Lecture XVI Notes

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In polar coordinates, equations relating x and y to $r(\theta)$ must be defined:

$$\boxed{x^2 + y^2 = r^2 \quad x = r\cos\theta \quad y = r\sin\theta}$$

On the region, R, defined by:

$$R = \{(r, \theta) | a \leq r \leq b, \alpha \leq \theta \leq \beta\}$$

The double integral for the area of a polar region is as follows:

$$\iint_R f(x, y)\, dA = \lim_{(n,m)\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(r_i^* \cos\theta_j^*, r_i^* \sin\theta_j^*)$$

This all simplifies down to:

$$\int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$$

For general polar regions, the same process as for Cartesian coordinates is used:

$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r\cos\theta, r\sin\theta) r\, dr\, d\theta$$