Final Project — Chapter 14

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19. #8

- (a) The linearization of f is a function which results in a plane tangent to the surface of f(x, y) at the given point.
- (b) The formula for the approximation may be expressed as:

$$f(x,y) \approx f(a,b) + \frac{\partial f}{\partial x}(x-a) + \frac{\partial f}{\partial y}(y-b)$$

(c) In geometric terms, the linear approximation may be used to approximate values near (a, b)

20. #13

(a) The directional derivative of f (towards the direction of $\overrightarrow{\mathbf{u}}$) may be expressed as:

$$D_{\overrightarrow{\mathbf{u}}} = \lim_{h \to 0} \frac{f(x_o + ha, y_o + hb) - f(x_o, y_o)}{h}$$

This represents the rate of change (slope) of f in the direction of $\overrightarrow{\mathbf{u}}$ In geometric terms, this represents the slope of the plane tangent to f at a certain point.

(b) $D_{\vec{\mathbf{u}}} = f_x(x_o, y_o)a + f_y(x_o, y_o)b$

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21. #9

(a)
$$\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2}$$

Test (Direct Plug-in): $\lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2} = \frac{2}{3}$
Test (along $y = mx$): $\lim_{(x,y)\to(1,1)} \frac{2mx^2}{(2m^2+1)x^2}$ if $m = 1 \Longrightarrow \frac{2}{3}$
Test (along $y = x^2$): $\lim_{(x,y)\to(1,1)} \frac{2x^3}{x^2+2x^4} = \frac{2}{3}$
 $\therefore \lim_{(x,y)\to(1,1)} \frac{2xy}{x^2+2y^2} = \frac{2}{3}$

22. #13

(a)
$$\frac{\partial f}{\partial x} = 32xy(5y^3 + 2x^2y)^7, \frac{\partial f}{\partial y} = 8(15y^2 + 2x^2)(5y^3 + 2x^2y)^7$$

23. #17

(a)
$$\frac{\partial S}{\partial u} = \arctan(v\sqrt{w}), \frac{\partial S}{\partial v} = \frac{u\sqrt{w}}{1+v^2w}, \frac{\partial S}{\partial w} = \frac{uv}{2\sqrt{w}(1+v^2w)}$$

24. #25

(a)
$$\frac{\partial z}{\partial x} = 6x + 2$$
, $\frac{\partial z}{\partial y} = -2y \Longrightarrow slope_x = 8$, $slope_y = 4$
 $z = 1 + 8(x - 1) + 4(y + 2) \Longrightarrow z = 8x + 4y + 1 \text{ OR } z - 4y - 8x = 1$

(b)
$$\frac{x-1}{8} = \frac{y+2}{4} = 1 - z$$

25. #31

(a)
$$z = \pm \sqrt{x^2 + 4y^2 - 4}$$

$$\pm \frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2 + 4y^2 - 4}}; \frac{\partial z}{\partial y} = \pm \frac{4y}{\sqrt{x^2 + 4y^2 - 4}}$$

$$\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} = 2 @ x = \pm 2, y = \pm \frac{1}{2}, z = \mp 1$$

26. #32

(a)
$$\frac{\partial u}{\partial s} = \frac{e^{2t}}{1+se^{2t}}, \frac{\partial u}{\partial t} = \frac{2se^{2t}}{1+se^{2t}}$$

$$du = \left(\frac{e^{2t}}{1 + se^{2t}}\right)ds + \left(\frac{2se^{2t}}{1 + se^{2t}}\right)dt$$

27. #35

(a) First find the partials and derivatives:

$$\frac{dx}{dp} = 6p + 1, \frac{dy}{dp} = e^p + pe^p, \frac{dz}{dp} = \sin(p) + p\cos(p)$$
$$\frac{\partial u}{\partial x} = 2xy^3, \frac{\partial u}{\partial y} = 3x^2y^2, \frac{\partial u}{\partial z} = 4z^3$$
$$du = (6p + 1)(2xy^3) + (e^p + pe^p)(3x^2y^2) + (\sin(p) + p\cos(p))(4z^3)$$

28. #39

(a)
$$\frac{\partial z}{\partial x} = 2xf(x^2 - y^2), \frac{\partial z}{\partial y} = 1 - 2yf(x^2 - y^2)$$
$$y(2xf(x^2 - y^2)) + x(1 - 2yf(x^2 - y^2)) \Longrightarrow x$$
$$\therefore y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x$$