Lecture X Notes

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1 Limits of Multivariable Functions – 14.2

Just like in Calculus I, there are three conditions for a function to be continuous:

1. $f(a)$ exists
2. $\lim_{x\to a} f(x) = f(a)$
3. $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

In three dimensions, the same rules apply, in addition to a new one:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L$$

if, for every number $\epsilon > 0$ there is a corresponding number $\partial > 0$ such that,
if $(x,y) \in D$ and $0 < \sqrt{(x-a)^2 + (y-b)^2} < \partial$ then $|f(x,y) - L| < \epsilon$

2 Partial Derivatives – 14.3

For the function, $z = f(x,y)$, the partial derivatives may be expressed through the use of the limit definition of a derivative, where:

$$f_x(x,y) = \lim_{\Delta x\to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$$
$$f_y(x,y) = \lim_{\Delta y\to 0} \frac{f(x, y+\Delta y) - f(x,y)}{\Delta y}$$

Some common notations are:

$$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$
$$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y) = \frac{\partial z}{\partial y} = f_2 = D_2 f = D_y f$$

2.1 Finding the Partial Derivative

To find the partial derivatives of the function $f(x, y)$, one would hold y constant, while differentiating with respect to x to find $\frac{\partial f}{\partial x}$, and hold x constant, while differentiating with respect to y to find $\frac{\partial f}{\partial y}$