

Final Project — Chapter 16

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41. #5

- (a) The Fundamental Theorem for Line Integrals states that, in a conservative vector field (where $\vec{F} = \nabla f$ exists), the integral of the derivative of the given function is equal to values of the function of the top bound minus the bottom:

$$\int_a^b \vec{F} d\mathbf{r} = \int_a^b \nabla f d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$

42. #6

- (a) If a line integral is independent of path, it yields the same value regardless of the path, meaning that, any line starting at the same point and ending at the same point will result in the same result of the line integral.
- (b) If the line integral is independent of path, this signifies that \vec{F} is a conservative vector field.

43. #7

- (a) Green's Theorem states that, given a line integral on a closed path, C , of a non-conservative vector field (otherwise the integrand becomes zero), this line integral is equal to the double integral of the partial derivative of the Q function with respect to x , minus the partial derivative of the P function with respect to y , all on the region D formed by the closed path.

$$\int_C P(x, y) dx + Q(x, y) dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

44. #1

- (a) The work is negative because it is done in a direction opposing the vector field.
- (b) Not Covered

45. #3

- (a) The work is as follows:

$$\begin{aligned} & \int_0^{\pi} 9 \cos^2(t) \sin(t) \sqrt{10} dt \\ u = \cos(t) & \implies du = -\sin(t) dt \\ & - \int_1^{-1} 9u^2 \sqrt{10} \\ & -\sqrt{10} \left(3u^3 \Big|_1^{-1} \right) \implies -\sqrt{10}(-6) \\ & 6\sqrt{10} \end{aligned}$$

46. #5

- (a) The work is as follows:

$$\begin{aligned} & \int_{-1}^1 -t^4 - 2t^2 + 1 dt \\ & -\frac{1}{5}t^5 - \frac{2}{3}t^3 + t \Big|_{-1}^1 \\ & \frac{2}{15} + \frac{2}{15} \implies \frac{4}{15} \end{aligned}$$

47. #7

- (a) The work is as follows:

$$x = 2t + 1, y = 4t, z = 3t - 1$$

$$\begin{aligned} & \int_0^1 116t^2 - 4t dt \\ & \frac{116}{3}t^3 - 2t^2 \Big|_0^1 \\ & \frac{110}{3} \end{aligned}$$

48. #11

(a) \vec{F} is conservative if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$$\frac{\partial P}{\partial y} = 2xe^{xy} + x^2ye^{xy}$$

$$\frac{\partial Q}{\partial x} = 2xe^{xy} + x^2ye^{xy}$$

$\therefore \vec{F}$ is conservative

$$\int e^y + x^2e^{xy} dy = e^y + xe^{xy} + C$$

$$\therefore f(x, y) = xe^{xy} + e^y + C$$

49. #13

(a) \vec{F} is conservative if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$$\frac{\partial Q}{\partial x} = 8x^3y - 6xy^2$$

$$\frac{\partial P}{\partial y} = 8x^3y - 6xy^2$$

$\therefore \vec{F}$ is conservative

$$\int_C \vec{F} d\mathbf{r} = \vec{F}(\mathbf{r}(1)) - \vec{F}(\mathbf{r}(0))$$

$$\vec{F}(\mathbf{r}(1)) - \vec{F}(\mathbf{r}(0)) = 0$$

50. #15

(a) Evaluating by line integral yields: $\{C_1 : x = t, y = t^2\}$, $\{C_2 : x = t, y = 1\}$, which yields the following line integral:

$$\int_{-1}^1 t(t^2)^2 - (2t)(t)^2(t^2) + t dt$$

$$\int_{-1}^1 t - t^5 dt = \frac{1}{2}t^2 - \frac{1}{6}t^6 \Big|_{-1}^1$$

$$\frac{1}{2}t^2 - \frac{1}{6}t^6 \Big|_{-1}^1 = \frac{1}{3} - \frac{1}{3} = 0$$

Evaluating by Green's Theorem yields:

$$\frac{\partial Q}{\partial x} = -2xy$$

$$\frac{\partial P}{\partial y} = 2xy$$

$$\iint_D -4xy \, dA \Rightarrow - \int_{-1}^1 \left(\int_{x^2}^1 4xy \, dy \right) dx$$

$$- \int_{-1}^1 2x - 2x^5 \, dx$$

$$x^2 - \frac{1}{3}x^6 \Big|_{-1}^1 = - \left(\frac{2}{3} - \frac{2}{3} \right) = 0$$

\therefore Green's Theorem is true for this line integral