Final Project – Chapter 15

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29. #2

(a) A rectangular D is defined with x and y being confined by numbers only. In a general region, D has one of two variables confined by functions: $h_1(y) \leq x \leq h_2(y)$, or $g_1(x) \leq y \leq g_2(x)$

(b) A Type I region is created when the y variable is bounded by functions: $g_1(x) \leq y \leq g_2(x)$. These may be evaluated by finding:

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

(c) A Type II region is created when the x variable is bounded by functions: $h_1(y) \leq x \leq h_2(y)$. These may be evaluated by finding:

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

(d) If necessary, a double integral may be split into two regions:

$$\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)dA$$

In addition to this, if a function of x is multiplied by a function of y, the two functions may be integrated separately:

$$\iint_D g(x)h(y)\,dA = \int_a^b g(x)\,dx \int_c^d h(y)\,dy$$

As with single integrals, a constant may be taken out of the integrand:

$$\iint_D cf(x,y)\,dA = c\iint_D f(x,y)\,dA$$

Also, added functions may be separated:

$$\iint_D [f(x,y) + g(x,y)]\, dA = \iint_D f(x,y)\, dA + \iint_D g(x,y)\, dA$$

If 1 is integrated, the area of D will be found:

$$\iint_D 1\, dA = A(D)$$

If there are two constants, m and M, and $m \leq f(x,y) \leq M$ for all (x,y) in D, then:

$$mA(D) \leq \iint_D f(x,y)\, dA \leq MA(D)$$

30. #6

(a) To find the surface area, one would simply use the surface area formula:

$$A(S) = \iint_D \sqrt{f_x(x,y)^2 + f_y(x,y)^2 + 1}\, dA$$

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31. #3

(a) The work is as follows:

$$\int_1^2 \int_0^2 (y + 2xe^y)\, dx\, dy$$

$$\int_1^2 (2y + 4e^y)\, dy$$

$$y^2 + 4e^y \Big|_1^2 \Longrightarrow 4e^2 - 4e + 3$$

32. #6

(a) The work is as follows:

$$\int_0^1 \int_x^{e^x} 3xy^2\, dy\, dx$$

$$\int_0^1 xe^{3x} - x^4\, dx$$

$$\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} - \frac{1}{5}x^5 \Big|_0^1 \Longrightarrow \frac{2}{9}e^3 - \frac{4}{45}$$

33. #9

(a) The work is as follows:

$$D = \{(r, \theta) | 2 \leq r \leq 4, 0 \leq \theta \leq \pi\}$$

$$\int_0^\pi \int_2^4 f(r\cos\theta, r\sin\theta) r \, dr \, d\theta$$

34. #19

(a) The work is as follows:

$$\int_0^1 1 \, dx \int_x^1 \cos(y^2) \, dy$$

$$\frac{1}{2y} \sin(y^2)$$

$$\frac{1}{2} \sin(1)$$

35. #23

(a) The work is as follows:

$$\int_0^1 \int_0^{\sqrt{x}} \frac{y}{1+x^2} \, dy \, dx$$

$$\frac{1}{2} \int_0^1 \frac{x}{1+x^2} \, dx$$

$$\frac{1}{4} \int_1^2 \frac{1}{u} \, du$$

$$\frac{1}{4} \ln(u) \Big|_1^2 = \frac{1}{4} \ln(2)$$

36. #27

(a) This would be much easier in polar coordinates. Therefore, the angle between the line $y = \sqrt{3}x$ and the x axis must be found. The angle turns out to be $\frac{\pi}{3}$, and the radius is 3. Then, the iterated integrals may be set up:

$$\int_0^{\frac{\pi}{3}} \int_0^3 r^4 \, dr \, d\theta$$

$$\int_0^{\frac{\pi}{3}} \frac{243}{5} \, d\theta$$

$$\frac{243}{5} \theta \Big|_0^{\frac{\pi}{3}} = \frac{81\pi}{5}$$

37. #35

(a) The work is as follows:

$$\int_0^2 \int_1^4 x^2 + 4y^2 \, dy \, dx$$

$$\int_0^2 3x^2 + 84 \, dx$$

$$x^3 + 84x \Big|_0^2 = 176$$

38. #37

    (a) The given points result in a pyramid. This pyramid has an isosceles base, with side lengths 2. This means the area of the base is equal to 2. Furthermore, the height of the pyramid is 1 using this in the formula for the volume of a pyramid results in: $\frac{1}{3}(2)(1) = \frac{2}{3}$

39. #53

    (a) The work is as follows:

$$\int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} (x, y, z) \, dx \, dy \, dz \Longrightarrow \int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) \, dx \, dy \, dz$$

40. #55

    (a) The work is as follows:

$$J = \frac{1}{2}$$

$$\int_2^4 \int_{-2}^{0} \frac{u}{2v} \, du \, dv$$

$$-\int_2^4 \frac{1}{v} \, dv = -\ln(2)$$