Final Project – Chapter 12

Michael Brodskiy
Professor: V. Cherkassky

June 20, 2020

Page 841

1. #15
   (a) Two vectors are parallel if they are scalar multiples of each other. For example, if $\overrightarrow{a} =< a_1, a_2, a_3 >$, a parallel vector would equal $c\overrightarrow{a}$, where c is a scalar. Therefore, if a vector is equal to another vector multiplied by a scalar, they are parallel. Also, two vectors are parallel if their cross product is equal to zero.
   (b) Two vectors are orthogonal (perpendicular) simply if their dot product is equal to zero.
   (c) It can be determined whether two planes are parallel by using the vectors normal to these two planes. If the two normal vectors are parallel (which can be determined using the method from (a)), then the planes are parallel as well.
2. #16
   (a) One can take the three points, $\overrightarrow{P}$, $\overrightarrow{Q}$, and $\overrightarrow{R}$, and form two vectors. Then, if the two vectors are scalar multiples of each other, then these points are collinear.
   (b) From four points, one can find three vectors, say $\overrightarrow{PQ}$, $\overrightarrow{PR}$, and $\overrightarrow{PS}$. By crossing, $\overrightarrow{PQ}$ x $\overrightarrow{PR}$, one can find a vector orthogonal, say, $\overrightarrow{n}$ to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Then, if the crossed vector, $\overrightarrow{n}$, is orthogonal to the third vector, $\overrightarrow{PS}$, (which can be determined if the dot product, $\overrightarrow{n} \cdot \overrightarrow{PS} = 0$) the four points lie on one plane.
3. #17
   (a) First, one must choose two points on the line, points A and B. Then, two vectors must be formed, $\overrightarrow{AB}$ and $\overrightarrow{AP}$ or $\overrightarrow{BP}$, where P is the point. Then, the distance can be found using the formula: $\frac{|\overrightarrow{AB} \text{ x } (\overrightarrow{AP} \text{ or } \overrightarrow{BP})|}{|\overrightarrow{AB}|}$
   (b) The distance from a point to a plane may be found using the formula: $\frac{|ax+by+cz+d|}{\sqrt{a^2+b^2+c^2}}$, where a-d are the coefficients of the standard form of the plane, and x-z are respective coordinates of the point.

(c) By determining one point on a line, say L_1, and two points on line L_2, the same process from part (a) may be used.

Page 843

4. #15

   (a) $\overrightarrow{v} \Longrightarrow \langle 1-4, 1-(-1), 5-2\rangle = \langle -3, 2, 3\rangle$
   $r = r_o + tv \Longrightarrow x = 4-3t, y = -1+2t, z = 2+3t \Longrightarrow r = \langle 4-3t, -1+2t, 2+3t\rangle$

5. #16

   (a) Because the given equations of lines are of symmetry form, they can all be rearranged in terms of t. This yields: $t = \frac{1}{3}(x-4) = \frac{1}{2}y = z+2 \Longrightarrow x = 4+3t, y = 2t, z = t-2$. These are the parametric equations for the parallel line, which means that the direction vectors are the same for both lines. Parametric equations for a line are found using $r = r_o + \overrightarrow{v}t$, which means that the coefficients of t are the same for both lines. To make the equations correct, the noncoefficient numbers need to be changed. This yields: $\langle 4+3t, 2t, t-2\rangle \Longrightarrow \langle 1+3t, 2t, t-1\rangle$. Therefore, the parametric equations of the line are $\langle 1+3t, 2t, t-1\rangle$

6. #17

   (a) The direction vector may be found by using the equation of the plane, or, more exactly, the coefficients of the variables. These coefficients give us the vector that is perpendicular to the plane: $\overrightarrow{v} = \langle 2, -1, 5\rangle$. This vector, because it is perpendicular to the plane, is parallel to the line. Then, one must only use the formula to find: $x = -2+2t, y = 2-t, z = 4+5t$, or $\langle -2+2t, 2-t, 4+5t\rangle$

7. #25

   (a) This problem may be solved using the simple formula: $k(a_1x + b_1y + c_1z + d_1) + l(a_2x+b_2y+c_2z+d_2) = 0$, where k and l are random constants, and $a_1 - d_2$ are the coefficients of the intersecting planes. This formula gives us: $k(x-z-1) + l(y+2z-3) = 0 \Longrightarrow kx-kz-k+ly+2lz-3l$. By forming a vector from the coefficients, we get: $\langle k, l, 2l-k\rangle$ To find the plane perpendicular to the given plane, simply dot the direction vectors, or $\langle k, l, 2l-k\rangle \cdot \langle 1, 1, -2\rangle \Longrightarrow 3k - 3l = 0$, or $k = l = 1$. Finally, this yields: $x - z - 1 + y + 2z - 3 = 0$, or $x + y + z - 4 = 0$

8. #26

   (a) First, the normal vector to these points needs to be found. This can be done by finding $\overrightarrow{AB}$ and $\overrightarrow{AC}$. $\overrightarrow{AB} = \langle -1-2, -1-1, 10-1\rangle, \overrightarrow{AC} = \langle 1-2, 3-1, -4-1\rangle$.
   $\overrightarrow{AB}$ x $\overrightarrow{AC} = \langle 1, 3, 1\rangle \Longrightarrow (x-2) + 3(y-1) + (z-1) = 0 \Longrightarrow x + 3y + z = 6$

(b) The vector perpendicular (and therefore parallel to the line) to the plane from part (a) is $\langle 1,3,1\rangle$ This means that the parametric equations for the line are: $x = -1+t, y = -1+3t, z = 10+t$. From here, the equations may be rearranged to find: $t = x+1 = \frac{1}{3}(y+1) = z-10$

(c) The normal vector to the first plane is: $\langle 1,3,1\rangle$. The acute angle between the two normal vectors is the same as the acute angle between the planes. Therefore, the angle is: $\theta = cos^{-1}(\frac{13}{\sqrt{319}}) = 43.2° \approx 43°$

(d) The plane formulas, when set equal to each other, yield: $x + 3y + z - 6 = 2x - 4y - 3z - 8$. Simplifying this yields: $x - 7y - 4z = 2$. One vector from this, which will be used later, is $\langle 2,0,0\rangle$. Next, finding the cross product of the normal vectors of the fields gets us: P_1 x $P_2 = -5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$. Using the formula $r = r_o + vt$, it is found that $r = \langle 2,0,0\rangle + t\langle -5,5,-10\rangle \implies \langle 2-5t, 5t, -10t\rangle$

9. #27

(a) First, a point from one plane needs to be chosen, say $(2,0,1)$ from the first plane. Next, plugging in known values it is found that: $\frac{|2(3)+0(y)+1(-4)-24|}{\sqrt{3^2+1+4^2}} = 4.315$

10. #37

(a) The radius of the ellipsoid in the z direction will be the same as that of y, as the ellipse will be rotated around the x axis. This means that the equation will be: $4x^2 + y^2 + z^2 = 16$