Lecture XV Notes

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1 Double Integrals over General Regions – 15.2

First of all, f must be continuous on a region D:

$$\{(x,y)|a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}$$

Such regions are categorized as Type I, and may be found using the following:

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

A Type II equation must meet the same criteria as Type I, but with functions of x:

$$\{(x,y)|c \leq y \leq d, h_1(x) \leq x \leq h_2(x)\}$$

Such regions may be solved using the following:

$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

If the region, D, is neither Type I nor II, then the regions may be broken into two parts, D_1 and D_2:

$$\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$$

Furthermore, if the function being integrated is a constant, c, then:

$$\iint_D c\,dA = cA(D)$$

Where $A(D)$ represents the area of the region D
This property may be used if $m \leq f(x,y) \leq M$ to achieve:

$$mA(D) \leq \iint_D f(x,y)\,dA \leq MA(D)$$