Homework 4

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1. (a) Given the Poisson distribution with given mean of $\alpha = .1$ interruptions per minute gives us a PMF of:

$$\boxed{P_\alpha(n) = \frac{(.1)^n e^{-.1}}{n!}}$$

   (b) The expectation value is simply given as the average value, such that:

$$\boxed{E[X] = .1}$$

   Furthermore, we know that the same value represents the standard deviation. Since we know that the standard deviation is the square root of the variance we get:

$$\sigma = \sqrt{.1}$$
$$\boxed{\sigma = .3162}$$

   (c) Given a 10 minute period, we may take $\alpha \to 10(.1) \to 1$. This gives us:

$$P_{10}(n) = \frac{(1)^n e^{-1}}{n!}$$

   We want the probability of no events, so we may take $n \to 0$ to get:

$$P_{10}(0) = \frac{(1)^0 e^{-1}}{0!}$$
$$\boxed{P_{10}(0) = .3679}$$

   (d) Given a 20 minute period, we may take $\alpha \to 20(.1) \to 2$. This gives us:

$$P_{20}(n) = \frac{(2)^n e^{-2}}{n!}$$

We want the probability of two or more events, so we may take the complement of 2, 1 or no events. This gives us:

$$P_{20}(>2) = 1 - \left(\frac{(2)^0 e^{-2}}{0!} + \frac{(2)^1 e^{-2}}{1!} + \frac{(2)^2 e^{-2}}{2!}\right)$$

(e)

(f)

2. (a)

(b)

(c)

3. First and foremost, we see that this CDF is valid, since the probabilities add up to 100%, or 1.

(a) We can find the first value as:

$$P[Y < 3] = F[2]$$

$$\boxed{P[Y < 3] = .25}$$

We can then find:

$$P[Y \leq 3] = F[3]$$

$$\boxed{P[Y \leq 3] = .5}$$

(b) We can see that:

$$P[Y < 4] = P[Y \leq 3] = F[3]$$

$$\boxed{P[Y < 4] = .5}$$

From here, we may find:

$$P[Y \geq 4] = 1 - P[Y < 4]$$

$$\boxed{P[Y \geq 4] = .5}$$

(c) Since there is no "bump" up at $y = 2$, we may find:

$$\boxed{P[Y = 2] = F[2] - F[1] = 0}$$

Similarly, we may find:

$$\boxed{P[1 \leq Y < 3] = F[2] - F[1] = 0}$$

(d) We know that the PMF may be expressed as:

$$PMF(Y) = F[Y] - F[Y-1]$$

Using this, we construct:

$$\boxed{PMF(Y) = \begin{cases} .25, & y = 1 \\ .25, & y = 3 \\ .5, & y = 4 \\ 0, & \text{otherwise} \end{cases}}$$

4. (a) We may observe that this is a geometric distribution, which means that the expectation may be written as:

$$E[K] = \frac{1}{p} = \frac{1}{.05}$$

$$\boxed{E[K] = 20}$$

From here, we may find the variance:

$$\text{Var}[K] = \frac{1-p}{p^2} = \frac{.95}{.05^2}$$

$$\boxed{\text{Var}[K] = 380}$$

And finally, we use this to find the standard deviation:

$$\sigma_K = \sqrt{\text{Var}[K]} = \sqrt{380}$$

$$\boxed{\sigma_K = 19.494}$$

(b) We may write the CDF using a sum and the formula for a geometric distribution:

$$\boxed{\text{CDF}_K[n] = \sum_{n=1}^{n_K} (.05)(.95)^n}$$

Where n represents the attempt number and n_K is the number of attempts until an error is encountered.

(c) Let us find the probability of observing the expectation value:

$$P[K = E[K]] = (.05)(.95)^{20}$$

This gives us:

$$\boxed{P[K = 20] = .017924}$$

(d) We can find the probability that more attempts than expected are taken as:

$$P[K > 20] = 1 - \sum_{n=1}^{20} (.05)(.95)^n$$

This gives us:

$$\boxed{P[K > 20] = .3906}$$

(e) On the other hand, we may find the probability that less attempts than expected are needed:

$$P[K < 20] = 1 - P[K > 20] - P[K = 20]$$
$$P[K < 20] = 1 - .3906 - .017924$$
$$\boxed{P[K < 20] = .5915}$$

(f)

(g)

(h)

5. (a)

(b)

(c)

6. (a)

(b)

(c)

7. (a)

(b)

(c)

(d)

(e)

(f)

8.

10. (a)

(b)