Homework 5

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February 13, 2025

1. (a) To be a valid CDF, we know that the terms continuously build until they sum to 1. In this case, all of the terms become 1 at $v = 10$. Thus, we can differentiate to find the PDF:

$$f_V(v) = \frac{d}{dv}[F_V(v)]$$
$$f_V(v) = 2c(v-2), 2 \leq v < 10$$

We then know:

$$\int_2^{10} 2c(v-2)\, dv = 1$$

We can solve to get:

$$2c\left[\frac{v^2}{2} - 2v\Big|_2^{10}\right] = 1$$
$$2c\left[(50-2) - (20-4)\right] = 1$$
$$64c = 1$$

Which finally gets us:

$$\boxed{c = 64}$$

(b) We can then find the probability that the response time is greater than 5[ms] as:

$$P(v > 5) = 1 - F_V(v)$$
$$P(v > 5) = 1 - \frac{1}{64}(5-2)^2$$
$$\boxed{P(v > 5) = \frac{55}{64}}$$

(c) We can then find the response time probability for between 5 and 8 milliseconds:

$$P(5 \le v < 8) = F_V(8) - F_V(5)$$
$$P(5 \le v < 8) = \frac{1}{64}[(8-2)^2 - (5-2)^2]$$
$$\boxed{P(5 \le v < 8) = \frac{27}{64}}$$

(d) We can find this to be:

$$P(v > 7 | 5 \le v \le 8) = \frac{P(7 < v \le 8)}{P(5 \le v \le 8)}$$

We find the probability of the numerator:

$$P(7 < v \le 8) = F_V(8) - F_V(7)$$
$$P(7 < v \le 8) = \frac{1}{64}[(8-2)^2 - (7-2)^2]$$
$$P(7 < v \le 8) = \frac{11}{64}$$

This gives us:

$$P(v > 7 | 5 \le v \le 8) = \frac{11/64}{27/64}$$
$$\boxed{P(v > 7 | 5 \le v \le 8) = \frac{11}{27}}$$

(e) To find the applicable value, we may write:

$$1 - F_V(a) = .36$$

We expand this to write:

$$1 - \frac{1}{64}(a-2)^2 = .36$$

We then solve:

$$a = \sqrt{64(.64)} + 2$$
$$a = \pm 6.4 + 2$$

Since the time has to be positive, we find:

$$\boxed{a = 8.4[\text{ms}]}$$

2. (a) To be a valid PDF, we know:

$$\int_{-\infty}^{\infty} ae^{-.2|x|}\,dx = 1$$

We expand:

$$\int_{-\infty}^{0} ae^{.2x}\,dx + \int_{0}^{\infty} ae^{-.2x}\,dx = 1$$

This gives us:

$$\frac{ae^{.2x}}{.2}\Big|_{-\infty}^{0} - \frac{ae^{-.2x}}{.2}\Big|_{0}^{\infty} = 1$$

We continue to solve:

$$(5a - 0) - (0 - 5a) = 1$$
$$10a = 1$$
$$\boxed{a = .1}$$

(b) We know that the expectation value can be expressed as:

$$E[x] = \frac{1}{\lambda}$$

Where λ is the coefficient of the exponent. Thus, we get:

$$E[x] = \frac{1}{.2}$$
$$\boxed{E[x] = 5}$$

(c)

(d)

(e)

3. (a)

(b)

(c)

(d)

(e)

4. (a)

(b)

(c)

(d)

(e)

6. (a)

   (b)

   (c) i.

       ii.

       iii.

       iv.

8. (a)

   (b)

   (c)

   (d)

9. (a)

   (b)

   (c)

   (d)

10. (a)

    (b)

    (c)

11.