Phase Transitions

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- First-Order Phase Transitions
  - Three Phases: Solid, Liquid, Gas (Vapor)
  - Liquid and Gas/Vapor are known as fluids
  - From liquid to gas, solid to gas, and solid to liquid, the process would be isothermal
  - Phase Changes:
    * Solid to Liquid $\rightarrow$ Melting
    * Liquid to Solid $\rightarrow$ Crystalization
    * Solid to Gas $\rightarrow$ Sublimation
    * Gas to Solid $\rightarrow$ Deposition
    * Liquid to Gas $\rightarrow$ Boiling
    * Gas to Liquid $\rightarrow$ Condensation

- Coexistence of two phases (l and g)

$$G = G_l + G_g = N_l\mu_l(P, \tau) + N_g\mu_g(P, \tau)$$
$$N_g + N_l = N \quad \text{(constant)}$$

  - This expression may be rewritten as:

$$G = N_l(\mu_l - \mu_g) + N\mu_g$$

  - If l and g coexist, then:
    * $\tau_l = \tau_g$ (thermal equilibrium)
    * $P_l = P_g$ (mechanical equilibrium)
    * $\mu_l = \mu_g$ (chemical equilibrium)
  - Thus, for coexistence of l and g, we may write:

$$\mu_l(\tau, P) = \mu_g(\tau, P)$$

- The pressure differential with respect to τ may be written as:

$$\frac{dP}{d\tau} = \frac{\left(\frac{\partial \mu_l}{\partial \tau}\right)_P - \left(\frac{\partial \mu_g}{\partial \tau}\right)_P}{\left(\frac{\partial \mu_g}{\partial \tau}\right)_\tau - \left(\frac{\partial \mu_l}{\partial \tau}\right)_\tau}$$

- Furthermore, implementing pressure and volume per molecules (s and v), we may write:

$$\frac{dP}{dt} = \frac{s_g - s_l}{v_g - v_l}$$

- We can recall that, in a reversible process:

$$\delta Q = \tau \, d\sigma$$
$$\Delta Q = \tau \Delta \sigma$$
$$\Delta Q = \tau(s_g - s_l)$$

  - Thus, we may described the heat gained from moving 1 molecule from g to l
  - This ΔQ term is referred to as L, or the latent heat of vaporization per molecule

- This leads us to the Clausius-Clapeyron Equation:

$$\frac{dP}{d\tau} = \frac{L}{\tau(v_g - v_l)}$$

  - This is used in liquid-gas coexistence
  - We assume the gas is ideal
  - We also assume $v_l << v_g$
  - Also, L is assumed to be constant
  - In assuming this, we may write:

$$\frac{dP}{d\tau} = \frac{LP}{\tau^2}$$

  - Solving this, we get:

$$P(\tau) = P_o e^{-\frac{L}{\tau}}$$