# Lecture XIII Notes

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## 1 Maxima and Minima – 14.7

In three dimensions, a point is determined to be critical if:

$$\frac{\partial f}{\partial x}(a,b) = 0, DNE \text{ and } \frac{\partial f}{\partial y}(a,b) = 0, DNE$$

From here, it needs to be determined whether or not there is an extrema at point $(a,b)$, and whether it is a minima or maxima.

### 1.1 The Second Derivative Test

First, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial x \partial y}$, $\frac{\partial^2 f}{\partial y \partial x}$ and $\frac{\partial^2 f}{\partial y^2}$ must be found. Then the determinant, $D$, of these must be found:

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$$f_{xx}f_{yy} - (f_{xy})^2$$

1. If $D > 0$, and $f_{xx}(a,b) > 0$, then $f(a,b)$ is a local minimum
2. If $D > 0$, and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a local maximum
3. If $D < 0$, then $f(a,b)$ is not a local maximum or minimum (saddle point)

### 1.2 Bounded Sets

In a closed, bounded set, $f$ attains a maximum at $(x_1, y_1)$, and a minimum at $(x_2, y_2)$