Heat and Work

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November 7, 2023

- Heat (Q) and Work (W)
  - Q is the energy transferred <u>to</u> S through thermal contact with R
  - W is work done by/on S through change in volume or external fields
- 1st Law of Thermodynamics

$$\delta Q = dU + \delta W$$

  - δ is the inexact differential
  - This means that:

$$\oint df = 0 \text{ or } \int_1^2 df = \int_1^2 df$$

    * df is independent of path

$$\oint \partial f \neq 0 \text{ and } \int_1^2 \partial f \text{ is path dependent}$$

  - Note it *may* equal 0, but does not have to
- U, σ are state functions, as they depend on state variables only $(\tau, V, N, \dots)$
  - Q, W are not state functions (*i.e.* they are dependent on path)
- Let us consider a reversible process — a process that does not increase the total entropy of $S + R$

$$\delta Q_{rev} = \tau d\sigma$$

- In general:

$$\delta Q \leq \tau d\sigma$$

- Q is low quality energy; adding δQ to S increases its entropy by $\delta Q/\tau$
- W is high quality energy
- Heat engines convert heat to work
  * Steam engine
  * Internal combustion
  * Power plant

- Carnot efficiency η_c, is the ratio o work generate by S to heat added to S in a reversible process

$$\eta_c = \left(\frac{W}{Q_h}\right)_{rev}$$

  - This can be rewritten in many forms, including:

$$\eta_c = \frac{Q_h - Q_l}{Q_h} = 1 - \frac{Q_l}{Q_h} = 1 - \frac{\tau_l}{\tau_h}$$

  - The actual efficiency is:

$$\eta \leq \eta_c$$

  - This can be obtained by assuming $\sigma_h \leq \sigma_l$

- Refrigerators use work to move heat
  - The Carnot efficiency for a refrigerator is:

$$\gamma_c = \left(\frac{Q_l}{W}\right)_{rev}$$

  - This can be expressed in more useful terms for us as:

$$\gamma_c = \frac{\tau_l}{\tau_h - \tau_l}$$

  - Similar to heat engines, we can say:

$$\gamma \leq \gamma_c$$

- Carnot Cycle

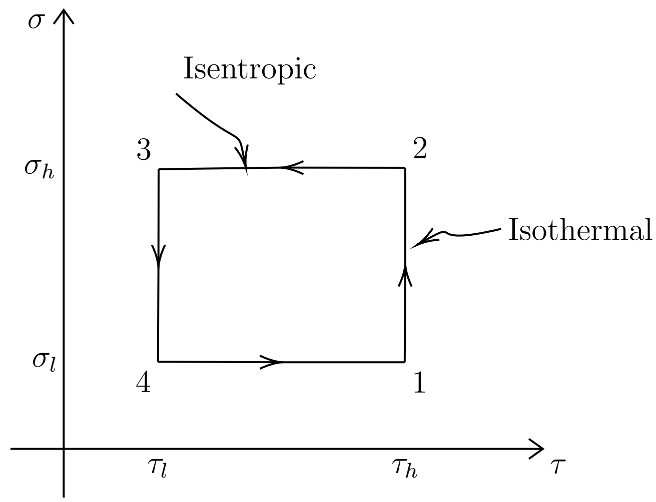

Figure 1: The Carnot Cycle

  - $1 \rightarrow 2$
    * $Q_{12} = \tau_h(\sigma_h - \sigma_l) > 0$
    * $W_{12} = \tau_h(\sigma_h - \sigma_l) - (U_2 - U_1)$
  - $2 \rightarrow 3$
    * $Q_{23} = 0$
    * $W_{23} = -(U_3 - U_2)$
  - $3 \rightarrow 4$
    * $Q_{34} = \tau_l(\sigma_l - \sigma_h) < 0$
    * $W_{34} = \tau_l(\sigma_l - \sigma_h) - (U_4 - U_3)$
  - $4 \rightarrow 1$
    * $Q_{41} = 0$
    * $W_{41} = -(U_1 - U_4)$
  - Total
    * Work:
$$(\tau_h - \tau_l)(\sigma_h - \sigma_l) > 0$$
    * Heat:
$$(\tau_h - \tau_l)(\sigma_h - \sigma_l) > 0$$
  - The efficiency may be defined as:
$$\eta = \frac{W_{tot}}{Q_{rec}} = 1 - \frac{\tau_l}{\tau_h}$$