$$g(N,s) = \frac{N!}{\left(\frac{1}{2}N+s\right)\left(\frac{1}{2}N-s\right)} = \frac{N!}{N_\uparrow! N_\downarrow!}$$

$$g(N,s) \approx \sqrt{\frac{2}{\pi N}} 2^N e^{-\frac{2s^2}{N}}$$

$$U(s) = -2smB$$

$$2s = N_\uparrow - N_\downarrow$$

$$\sigma(N,s) = \ln(g(N,s))$$
$$S = k_B \sigma$$

$$\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U}\right)_{N,V}$$
$$\tau = k_B T$$

Accessible States $(s = s_1 + s_2)$:

$$g(s) = \sum_s g_1(s_1) g_2(s - s_1)$$

System/Reservoir State Probability:

$$P(\varepsilon_s) = \frac{1}{z} e^{-\frac{\varepsilon_s}{\tau}}$$

$$z = \sum_s e^{-\frac{\varepsilon_s}{\tau}}$$

$$P = -\left(\frac{\partial U}{\partial V}\right)_\sigma = \tau \left(\frac{\partial \sigma}{\partial V}\right)_U = -\left(\frac{\partial F}{\partial V}\right)_\tau$$

$F = U - \tau\sigma$ min. in eq. with const τ, V

$F = -\tau \ln(z)$ to derive P, σ

Ideal Monatomic Gas

Given N atoms: $z_N = \frac{(n_Q V)^N}{N!}$

If $n = \frac{N}{V} << n_Q$, $n_Q = \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}}$

$$PV = N\tau$$

$$\sigma = N\left[\ln\left(\frac{n_Q}{n}\right) + \frac{5}{2}\right]$$

$$C_V = \frac{3}{2}N$$

A process is reversible if the system remains infinitesimally close to the equilibrium state at all times during the process.

Average in Mode at freq. ω: $\langle s \rangle = \frac{1}{e^{\frac{\hbar\omega}{\tau}} - 1}$

Energy Density at τ: $\langle s \rangle = \frac{U}{V} = \frac{\pi^2}{15\hbar^3 c^3}\tau^4$

Radiant energy per vol: $U_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\frac{\hbar\omega}{\tau}} - 1}$

Flux Density: $J_U = \sigma_B T^4$, $\sigma_B = \frac{\pi^2 k B^4}{60\hbar^3 c^2}$

Heat Capacity of Dielectric Solid: $C_V = \frac{12\pi^4 N k_B}{5}\left(\frac{T}{\theta}\right)^3 \to \theta = \left(\frac{\hbar\omega}{k_B}\right)\left(\frac{6\pi^2 N}{V}\right)^{\frac{1}{3}}$

$$\mu = \left(\frac{\partial F}{\partial N}\right)_{\tau,V} = \left(\frac{\partial U}{\partial N}\right)_{\sigma,V} = -\tau\left(\frac{\partial \sigma}{\partial N}\right)_{U,V}$$

In diffusive equilibrium if: $\mu_1 = \mu_2$

$$\mu = \mu_{int} + \mu_{ext}$$

$$\mu_{int} = \tau \ln\left(\frac{n}{n_Q}\right)$$

$$\mu_{ext} = \frac{U_{ext}}{N}$$

$$\mathfrak{z} = \sum_N \sum_s e^{\frac{N\mu-\varepsilon_s}{\tau}}$$

Gibbs Factor: $P(N, \varepsilon_s) = \dfrac{e^{\frac{N\mu-\varepsilon_s}{\tau}}}{\mathfrak{z}}$

Prob. chem. potential μ and temp τ has N particles in q.s. s of energy ε_s

$$\lambda = e^{\frac{\mu}{\tau}} \to \mathfrak{z} = \sum \lambda^N e^{-\frac{\varepsilon_s}{\tau}}$$

Therm. Average: $\langle N \rangle = \lambda \dfrac{\partial}{\partial \lambda} \ln(\mathfrak{z})$

Quant. Particle in Box: $\varepsilon = \dfrac{\hbar^2 \pi^2}{2mL^2}(n_x^2+n_y^2+n_z^2)$

$$U = \tau^2 \left(\frac{\partial \ln(z)}{\partial \tau}\right)_V$$

$$W = PA\Delta x = P\Delta V$$

$$\tau\, d\sigma = dU + P\, dV$$

$$\int_{-\infty}^{\infty} e^{-\alpha^2 n^2}\, dn = \frac{\sqrt{\pi}}{2\alpha}$$

$$\sigma = -\left(\frac{\partial F}{\partial \tau}\right)_{V,N}$$

$$\langle N \rangle = \sum_N \sum_s N(N, \varepsilon_s) = \tau \left(\frac{\partial \ln(\mathfrak{z})}{\partial \mu}\right)_{\tau,V}$$

$$\langle \varepsilon_s \rangle = \sum_N \sum_s \varepsilon_s(N, \varepsilon_s) = \tau^2 \left(\frac{\partial \ln(\mathfrak{z})}{\partial \tau}\right)_{\mu,V} + \tau\mu \left(\frac{\partial \ln(\mathfrak{z})}{\partial \mu}\right)_{\tau,V}$$

$f(\varepsilon_n)$ avg. occupancy

<u>Bose-Einstein:</u>

$$f(\varepsilon_n) = \frac{1}{e^{\frac{\varepsilon_n-\mu}{\tau}} - 1}$$

<u>Fermi-Dirac:</u>

$$f(\varepsilon_n) = \frac{1}{e^{\frac{\varepsilon_n-\mu}{\tau}} + 1}$$

<u>Classical Limit:</u>

$$f(\varepsilon_n) = e^{\frac{\mu-\varepsilon_n}{\tau}}$$

	ΔU	$\Delta\sigma$	W	Q
Rev. Isothermal	0	$N\ln\left(\frac{V_2}{V_1}\right)$	$-N\tau\ln\left(\frac{V_2}{V_1}\right)$	$N\tau\ln\left(\frac{V_2}{V_1}\right)$
Rev. Isentropic	$-\frac{3}{2}N\tau_1\left[1-\left(\frac{V_1}{V_2}\right)^{\frac{2}{3}}\right]$	0	$-\frac{3}{2}N\tau_1\left[1-\left(\frac{V_1}{V_2}\right)^{\frac{2}{3}}\right]$	0
Irrev. Expansion	0	$N\ln\left(\frac{V_2}{V_1}\right)$	0	0

<u>Constants</u>

$k_B = 1.381 \cdot 10^{-23}\left[\frac{\mathrm{J}}{\mathrm{K}}\right]$ $\sigma_B = 5.67 \cdot 10^{-8}\left[\frac{\mathrm{J}}{\mathrm{m^2sK^4}}\right]$ $c = 3 \cdot 10^8\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$