Final Project – Chapter 13

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11. #6

   (a) Curvature is defined as κ (kappa) $= \frac{|d\mathbf{T}|}{|ds|}$, where $\mathbf{T}$ is the total tangent unit vector and s is the arc length of the function.

   (b) $\kappa = \frac{|\overrightarrow{T}'(t)|}{|\overrightarrow{r}'(t)|}$

   (c) $\kappa = \frac{|\overrightarrow{r}'(t) \text{ x } \overrightarrow{r}''(t)|}{|\overrightarrow{r}'(t)|^3}$

   (d) $\kappa = \frac{f''(x)}{(1+f'(x)^2)^{\frac{3}{2}}}$

12. #7

   (a) Normal: $\overrightarrow{N} = \frac{\overrightarrow{T}'(t)}{|\overrightarrow{T}'(t)|}$ Binormal: $\overrightarrow{B} = \overrightarrow{T} \text{ x } \overrightarrow{N}$

   (b) The normal plane is given by the normal vectors, $\overrightarrow{N}$ and $\overrightarrow{B}$. The osculating plane contains vectors $\overrightarrow{T}$ and $\overrightarrow{N}$. It wraps around ("*kisses*") the plane at a point. The osculating circle is a circle with radius $\frac{1}{\kappa}$, the inverse of the curvature. Much like the osculating plane, the edges of the circle wraps around the curve.

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13. #3

   (a) Because the equation of a circle is given, the function may be parametrized easily by setting $x = 4\cos(t)$ and $y = 4\sin(t)$. Using the parametric equations, $z = 5 - 4\cos(t)$. This gives: $\overrightarrow{r}(t) = \langle 4\cos(t), 4\sin(t), 5 - 4\cos(t)\rangle$

14. #5

   (a) $\int_0^1 \overrightarrow{r}(t)dt = \int_0^1 t^2\hat{\mathbf{i}}dt + \int_0^1 t\cos\pi t\hat{\mathbf{j}}dt + \int_0^1 \sin\pi t\hat{\mathbf{k}}dt \Longrightarrow \langle \frac{1}{3}t^3, \frac{t}{\pi}\sin\pi t + \frac{1}{\pi^2}\cos\pi t, -\frac{1}{\pi}\cos\pi t\rangle\Big|_0^1 \Longrightarrow \int_0^1 \overrightarrow{r}(t)dt = \langle \frac{1}{3}, -\frac{2}{\pi^2}, \frac{2}{\pi}\rangle$

15. #8

(a) $\int_0^1 \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}dt \Longrightarrow \overrightarrow{r}'(t) = \langle 3\sqrt{t}, -2\sin 2t, 2\cos 2t\rangle$
$\Longrightarrow \int_0^1 \sqrt{9t + 4\sin^2 2t + 4\cos^2 2t} \Rightarrow u = 9t + 4 \Rightarrow \frac{1}{9}\int_4^1 3\sqrt{u}du = \frac{2}{27}(13)^{\frac{3}{2}} - \frac{16}{27}$

16. #11

(a) $\overrightarrow{r}'(t) = \langle 3\sin^2(t)\cos(t), -3\cos^2(t)\sin(t), 2\cos(t)\sin(t)\rangle$, $|\overrightarrow{r}'(t)| = \sin(t)\cos(t)\sqrt{13} \Longrightarrow$
$\overrightarrow{T}(t) = \langle \frac{3}{\sqrt{13}}\sin(t), -\frac{3}{\sqrt{13}}\cos(t), \frac{2}{\sqrt{13}}\rangle$

(b) $\overrightarrow{T}'(t) = \langle \frac{3}{\sqrt{13}}\cos(t), \frac{3}{\sqrt{13}}\sin(t), 0\rangle$, $|\overrightarrow{T}'(t)| = \sqrt{\frac{9}{13}\sin^2(t) + \frac{9}{13}\cos^2(t)} = \frac{3}{\sqrt{13}} \Longrightarrow$
$\overrightarrow{N}(t) = \langle \cos(t), \sin(t), 0\rangle$

(c) $\overrightarrow{B}(t) = \overrightarrow{T}(t)$ x $\overrightarrow{N}(t) \Longrightarrow \overrightarrow{B}(t) = -2\sin(t)\hat{\mathbf{i}} + 2\cos(t)\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

(d) $\kappa = \frac{|\overrightarrow{T}'(t)|}{|\overrightarrow{r}'(t)|} \Rightarrow \frac{3}{\sqrt{13}}\frac{1}{\sqrt{13}\sin(t)\cos(t)} \Rightarrow \frac{3}{13\sin(t)\cos(t)} \Rightarrow \kappa = \frac{6}{13\sin(2t)}$

17. #13

(a) $\kappa = \frac{f''(x)}{(1+f'(x)^2)^{\frac{3}{2}}} \Longrightarrow \frac{12x^2}{(1+16x^6)^{\frac{3}{2}}} \Longrightarrow \frac{12}{(17)^{\frac{3}{2}}}$

18. #19

(a) $r_o = \langle 0,0,0\rangle$, $v_o = \langle 1,-1,3\rangle$, $\overrightarrow{a}(t) = \langle 6t, 12t^2, -6t\rangle$

$$\overrightarrow{v}(t) = \overrightarrow{v}_o + \int \overrightarrow{a}(t) = \langle 3t^2 + 1, 4t^3 - 1, -3 - 3t^2$$

$$\overrightarrow{r}(t) = \overrightarrow{r}_o + \int \overrightarrow{v}(t) = \langle t^3 + t, t^4 - t, -t^3 - 3t\rangle$$

$$\therefore \overrightarrow{r}(t) = (t^3 + t)\hat{\mathbf{i}} + (t^4 - t)\hat{\mathbf{j}} + (-t^3 - t)\hat{\mathbf{k}}$$