# Chemical Potential and Gibbs Distribution

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- Chemical Potential, $\mu(\tau, V, N)$
  - Given a reservoir, $R$ with fundamental temperature $\tau$, and two systems in contact, $S_1$ and $S_2$, with $\tau, V_1, N_1$ and $\tau, V_2, N_2$, respectively
  - $R$ is in thermal equilibrium with $S_1$ and $S_2$
  - $R$ is in mechanical equilibrium with $S_1$ and $S_2$
  - $S_1$ is in mechanical equilibrium with $S_2$, which means $V_1$ and $V_2$ are constant
  - $N_1$ and $N_2$ can change, but $N_1 + N_2 = N$ is constant
  - We find that the partial of $F$ with respect to $N$ plays an important role in this situation
  - We find that the chemical potential can be represented as:

$$\mu(\tau, V, N) \equiv \left(\frac{\partial F(\tau, V, N)}{\partial N}\right)_{\tau, V}$$

    * In chemical equilibrium, $\mu_1 = \mu_2$
  - We can find that, away from equilibrium:

$$dF_{S_1+S_2} = (\mu_1 - \mu_2)dN_1$$

  - This implies that particles flow from greater energy to lower energy (this makes sense, as they would want to settle at minimal potential)

- The chemical potential must be defined for all particle species:

$$\mu_i = \left(\frac{\partial F(\tau, V, N_1, N_2, N_3)}{\partial N_i}\right)_{\tau \forall N_{k \neq i}}$$

  - The chemical potential of an ideal gas then becomes:

$$F = -\tau N \left(\ln\left(\frac{n_Q}{n}\right) + 1\right)$$

    * Note: we also know $\frac{n_Q}{n} = \frac{N}{V}$

  - By taking the partial with respect to $N$, we get:

$$\mu = -\tau \ln \left( \frac{n_Q}{n} \right)$$

  - From this, we can tell:
    * $\mu < 0$ because $n << n_Q$ in classical regime
    * if $n$ increases, then $\mu$ increases
  - Using $F = U - \tau\sigma$, we can determine internal and external chemical potentials as:

$$\mu_{ext} = \left( \frac{\partial F_{ext}}{\partial N} \right)_{\tau,V} = \left( \frac{\partial U_{ext}}{\partial N} \right)_{\tau,V} = \frac{U_{ext}}{N}$$

$$\mu_{int} = \left( \frac{\partial F_{int}}{\partial N} \right)_{\tau,V}$$

- We know that in chemical equilibrium, $\mu_{1|total} = \mu_{2|total}$, or, more generally:

$$\mu_{1|int} + \frac{U_{1|ext}}{N_1} = \mu_{2|int} + \frac{U_{2|ext}}{N_2}$$