Homework 6

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2. (a) We can find the density of states $d\varepsilon$ as:

$$D(\varepsilon)\,d\varepsilon = \frac{V\varepsilon^2}{\pi^2\hbar^3c^3}\,d\varepsilon$$

We can then find:

$$N = \int_0^{\varepsilon_F} \frac{V\varepsilon^2}{\pi^2\hbar^3c^3}\,d\varepsilon$$
$$N = \frac{V\varepsilon_F^3}{3\pi^2\hbar^3c^3}$$
$$N(3\pi^2\hbar^3c^3) = V\varepsilon_F^3$$
$$\varepsilon_F^3 = \frac{N}{V}(3\pi^2\hbar^3c^3)$$
$$\varepsilon_F^3 = 3n(\pi^2\hbar^3c^3)$$
$$\varepsilon_F^3 = \frac{3n}{\pi}(\pi\hbar c)^3$$

And finally:

$$\boxed{\varepsilon_F = \left(\frac{3n}{\pi}\right)^{\frac{1}{3}}(\pi\hbar c)}$$

(b) The energy can be defined in a similar manner:

$$U = \int_0^{\varepsilon_F} \varepsilon D(\varepsilon)\,d\varepsilon$$

Which gives us:

$$U = \int_0^{\varepsilon_F} \frac{V\varepsilon^3}{\pi^2\hbar^3c^3}\,d\varepsilon$$

$$U = \frac{V\varepsilon_F^4}{4\pi^2\hbar^3c^3}$$

$$U = \left(\frac{V\varepsilon_F^3}{3\pi^2\hbar^3c^3}\right)\frac{3}{4}\varepsilon_F$$

And finally:

$$\boxed{U_o = \frac{3}{4}N\varepsilon_F}$$

3. (a) We know that the ground state energy may be expressed as:

$$U_o = \frac{3N\hbar^2}{10M}\left(\frac{3\pi^2N}{V}\right)^{\frac{2}{3}}$$

We also know the relation:

$$P = -\frac{\partial U}{\partial V}$$

This gives us:

$$P = -\left(-\frac{2}{3}\frac{3\hbar^2(3\pi^2)^{\frac{2}{3}}}{10M}\left(\frac{N}{V}\right)^{\frac{5}{3}}\right)$$

$$P = \frac{\hbar^2(3\pi^2)^{\frac{2}{3}}}{5M}\left(\frac{N}{V}\right)^{\frac{5}{3}}$$

$$\boxed{P = \frac{(3\pi^2)^{\frac{2}{3}}}{5}\frac{\hbar^2}{M}\left(\frac{N}{V}\right)^{\frac{5}{3}}}$$

(b) We can define:

$$\delta\sigma(\tau) = \int \frac{1}{\tau}\,dU$$

We know the heat capacity of an electron gas as:

$$C_V = \frac{1}{2}\pi^2N\left(\frac{\tau}{\tau_F}\right)$$

We know the heat capacity is equal to $\frac{\partial U}{\partial \tau}$, which allows us to write:

$$\sigma(\tau) = \int_0^\tau \frac{1}{\tau}C_v\,d\tau$$

$$\sigma(\tau) = \int_0^\tau \frac{1}{2}\pi^2N\frac{1}{\tau_F}\,d\tau$$

$$\boxed{\sigma(\tau) = \frac{1}{2}\pi^2 N \frac{\tau}{\tau_F}}$$

Notice: $\sigma = C_V$

5. (a)

   (b)

6. (a)

   (b)

   (c)

   (d)

   (e)

7.