Lecture XIV Notes

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1 Multiple Integrals – 15.1

In calculus I, integrals were defined as the area, A, under a curve, where:

$$A = \lim_{x\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

This formula was simplified to what is known as the integral:

$$\int_a^b f(x)\,dx$$

The volume of a multivariable function, then, is given by:

$$V \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

$$V = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A$$

$$V = \iint_R f(x,y)\,dA$$

1.1 Double Integral Properties

Double integrals have many of the same properties as single integrals:

$$1.\ \iint_R [f(x,y)+g(x,y)]\,dA = \iint_R f(x,y)\,dA + \iint_R g(x,y)\,dA$$

If c is a constant:

$$\iint_R cf(x,y)\,dA = c\iint_R f(x,y)\,dA$$

If $f(x,y) \geq g(x,y)$ for all (x,y), then:

$$\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$$

1.2 Iterated Integrals

The double integral, $\iint_R f(x,y)\,dA$, may be broken up in order to be calculated with ease:

$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

This may be solved using the method of partial integrals, where, much like partial derivatives, only one of the variables is treated as a variable at once. As a result of this, just like with partial derivatives, the order that the operation is done in yields the same result no matter what:

$$\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

1.3 Fubini's Theorem

If $f(x,y)$ is continuous on $R = \{(x,y)|a \leq x \leq b, c \leq y \leq d\}$

$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$