Lecture XV Notes

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1 Double Integrals over General Regions – 15.2

First of all, f must be continuous on a region D:

$$\{(x,y)|a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

Such regions are categorized as Type I, and may be found using the following:

$$\iint_D f(x,y) \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y) \, dy \, dx$$

A Type II equation must meet the same criteria as Type I, but with functions of x:

$$\{(x,y)|c \le y \le d, h_1(x) \le x \le h_2(x)\}$$

Such regions may be solved using the following:

$$\iint_D f(x,y) \, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y) \, dx \, dy$$

If the region, D, is neither Type I nor II, then the regions may be broken into two parts, D_1 and D_2 :

$$\iint_D f(x,y) \, dA = \iint_{D_1} f(x,y) \, dA + \iint_{D_2} f(x,y) \, dA$$

Furthermore, if the function being integrated is a constant, c, then:

$$\iint_{D} c \, dA = cA(D)$$

Where A(D) represents the area of the region D

This property may be used if $m \leq f(x, y) \leq M$ to achieve:

$$mA(D) \le \iint_D f(x, y) dA \le MA(D)$$