Lecture II Notes

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1 Dot Product – 12.3

A dot product is written using "$\cdot$," or $\vec{a}\cdot\vec{b}$. This is pronounced "a dot b."

A dot product can be found by multiplying each respective term with the other term, and adding each product together, or $\vec{a}\cdot\vec{b} \longrightarrow a_1b_1 + a_2b_2 + a_3b_3$

Practice Problem: Given $\vec{a} =< 3, 4, 5 >, \vec{b} =< 2, 7, -1 >$, find $\vec{a}\cdot\vec{b}$

$$(3)(2) + (4)(7) + (-1)(5) = 27$$

1.1 Dot Product Properties

1. $\vec{a}\cdot\vec{a} = |a|^2$
2. $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$
3. If $\vec{b}$ equals $< 0, 0, 0 >, \vec{a}\cdot\vec{b} = 0$
4. $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
5. If c is a scalar, $(c\vec{a})\vec{b} = c(\vec{a}\cdot\vec{b})$
6. If the angle between two vectors is known, $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|cos(\theta)$[1]
   (a) $cos(\theta) = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$
7. The angle between a certain axis and a vector is equal to the dot product of the vector and the respective unit vector of the axis, divided by the magnitude of the vector. For the x axis, this would be: $cos(\theta_x) = \frac{\vec{a}\cdot i}{|\vec{a}|}$

[1] Vectors $\vec{a}$ and $\vec{b}$ are orthogonal (perpendicular) if and only if $\vec{a}\cdot\vec{b} = 0$

(a) $cos^2(\theta_x) + cos^2(\theta_y) + cos^2(\theta_z) = 1$

8. The component of projection of one vector upon another is given by: $Comp_{\vec{a}\vec{b}} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$[2]

   (a) To find the projection of one vector onto the other is, for the x axis: $(\hat{i})Comp_{\vec{a}\vec{b}}$. For another axis, multiply by the respective unit vector instead of $\hat{i}$.

2 Cross Product – 12.4

While the dot product yields a scalar value, the cross product yields a vector, and is denoted by $\vec{a}$ x $\vec{b}$, and is pronounced "a cross b". First, one must know how to work with the determinant. A determinant in a 3x3 matrix may be found using the formula:
$det_3 = a_1(b_2c_3 - c_2b_3) - b_1(a_2c_3 - c_2a_3) + c_1(a_2b_3 - b_2a_3)$
As such, the matrix would look like this:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The cross product[3] would be found using the same process as a determinant, with unit vectors as the first row:

$$\vec{a} \text{ x } \vec{b} =$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \\ \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{vmatrix}$$

A triple product may be found using a determinant when the problem is of form: $\vec{a} \cdot (\vec{b} \text{ x } \vec{c})$. The matrix for such a triple product would look as follows:

$$\begin{vmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \\ \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \\ \vec{c}_1 & \vec{c}_2 & \vec{c}_3 \end{vmatrix}$$

[2] Order matters! If it is the projection of $\vec{b}$ onto $\vec{a}$, then $|\vec{b}|$ is canceled.
[3] Note: cross product may only be found in $\mathbb{R}^3$