Lecture XI Notes

Michael Brodskiy
Professor: V. Cherkassky

July 1, 2020

0.1 Second Order Partial Derivatives

Although the order of second partial differentiation usually does not matter, sometimes the second order partial derivatives may not be equal. Differentiating $\frac{\partial f}{\partial x}$ by $\frac{\partial}{\partial y}$ results in $\frac{\partial^2 f}{\partial y \partial x}$. If one differentiates in the reversed order, this yields $\frac{\partial^2 f}{\partial x \partial y}$, where the order of partial differentiation is read from right to left.

Example Where $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$*:*

$$f(x,y) = x^3 + x^2y^3 - 2y^2$$
$$\frac{\partial f}{\partial x} = 3x^2 + 2xy^3$$
$$\frac{\partial f}{\partial y} = 3x^2y^2 - 4y$$
$$\frac{\partial^2 f}{\partial y \partial x} = 6xy^2$$
$$\frac{\partial^2 f}{\partial x \partial y} = 6xy^2$$
$$\therefore \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$ [1]

0.2 Clairaut's Theorem

If $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ are continuous at point (a,b), then $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ at (a,b)[2]

[1] In most cases, $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

[2] This rule applies to any nth order derivative

0.3 Laplace's Differential Equations

Named after Pierre Laplace, the partial differential equation looks like this:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Any function that fits the criteria is called a harmonic function

0.4 Wave Equations

Any wave, whose displacement is defined by the function $u(x,t)$ must satisfy the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

1 Tangent Plane Approximation and Total Differentials – 14.4

The tangent plane approximation function is given by:

$$z - z_o = f_x(x_o, y_o)(x - x_o) + f_y(x_o, y_o)(y - y_o)$$

The definition of a total differential is:

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

dz represents the change in the plane approximation