

# Final Project – Chapter 12

Michael Brodskiy

Professor: V. Cherkassky

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1. #15

- (a) Two vectors are parallel if they are scalar multiples of each other. For example, if  $\vec{a} = \langle a_1, a_2, a_3 \rangle$ , a parallel vector would equal  $c\vec{a}$ , where  $c$  is a scalar. Therefore, if a vector is equal to another vector multiplied by a scalar, they are parallel. Also, two vectors are parallel if their cross product is equal to zero.
- (b) Two vectors are orthogonal (perpendicular) simply if their dot product is equal to zero.
- (c) It can be determined whether two planes are parallel by using the vectors normal to these two planes. If the two normal vectors are parallel (which can be determined using the method from (a)), then the planes are parallel as well.

2. #16

- (a) One can take the three points,  $\vec{P}$ ,  $\vec{Q}$ , and  $\vec{R}$ , and form two vectors. Then, if the two vectors are scalar multiples of each other, then these points are collinear.
- (b) From four points, one can find three vectors, say  $\vec{PQ}$ ,  $\vec{PR}$ , and  $\vec{PS}$ . By crossing,  $\vec{PQ} \times \vec{PR}$ , one can find a vector orthogonal, say,  $\vec{n}$  to both  $\vec{PQ}$  and  $\vec{PR}$ . Then, if the crossed vector,  $\vec{n}$ , is orthogonal to the third vector,  $\vec{PS}$ , (which can be determined if the dot product,  $\vec{n} \cdot \vec{PS} = 0$ ) the four points lie on one plane.

3. #17

- (a) First, one must choose two points on the line, points  $A$  and  $B$ . Then, two vectors must be formed,  $\vec{AB}$  and  $\vec{AP}$  or  $\vec{BP}$ , where  $P$  is the point. Then, the distance can be found using the formula:  $\frac{|\vec{AB} \times (\vec{AP} \text{ or } \vec{BP})|}{|\vec{AB}|}$
- (b) The distance from a point to a plane may be found using the formula:  $\frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}$ , where  $a-d$  are the coefficients of the standard form of the plane, and  $x-z$  are respective coordinates of the point.

- (c) By determining one point on a line, say  $L_1$ , and two points on line  $L_2$ , the same process from part (a) may be used.

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4.  $\vec{v} \implies \langle 1 - 4, 1 - (-1), 5 - 2 \rangle = \langle -3, 2, 3 \rangle$   
 $r = r_o + tv \implies x = 4 - 3t, y = -1 + 2t, z = 2 + 3t \implies r = \langle 4 - 3t, -1 + 2t, 2 + 3t \rangle$