Lecture XIX Notes

Michael Brodskiy
Professor: V. Cherkassky

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1 Vector Fields – 16.1

If the region D lies in $\mathbb{R}^2$, then the function for the field is as follows:

$$\bar{\mathbf{F}}(x,y) = P(x,y)\hat{\mathbf{i}} + Q(x,y)\hat{\mathbf{j}}$$

If D lies in $\mathbb{R}^3$:

$$\bar{\mathbf{F}}(x,y) = P(x,y)\hat{\mathbf{i}} + Q(x,y)\hat{\mathbf{j}} + R(x,y)\hat{\mathbf{k}}$$

The formula for Newton's law of gravitation is as follows:

$$|\bar{\mathbf{F}}| = \frac{GmM}{r^2}$$

This may be transformed into a vector field by using the following steps:

$$|\bar{\mathbf{F}}| = \frac{GmM}{r^2}$$

$$|\bar{\mathbf{F}}| = \frac{GmM}{r^3}\hat{\mathbf{r}}$$

$$\hat{\mathbf{r}} = \langle x, y, z\rangle,\ r = \sqrt{x^2+y^2+z^2}$$

$$|\bar{\mathbf{F}}| = \frac{GmMx\hat{\mathbf{i}}}{(x^2+y^2+z^2)^{\frac{3}{2}}} + \frac{GmMy\hat{\mathbf{j}}}{(x^2+y^2+z^2)^{\frac{3}{2}}} + \frac{GmMz\hat{\mathbf{k}}}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

Calling back to the gradient, which is written as $\nabla f(x,y) = \frac{\partial f}{\partial x}\hat{\mathbf{i}} + \frac{\partial f}{\partial y}\hat{\mathbf{j}}$, where $f(x,y)$ is a scalar function. This may be used in conjunction with vector fields.

Any vector field which has $\bar{\mathbf{F}} = \nabla f$ is called a conservative vector field. In such a case, f is called a potential function for $\bar{\mathbf{F}}$

In Physics: Newton's formula for gravitation is a conservative vector field. In addition to this, the potential function, f, for conservative vector fields was used to find potential electrical and magnetic field, in addition to the aforementioned gravitational field.

2 Line Integrals – 16.2

A line integral may be found using the formula:

$$\int_C f(x,y)\,ds$$

As used in an earlier chapter, $ds = \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}$

Therefore, the line integral may be found by using:

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Much like any other integral, when applied to a piecewise function, the integral may be broken up:

$$\int_C f(x,y)\,ds = \int_{C_1} f(x,y)\,ds + \int_{C_2} f(x,y)\,ds + \int_{C_3} f(x,y)\,ds + \cdots \int_{C_n} f(x,y)\,ds +$$