Homework 6

Michael Brodskiy
Professor: I. Salama

February 22, 2025

1. We begin by sketching the CDF:

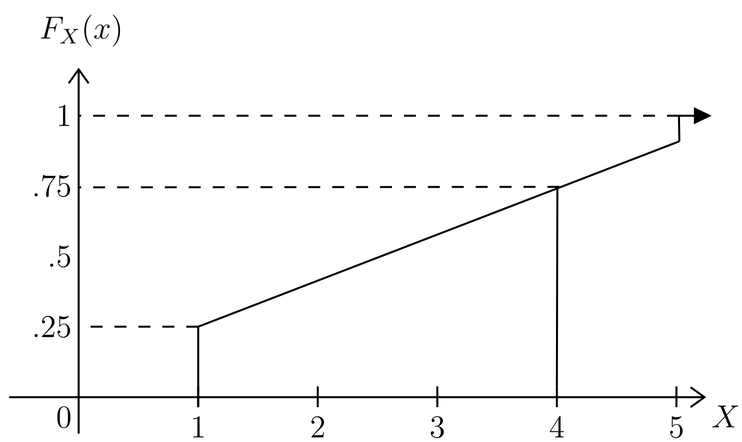

Figure 1: CDF of Given Function

   (a) From the CDF, we may conclude:

$$P[X < 1] = F_X(X^- = 1) - F_X(X = 0)$$
$$\boxed{P[X < 1] = 0}$$

   And then:

$$P[X \leq 1] = F_X[X = 1] - F_X[X = 0]$$
$$P[X \leq 1] = .25 - 0$$
$$\boxed{P[X \leq 1] = .25}$$

(b) Since the given probability occurs in the continuous portion, we can find that:

$$P[X < 2] = P[X \leq 2]$$

Furthermore, we can write:

$$P[X \leq 2] = F_X[X = 2] - F_X[X = 0]$$
$$P[X \leq 2] = \frac{25}{60} - 0$$
$$\boxed{P[X \leq 2] = P[X < 2] = .416\bar{6}}$$

(c) We continue to find:

$$P[X > 5] = 1 - F_X(X^- = 5)$$
$$P[X > 5] = 1 - .916\bar{6}$$
$$\boxed{P[X > 5] = .083\bar{3}}$$

Since the right side is not in the continuous portion, we get:

$$P[X \geq 5] = 1 - F_X(X^+ = 5)$$
$$P[X \geq 5] = 1 - 1$$
$$\boxed{P[X \geq 5] = 0}$$

(d) With the given range, we may find:

$$P[1 < X < 2] = F_X(X^- = 2) - F_X(X^+ = 1)$$
$$P[1 < X < 2] = .416\bar{6} - .25$$
$$\boxed{P[1 < X < 2] = .16\bar{6}}$$

2. (a) We may begin by finding a PDF for the continuous region:

$$f_{W,2}(w) = (1 - f_{W,1})\lambda e^{-\lambda w}$$

Furthermore, we know that:

$$f_{W,1}(w) = .3$$

Given the expectation value given successful communications, we may know:

$$E[W] = \frac{1}{\lambda}$$

This gives us:

$$\lambda = \frac{1}{3}[\text{min}]$$

Thus, the pdf becomes:

$$\boxed{f_W(w) = \begin{cases} \dfrac{3}{10}, & w = 0 \\ \\ \dfrac{7}{30}e^{-\frac{1}{3}w} & w > 0 \end{cases}}$$

(b) We may find the CDF by integrating to get:

$$F_W(w) = \begin{cases} \dfrac{3}{10}, & w = 0 \\ \\ 1 - .7e^{-\frac{1}{3}w} & w > 0 \end{cases}$$

We may note that this is continuous, so we may simply write:

$$\boxed{F_W(w) = 1 - .7e^{-\frac{1}{3}w} \quad w \geq 0}$$

(c) We can write the expectation value as:

$$E[W] = \int_0^\infty w f_W(w)\, dw$$

This gives us:

$$E[W] = .3(w \to 0) + \int_{0^+}^\infty \frac{7w}{30}e^{-\frac{1}{3}w}\, dw$$

We evaluate to get:

$$E[W] = .3(0) + 2.1$$

$$\boxed{E[W] = 2.1[\text{min}]}$$

We can then find the variance using the law of total variance as:

$$\text{Var}[W] = E[\text{Var}[W|S]] + \text{Var}[E[W|S]]$$

Where S corresponds to success/failure. Thus, we use the information given:

$$E[W|S] = 3 \quad \text{and} \quad \text{Var}[W|S] = \frac{1}{\lambda^2} = 9$$

From here, we can get:

$$\text{Var}[E[W|X]] = (3 - 2.1)^2 \cdot S \cdot S^c$$
$$\text{Var}[E[W|X]] = (3 - 2.1)^2 \cdot (.7) \cdot (.3)$$
$$\text{Var}[E[W|X]] = .1701$$

This gives us:

$$\text{Var}[W] = (9)(.7) + .1701$$
$$\boxed{\text{Var}[W] = 6.4701[\text{min}^2]}$$

(d) To define this probability, we simply use the CDF to write:

$$P[W < 1] = F_W(w = 1^-)$$

We can find this using:

$$P[W < 1] = 1 - .7e^{-1/3}$$
$$\boxed{P[W < 1] = .4984}$$

3. (a)
   (b)
   (c)
   (d)
4. (a)
   (b)
   (c)
   (d)
   (e)
   (f)
   (g)
5. (a)
   (b)
   (c)
   (d)
   (e)
6. (a)
   (b)

(c)

(d)

7. (a)

(b)

(c)