# Lecture V Notes

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## 1 Vectors in Space – 13.1

A vector function: $\overrightarrow{r}(t) = \langle f(t), g(t), h(t)\rangle = f(t)\hat{\mathbf{i}} + g(t)\hat{\mathbf{j}} + h(t)\hat{\mathbf{k}}$

Example:

$$\overrightarrow{r}(t) = \langle t^2, ln(2-t), \sqrt{t}\rangle$$

The domains of each component function must be considered to find the domain of the vector function, $0 \leq x < 2$

For a vector function:

$$\lim_{t\to a} \overrightarrow{r}(t) = \langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\rangle$$

Example: Find the limit

$$\lim_{t\to 0}(e^{-3t}\hat{\mathbf{i}} + \frac{t^2}{\sin^2 t}\hat{\mathbf{j}} + \cos(2t)\hat{\mathbf{k}}) = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

## 2 Derivatives and Integrals of Vector Functions – 13.2

$$\overrightarrow{r}'(t) = \lim_{h\to 0} \frac{\overrightarrow{r}(t+h) - \overrightarrow{r}(t)}{h}$$

$\overrightarrow{r}'(t)$ is a vector tangent to $\overrightarrow{r}(t)$, where $\overrightarrow{r}'(t) = \langle f'(t), g'(t), h'(t)\rangle$
The unit tangent vector, $\overrightarrow{T}(t)$, may found using the formula: $\overrightarrow{T}(t) = \frac{\overrightarrow{r}'(t)}{|\overrightarrow{r}'(t)|}$
The integral of the vector function $\overrightarrow{r}(t)$ is:

$$\int_a^b \overrightarrow{r}(t)dt = \left(\int_a^b f(t)dt\right)\hat{\mathbf{i}} + \left(\int_a^b g(t)dt\right)\hat{\mathbf{j}} + \left(\int_a^b h(t)dt\right)\hat{\mathbf{k}}$$