Fermi and Bose Gases

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November 3, 2023

- Fermi and Bose Gases
  - Some important quantities we have already discussed are:

$$n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}}$$

  - The quantum concentration

$$f(\varepsilon,\mu,\tau) = \frac{1}{e^{\frac{\varepsilon-\mu}{\tau}} \pm 1}$$

  - Occupancy of orbitals (plus one for Fermi-Bose, minus one for Bose-Einstein)
  - The following relations are important:

$$\begin{gathered} n << n_Q \quad \text{Classical Regime} \quad \tau >> \tau_o \\ n = n_Q \quad \text{Quantum Gas (regime)} \quad \tau = \tau_o \\ n >> n_Q \quad \text{Degenerate Gas (regime)} \quad \tau << \tau_o \end{gathered}$$

  - We may write:

$$\tau_o = \frac{2\pi\hbar^2}{M} n^{\frac{2}{3}}$$

- Classical Gas
  - We know the following for a classical gas:

$$\mu = \tau \ln\left(\frac{n}{n_Q}\right)$$

$$U = \frac{3}{2}N\tau$$

$$\sigma = N\left[\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right]$$

$$C_V = \frac{3}{2}N$$

- Energy Density
  - The density of states (orbitals) may be written as:

$$D(\varepsilon) = \frac{V}{2\pi^2}\left(\frac{2M}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{\varepsilon}$$

  - $D(\varepsilon)\,d\varepsilon$ is the # of orbitals with energies in range $[\varepsilon, \varepsilon + d\varepsilon]$
  - Some quantities that may be derived from this include:

$$N = \int_0^\infty d\varepsilon D(\varepsilon) f(\varepsilon, \mu, \tau)$$
$$U = \int_0^\infty d\varepsilon D(\varepsilon) f(\varepsilon, \mu, \tau)\varepsilon$$

  - Fermi Energy/Temperature may be defined as:

$$\varepsilon_F = \left(3\pi^2\right)^{\frac{2}{3}} \frac{\hbar^2}{2M}\left(\frac{N}{V}\right)^{\frac{2}{3}} \equiv \tau_F$$

- Degenerate Gas ($\tau << \tau_o$)
  - The chemical potential is:

$$\mu = \varepsilon_F\left(1 - \frac{\pi^2\tau^2}{12\varepsilon_F^2}\right)$$

  - The energy is:

$$U = \frac{3}{5}N\varepsilon_F\left(1 + \frac{5\pi^2\tau^2}{12\varepsilon_F^2}\right)$$

  - The specific heat is“

$$C_v = \frac{\pi^2 N\tau}{2\tau_F}$$

  - Entropy

$$\sigma = \frac{\pi^2 N\tau}{2\varepsilon_F}$$

- For a Bose-Einstein gas, the values become slightly different:
  - The density of states becomes:

$$D(\varepsilon) = \frac{V}{4\pi^2}\left(\frac{2M}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{\varepsilon}$$

- The total particles per volume become:

$$N = N_o(\tau) + N_e(\tau)$$

- Where N_o are in the condensed phase (lowest orbital), and N_e represents those in a normal phase (non-lowest orbital)

$$N_o(\tau) = \frac{1}{e^{-\frac{\mu}{\tau}} - 1}$$

$$N_e(\tau) = \int_0^{\infty} f(\varepsilon, \mu, \tau) D(\varepsilon) \, d\varepsilon$$