Final Project – Chapter 14

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July 4, 2020

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19. #8

    (a) The linearization of f is a function which results in a plane tangent to the surface of $f(x, y)$ at the given point.

    (b) The formula for the approximation may be expressed as:

    $$f(x, y) \approx f(a, b) + \frac{\partial f}{\partial x}(x - a) + \frac{\partial f}{\partial y}(y - b)$$

    (c) In geometric terms, the linear approximation may be used to approximate values near (a, b)

20. #13

    (a) The directional derivative of f (towards the direction of $\overrightarrow{\mathbf{u}}$) may be expressed as:

    $$D_{\overrightarrow{\mathbf{u}}} = \lim_{h \to 0} \frac{f(x_o + ha, y_o + hb) - f(x_o, y_o)}{h}$$

    This represents the rate of change (slope) of f in the direction of $\overrightarrow{\mathbf{u}}$ In geometric terms, this represents the slope of the plane tangent to f at a certain point.

    (b) $D_{\overrightarrow{\mathbf{u}}} = f_x(x_o, y_o)a + f_y(x_o, y_o)b$

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21. #9

    (a) $\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2}$
    Test (Direct Plug-in): $\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2} = \frac{2}{3}$
    Test (along $y = mx$): $\lim_{(x,y)\to(1,1)} \frac{2mx^2}{(2m^2+1)x^2}$ if $m = 1 \Longrightarrow \frac{2}{3}$
    Test (along $y = x^2$): $\lim_{(x,y)\to(1,1)} \frac{2x^3}{x^2+2x^4} = \frac{2}{3}$

    $$\therefore \lim_{(x,y)\to(1,1)} \frac{2xy}{x^2 + 2y^2} = \frac{2}{3}$$

22. #13

    (a) $\frac{\partial f}{\partial x} = 32xy(5y^3 + 2x^2y)^7, \frac{\partial f}{\partial y} = 8(15y^2 + 2x^2)(5y^3 + 2x^2y)^7$

23. #17

    (a) $\frac{\partial S}{\partial u} = \arctan(v\sqrt{w}), \frac{\partial S}{\partial v} = \frac{u\sqrt{w}}{1+v^2w}, \frac{\partial S}{\partial w} = \frac{uv}{2\sqrt{w}(1+v^2w)}$

24. #25

    (a) $\frac{\partial z}{\partial x} = 6x + 2, \frac{\partial z}{\partial y} = -2y \Longrightarrow slope_x = 8, slope_y = 4$

    $$z = 1 + 8(x-1) + 4(y+2) \Longrightarrow z = 8x + 4y + 1 \text{ OR } z - 4y - 8x = 1$$

    (b) $\frac{x-1}{8} = \frac{y+2}{4} = 1 - z$

25. #31

    (a) $z = \pm\sqrt{x^2 + 4y^2 - 4}$

    $$\pm\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+4y^2-4}}; \frac{\partial z}{\partial y} = \pm\frac{4y}{\sqrt{x^2+4y^2-4}}$$

    $$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} = 2 \text{ @ } x = \pm 2, y = \pm\frac{1}{2}, z = \mp 1$$

26. #32

    (a) $\frac{\partial u}{\partial s} = \frac{e^{2t}}{1+se^{2t}}, \frac{\partial u}{\partial t} = \frac{2se^{2t}}{1+se^2t}$

    $$du = (\frac{e^{2t}}{1+se^{2t}})ds + (\frac{2se^{2t}}{1+se^2t})dt$$

27. #35

    (a) First find the partials and derivatives:

    $$\frac{dx}{dp} = 6p + 1, \frac{dy}{dp} = e^p + pe^p, \frac{dz}{dp} = \sin(p) + p\cos(p)$$

    $$\frac{\partial u}{\partial x} = 2xy^3, \frac{\partial u}{\partial y} = 3x^2y^2, \frac{\partial u}{\partial z} = 4z^3$$

    $$du = (6p+1)(2xy^3) + (e^p + pe^p)(3x^2y^2) + (\sin(p) + p\cos(p))(4z^3)$$

28. #39

    (a)

    $$\frac{\partial z}{\partial x} = 2xf(x^2 - y^2), \frac{\partial z}{\partial y} = 1 - 2yf(x^2 - y^2)$$

    $$y(2xf(x^2 - y^2)) + x(1 - 2yf(x^2 - y^2)) \Longrightarrow x$$

    $$\therefore y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x$$