Lecture XXII Notes

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1 Green's Theorem – 16.4

For this theorem, let C be a curve that is:

1. positively oriented (moving counterclockwise)
2. piecewise smooth
3. simple
4. closed

D is the region enclosed by the curve.
If P and Q have continuous first-order partial derivatives on D, then:

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

For Green's Theorem, D must have boundaries.
Notation (If C is closed):

$$\int_C P\,dx + Q\,dy = \oint_C P\,dx + Q\,dy$$

If $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1$, then Green's theorem yields the area of the region D:

$$\iint_D 1\,dA = A(D)$$