Lecture XVII Notes

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1 Mass and Density – 15.4

The mass may be expressed as

$$m = \iint_D \rho(x,y)\,dA$$

Total charge is expressed as:

$$Q = \iint_D \sigma(x,y)\,dA$$

The (first) moments on the x and y axis, respectively, may be found by using:

$$M_x = \iint_D y\rho(x,y)\,dA \text{ and } M_y = \iint_D x\rho(x,y)\,dA$$

The center of mass, $(\overline{x},\overline{y})$ may be found by rearranging the formula:

$$m\overline{x} = M_y \text{ and } m\overline{y} = M_x$$

$$\overline{x} = \frac{M_y}{m} = \frac{\iint_D x\rho(x,y)\,dA}{\iint_D \rho(x,y)}\,dA$$

$$\overline{y} = \frac{M_x}{m} = \frac{\iint_D y\rho(x,y)\,dA}{\iint_D \rho(x,y)}\,dA$$

The moment of inertia (or the second moment) is mr^2, where r is the distance from a particle to the axis. About the x axis, the moment of inertia is equal to:

$$I_x = \iint_D y^2\rho(x,y)\,dA$$

Across the y axis the moment of inertia is equal to:

$$I_y = \iint_D x^2\rho(x,y)\,dA$$

In addition to this, the polar moment of inertia (about the origin) is equal to:

$$I_0 \iint_D (x^2 + y^2)\rho(x, y)\, dA$$

This means that $I_0 = I_x + I_y$

2 Surface Area – 15.5

The surface area, $A(S)$ is defined by:

$$A(S) = \lim_{(m,n)\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} \Delta T_i j$$

$T_i j$ is defined by $|a \text{ x } b|$, where:

$$a = \Delta x\hat{\mathbf{i}} + f_x(x_i, y_i)\Delta x\hat{\mathbf{k}}$$
$$b = \Delta y\hat{\mathbf{j}} + f_y(x_i, y_j)\Delta y\hat{\mathbf{k}}$$

Therefore, as an iterated integral, the surface area may be expressed as:

$$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dA$$