Lecture XX Notes

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1 Line Integrals (Continued) – 16.2

A line integral defined by the function $f(x, y, z)$ may be integrated by using the formula:

$$\int_C f(x,y,z)\,ds = \int_C f(x(t),y(t),z(t))\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2}\,dt$$

Given a vector function, $\overrightarrow{r}(t)$, the above formula may be simplified to yield:

$$\int_C f(x,y,z)\,ds = \int_C f(\overrightarrow{r}(t))|\overrightarrow{r}'(t)|\,dt$$

Line integrals may also appear with respect to dx and dy. This may be simplified by finding dx and dy in terms of dt:

$$\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))x'(t)\,dt$$

$$\int_C f(x,y)\,dy = \int_a^b f(x(t),y(t))y'(t)\,dt$$

When line integrals appear in such a form it is abbreviate them:

$$\int_C P(x,y)\,dx + \int_C Q(x,y)\,dy = \int_C P(x,y)\,dx + Q(x,y),dy$$

In physics, the formula for work was defined as $W = Fd\cos(\theta)$
This may be found by integrating:

$$W = \int_a^b \overrightarrow{F}(\overrightarrow{r}(t)) \cdot \frac{\overrightarrow{r}'(t)}{|\overrightarrow{r}'(t)|}\,ds$$

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$$W = \int_a^b \overrightarrow{F} \cdot \overrightarrow{r}\, dt$$

This yields the work done on a particle by a field.