Homework 4

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1. (a) Given the Poisson distribution with given mean of $\alpha = .1$ interruptions per minute gives us a PMF of:

$$\boxed{P_\alpha(n) = \frac{(.1)^n e^{-.1}}{n!}}$$

(b) The expectation value is simply given as the average value, such that:

$$\boxed{E[X] = .1}$$

Furthermore, we know that the same value represents the standard deviation. Since we know that the standard deviation is the square root of the variance we get:

$$\sigma = \sqrt{.1}$$
$$\boxed{\sigma = .3162}$$

(c) Given a 10 minute period, we may take $\alpha \to 10(.1) \to 1$. This gives us:

$$P_{10}(n) = \frac{(1)^n e^{-1}}{n!}$$

We want the probability of no events, so we may take $n \to 0$ to get:

$$P_{10}(0) = \frac{(1)^0 e^{-1}}{0!}$$
$$\boxed{P_{10}(0) = .3679}$$

(d) Given a 20 minute period, we may take $\alpha \to 20(.1) \to 2$. This gives us:

$$P_{20}(n) = \frac{(2)^n e^{-2}}{n!}$$

We want the probability of two or more events, so we may take the complement of 2, 1 or no events. This gives us:

$$P_{20}(> 2) = 1 - \left(\frac{(2)^0 e^{-2}}{0!} + \frac{(2)^1 e^{-2}}{1!} + \frac{(2)^2 e^{-2}}{2!} \right)$$

(e)

(f)

2. (a)

   (b)

   (c)

3. First and foremost, we see that this CDF is valid, since the probabilities add up to 100%, or 1.

   (a) We can find the first value as:

$$P[Y < 3] = F[2]$$

$$\boxed{P[Y < 3] = .25}$$

   We can then find:

$$P[Y \leq 3] = F[3]$$

$$\boxed{P[Y \leq 3] = .5}$$

   (b) We can see that:

$$P[Y < 4] = P[Y \leq 3] = F[3]$$

$$\boxed{P[Y < 4] = .5}$$

   From here, we may find:

$$P[Y \geq 4] = 1 - P[Y < 4]$$

$$\boxed{P[Y \geq 4] = .5}$$

   (c) Since there is no "bump" up at $y = 2$, we may find:

$$\boxed{P[Y = 2] = F[2] - F[1] = 0}$$

   Similarly, we may find:

$$\boxed{P[1 \leq Y < 3] = F[2] - F[1] = 0}$$

(d) We know that the PMF may be expressed as:

$$PMF(Y) = F[Y] - F[Y-1]$$

Using this, we construct:

$$\boxed{PMF(Y) = \begin{cases} .25, & y = 1 \\ .25, & y = 3 \\ .5, & y = 4 \\ 0, & \text{otherwise} \end{cases}}$$

4. (a) We may observe that this is a geometric distribution, which means that the expectation may be written as:

$$E[K] = \frac{1}{p} = \frac{1}{.05}$$

$$\boxed{E[K] = 20}$$

From here, we may find the variance:

$$\text{Var}[K] = \frac{1-p}{p^2} = \frac{.95}{.05^2}$$

$$\boxed{\text{Var}[K] = 380}$$

And finally, we use this to find the standard deviation:

$$\sigma_K = \sqrt{\text{Var}[K]} = \sqrt{380}$$

$$\boxed{\sigma_K = 19.494}$$

(b) We may write the CDF using a sum and the formula for a geometric distribution:

$$\boxed{\text{CDF}_K[n] = \sum_{n=1}^{n_K} (.05)(.95)^{n-1}}$$

Where n represents the attempt number and n_K is the number of attempts until an error is encountered.

(c) Let us find the probability of observing the expectation value:

$$P[K = E[K]] = (.05)(.95)^{19}$$

This gives us:

$$\boxed{P[K = 20] = .018868}$$

(d) We can find the probability that more attempts than expected are taken as:

$$P[K > 20] = 1 - \sum_{n=1}^{20} (.05)(.95)^{n-1}$$

This gives us:

$$\boxed{P[K > 20] = .3585}$$

(e) On the other hand, we may find the probability that less attempts than expected are needed:

$$P[K < 20] = 1 - P[K > 20] - P[K = 20]$$
$$P[K < 20] = 1 - .3585 - .018868$$
$$\boxed{P[K < 20] = .6226}$$

(f) Using the standard deviation we obtained above, we may round the difference (or sum) of it and the expectation value to get::

$$P[(20 - 19.494) \leq K \leq (20 + 19.494)] = P[1 \leq K \leq 39]$$

From here, we may use our formula to write:

$$P[1 \leq K \leq 39] = \sum_{1}^{39} (.05)(.95)^{n-1}$$

Plugging this in to a solver, we get:

$$\boxed{P[1 \leq K \leq 39] = .8647}$$

(g) By Bayes' rule, we know that, since the two events are independent, the probability that there is success $\rightarrow$ success $\rightarrow$ failure is simply the probability given by two successes and a failure. We can then find the probability that the next three events are two successes and one failure to get:

$$\boxed{P[2, 1] = (.95)^2(.05) = .045125}$$

(h) Since we know that an error is expected to occur after twenty attempts, we would expected for there to be another error twenty attempts after an initial one. As such, with a 2-second request time, we may simply write the time between errors as:

$$\boxed{t = 20(2) = 40[\mathrm{s}]}$$

5. (a)

(b)

(c)

6. (a)

   (b)

   (c)

7. (a)

   (b)

   (c)

   (d)

   (e)

   (f)

8.

10. (a)

    (b)