Homework 7

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1. (a) First and foremost, we know:

$$Q_h = Q_l + W$$
$$W = Q_l - Q_h$$

Per reversible conditions:

$$\sigma_h = \sigma_l$$
$$\frac{Q_h}{\tau_h} = \frac{Q_l}{\tau_l}$$
$$Q_h = \frac{\tau_h Q_l}{\tau_l}$$

Thus, we can combine to write:

$$W = Q_h - \frac{\tau_l}{\tau_h} Q_h$$
$$W = Q_h \left(1 - \frac{\tau_l}{\tau_h}\right)$$
$$W = Q_h \left(\frac{\tau_h - \tau_l}{\tau_h}\right)$$

And finally:

$$\boxed{\frac{W}{Q_h} = \frac{\tau_h - \tau_l}{\tau_h}}$$

If the heat pump is not reversible, we know the efficiency is:

$$\frac{W}{Q_h} < \frac{\tau_h - \tau_l}{\tau_h}$$

(b)

(c)

2. (a)

(b)

3. (a)

(b)

(c)

(d)

5. First and foremost, we may write:

$$W = (\tau_h - \tau_l)\sigma_l$$

Given the assumption that the process is reversible, we may write:

$$\sigma_l = \frac{Q_l}{\tau_l}$$

This gives us:

$$W = \left(\frac{\tau_h}{\tau_l} - 1\right) Q_l$$

We can apply the given parameters to find:

$$W = \left(\frac{500}{20} - 1\right) 1500$$

$$\boxed{W = 36[\text{GW}]}$$

Given improvements, we can write:

$$W = \left(\frac{600}{20} - 1\right) 1500$$

$$\boxed{W = 43.5[\text{GW}]}$$

Clearly, there is nearly a 20% increase in the ability to produce energy

6. (a)

(b)

7. We know from (1) that:

$$\frac{W}{Q_l} = \left(\frac{\tau_h}{\tau_l} - 1\right)$$

And also that:

$$W + Q_l = Q_h$$

We can express the work done by the light bulb as $W = Q_l$. The total work of the two processes needs to equal zero, which allows us to write:

$$\left(\frac{\tau_h}{\tau_l} = 1\right) Q_l - Q_l = 0$$

This simplifies to:

$$\tau_h = 2\tau_l$$