Final

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1. (a) First, we may write the autocorrelation function as:

$$R_{XX}[n,k] = E[X_n X_{n+k}]$$

Given their independence, we may write:

$$R_{XX}[n,k] = E[X_n]E[X_{n+k}]$$

We are given the expectation value of X, which allows use to write:

$$R_{XX}[n,k] = (\mu_x)^2$$
$$R_{XX}[n,k] = (2)^2$$
$$\boxed{R_{XX}[n,k] = 4}$$

The autocovariance may be written as:

$$C_{XX}[n,k] = R_{XX}[n,k] - E[X_n]E[X_{n+k}]$$
$$\boxed{C_{XX}[n,k] = 0, \quad k \neq 0}$$

Since the functions are i.i.d., we find that the autocovariance is zero, as the autocorrelation is equivalent to the product of the means.

(b) We may express the expected power as:

$$E[X_n^2] = \text{Var}(X_n) + (E[X_n])^2$$

We may calculate the variance for an exponential distribution as:

$$\text{Var}(\text{Exp}(\mu = 2)) = (\mu)^2$$
$$\text{Var}(X_n) = 4$$

As such, we get:

$$E[X_n^2] = 4 + 4$$
$$\boxed{E[X_n^2] = 8}$$

(c) We can express the expected value of the output sequence as:

$$E[Y_n] = \frac{1}{2}(E[X_n] - E[X_{n-2}])$$

Since each X_n has the same mean, we find:

$$\boxed{E[Y_n] = 0}$$

(d) We may rewrite the expression as:

$$R_{YY}[n,k] = E[Y_n Y_{n+k}]$$
$$R_{YY}[n,k] = \frac{1}{4}E[(X_n - X_{n-2})(X_{n+k} - X_{n+k-2})]$$

Since the mean for every X is the same, we can drop the subscripts to get:

$$R_{YY}[n,k] = \frac{1}{4}E[(X - X)(X - X)]$$
$$R_{YY}[n,k] = \frac{1}{4}E[(0)(0)]$$
$$\boxed{R_{YY}[n,k] = 0}$$

The autocovariance may be written as:

$$C_{YY}[n,k] = R_{YY}[n,k] - E[Y_n Y_{n+k}]$$

We may observe that both of the terms defined above are equivalent to zero, which leads us to conclude:

$$\boxed{C_{YY}[n,k] = 0}$$

(e) The sequence Y_n <u>is wide-sense stationary</u>, since its mean is constant for all indices and the autocorrelation is dependent solely on the time shift ($R_{YY}[n,k] = R_{YY}[n-k]$)

(f) The <u>components of Y_n are uncorrelated</u>. We may conclude this since, for the entire sequence, $R_{YY}[n,k] = 0$.

(g) i.

ii.

iii.

2. (a) First and foremost, we know that, for a Gaussian distribution, we have:

$$\boxed{E[X(t)] = 0}$$

Now, we may find the power by using the autocorrelation and taking $\tau \to 0$:

$$E[X^2(t)] = 25e^{-20000\tau^2}$$
$$E[X^2(t)] = 25e^{-20000(0)^2}$$
$$\boxed{E[X^2(t)] = 25}$$

(b) We can find this probability by converting to a Z-score:

$$Z = \frac{X(1[\text{ms}])}{\sigma}$$

This gives us:

$$P\left[Z < \frac{2}{\sqrt{25}}\right] = P\left[\frac{X(1[\text{ms}])}{5} < .4\right]$$
$$\boxed{P\left[Z < .4\right] = .6554}$$

(c) We may begin by expanding

(d)

(e)

(f)

3. (a)

(b)

(c)

(d)