

Fermi and Bose Gases

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- Fermi and Bose Gases

- Some important quantities we have already discussed are:

$$n_Q = \left(\frac{M\tau}{2\pi\hbar^2} \right)^{\frac{3}{2}}$$

- The quantum concentration

$$f(\varepsilon, \mu, \tau) = \frac{1}{e^{\frac{\varepsilon-\mu}{\tau}} \pm 1}$$

- Occupancy of orbitals (plus one for Fermi-Bose, minus one for Bose-Einstein)
- The following relations are important:

$$\begin{array}{lll} n \ll n_Q & \text{Classical Regime} & \tau \gg \tau_o \\ n = n_Q & \text{Quantum Gas (regime)} & \tau = \tau_o \\ n \gg n_Q & \text{Degenerate Gas (regime)} & \tau \ll \tau_o \end{array}$$

- We may write:

$$\tau_o = \frac{2\pi\hbar^2}{M} n^{\frac{2}{3}}$$

- Classical Gas

- We know the following for a classical gas:

$$\mu = \tau \ln \left(\frac{n}{n_Q} \right)$$

$$U = \frac{3}{2} N \tau$$

$$\sigma = N \left[\ln \left(\frac{n_Q}{n} \right) + \frac{5}{2} \right]$$

$$C_V = \frac{3}{2} N$$

- Energy Density

- The density of states (orbitals) may be written as:

$$D(\varepsilon) = \frac{V}{2\pi^2} \left(\frac{2M}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{\varepsilon}$$

- $D(\varepsilon) d\varepsilon$ is the # of orbitals with energies in range $[\varepsilon, \varepsilon + d\varepsilon]$
- Some quantities that may be derived from this include:

$$N = \int_0^\infty d\varepsilon D(\varepsilon) f(\varepsilon, \mu, \tau)$$

$$U = \int_0^\infty d\varepsilon D(\varepsilon) f(\varepsilon, \mu, \tau) \varepsilon$$

- Fermi Energy/Temperature may be defined as:

$$\varepsilon_F = (3\pi^2)^{\frac{2}{3}} \frac{\hbar^2}{2M} \left(\frac{N}{V} \right)^{\frac{2}{3}} \equiv \tau_F$$

- Degenerate Gas ($\tau \ll \tau_o$)

- The chemical potential is:

$$\mu = \varepsilon_F \left(1 - \frac{\pi^2 \tau^2}{12 \varepsilon_F^2} \right)$$

- The energy is:

$$U = \frac{3}{5} N \varepsilon_F \left(1 + \frac{5\pi^2 \tau^2}{12 \varepsilon_F^2} \right)$$

- The specific heat is“

$$C_v = \frac{\pi^2 N \tau}{2 \tau_F}$$

- Entropy

$$\sigma = \frac{\pi^2 N \tau}{2 \varepsilon_F}$$

- For a Bose-Einstein gas, the values become slightly different:

- The density of states becomes:

$$D(\varepsilon) = \frac{V}{4\pi^2} \left(\frac{2M}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{\varepsilon}$$