Lecture 2 — Random Variables

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- A random variable is a function that maps the outcomes of a random experiment into a set of real numbers
- The Probability Mass Function (PMF)
  - The PMF is a probability measure that gives us probabilities of the possible values for a random variable
  - The PMF may be defined as:

$$P_x(x) = \begin{cases} P(X = x), & \text{if } x \in S_x \\ 0, & \text{Otherwise} \end{cases}$$

  - The probability mass function can be obtained using the probabilities of the corresponding sample space outcomes
- The Bernoulli Random Variable
  - We may write this as $X = \text{Bernoulli}(p)$
  - This can be expressed as:

$$P_x(x) = \begin{cases} 1-p, & x = 0 \\ p, & x = 1 \\ 0, & \text{otherwise} \end{cases}$$

- The Binomial Random Variable
  - We may write this as $X = \text{Binomial}(n, p)$
  - This can be expressed as:

$$P_x(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & 0 \leq k \leq n \\ 0, & \text{otherwise} \end{cases}$$

- The Geometric Random Variable
  - We may write this as: $X = \text{Geometric}(p)$
  - This can be expressed as:

$$P_x(x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, 3 \cdots \\ 0, & \text{otherwise} \end{cases}$$

- The Poisson Random Variable
  - We may write this as: $X = \text{Poisson}(a)$
  - This can be expressed as:

$$P_K(k) = \begin{cases} \frac{a^k}{k!} e^{-a}, & k = 0, 1, 2, 3 \cdots \\ 0, & \text{otherwise} \end{cases}$$

  - The Poisson random variable can be described using the average arrival rate $a = \lambda T$

- The Pascal or Negative Binomial Random Variable
  - We may write this as: $X = \text{Pascal}(k, p)$
  - This can be expressed as:

$$P_X(x) = \begin{cases} \binom{x-1}{k-1} p^k (1-p)^{x-k}, & x = k, k+1, \cdots \\ 0, & \text{otherwise} \end{cases}$$

- The Discrete Uniform Random Variable
  - We may write this as: $N = \text{Uniform}(k, l)$
  - This can be expressed as:

$$P_N(n) = \begin{cases} \frac{1}{l-k-1}, & l-k = 2, 3, 4, \cdots \\ 0, & \text{otherwise} \end{cases}$$