Lecture VIII Notes

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June 25, 2020

1 Projectile Motion – 13.4

Given an acceleration, $\vec{a}(t)$, one may find the velocity and position formulas. In projectile motion, the acceleration is always $\vec{a}(t) = g = 9.80665 \left[\frac{m}{s^2}\right]$

From here, one may find that: $\Delta v = \int \vec{a}(t)dt \Longrightarrow v - v_o = \vec{a}(t)t \Longrightarrow \vec{v}(t) = v_o + \vec{a}(t)t$

Furthermore, one may find the position vector by integrating once more: $\Delta r = \int v_o + \vec{a}(t)tdt \Longrightarrow r - r_o = v_o t + \frac{1}{2}\vec{a}(t)t^2 \Longrightarrow \vec{r}(t) = r_o + v_o t + \frac{1}{2}\vec{a}(t)t^2$

In projectile motion, acceleration only acts in the vertical direction, and, therefore:
$\vec{r}_y(t) = r_{oy} + v_{oy}t - \frac{1}{2}gt^2$
$\vec{v}_y(t) = v_{oy} + gt$
$\vec{r}_x(t) = r_{ox} + v_{ox}t$
$\vec{v}_x(t) = v_{ox}$

One may find the acceleration through a different method by using the curvature, κ. $\kappa = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} \Longrightarrow |\vec{T}'(t)| = \kappa|\vec{v}(t)|$
Simplifying this leaves us with: $\vec{a} = |\vec{v}'|\vec{T} + \kappa|\vec{v}|^2\vec{N}$
This makes sense, because on a straight line, $\kappa = 0$, and, therefore, on a straight line, $\vec{a} = |\vec{v}'|\vec{T}$

The components of acceleration, a_N and a_T can be found using the formulas: $a_N = \frac{|\vec{r}'(t) \text{ x } \vec{r}''(t)|}{|\vec{r}'(t)|}$ and $a_T = \frac{\vec{r}'(t) \cdot \vec{r}''(t)}{|\vec{r}'(t)|}$