\boxed{P[A \cap B] = .006}$$

(b) We may calculate the probability of no good units in five systems by simply raising part (a) to the power of 5 to get:

$$\boxed{P_5[A \cap B] = 7.776 \cdot 10^{-12}}$$

(c) The probability that there is a working unit in a single computer can be given by the complement of no working units. Thus, we see:

$$P[(A \cap B)^c] = 1 - .006 = .994$$

We can then sum the possible combinations for the other two computers (at least one working or none working) to get:

$$P_3[(A \cap B)^c] = .994(.994^2 + (.994)(.006) + .006^2)$$
$$\boxed{P_3[(A \cap B)^c] = .9881}$$

2. (a) The probability that the system does not fail may be expressed as:

$$P[F^c] = (1 - p)^{20}$$

We can thus take the complement and say that failure will occur for:

$$P[F] = 1 - (1-p)^{20}$$

From the problem, the system failure rate is 20%, which gives us:

$$1 - (1-p)^{20} = .2$$

From here, we solve:

$$\begin{aligned} (1-p)^{20} &= .8 \\ 1-p &= \sqrt[20]{.8} \\ p &= 1 - \sqrt[20]{.8} \\ &\boxed{p = .011095} \end{aligned}$$

(b) Similar to part (a), we now solve for a failure rate of .1:

$$\begin{aligned} 1 - (1-p)^{20} &= .1 \\ (1-p)^{20} &= .9 \\ p &= 1 - \sqrt[20]{.9} \\ &\boxed{p = .0052542} \end{aligned}$$

4. (a) We can calculate this by multiplying the success rates together:

$$P_3 = (.9)^2(.8)$$
$$\boxed{P_3 = .648}$$

(b) We want to calculate the probability of at least one of the components succeeding. This may be expressed as:

$$P_{1+} = \underbrace{2[(.9)(.1)(.2)]}_{\text{One router}} + \overbrace{[(.1)^2(.8)]}^{\text{Only switch}} + \underbrace{[(.9)^2(.2)]}_{\text{Both routers}} + \overbrace{2[(.9)(.8)(.1)]}^{\text{One router/one switch}} + .648$$
$$\boxed{P_{1+} = .998}$$

(c) To find the odds of exactly one of the components passing, we simply subtract cases from above in which multiple devices pass. This gives us:

$$P_1 = .998 - 2[(.9)(.8)(.1)] - [(.9)^2(.2)] - .648$$
$$\boxed{P_1 = .044}$$

(d) Since we know the probability that no components fail, we simply take the complement to find the probability that a component does fail:

$$P_3^c = 1 - .648$$
$$\boxed{P_3^c = .352}$$

(e) Per our formulas, we can find $P[\text{all}|\text{one}]$ as:

$$P[\text{all}|\text{one}] = \frac{.648}{.998}$$
$$\boxed{P[\text{all}|\text{one}] = .6493}$$

5. (a)
   (b)
   (c)
   (d)
6. (a)
   (b)
7.
8.
9. (a)
   (b)
   (c)
   (d)
10. (a)
    (b)
    (c)